Review

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Survey about Classical and Innovative Definitions of the Power Quantities Under Nonsinusoidal Conditions

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Abstract:
Today, electric power and energy measurements are widely required, practically in all the research, industrial and consumer applications. Power measurements are of importance primarily for the test, monitoring and maintenance of energy supply networks and electric equipment. The measurement of both electric power and energy is a still open research problem in the electrical engineering community. Phenomena like harmonic distortion, noise, transients, over-voltages and voltage dips have increased the difficulty in achieving accurate measurements, compared with the case of sinusoidal signals. Many of the non-active power component definitions that have been proposed cannot be implemented in the traditional electro-mechanical or solid-state meters, but require the adoption of more expensive and time-consuming digital techniques; recently, some new approaches for the definition of power quantities have been investigated. In this paper, a survey of the classical and innovative definitions is proposed, with the aim of summarize the different points of view outlined by the researchers. A case study of power factor correction in nonsinusoidal conditions is also presented, to give a numerical comparison about the power quantities measured according to the various approaches.

Keywords: power measurement, harmonic distortion, apparent power, nonactive power

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1 Introduction

Power measurements are of importance primarily for the test, monitoring and maintenance of energy supply networks and electric equipment (e. g. to measure the efficiency of electric machines). It is well known that the lack of quality in the electric supply can involve faults, overload and malfunctions in many electric devices. So, the electric power measurements should take into account the quality of the power itself, in terms of distortion from the ideal sinusoidal conditions.

In most cases, the currents and voltages are nonsinusoidal and periodic functions of time with a period T and can be expressed as a Fourier series:

$$\begin{aligned} v(t) &= \sqrt{2} \cdot \sum_{i=1}^{n} V_i \cos(i\omega t + \phi_{Vi}) = \\ &= \frac{1}{\sqrt{2}} \sum_{i=1}^{n} V_i \left[e^{j(i\omega t + \phi_{Vi})} + e^{-j(i\omega t + \phi_{Vi})} \right] \end{aligned} \quad (1)$$

$$\begin{aligned} i(t) &= \sqrt{2} \cdot \sum_{k=1}^{n} I_k \cos(k\omega t + \phi_{Ik}) = \\ &= \frac{1}{\sqrt{2}} \sum_{k=1}^{n} I_k \left[e^{j(k\omega t + \phi_{Ik})} + e^{-j(k\omega t + \phi_{Ik})} \right] \end{aligned} \quad (2)$$

and the active power, always defined as the mean power dissipated in one fundamental period, is:

$$P = \sum_{i=1}^{n} {}_i V_i I_i \cos(\phi_{Vi} - \phi_{Ii}) \quad (3)$$

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Differently from the active power, the apparent and reactive powers and the power factor are not based on a single, defined, physical phenomenon, but are conventionally defined quantities that are useful in sinusoidal or quasi sinusoidal conditions. Their definitions under nonsinusoidal conditions are still matter of discussions, and different approaches have been proposed in the years with the aim of identifying new quantities able to describe exhaustively the properties of the power [1–9]. Most of the formulated theories were based on a particular type of reactive energy compensation in mind, which has influenced the definitions.

By analogy with the sinusoidal conditions, the apparent power S under nonsinusoidal conditions is still usually defined as the product of the rms values of current and voltage, including all the harmonic components:

$$S = VI = \sqrt{\sum_{i=1}^{n} (V_i)^2} \times \sqrt{\sum_{k=1}^{m} (I_k)^2} \tag{4}$$

Starting from the given definitions of P and S, in the past years many researchers focused their attention on the definitions of the apparent power Q under nonsinusoidal conditions, proposing interesting and valid theories [3–13]. Unfortunately they reached different conclusions, also because different were the objectives of their works (system modelling, measurement, load control, energy pricing, filtering and so on)[14–20].

The first relevant works in this field were performed by *Budeanu* [3] and *Fryze* [4]; differently from *Budeanu* who operated in the frequency domain considering the Fourier series of voltage and current waveforms, *Fryze* developed his theory in the time domain, generalizing the concepts of the direct (in phase with the voltage) and quadrature current components.

Successively, this topic was further investigated by other researchers, that proposed different interpretations of the power quantities: (i) starting from Budeanu's theory *Sheperd – Zakikhani* [5] and *Sharon* [6] operated in the frequency domain; (ii) starting from Fryze's theory, *Kusters and Moore* [7], and *Page* [8] worked in the time domain; (iii) *Czarnecki* [9] operated in both frequency and time domains.

2 Budeanu's approach

Budeanu proposed [3] a definition for the reactive power Q under nonsinusoidal conditions, synthesized by the following relation with reference to the voltage and current waveforms expressed by the (1) and (2), and the mean power expressed by the (4):

$$Q = \sum_{i=1}^{n} {}_i V_i I_i \sin(\phi_{Vi} - \phi_{Ii}) \tag{5}$$

The main disadvantage for this quantity is that it makes not possible to satisfy the relation $S^2 = P^2 + Q^2$(power triangle), mainly because the apparent power S (4) is a quantity obtained from the voltage and current rms values, without considering the phases of the harmonic components; this is evidenced by the following relationship:

$$S^2 = \sum_{i=1}^{n} V_i^2 \sum_{i=1}^{n} I_i^2 \geq \left(\sum_{i=1}^{n} V_i I_i \cos(\phi_{Vi} - \phi_{Ii}) \right)^2 + \\ + \left(\sum_{i=1}^{n} V_i I_i \sin(\phi_{Vi} - \phi_{Ii}) \right)^2 \tag{6}$$

To ride out this problem, Budeanu introduced a new quantity, the distortion power D, so that:

$$S^2 = P^2 + Q^2 + D^2 \tag{7}$$

In this way the new reactive power $Q_B^2 = Q^2 + D^2$ defined by Budeanu satisfies the power triangle relation, but with a fictitious term, the distorted power D. This quantity consists of the heteronym cross products between the harmonic voltage and current components of different order, that are null in sinusoidal conditions. In terms of non-isofrequential products, for $n \neq m$, Budeanu suggested the following relation:

$$D^2 = \sum_{i=1}^{n} \sum_{k=1}^{m} \left[V_i^2 I_k^2 + V_k^2 I_i^2 - 2\, V_i I_k V_k I_i \cos(\phi_{Vi} - \phi_{Ik}) \right] \tag{8}$$

3 Fryze's approach

Fryze [4] operated in the time domain, separating the instantaneous current waveforms in two components: the instantaneous active current $i_{dn}(t)$, in phase with the voltage waveform, and the instantaneous non-active current $i_{qn}(t)$, consisting of the residual part of the current.

