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Mechanical properties of cables made with helically wound carbon-nanotube fibers for advanced structural applications

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Abstract. Big structures, such as super long suspension bridges, require materials that overcome the specific strength of steel, since there is a material-dependent limit size beyond which they shall collapse under their own weight. Carbon NanoTube Fibers (CNTFs) hold great promise for advanced applications, for their exceptional strength and stiffness per unit specific weight. We propose a theoretical model to describe the mechanical response of cables made of CNTFs. The mechanical response is studied via a variational approach and closed-form expressions are obtained for the stiffness parameters of the cable and the stress in the constituent CNTFs. This study shows that the stiffness and strength of the cable present opposite variations with respect to parameters associated with the geometric arrangement of the CNTFs, suggesting the existence of an optimal compromise yet to be experimentally verified.

Introduction

Challenging engineering applications require materials with specific strength much higher than metals [1,2]. Carbon NanoTubes (CNTs) may be used in new applications, yet to be fully appreciated, as they possess exceptional physical properties [3-4] not only to replace metals in current engineering applications, but also to design structures not even conceivable with the materials currently available [1,2]. The problem is how to employ very small CNTs (1nm diameter, 10μm length) in large structural elements. When assembled into a matrix, their strength is impaired by non-optimal orientations, defects and the intrinsic weakness of the matrix. CNT Fibers (CNTFs), obtained via solution spinning [3], are made of highly-aligned densely-packed CNTs and can currently be manufactured with diameter of about 100μm and, in principle, no limit in length. Since there is no matrix embedding the CNTs, the structural capacity of such CNTFs could theoretically approach that of the CNTs. Indeed, technological improvements have permitted an increase in specific strength of about 23% per year over the last 20 years [3], and a similar positive trend is expected for the years to come. Recent studies [5] have shown that stiffness and strength of CNTFs [4] depend on the lateral bond and on the longitudinal offset between the constituent CNTs. New production techniques are expected to allow denser packaging of longer CNTs with optimal offset and, consequently, higher performing CNTFs [6].

Here, we present a theoretical model, propaedeutic for an experimental campaign, to define the mechanical properties of cables made of CNTFs. The cable mechanical properties are derived via a variational approach, in which the CNTFs are rod-like elements with macroscopic properties derived by the micromechanics of the constituent CNTs. Some of the main results of this study are summarized in the Conclusion section, together with indications on future developments.

Mechanical model

Consider a cable made of M layers of CNTFs wound around a straight central fiber. Figure 1(a) shows the cases with $M = 1,2,3$. The tensile and torsional properties of the cable clearly depend upon the geometric arrangement of the CNTFs.

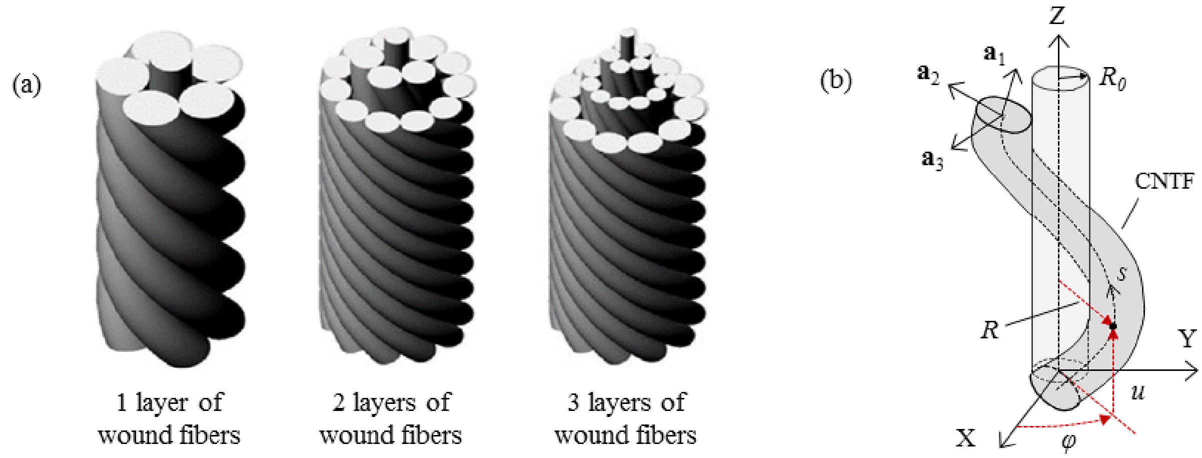

Figure 1: (a) cable with 1,2,3 layers of wound fibers; (b) problem of a fiber wound on a cylinder.

