

Contents

List of Figures	<i>xv</i>
Nomenclature	<i>xxi</i>
Preface to First Edition	<i>xxvi</i>
Preface to Second Edition	<i>xxvii</i>
1 Inverse Heat Conduction Problems: An Overview	1
1.1 Introduction	1
1.2 Basic Mathematical Description	3
1.3 Classification of Methods	4
1.4 Function Estimation Versus Parameter Estimation	5
1.5 Other Inverse Function Estimation Problems	6
1.6 Early Works on IHCPs	7
1.7 Applications of IHCPs: A Modern Look	7
1.7.1 Manufacturing Processes	7
1.7.1.1 Machining Processes	7
1.7.1.2 Milling and Hot Forming	9
1.7.1.3 Quenching and Spray Cooling	9
1.7.1.4 Jet Impingement	11
1.7.1.5 Other Manufacturing Applications	11
1.7.2 Aerospace Applications	11
1.7.3 Biomedical Applications	12
1.7.4 Electronics Cooling	13
1.7.5 Instrumentation, Measurement, and Non-Destructive Testing	13
1.7.6 Other Applications	14
1.8 Measurements	15
1.8.1 Description of Measurement Errors	15
1.8.2 Statistical Description of Errors	15
1.9 Criteria for Evaluation of IHCP Methods	17
1.10 Scope of Book	17
1.11 Chapter Summary	18
References	18
2 Analytical Solutions of Direct Heat Conduction Problems	25
2.1 Introduction	25
2.2 Numbering System	26
2.3 One-Dimensional Temperature Solutions	26
2.3.1 Generalized One-Dimensional Heat Transfer Problem	26
2.3.2 Cases of Interest	27
2.3.3 Dimensionless Variables	28
2.3.4 Exact Analytical Solution	28
2.3.5 The Concept of Computational Analytical Solution	29

2.3.5.1	Absolute and Relative Errors	30
2.3.5.2	Deviation Time	31
2.3.5.3	Second Deviation Time	31
2.3.5.4	Quasi-Steady, Steady-State and Unsteady Times	32
2.3.5.5	Solution for Large Times	32
2.3.5.6	Intrinsic Verification	33
2.3.6	X12B10T0 Case	33
2.3.6.1	Computational Analytical Solution	34
2.3.6.2	Computer Code and Plots	34
2.3.7	X12B20T0 Case	34
2.3.7.1	Computational Analytical Solution	36
2.3.7.2	Computer Code and Plots	36
2.3.8	X22B10T0 Case	37
2.3.8.1	Computational Analytical Solution	38
2.3.8.2	Computer Code and Plots	39
2.3.9	X22B20T0 Case	40
2.3.9.1	Computational Analytical Solution	40
2.3.9.2	Computer Code and Plots	41
2.4	Two-Dimensional Temperature Solutions	42
2.4.1	Dimensionless Variables	43
2.4.2	Exact Analytical Solution	43
2.4.3	Computational Analytical Solution	44
2.4.3.1	Absolute and Relative Errors	45
2.4.3.2	One- and Two-Dimensional Deviation Times	46
2.4.3.3	Quasi-Steady Time	48
2.4.3.4	Number of Terms in the Quasi-Steady Solution with Eigenvalues in the Homogeneous Direction	50
2.4.3.5	Number of Terms in the Quasi-Steady Solution with Eigenvalues in the Nonhomogeneous Direction	50
2.4.3.6	Deviation Distance Along x	51
2.4.3.7	Deviation Distance Along y	53
2.4.3.8	Number of Terms in the Complementary Transient Solution	54
2.4.3.9	Computer Code and Plots	56
2.5	Chapter Summary	57
	Problems	58
	References	59
3	Approximate Methods for Direct Heat Conduction Problems	61
3.1	Introduction	61
3.1.1	Various Numerical Approaches	61
3.1.2	Scope of Chapter	61
3.2	Superposition Principles	62
3.2.1	Green's Function Solution Interpretation	63
3.2.2	Superposition Example – Step Pulse Heating	63
3.3	One-Dimensional Problem with Time-Dependent Surface Temperature	64
3.3.1	Piecewise-Constant Approximation	64
3.3.1.1	Superposition-Based Numerical Approximation of the Solution	65
3.3.1.2	Sequential-in-time Nature and Sensitivity Coefficients	65
3.3.1.3	Basic “Building Block” Solution	66
3.3.1.4	Computer Code and Example	66
3.3.1.5	Matrix Form of the Superposition-Based Numerical Approximation	68
3.3.2	Piecewise-Linear Approximation	69
3.3.2.1	Superposition-Based Numerical Approximation of the Solution	70
3.3.2.2	Sequential-in-time Nature and Sensitivity Coefficients	71
3.3.2.3	Basic “Building Block” Solutions	72
3.3.2.4	Computer Code and Examples	72