Starting from the Budeanu's approach, considering only the homologous components of the voltage and current waveforms:

$$v_n(t) = \sqrt{2}\cdot V_n \cos(n\omega t + \phi_{Vn}) \tag{9}$$

$$i_n(t) = \sqrt{2}\cdot I_n \cos(n\omega t + \phi_{In}) \tag{10}$$

the direct and the quadrature currents are:

$$I_{dn} = \frac{P_n}{V_n} = I_n \cos(\phi_{Vn} - \phi_{In}) \tag{11}$$

$$I_{qn} = \frac{Q_n}{V_n} = I_n \sin(\phi_{Vn} - \phi_{In}) \tag{12}$$

Therefore:

$$I_n^2 = I_{dn}^2 + I_{qn}^2 \tag{13}$$

and:

$$I^2 = \sum_{=1}^{n} I_i^2 = \sum_{i=1}^{n} \left(\frac{P_i}{V_i}\right)^2 + \sum_{i=1}^{n} \left(\frac{Q_i}{V_i}\right)^2 \tag{14}$$

The apparent power (16) can be expressed as:

$$\begin{aligned} S^2 = V^2 I^2 = V^2 \sum_{i=1}^{n} \left(\tfrac{P_i}{V_i}\right)^2 + V^2 \sum_{i=1}^{n} \left(\tfrac{Q_i}{V_i}\right)^2 = \\ = P^2 + Q^2 \end{aligned} \tag{15}$$

Fryze proposed the following definition for the reactive power:

$$Q_f = \sqrt{V^2 \sum_{i=1}^{n} \left(\frac{Q_i}{V_i}\right)^2} \tag{16}$$

From the (12), to obtain $PF = 1$, the instantaneous quadrature current must be nulled.

4 Sheperd – Zakikhani's approach

The Sheperd – Zakikhani's [5] approach is based on the frequency domain analysis, by introducing a division into "common" and "non-common" harmonics of voltage and current waveforms, to handle nonlinear loads in which the order of voltage and current harmonics may be different. The apparent power is:

$$S^2 = \left(\sum_{i=1}^{N} V_i^2 + \sum_{j=1}^{M} V_j^2\right) \cdot \left(\sum_{i=1}^{N} I_i^2 + \sum_{j=1}^{L} I_j^2\right) \tag{17}$$

where N is the set of harmonics whose order is common to both voltage and current waveforms, while M and L are the set of non-common harmonics of voltage and current waveforms respectively.

The active power P is still defined by the (3). Sheperd and Zakikhani suggested to split S in three terms: S_R, S_X and S_D that is a rest term, so $S = S_R + S_X + S_D$:

$$S_R^2 = \sum_{i=1}^{N} V_i^2 \sum_{i=1}^{N} I_i^2 \cos^2(\phi_{Vn} - \phi_{In}) \tag{18}$$

$$S_X^2 = \sum_{i=1}^{N} V_i^2 \sum_{i=1}^{N} I_i^2 \sin^2(\phi_{Vn} - \phi_{In}) \tag{19}$$

$$S_D^2 = \sum_{i=1}^{N} V_i^2 \sum_{k=1}^{L} I_k^2 + \sum_{j=1}^{M} V_k^2 \cdot \left(\sum_{i=1}^{N} I_i^2 + \sum_{k=1}^{L} I_j^2 \right) \tag{20}$$

For a linear load, the distortion power S_D is zero, while S_R and S_X can be considered as related to two hypothetical currents: $i_R(t)$, that is in phase with the voltage waveform and flows into the pure resistive impedance branch of the equivalent circuit of the load, and $i_r(t)$, that is displaced by half a period and flows into the pure reactive impedance branch of the equivalent circuit of the load:

$$i_R(t) = I_0 + \sum_{i=1}^{n} I_i \cos(\phi_{Vi} - \phi_{Ii}) \cos(i\omega t + \phi_{Vi}) \tag{21}$$

$$i_r(t) = \sum_{i=1}^{n} I_i \sin(\phi_{Vi} - \phi_{Ii}) \sin(i\omega t + \phi_{Vi}) \tag{22}$$

5 Sharon's approach

Sharon [6] suggested to represent the apparent power using three orthogonal components:

-- the active power:

$$P = \sum_{i=1}^{n} V_i I_i \cos(\phi_{Vi} - \phi_{Ii}) \tag{23}$$

-- the quadrature reactive power:

$$S_0^2 = V^2 \sum_{i=1}^{n} I_i^2 \sin^2(\phi_{Vi} - \phi_{Ii}) \tag{24}$$

-- and the complementary reactive power:

$$\begin{aligned} S_C^2 = &\left(\sum_{i=1}^{n} V_i^2 \right) \left(\sum_{k=1}^{m} I_i^2 \cos^2(\varphi_{Vk} - \varphi_{Ik}) \right) + V^2 \left(\sum_{j=1}^{l} I_j^2 \right) \\ &+ \frac{1}{2} \sum_{h=1}^{p} \sum_{r=1}^{q} (V_h I_r \cos(\varphi_{Vh} - \varphi_{Ir}) + V_r I_h \cos(\varphi_{Vr} - \varphi_{Ih}))^2 \end{aligned} \tag{25}$$

where h and r are the order of the uncommon harmonics.

6 Kuster and Moore's approach

Kuster and Moore [7] developed their theory starting from Fryze's time domain approach; they proposed a further decomposition of Fryze's non-active current in two contributions: the capacitive $i_{qC}(t)$ and inductive $i_{qL}(t)$ currents:

$$i_{qC}(t) = \frac{\dot{v} \left(\frac{1}{T} \int_0^T \dot{v} \cdot i \, dt \right)}{\dot{V}^2} \tag{26}$$

$$i_{qL}(t) = \frac{\hat{v} \left(\frac{1}{T} \int_0^T \hat{v} \cdot i \, dt \right)}{\hat{V}^2} \tag{27}$$

where: $\dot{v} = \frac{dv}{dt}$ and $\hat{v} = \int v dt$ are the derivative and the integral of the instantaneous voltage v; $\dot{V}$, $\hat{V}$ and $V$are the rms values of $\dot{v}$,$\hat{v}$ and v. As by Fryze, the active current i_a is:

$$i_a(t) = \frac{v\ \left(\frac{1}{T}\int_0^T v \cdot i\,dt\right)}{V^2} \tag{28}$$

The residual currents are:

$$i_{qCr}(t) = i(t) - i_a(t) - i_{qC}(t) \tag{29}$$

$$i_{qLr}(t) = i(t) - i_a(t) - i_{qL}(t) \tag{30}$$

The definition of the apparent power depends on the load: if a capacitive load is predominant, S is:

$$S^2 = P^2 + Q_C^2 + Q_{Cr}^2 \tag{31}$$

otherwise, if an inductive load is predominant, S is:

$$S^2 = P^2 + Q_L^2 + Q_{Lr}^2 \tag{32}$$

where $Q_C = V \times I_{qC}$ and $Q_L = V \times I_{qL}$; Q_C and Q_L can be positive or negative, and they can be compensated with inductors or capacitors.