The basic problem, represented in Figure 1(b), is that of a fiber, modelled as a circular cross-sectioned rod of radius R_f, wound around a straight cylinder of radius R_0 ($R=R_0+R_f$), whose ends are displaced in a given position. The fiber deformation is described via classical indicators of strain [7-9], i.e., the axial extension ε_f, bending curvature k_{Bf}, and torsion curvature k_{Tf}, while the equilibrium state is found with a variational approach. The strain energy of the fiber is the line integral over the fiber length L_f of the strain energy density W_f, which depends upon the fiber axial stiffness $\mathcal{E}_f$, bending stiffness $\mathcal{B}_f$, and torsional stiffness $\mathcal{T}_f$ (derivable from a micro-mechanical model of the CNTFs [5], or from experimental tests [4]), and is written in the form

$$2W_f = \mathcal{E}_f \varepsilon_f^2 + \mathcal{B}_f k_{Bf}^2 + \mathcal{T}_f k_{Tf}^2. \quad (1)$$

For a stretched and twisted cable, we assume that the problem symmetry is respected: all the fibers deform together, cross-sections remain plane, and frictional forces are negligible. With respect to the fixed reference frame X,Y,Z in Figure 1(b), the actual position of a point of the fiber center-line at the arc-length $s \in (0,L_f)$ is expressed in terms of the cylindrical coordinates $\varphi(s)$ and $u(s)$, while $\theta(s)$ describes the rotation of the fiber cross-section about the fiber center-line, with tangent $\mathbf{a}_1$. When $\varphi(s)$, $u(s)$, $\theta(s)$ are prescribed at the fiber ends, s=0 and $s=L_f$, a stationary point for the fiber strain energy corresponds to a configuration in which the fiber center-line is a helix, the cross-sectional twist θ' is constant (prime denotes s-derivative), and

$$k_{Bf} = \varphi' cos\gamma, \;\; k_{Tf} = \theta' + \varphi' sin\gamma, \quad (2)$$

where $tan\, \gamma = u'/(R\varphi')$ defines the constant pitch angle γ in terms of the other variables.

It is important to observe that the torsion curvature k_{Tf} is composed of two contributions: the cross-sectional twist θ' and the tortuosity-induced twist $\varphi' sin\, \gamma$. The fiber is torsion-free when the cross-sectional twist balances the tortuosity-induced twist. This condition plays an important role in the manufacturing of a cable made of helically wound CNTFs, which consists in inducing a cross-sectional "pre-twist" in a straight fiber that is subsequently transformed in "tortuosity" for its helical configuration, as in the manufacturing of a hemp rope [6].

A 7-wire strand (1-layer cable) is represented in Figure 2. For its manufacturing, straight fibers are pulled and pre-twisted and, subsequently, are brought closer and wound together in such a way that section pre-twist turns into center-line tortuosity. Adding a second layer of fibers one obtains the 2-layer cable of Figure 1(a); repeating the construction, a *M*-layer cable can be manufactured.

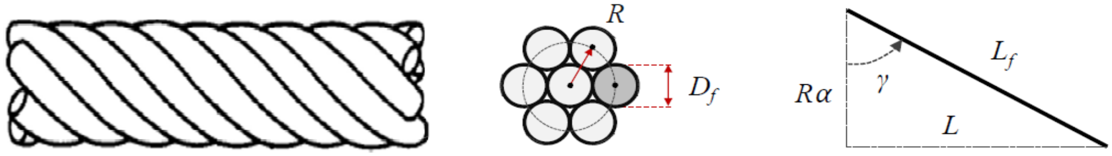

Figure 2: A 7-wire strand (1 layer of helically wound fibers) and geometric parameters.

The first layer of helical fibers in a 7-wire strand is the prototypical model for the *i-th* layer in a *M*-layer cable, for which each layer is wound around a cylinder (i.e., the straight central fiber for the strand). Assume that the strand has been manufactured: each fiber is arranged according to an helix of length L_f, helical radius R, slope γ, $L=L_f \sin\gamma$, and coordinate φ that spans the angle α from the first end to the second end of the fiber, such that $\alpha R = L_f \cos\gamma$. A system of forces, equipollent to an axial force and a twisting moment, are applied at the strand ends. All fibers deform together and their varied state is a new helix with new parameters $L_f+\Delta L_f$, $L+\Delta L$, $R+\Delta R$, $\alpha+\Delta\alpha$, $\gamma+\Delta\gamma$.