3.3.2.5	Matrix Form of the Superposition-Based Numerical Approximation	74
3.4	One-Dimensional Problem with Time-Dependent Surface Heat Flux	75
3.4.1	Piecewise-Constant Approximation	76
3.4.1.1	Superposition-Based Numerical Approximation of the Solution	76
3.4.1.2	Heat Flux-Based Sensitivity Coefficients	77
3.4.1.3	Basic “Building Block” Solution	77
3.4.1.4	Computer Code and Example	77
3.4.1.5	Matrix Form of the Superposition-Based Numerical Approximation	79
3.4.2	Piecewise-Linear Approximation	79
3.4.2.1	Superposition-Based Numerical Approximation of the Solution	81
3.4.2.2	Heat Flux-Based Sensitivity Coefficients	81
3.4.2.3	Basic “Building Block” Solutions	82
3.4.2.4	Computer Code and Examples	82
3.4.2.5	Matrix Form of the Superposition-Based Numerical Approximation	84
3.5	Two-Dimensional Problem with Space-Dependent and Constant Surface Heat Flux	85
3.5.1	Piecewise-Uniform Approximation	85
3.5.1.1	Superposition-Based Numerical Approximation of the Solution	87
3.5.1.2	Heat Flux-Based Sensitivity Coefficients	87
3.5.1.3	Basic “Building Block” Solution	87
3.5.1.4	Computer Code and Examples	88
3.5.1.5	Matrix Form of the Superposition-Based Numerical Approximation	90
3.6	Two-Dimensional Problem with Space- and Time-Dependent Surface Heat Flux	90
3.6.1	Piecewise-Uniform Approximation	91
3.6.1.1	Numerical Approximation in Space	91
3.6.2	Piecewise-Constant Approximation	91
3.6.2.1	Numerical Approximation in Time	92
3.6.3	Superposition-Based Numerical Approximation of the Solution	92
3.6.3.1	Sequential-in-time Nature and Sensitivity Coefficients	94
3.6.3.2	Basic “Building Block” Solution	95
3.6.3.3	Computer Code and Example	95
3.6.3.4	Matrix Form of the Superposition-Based Numerical Approximation	97
3.7	Chapter Summary	98
	Problems	99
	References	101
4	Inverse Heat Conduction Estimation Procedures	103
4.1	Introduction	103
4.2	Why is the IHCP Difficult?	104
4.2.1	Sensitivity to Errors	104
4.2.2	Damping and Lagging	104
4.2.2.1	Penetration Time	104
4.2.2.2	Importance of the Penetration Time	105
4.3	Ill-Posed Problems	105
4.3.1	An Exact Solution	105
4.3.2	Discrete System of Equations	107
4.3.3	The Need for Regularization	108
4.4	IHCP Solution Methodology	108
4.5	Sensitivity Coefficients	108
4.5.1	Definition of Sensitivity Coefficients and Linearity	108
4.5.2	One-Dimensional Sensitivity Coefficient Examples	110
4.5.2.1	X22 Plate Insulated on One Side	110
4.5.2.2	X12 Plate Insulated on One Side, Fixed Boundary Temperature	113
4.5.2.3	X32 Plate Insulated on One Side, Fixed Heat Transfer Coefficient	113