Kuster and Moore finally introduced the residual powers:

$$Q_{Cr}^2 = S^2 - P^2 - Q_C^2 \tag{33}$$

$$Q_{Lr}^2 = S^2 - P^2 - Q_L^2 \tag{34}$$

In sinusoidal conditions, the residual components of the reactive power are null, while the inductive and capacitive components of the reactive power are opposite.

7 Page's approach

Page further extended Kuster and Moore's theory by fixing a sign to the rms values of the reactive currents; the sign of inductive reactive current is the same of the mean value of the product of the integral of the voltage and the current; similarly, the sign of capacitive reactive current is the same of the mean value of the product of the derivative of the voltage and the current [8].

8 Czarnecki's approach

Czarnecki started from *Fryze's* work and defined for each n^{th} harmonic frequency an equivalent susceptance [9]; this theory can be applied in both frequency and time domains.

The application of the following periodic voltage:

$$v\,(t) = \sqrt{2}\mathrm{Re}\sum_{i=1}^{n} \dot{V}_i\, e^{j\,i\omega t} \tag{35}$$

at a time invariant system, generates an instantaneous current:

$$i\,(t) = \sqrt{2}\mathrm{Re}\sum_{i=1}^{n} (G_i + jB_i)\,\dot{V}_i\, e^{j\,i\omega t} \tag{36}$$

where: w is the pulsation of $v(t)$; V_i is the amplitude of the i^{th} harmonic component; $G_i + jB_i = \dot{Y}_i$ is the admittance at the iw pulsation.

The equivalent conductance of the load is:

$$G_e = \frac{P}{\dot{V}^2} \tag{37}$$

Czarnecki defined three current components: the active current i_a, the complementary current i_s and the reactive current i_r, respectively as:

$$i_a = \sqrt{2}\mathrm{Re}\sum_{i=1}^{n} G_e\,\dot{V}_i\,e^{j\,i\omega t} \tag{38}$$

$$i_s = \sqrt{2}\mathrm{Re}\sum_{i=1}^{n} (G_i - G_e)\,\dot{V}_i\,e^{j\,i\omega t} \tag{39}$$

$$i_r = \sqrt{2}\mathrm{Re}\sum_{i=1}^{n} j\,B_i\,\dot{V}_i\,e^{j\,i\omega t} \tag{40}$$

The three current components are mutually orthogonal, so $I^2 = I_a^2 + I_s^2 + I_r^2$ and $S^2 = P^2 + D_s^2 + Q_r^2$, with $S = VI$, $P = VI_a$, $D_s = VI_s$ and $Q_r = VI_r$.

The active component of the current is the same of Fryze's current, and it is related to the active power; i_s is related to the variation of the equivalent conductance with the frequency. Finally i_r is due to the energy exchange between the harmonic components.

9 The IEEE 1459 approach

The IEEE Standard 1459–2010 [21] contains the definitions for the measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced or unbalanced conditions, with the aim of providing organizations with criteria for designing and using metering instrumentations. Besides the classical definitions under sinusoidal conditions for power quantities as the instantaneous, active, reactive and apparent powers and power factor, the IEEE 1459–2010 defines the power quantities in a frequency domain approach, and relates them with the total harmonic distortion index THD of voltage and current waveforms.

Starting from the definitions of the voltage and current waveforms under nonsinusoidal conditions (13) and (14), the instantaneous power is:

$$p(t) = v(t)\cdot i(t) = p_a(t) + p_q(t) \tag{41}$$

where:

$$p_a(t) = V_0 I_0 + \sum_{i=1}^{n} V_i I_i \cos(\phi_{Vi} - \phi_{Ii})\;\left[1 - \cos(2i\omega\,t + 2\phi_{Vi})\right] \tag{42}$$

$$\begin{aligned} p_q(t) = &-\sum_{i=1}^{n} V_i I_i \sin(\phi_{Vi} - \phi_{Ii})\sin(2i\omega t + 2\phi_{Vi}) + \\ &+2\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{m} V_i I_j \sin(i\omega t + \phi_{Vi})\sin(j\omega t + \phi_{Ij}) + \\ &+\sqrt{2}V_0\sum_{j=1}^{m} I_j \sin(j\omega t + \phi_{Ij}) + \sqrt{2}I_0\sum_{i=1}^{n} V_i \sin(i\omega t + \phi_{Vi}) \end{aligned} \tag{43}$$

The active power is the sum of two contributions, at the fundamental frequency P_1 and at the other frequencies P_H:

$$P = P_1 + P_H \tag{44}$$

where

$$P_1 = V_1 I_1 \cos(\phi_{V1} - \phi_{I1}) \tag{45}$$

is the fundamental active power:

$$P_H = V_0 I_0 + \sum_{i=2}^{n} V_i\,I_i \cos(\phi_{Vi} - \phi_{Ii}) \tag{46}$$

is the harmonic (nonfundamental) active power.

The IEEE 1459–2010 also defines the fundamental reactive power:

$$Q_1 = V_1 I_1 \sin(\phi_{V1} - \phi_{I1}) \tag{47}$$

while the apparent power is still classically defined as:

$$S = VI \tag{48}$$

The IEEE 1459–2010 proposes to represent the apparent power separating the rms voltage and current into fundamental and harmonic terms:

$$\begin{aligned} S^2 &= (VI)^2 = (V_1^2 + V_H^2)(I_1^2 + I_H^2) = \\ &= (V_1 I_1)^2 + (V_1 I_H)^2 + (V_H I_1)^2 + (V_H I_H)^2 = \\ &= S_1^2 + S_N^2 \end{aligned} \tag{49}$$

where:

$$S_1 = V_1 I_1 \tag{50}$$

is the fundamental apparent power; S_N is the nonfundamental apparent power, expressed by three distinctive terms:

-- the current distortion power

$$D_I = V_1 I_H = S_1(THD_I) \tag{51}$$

-- the voltage distortion power

$$D_V = V_H I_1 = S_1(THD_V) \tag{52}$$

-- the harmonic apparent power

$$S_H = V_H I_H = S_1(THD_V)(THD_I) \tag{53}$$

$$S_N^2 = D_I^2 + D_V^2 + S_H^2 \tag{54}$$

Moreover, S_H can be defined as:

$$S_H = \sqrt{P_H^2 + D_H^2} \tag{55}$$

the harmonic distortion power is:

$$D_H = \sqrt{S_H^2 - P_H^2} \tag{56}$$

and the nonactive power is defined as:

$$N = \sqrt{S^2 - P^2} \tag{57}$$

The IEEE 1459–2010 defines two power factors:

-- the fundamental power factor

$$PF_1 = \cos(\phi_{V1} - \phi_{I1}) = \frac{P_1}{S_1} \tag{58}$$

-- and the power factor

$$PF = \frac{P}{S} = \frac{P_1+P_H}{\sqrt{S_1^2+S_N^2}} = \frac{\frac{P_1}{S_1}\left(1+\frac{P_H}{P_1}\right)}{\sqrt{1+\left(\frac{S_N}{S_1}\right)^2}} = \\ = \frac{PF_1\left(1+\frac{P_H}{P_1}\right)}{\sqrt{1+THD_I^2+THD_V^2+(THD_I THD_V)^2}} \tag{59}$$