The macroscopic strain of an arrangement of this kind, when pulled and twisted, is described via the axial strain $\varepsilon=\Delta L/L$ and twist rate $\beta=\Delta\alpha/L$. For small variations, ΔL_f, ΔL, ΔR, $\Delta\alpha$, $\Delta\gamma$, it is possible to demonstrate that the extension, ε_f, and curvatures, k_{Bf} and k_{Tf}, of all fibers can be expressed as linear combinations of the strain indicators ε and β, via coefficients that depend on the slope γ and radius R of the unstrained state. Therefore, considering that the strain energy density W of the arrangement is the sum of those of the constituent CNTFs, W turns out to be a quadratic function of the strain indicators ε and β. This holds for a 7-wire strand as well as for a cable made of M concentric layers of helically wound CNTFs.

Denoting as $W_{(2)}$ the quadratic part of the cable strain energy density, one can write

$$2W_{(2)} = \mathcal{E}\varepsilon^2 + \mathcal{T}\beta^2 + 2X\varepsilon\beta \,, \tag{3}$$

where $\mathcal{E}$ represents the axial stiffness of the cable, $\mathcal{T}$ is the torsional stiffness, and X accounts for the extension-torsion coupling associated with the helical configuration. These parameters depend on the slope, diameter, and Poisson's ratio of the helical CNTFs, but their expressions are generally complicated. Compact analytical formulas can be obtained in paradigmatic cases, e.g., when all fibers have the same slope γ, diameter D_f, and Poisson's ratio ν_f, and the contribution of the fiber bending stiffness $\mathcal{B}_f$ and torsional stiffnesses $\mathcal{T}_f$ are negligible with respect to that of the fiber axial stiffness $\mathcal{E}_f$, as is expected for CNTFs [4,5].

Under the aforementioned hypotheses, the dependence of the cable axial stiffness $\mathcal{E}$ on the axial stiffness $\mathcal{E}_f$ and slope γ of the helical fibers can be expressed [6] as

$$\mathcal{E} = \rho\, N_f \mathcal{E}_f sin^3\gamma, \tag{4}$$

where N_f represents the total number of fibers in the cable and ρ is a coefficient (close to 1) that depends on ν_f. Similar expressions hold for the other stiffness parameters of the cable [6].

Observe that the cable axial stiffness $\mathcal{E}$ increases with the angle γ, and is maximized for $\gamma=90°$ (straight fibers). However, helical fibers are expected to provide a cable with a superior internal cohesion because of the radial contact forces that take place among the cable layers.

Results

Recent experiments [4] have measured the mechanical properties of CNTFs with $D_f=22\mu m$, made of CNTs of diameter $d=1.5$nm and lengths 2.2-6.3µm. The axial stiffness $\mathcal{E}_f$ of these fibers is in the range 27.5-54.4 Nm/m, and is dominant over the bending stiffness, being $\mathcal{B}_f/(\mathcal{E}_f D_f^2)$ within the interval 0.01-0.05. Relying on these measures, we have performed a parametric analysis with our theoretical model, to evaluate the mechanical properties of a cable made of M layers of helical CNTFs as a function of representative properties, i.e., fiber stiffnesses, Poisson's ratio, slope γ.

To illustrate, Figure 3 reports results obtained for the cable axial stiffness $\mathcal{E}$ normalized via the reference parameter $\mathcal{E}_{ref} = N_f \mathcal{E}_f$. It is found that the CNTF bending and torsional stiffnesses $\mathcal{B}_f$ and $\mathcal{T}_f$ have a negligible effect; Poisson's ratio has a mild influence for larger γ (say, above 65-70°), while the helical slope γ, the number of fibers N_f, and the fiber axial stiffness $\mathcal{E}_f$ play the major role. Similar results hold for the other stiffness parameters of the cable.

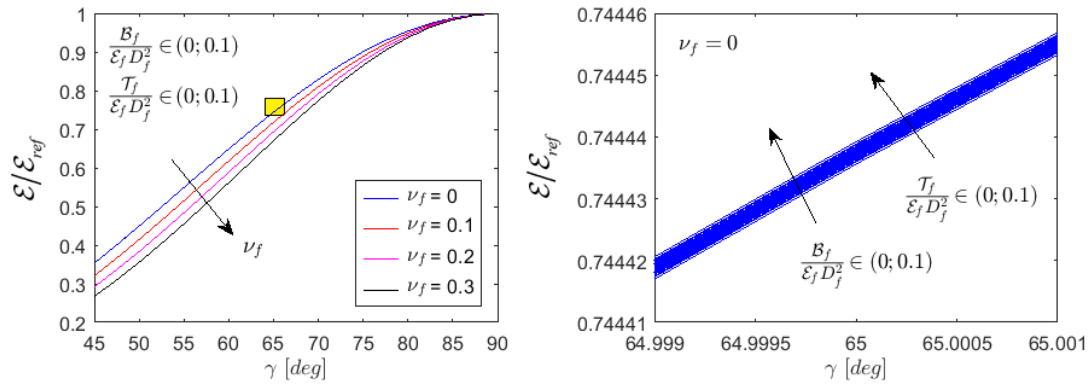

Figure 3: Cable axial stiffness as a function of the fiber stiffness, Poisson's ratio, and slope γ.