4.5.3	Two-Dimensional Sensitivity Coefficient Example	114
4.6	Stolz Method: Single Future Time Step Method	116
4.6.1	Introduction	116
4.6.2	Exact Matching of Measured Temperatures	116
4.7	Function Specification Method	118
4.7.1	Introduction	118
4.7.2	Sequential Function Specification Method	119
4.7.2.1	Piecewise Constant Functional Form	119
4.7.2.2	Piecewise Linear Functional Form	125
4.7.3	General Remarks About Function Specification Method	127
4.8	Tikhonov Regularization Method	127
4.8.1	Introduction	127
4.8.2	Physical Significance of Regularization Terms	128
4.8.2.1	Continuous Formulation	128
4.8.2.2	Discrete Formulation	128
4.8.3	Whole Domain TR Method	129
4.8.3.1	Matrix Formulation	129
4.8.4	Sequential TR Method	133
4.8.5	General Comments About Tikhonov Regularization	134
4.9	Gradient Methods	134
4.9.1	Conjugate Gradient Method	134
4.9.1.1	Fletcher-Reeves CGM	136
4.9.1.2	Polak-Ribiere CGM	136
4.9.2	Adjoint Method (Nonlinear Problems)	137
4.9.2.1	Some Necessary Mathematics	138
4.9.2.2	The Continuous Form of IHCP	139
4.9.2.3	The Sensitivity Problem	139
4.9.2.4	The Lagrangian and the Adjoint Problem	140
4.9.2.5	The Gradient Equation	141
4.9.2.6	Summary of IHCP solution by Adjoint Method	142
4.9.2.7	Comments About Adjoint Method	142
4.9.3	General Comments about CGM	143
4.10	Truncated Singular Value Decomposition Method	143
4.10.1	SVD Concepts	143
4.10.2	TSVD in the IHCP	144
4.10.3	General Remarks About TSVD	145
4.11	Kalman Filter	145
4.11.1	Discrete Kalman Filter	146
4.11.2	Two Concepts for Applying Kalman Filter to IHCP	147
4.11.3	Scarpa and Milano Approach	148
4.11.3.1	Kalman Filter	148
4.11.3.2	Smoother	149
4.11.4	General Remarks About Kalman Filtering	150
4.12	Chapter Summary	150
	Problems	151
	References	154
5	Filter Form of IHCP Solution	157
5.1	Introduction	157
5.2	Temperature Perturbation Approach	157
5.3	Filter Matrix Perspective	159
5.3.1	Function Specification Method	159
5.3.2	Tikhonov Regularization	161

5.3.3	Singular Value Decomposition	162
5.3.4	Conjugate Gradient	163
5.4	Sequential Filter Form	164
5.5	Using Second Temperature Sensor as Boundary Condition	166
5.5.1	Exact Solution for the Direct Problem	166
5.5.2	Tikhonov Regularization Method as IHCP Solution	168
5.5.3	Filter Form of IHCP Solution	168
5.6	Filter Coefficients for Multi-Layer Domain	171
5.6.1	Solution Strategy for IHCP in Multi-Layer Domain	171
5.6.1.1	Inner Layer	171
5.6.1.2	Outer Layer	172
5.6.1.3	Combined Solution	173
5.6.2	Filter Form of the Solution	173
5.7	Filter Coefficients for Non-Linear IHCP: Application for Heat Flux Measurement Using Directional Flame Thermometer	174
5.7.1	Solution for the IHCP	176
5.7.1.1	Back Layer (Insulation)	176
5.7.1.2	Front Layer (Inconel plate)	176
5.7.1.3	Combined Solution	176
5.7.2	Filter form of the solution	177
5.7.3	Accounting for Temperature-Dependent Material Properties	177
5.7.4	Examples	179
5.8	Chapter Summary	183
	Problems	184
	References	184
6	Optimal Regularization	187
6.1	Preliminaries	187
6.1.1	Some Mathematics	187
6.1.2	Design vs. Experimental Setting	188
6.2	Two Conflicting Objectives	188
6.2.1	Minimum Deterministic Bias	188
6.2.2	Minimum Sensitivity to Random Errors	188
6.2.3	Balancing Bias and Variance	189
6.3	Mean Squared Error	189
6.4	Minimize Mean Squared Error in Heat Flux R_q^2	190
6.4.1	Definition of R_q^2	190
6.4.2	Expected Value of R_q^2	191
6.4.3	Optimal Regularization Using $E(R_q^2)$	193
6.5	Minimize Mean Squared Error in Temperature R_T^2	196
6.5.1	Definition of R_T^2	196
6.5.2	Expected Value of R_T^2	197
6.5.3	Morozov Discrepancy Principle	197
6.6	The L-Curve	199
6.6.1	Definition of L-Curve	199
6.6.2	Using Expected Value to Define L-Curve	199
6.6.3	Optimal Regularization Using L-Curve	200
6.7	Generalized Cross Validation	201
6.7.1	The GCV Function	202
6.8	Chapter Summary	205
	Problems	205
	References	206