IEEE 1459–2010 discusses in a note [16, par.3.1.2.7] the meaning of the apparent power (61). "*Apparent power is the amount of active power that cannot be supplied to a load, or to a cluster of loads, under ideal conditions. The ideal conditions may assume the load supplied with sinusoidal voltage and current. The loads are compensated by means of active or passive devices such that the line current is sinusoidal and in phase with the voltage that, ideally, is also adjusted to be sinusoidal. The rms value of the current I is kept equal with the line rms value of the actual current. The load voltage is adjusted to a value that yields unchanged load performance (i.e. the same amount of useful energy is converted and delivered by the load). If the performance criterion is the electrothermal conversion of the electric energy, then the rms value of the voltage at the terminals where the measurement is implemented must be kept constant*." The apparent power S is also considered to evaluate the power losses DP in a feeder supplying the apparent power S.

10 A new approach based on the instantaneous power

In [22, 23], a new approach for the definition of power quantities has been discussed, by adopting instantaneous power waveforms for the definition of apparent power and reactive power. A different definition of S is proposed; by introducing the measurement of P_{rms}, it allows the evaluation of the actual power waveform features, avoiding losses of information related to the absence of the harmonic phases in the traditional definition of S. In contrast to S, which is directly obtainable from the $p(t)$ waveform only under sinusoidal conditions, AS can be derived from $p(t)$ under all conditions.

To explain the meaning of P_{rms} in nonsinusoidal conditions, voltage and current waveforms containing up to the 4th harmonic are considered:

$$v(t) = \frac{1}{\sqrt{2}}\left\{\begin{array}{c} V_1\left[e^{j(1\omega t+\phi_{V1})}+e^{-j(1\omega t+\phi_{V1})}\right]+ \\ +V_2\left[e^{j(2\omega t+\phi_{V2})}+e^{-j(2\omega t+\phi_{V2})}\right]+ \\ +V_3\left[e^{j(3\omega t+\phi_{V3})}+e^{-j(3\omega t+\phi_{V3})}\right]+ \\ +V_4\left[e^{j(4\omega t+\phi_{V4})}+e^{-j(4\omega t+\phi_{V4})}\right] \end{array}\right\} \tag{60}$$

$$i(t) = \frac{1}{\sqrt{2}}\left\{\begin{array}{c} I_1\left[e^{j(1\omega t+\phi_{I1})}+e^{-j(1\omega t+\phi_{I1})}\right]+ \\ +I_2\left[e^{j(2\omega t+\phi_{I2})}+e^{-j(2\omega t+\phi_{I2})}\right]+ \\ +I_3\left[e^{j(3\omega t+\phi_{I3})}+e^{-j(3\omega t+\phi_{I3})}\right]+ \\ +I_4\left[e^{j(4\omega t+\phi_{I4})}+e^{-j(4\omega t+\phi_{I4})}\right] \end{array}\right\} \tag{61}$$

Each component of the instantaneous power $p(t)$ *is*

$$p_{ij} = V_i I_j \left\{\begin{array}{c} \cos\left[(i+j)\omega t+\phi_{Vi}+\phi_{Ij}\right]+ \\ +\cos\left[(i-j)\omega t+\phi_{Vi}-\phi_{Ij}\right] \end{array}\right\} \tag{62}$$

The instantaneous power can be expressed as:

$$p(t) = \sum_{i=1}^{4}\sum_{j=1}^{4} V_i I_j \left\{\begin{array}{c} \cos\left[(i+j)\omega t+\phi_{Vi}+\phi_{Ij}\right]+ \\ +\cos\left[(i-j)\omega t+\phi_{Vi}-\phi_{Ij}\right] \end{array}\right\} \tag{63}$$

In this case, the mean value of $p(t)$ P_m is

$$\begin{array}{c} P_m = V_1 I_1\{\cos[\phi_{V1}-\phi_{I1}]\}+ \\ +V_2 I_2\{\cos[\phi_{V2}-\phi_{I2}]\}+ \\ +V_3 I_3\{\cos[\phi_{V3}-\phi_{I3}]\}+ \\ +V_4 I_4\{\cos[\phi_{V4}-\phi_{I4}]\} \end{array} \tag{64}$$

Considering (62), the product of the instantaneous voltage and current waveforms generates, in the instantaneous power, sinusoidal components at $+/-\omega t$, which are present in $P_{12}, P_{21}, P_{23}, P_{32}, P_{34}, P_{43}$,; if a voltage and current waveforms with a given harmonic content are considered, each product of the ith voltage harmonic and the jth voltage harmonic, for j=i+/−1, generates a power component at the fundamental frequency of the power system that behaves as a subharmonic component for $p(t)$.

The rms value of the instantaneous power component at *mwt* is

$$P_{rms}(m) = \sqrt{\frac{1}{T}\int_0^T \left[\begin{array}{l} \sum_{\substack{i=1 \\ k=m-i}}^{m-1} V_i I_k \{\cos[\omega t(i+k) + \phi_{Vi} + \phi_{Ik}]\} + \\ + \sum_{\substack{i=1 \\ k=i+m}}^{n-m} V_i I_k \{\cos[\omega t(i-k) + \phi_{Vi} - \phi_{Ik}]\} + \\ + \sum_{\substack{i=k+m \\ k}}^{n-m} V_i I \{\cos[\omega t(i-k) + \phi_{Vi} - \phi_{Ik}]\} \end{array} \right]^2 dt} \tag{65}$$

For the considered case with n=4, the rms value of the instantaneous power component at *wt* is

$$P_{rms}(1) = \sqrt{\frac{1}{T}\int_0^T \left\{ \begin{array}{c} V_1 I_2 \{\cos[-\omega t + \phi_{V1} - \phi_{I2}]\} + \\ +V_2 I_1 \{\cos[\omega t + \phi_{V2} - \phi_{I1}]\} + \\ +V_2 I_3 \{\cos[-\omega t + \phi_{V2} - \phi_{I3}]\} + \\ +V_3 I_2 \{\cos[\omega t + \phi_{V3} - \phi_{I2}]\} + \\ +V_3 I_4 \{\cos[-\omega t + \phi_{V3} - \phi_{I4}]\} + \\ +V_4 I_3 \{\cos[\omega t + \phi_{V4} - \phi_{I3}]\} \end{array} \right\}^2 dt} \tag{66}$$

In addition to the *2wt* sinusoidal component due to the product of the fundamental voltage and current components, there are some more sinusoidal components with this pulsation in P_{13}, P_{24}, P_{31}, P_{42}. If a voltage and current waveforms with a given harmonic content are considered, each product of the ith voltage harmonic and the jth voltage harmonic for j=i+/−2 generates a power component interfering with the fundamental component of *p(t)*; therefore, its amplitude is different form $\frac{V_1 I_1}{2}$, as in (67):

$$P_{rms}(2) = \sqrt{\frac{1}{T}\int_0^T \left\{ \begin{array}{l} V_1 I_1 \{\cos[2\omega t + \phi_{V1} + \phi_{I1}]\} + \\ +V_1 I_3 \{\cos[-2\omega t + \phi_{V1} - \phi_{I3}]\} + \\ +V_2 I_4 \{\cos[-2\omega t + \phi_{V2} - \phi_{I4}]\} + \\ +V_3 I_1 \{\cos[2\omega t + \phi_{V3} - \phi_{I1}]\} + \\ +V_4 I_2 \{\cos[2\omega t + \phi_{V4} - \phi_{I2}]\} \end{array} \right\}^2 dt} \tag{67}$$

So,

$$P_{rms} = \sqrt{P_m^2 + \frac{1}{2}\sum_{1\ m}^{8} [P_{rms}(m)]^2} \tag{68}$$

and *AS* is defined starting from the components of the amplitude spectrum of *p(t)* as

$$AS = \sqrt{2\sum_{1\ m}^{8} [P_{rms}(m)]^2} \tag{69}$$

After discussing P_{rms} and *AS*, a new definition of the reactive power *Q* can be proposed, the distortion power *D* and the nonactive power *N*, by involving the amplitude spectrum components of *p(t)*. The amplitude spectrum of *p(t)* consists of three kinds of components: the DC component P_m, the even-order power harmonics, and the odd-order ones. The even-order components can be considered to define the actual reactive power *AQ* because they are related to the mean power addends in (64)

$$AQ = \sqrt{2\sum_{1\ m}^{n} [P_{rms}(2m)]^2 - P_m^2} \tag{70}$$

The actual distortion power *AD* is obtained from the odd-order components, having no effect on the mean power

$$AD = \sqrt{2\sum_{0\ m}^{n-1} [P_{rms}(2m+1)]^2} \tag{71}$$

while the actual nonactive power *AN* is

$$AN = \sqrt{AQ^2 + AD^2} \tag{72}$$

so

$$AS = \sqrt{P_m^2 + AQ^2 + AD^2} \tag{73}$$

The sign of the reactive power can be conventionally considered as positive for ohmic-inductive loads in which the current lags the voltage, while it assumed as negative for ohmic-capacitive loads in which the current leads the voltage.

11 Remarks about Pros and Cons of each approach

Each of the presented approaches has been proposed with the aim of extending the properties regarding power parameters from sinusoidal to nonsinusoidal conditions. The most important properties and interpretations, with respect to the sinusoidal conditions, are as follows: (i) the reactive power is expressed as $V\ I\ sin\varphi$; (ii) the reactive power is a quantity with a sign (+ o −); (iii) the algebraic sum of the reactive powers in a system is null; (iv) the reactive power is proportional to the difference between the mean values of the energy stored in the electrostatic and in the magnetic fields in a period; (v) the reactive power is the amplitude of the oscillatory power; (vi) the reactive power can be compensated and its value can be utilized to design a linear capacitive or inductive compensating elements; (vii) the compensation of the reactive power increases the power factor, so the power factor is unity when the reactive power is zero; (viii) the geometric sum of the reactive and active powers is the apparent power; (ix) the voltage drops in the power lines are only due to the reactive power.

It is evident that, in nonsinusoidal conditions, it is quite difficult to satisfy all those requirements with the introduction of few power parameters; from a theoretical point of view, to completely define the behavior of a power system node, it is necessary to measure amplitudes, phases and frequencies of all voltage and current harmonics but, nevertheless, all those information should be processed to leading to shared definitions for the power quantities. By analyzing the presented approaches, it is evident that no one of them satisfies all the requirements above; this is because the reactive power has several interpretations, related to its mathematical representation and to the implicated physical phenomena [14–20], with particular respect to the research field in which the definitions themselves have been elaborated (system modelling, measurement, load control, energy pricing, filtering and so on).

In the following, some remarks about pros and cons of each approach are presented.

The main advantage of Budeanus's approach is that the sum of all the reactive powers in the same point of a power system is null. The disadvantages are various: neither to the reactive nor to the distortion power components could be assigned any physical significance; in a power system, when the reactive power is zero, the PF can be different form zero; the measurement of Q and D requires complex elaborations impossible to execute using analog instruments.

With Fryze's approach, the reactive power Q_f is the single orthogonal component accounting for the difference between apparent and average power. Q_f is a suitable quantity, because the case of $Q_n = 0$, " n, corresponds to $S = P$ (unity power factor). The Fryze's approach doesn't require the measurement of the voltage and current harmonic components, and it isn't related to the definition of power addends; nevertheless, its reactive power definition cannot be used to individuate the nature of the loads and it doesn't give information useful for the load compensation.

In Sheperd-Zakikhani's approach, all the apparent power components are without a sign; Sheperd and Zakikhani considered their approach to be closer to physical reality, for compensation of reactive power for maximum power factor, with passive components. This is possible only if $S_X^2$is minimized, because S_D^2 only contains non-common harmonics that cannot be compensated by passive components. For linear systems S_D^2= 0 because there are no non-common harmonics, but, in some practical situations, even if the voltage is nearly sinusoidal, a distorted current gives a large contribution to S_R and S_X due to the cross products between the fundamental voltage component and the current harmonics, so S_R^2 is not equal to P^2 as defined in (3), even if S_R^2 contains P^2, and S_X is different from Budeanu's Q_B.

Kuster and Moore's approach is useful when the measurement of the sign of reactive power components is needed, to compensating only capacitive reactive power or inductive reactive power.

As an improvement of the Fryze's work, in Czarnecki's approach the active component of the current is the same of Fryze's current, and it is related to the active power; is is related to the variation of the equivalent conductance with the frequency. Finally ir is due to the energy exchange between the harmonic components.

The IEEE 1459 approach is pragmatically devoted to simplifying and defining criteria for designing and using metering instrumentations. It proposes to focusing on two contributions to active and reactive power, due to the fundamental frequency and all other frequencies respectively, by introducing the measurement of harmonic distortion.

As a general remark, the effort toward the definition of more accurate power quantities has involved an increasing complexity in the implementation of the measurement algorithms, whose more complex is the IEEE 1459 approach because of the number of defined quantities.

The main advantage of the new approach [23] consists in modifying the Budeanu's definitions by relating them to measurements obtained from the actual shape of the instantaneous power waveform that can be obtained with low-cost devices too, by measuring the rms and mean values of the instantaneous power. The measurement of all the power quantities from the actual instantaneous power can be interesting from a metrological point of view, because all the power parameters are obtained by taking into account also the effects of the harmonics phases; in this way, the measured power quantities are univocally defined from the power that is absorbed by users.