7	Evaluation of IHCP Solution Procedures	209
7.1	Introduction	209
7.2	Test Cases	210
7.2.1	Introduction	210
7.2.2	Step Change in Surface Heat Flux	211
7.2.3	Triangular Heat Flux	211
7.2.4	Quartic Heat Flux	213
7.2.5	Random Errors	216
7.2.6	Temperature Perturbation	217
7.2.7	Test Cases with Units	217
7.3	Function Specification Method	217
7.3.1	Step Change in Surface Heat Flux	218
7.3.2	Triangular Heat Flux	219
7.3.3	Quartic Heat Flux	219
7.3.4	Temperature Perturbation	221
7.3.5	Function Specification Test Case Summary	223
7.4	Tikhonov Regularization	224
7.4.1	Step Change in Surface Heat Flux	224
7.4.2	Triangular Heat Flux and Quartic Heat Flux	225
7.4.3	Temperature Perturbation	226
7.4.4	Tikhonov Regularization Test Case Summary	228
7.5	Conjugate Gradient Method	229
7.5.1	Step Change in Surface Heat Flux	229
7.5.2	Triangular Heat Flux and Quartic Heat Flux	230
7.5.3	Temperature Perturbation	232
7.5.4	Conjugate Gradient Test Case Summary	234
7.6	Truncated Singular Value Decomposition	234
7.6.1	Step Change in Surface Heat Flux	235
7.6.2	Triangular and Quartic Heat Flux	236
7.6.3	Temperature Perturbation	237
7.6.4	TSVD Test Case Summary	238
7.7	Kalman Filter	239
7.7.1	Step Change in Surface Heat Flux	239
7.7.2	Triangular and Quartic Heat Flux	239
7.7.3	Temperature Perturbation	241
7.7.4	Kalman Filter Test Case Summary	244
7.8	Chapter Summary	245
	Problems	247
	References	248
8	Multiple Heat Flux Estimation	249
8.1	Introduction	249
8.2	The Forward and the Inverse Problems	249
8.2.1	Forward Problem	249
8.2.2	Inverse Problem	250
8.2.3	Filter Form of the Solution	251
8.3	Examples	254
8.4	Chapter Summary	258
	Problems	258
	References	259

9	Heat Transfer Coefficient Estimation	261
9.1	Introduction	261
9.1.1	Recent Literature	261
9.1.2	Basic Approach	262
9.2	Sensitivity Coefficients	263
9.3	Lumped Body Analyses	266
9.3.1	Exact Matching of the Measured Temperatures	266
9.3.2	Filter Coefficient Solution	268
9.3.3	Estimating Constant Heat Transfer Coefficient	269
9.4	Bodies with Internal Temperature Gradients	272
9.5	Chapter Summary	274
	Problems	274
	References	275
10	Temperature Measurement	277
10.1	Introduction	277
10.1.1	Subsurface Temperature Measurement	277
10.1.2	Surface Temperature Measurement	278
10.2	Correction Kernel Concept	278
10.2.1	Direct Calculation of Surface Heat Flux	278
10.2.2	Temperature Correction Kernels	279
10.2.2.1	Determining the Correction Kernel	279
10.2.2.2	Correcting Temperature Measurements	280
10.2.3	2-D Axisymmetric Model	280
10.2.4	High Fidelity Models and Thermocouple Measurement	285
10.2.4.1	Three-Dimensional Models	285
10.2.4.2	Location of Sensed Temperature in Thermocouples	286
10.2.5	Experimental Determination of Sensitivity Function	286
10.3	Unsteady Surface Element Method	287
10.3.1	Intrinsic Thermocouple	288
10.4	Chapter Summary	291
	Problems	292
	References	293
	Appendix A Numbering System	295
	Appendix B Exact Solution	301
	Appendix C Green's Functions Solution Equation	309
	Index	321