12 Case study: power factor correction of an ohmic- inductive load under non sinusoidal conditions

The power factor correction in nonsinusoidal conditions is a typical issue that can be investigated for comparing the approaches. A case study is proposed, towards two different goals, consisting in numerical comparisons among the power quantities that can be measured according the various approaches in the same real case, as well as the evaluation of the possible advantages that can be obtained with the new approach. Firstly, an ohmic- inductive load of 42,32 W and 101 mH has been considered; then, a capacitor of 13 mF has been parallel connected to the load (Figure 1).

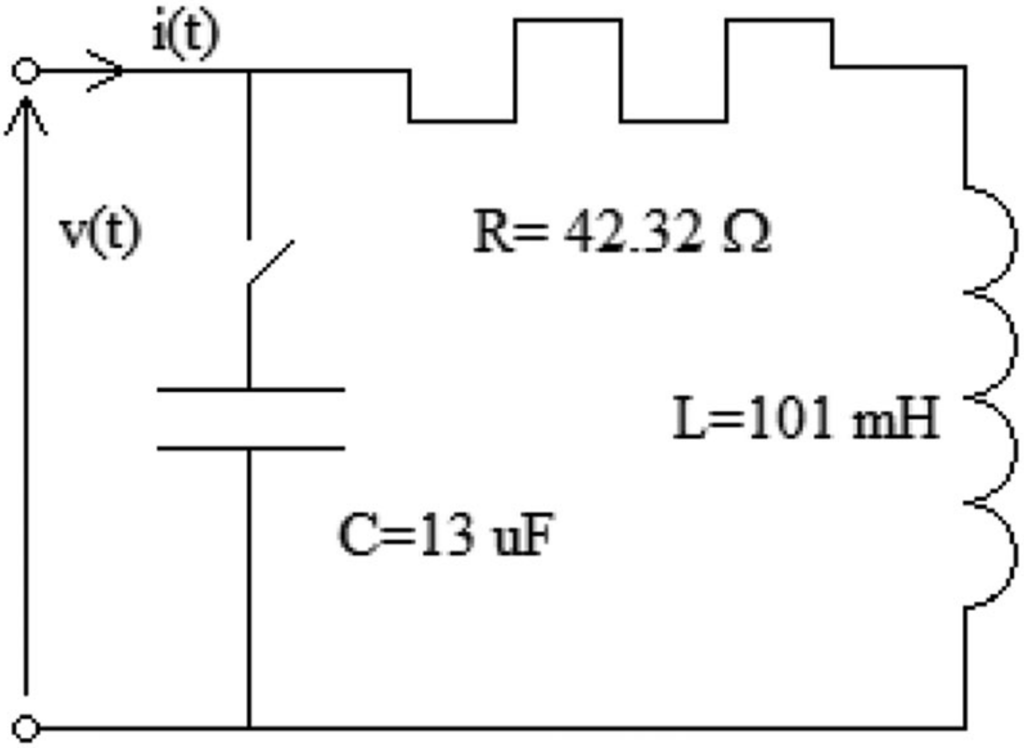

Figure 1: The considered load.

In sinusoidal conditions at 50 Hz, the power factor varies from 0.800 to 0.902. In both cases the same set of voltage harmonics has been applied [24], and the phase of the third component has been varied in the range −180 to 135 degrees, while the amplitudes and the phases of the remaining components are fixed (Table 1). The power parameters measured according to Budeanu, Fryze, Sharon, Kusters and Moore, Shepherd and Zakinhani, Czarnecki and IEEE are presented in Table 2 with disconnected capacitor, and in Table 3 with connected capacitor. As a first remark, for all these approaches, no effect of the phase variation of the third harmonic voltage has been measured.

Table 1: Features of voltage and current harmonics.

Harmonic order	Voltage amplitude [Vrms]	Voltage phase [degree]	Current amplitude [Arms]		Current phase [degree]	
			Disconnected capacitor	Connected capacitor	Disconnected capacitor	Connected capacitor
1	230.00	0.00	4.364	3.878	−36.59	−25.37
2	2.30	0.00	0.030	0.018	–56.04	−20.66
		–180			114.18	−139,20
		–135			159.82	−94.20
3	6.90	–90	0.067	0.0361	–155.82	−49.20
		–45			–110.82	−4.20
		0			–65.82	40.80
		45			–20.82	85.80
		90			24.18	130.80
		135			69.18	175.80
4	2.30	0.00	0.017	0.021	–71.39	75.32
5	6.90	0.00	0.042	0.101	−74.92	83.70
6	1.15	0.00	0.006	0.022	−77.35	86.64
7	2.30	0.00	0.010	0.056	−79.11	88.00

Table 2: Power parameter with disconnected capacitor.

Budeanu	Fryze	Sharon	Kusters & Moore	Shepherd & Zakikhani	Czarnecki	IEEE
P=806.22 [W] S=1004.94 [VA] PF=0.80 Q=–599.08 [var] D=32.13 [var]	P=806.22 [W] S=1004.94 [VA] PF=0.80 Q=599.94 [var]	P=806.22 [W] S=1004.94 [VA] PF=0.80 SQ=599.17 [var] SC=30.39 [var]	P=806.22 [W] S=1004.94 [VA] PF=0.80 QC=593.92 [var] QCR=83.21 [var] QL=−598,78 [var] QLR=33.48[var]	P=806.22 [W] S=1004.94 [VA] PF=0.80 Sr=806.79 [VA] Sx=599.17 [VA] Sd=0 [VA]	P=806.22 [W] S=1004.94 [VA] PF=0.80 Qr=596,90[var] D=−60,35 [var]	P=806.22 [W] S=1004.94 [VA] PF=0.80 N=599.94 [var] Sn=50.45 [var] P1=805.89 [W] S1=1003.98 [VA] Q1=–598.25[var] Ph=0.32 Sh=0.93 Nh=0.87

Table 3: Power parameter with connected capacitor.

Budeanu	Fryze	Sharon	Kusters & Moore	Shepherd & Zakikhani	Czarnecki	IEEE
P=806.22 [W] S=893.35 [VA] PF=0.902 Q=–381.16 [var] D=52.95 [var]	P=806.22 [W] S=893.35 [VA] PF=0.902 Q=384.82 [var]	P=806.22 [W] S=893.35 [VA] PF=0.902 SQ=383,62 [var] SC=30,39 [var]	P=806.22 [W] S=893.35 [VA] PF=0.902 QC=–365.84 [var] QCR=118.63 [var] QL=382.34[var] QLR=41.60[var]	P=806.22 [W] S=893.35 [VA] PF=0.902 Sr=806.79 [VA] Sx=383.62[VA] Sd=0 [VA]	P=806.22 [W] S=893.35 [VA] PF=0.902 Qr=380,06[var] D=−60,35 [var]	P=806.22 [W] S=893.35 [VA] PF=0.902 N=384.82 [var] Sn=50.29 [var] P1=805.89 [W] S1=891.93 [VA] Q1=–382.20[var] Ph=0.32 Sh=1.13

Nh=1.29

By comparing the measured quantities, it can be found that Fryze's Q is equal to IEEE' N, and they are also equal to the square root of the sum of following squared quantities: (i) Budeanu's Q and D; (ii) Sharon's SQ and SC; (iii) Czarnecki's Q_r and D. Moreover, Sharon's Q is equal to Shepherd & Zakikhani's S_x. Before presenting the results obtained with the new approach, some observations about the reactive power can be introduced.

The reactive power is usually related to the reactive energy that is absorbed by a load; if the oscillating power is considered, defined as the difference between the instantaneous power *p(t)* and the active power P_m:

$$p'(t) = p(t) - P_m \tag{74}$$

the mean value of *p'(t)* has been measured, to investigate its dependence from the phase of third harmonic voltage; the results are in Table 4. In Figure 2 and Figure 3 the oscillating power waveforms are also presented, for both cases with disconnected and connected capacitor. The proposed new approach reveals to be sensitive to the effect of the phase of the third harmonic voltage, as shown by Table 5 and Table 6 in which the new power parameters are presented. In Figure 4 and Figure 5 a comparison is proposed among the actual reactive power AQ, the actual non active power AN, the actual power factor APF and the mean value of p'(t).

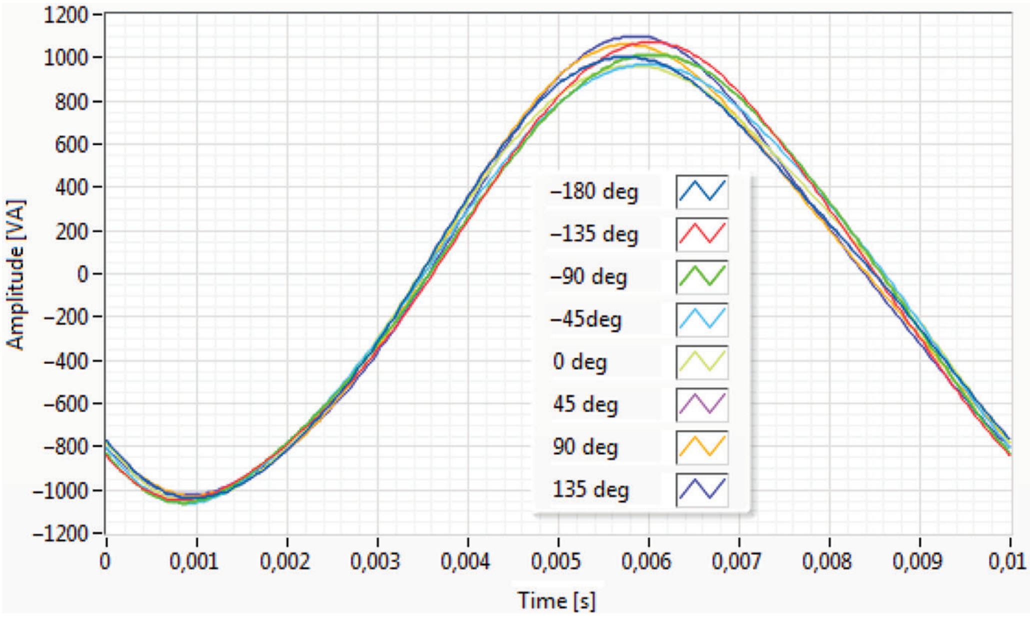

Figure 2: Oscillating power, with disconnected capacitor, by varying the third harmonic voltage phase.

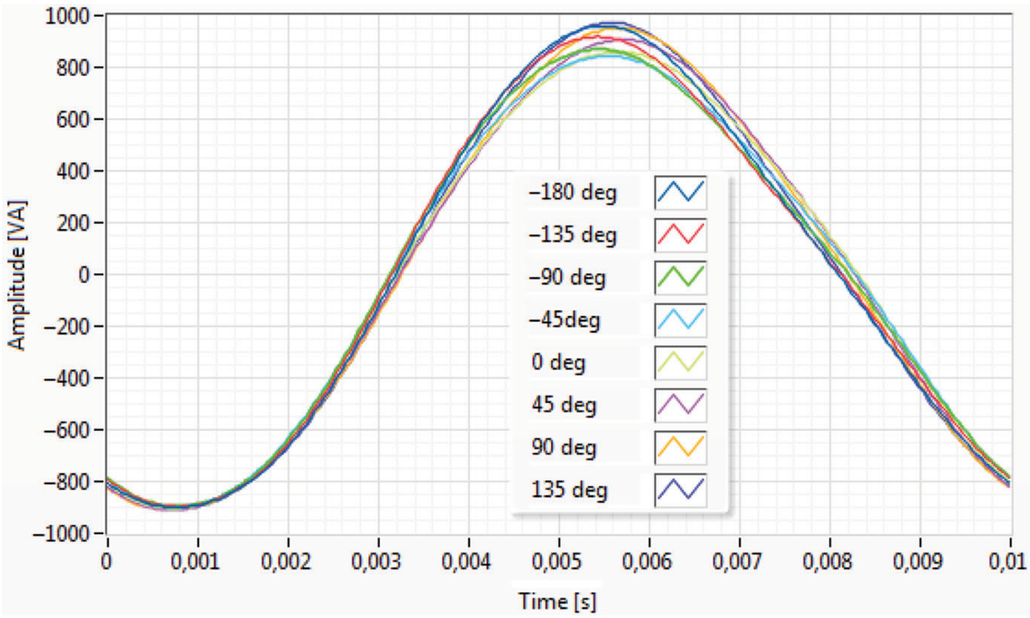

Figure 3: Oscillating power, with connected capacitor, by varying the third harmonic voltage phase.

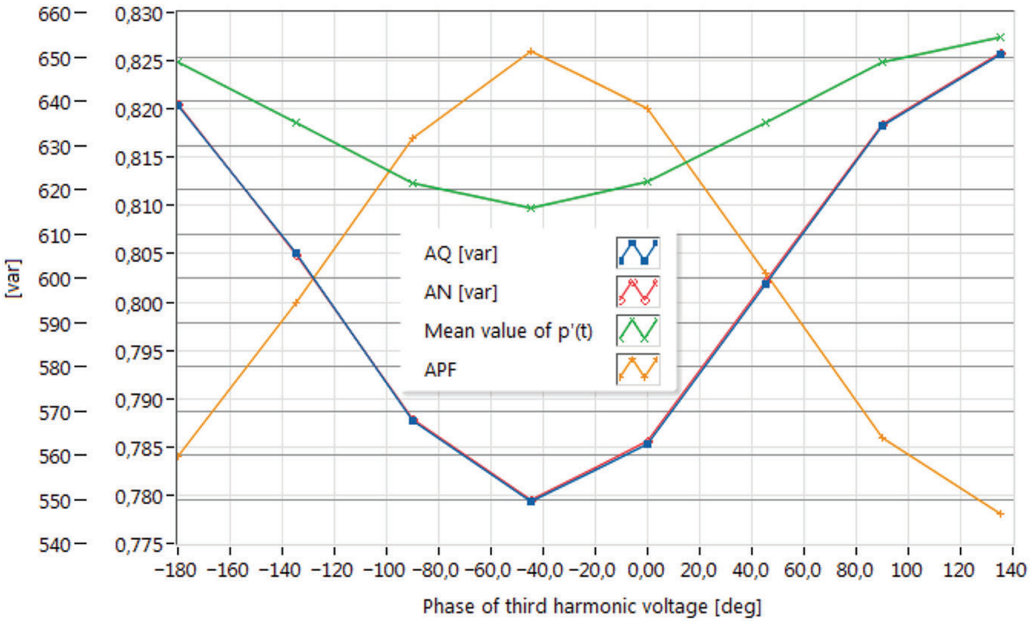

Figure 4: Actual reactive power AQ, non active power AN, power factor APF and mean value of p′(t) with disconnected capacitor, by varying the third harmonic voltage phase.

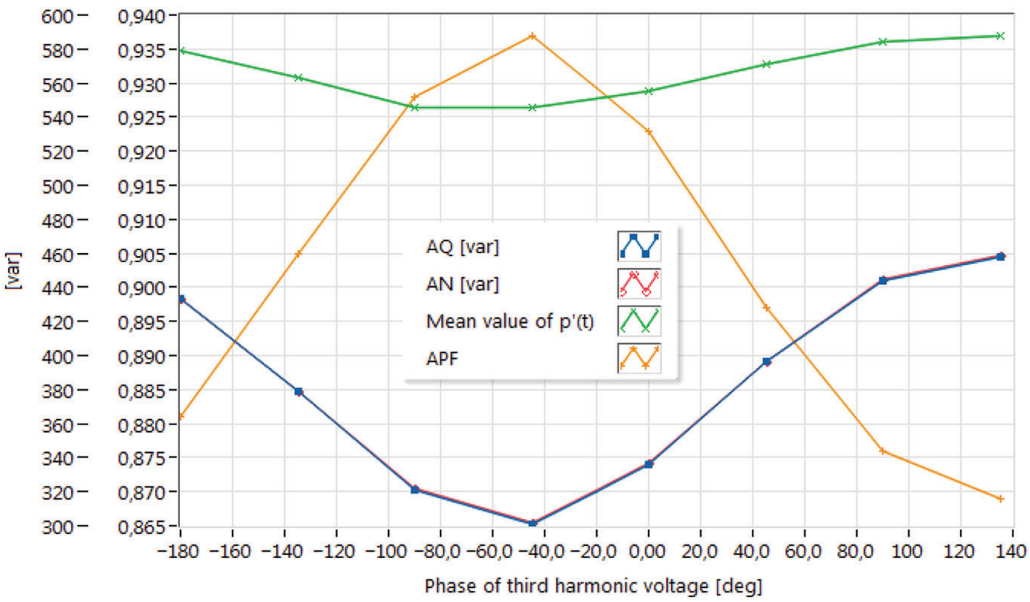

Figure 5: Actual reactive power AQ, non active power AN, power factor APF and mean value of p′(t) with connected capacitor, by varying the third harmonic voltage phase.

Table 4: Mean values of oscillating power p(t)′.

Third harmonic voltage phase [deg]	**−180**	**−135**	**−90**	**−45**	**0**	**45**	**90**	**135**
Mean value of p′(t) [var] Disconnected capacitor	648.85	635.02	621.26	615.88	621.64	635.10	648.63	654.40

Mean value of p′(t) [var] Connected capacitor	579.15	563.02	549.05	545.79	554.86	570.71	584.24	587.79

Table 5: New power parameters, with disconnected capacitor.

Third harmonic voltage phase [deg]	**−180**	**−135**	**−90**	**−45**	**0**	**45**	**90**	**135**
Pm [W]	806.22	806.22	806.22	806.22	806.22	806.22	806.22	806.22
Prms [VA]	1085.97	1076.32	1066.03	1061.20	1064.72	1074.66	1084.65	1089.39
AS [VA]	1028.92	1008.43	986.33	975.87	983.81	1004.46	1026.12	1036.13
AQ [var]	638.88	605.77	567.73	549.34	562.30	598.66	634.35	650.42
AD [var]	22.95	23.09	23.40	22.69	23.80	23.66	23.35	23.06
AN [var]	639.29	605.33	568.21	549.85	563.30	599.13	634.75	650.83
APF	0.783	0.799	0.817	0.826	0.820	0.803	0.786	0.778

Table 6: New power parameters, with connected capacitor.

Third harmonic voltage phase [deg]	**−180**	**−135**	**−90**	**−45**	**0**	**45**	**90**	**135**
Pm [W]	806.22	806.22	806.22	806.22	806.22	806.22	806.22	806.22
Prms [VA]	1033.92	1023.16	1013.29	1010.18	1015.70	1026.51	1036.25	1039.30
AS [VA]	915.43	890.93	868.06	860.78	873.68	898.60	920.68	927.53
AQ [var]	433.12	378.58	321.07	300.82	335.94	396.28	444.08	458.14
AD [var]	21.04	21.22	21.51	21.73	21.75	21.56	21.28	21.06
AN [var]	433.63	379.17	321.79	301.60	336.65	396.86	444.59	458.62
APF	0.881	0.905	0.928	0.937	0.923	0.897	0.876	0.869

13 Conclusions

In this paper, a survey of classical and innovative definitions has been proposed. In the past years, many researchers investigated the meaning of power quantities, by focusing their attention on the definitions of the apparent power Q under nonsinusoidal conditions, proposing interesting and valid theories [25]. Nevertheless, they reached different conclusions, because they started from different points of view, towards different objectives (system modelling, measurement, load control, energy pricing, filtering and so on).

All the approaches, from Budeanu's to IEEE 1459, are based on the definition of apparent power as the product of rms values of voltage and current, so they all lead to the same results in terms of power factor, although they suggests various methods for evaluating nonactive or reactive power. In a different way, the new approach starts from the measurement of apparent power from the instantaneous power waveform, with the following advantages: (i) the measurement of reactive and nonactive power are relates to the actual mean value of the oscillating power, that varies also depending on the relative phases on the voltage harmonics while, in all the previous approaches, this effect is not considered; (ii) the power factor that can be obtained is a more accurate measurement of the actual reactive energy absorbed by a load in the actual conditions in which it is fed by the power system; (iii) the new approach can be used to improve the power factor correction systems, because of its sensibility to the actual shape of the instantaneous power [26–29]

From a metrological point of view, the new approach allows the evaluation of all the power parameters starting from the actual features of voltage and current waveforms, because the apparent power is now defined starting from the actual shape of the instantaneous power instead of being the product of rms values of voltage and current; this gets more interesting if the meaning of the power factor is reconsidered, because with the new approach it becomes a ratio between two actual properties of the instantaneous power waveform.

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