When the cable is pulled by a force F and the rotation of its end sections is prevented, the constituent helical fibers undergo a tensile force $\mathcal{E}_f \varepsilon_f$, which is independent of the position of the layer they belong to in the cable and is shown (normalized by F/N_f) in Figure 4. The conclusions about the dependence of the cable axial stiffness on the fiber parameters also hold true for the stress parameter $(\mathcal{E}_f \varepsilon_f)/(F/N_f)$, which represents the ratio between the tensile force in the helical fibers of the cable and the value of such force if all the fibers were straight.

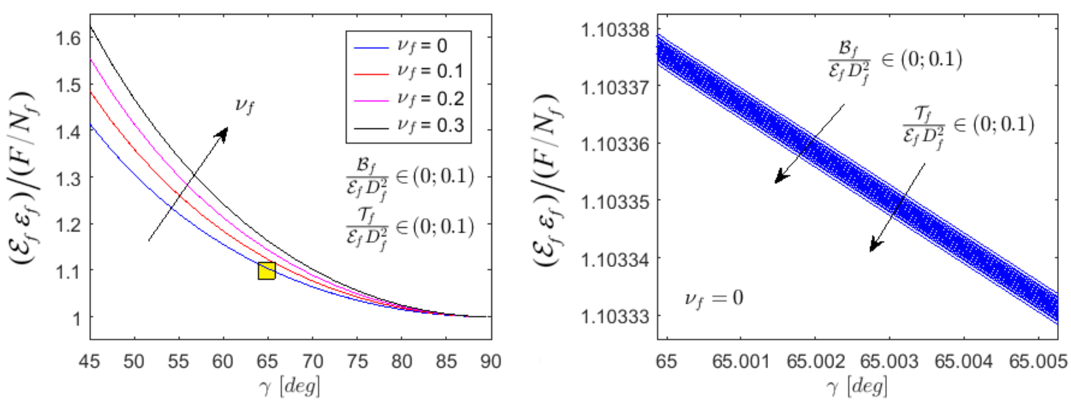

Figure 4: Tensile force in cable fibers as a function of fiber stiffness, Poisson's ratio, and slope γ.

For a cable made of M layers of helical CNTFs, another important stress indicator is represented by the radial contact forces among the cable layers. Indeed, a pressure orthogonal to the CNTFs can increase the lateral bond among the constituent CNTs and, consequently, the mechanical properties of the CNTFs. Furthermore, radial contact forces compress the CNTFs together, so that if a fiber breaks, the frictional forces associated with the compression allows to re-establish the load bearing capacity of the broken fiber at a certain distance from the failure point. Therefore, in a cable made of many fibers, a limited number of fiber failures occurring at different sections in the cable does not compromise the overall bearing capacity of the cable. In conclusion, the cable resilience is enhanced by the radial contact forces produced by a helical arrangement of fibers.

For the considered cable, the radial force that is exerted on each fiber of the *i-th* layer at the radial position R_i (distance between the *i-th* layer and the cable axis) can be evaluated as the sum of all actions that are exerted on this layer by the other layers $i+1,\ldots, M$. Ignoring the effects of the fiber bending/torsional stiffness and considering only the dominant contribution of the fiber axial stiffness, it is possible to show [6] that if the cable is pulled and the rotation of its end sections is prevented, the radial contact forces can be evaluated via the simplified formula

$$\frac{F_R D_f}{\mathcal{E}_f \varepsilon_f} = cos^2\gamma\left(\frac{R_M}{R_i} - 1\right), \tag{5}$$

where R_M represents the radius of the cable and F_R is the radial force at the *i-th* layer.

The dimensionless parameter $(F_R D_f)/(F/N_f)$ associated with the radial forces among the cable layers represents a measure of the resilience of the cable and of the increase of its tensile strength obtainable by helically arranged fibers. This parameter is reported in Figure 5 as a function of the slope γ. In the same Figure, for comparison, we juxtapose the graph of Figure 3, which is associated with the stiffness of the cable. The case shown corresponds to a cable composed of $M = 100$ layers of CNTFs and considers the state of stress at the layer $R_i = 30D_f$.

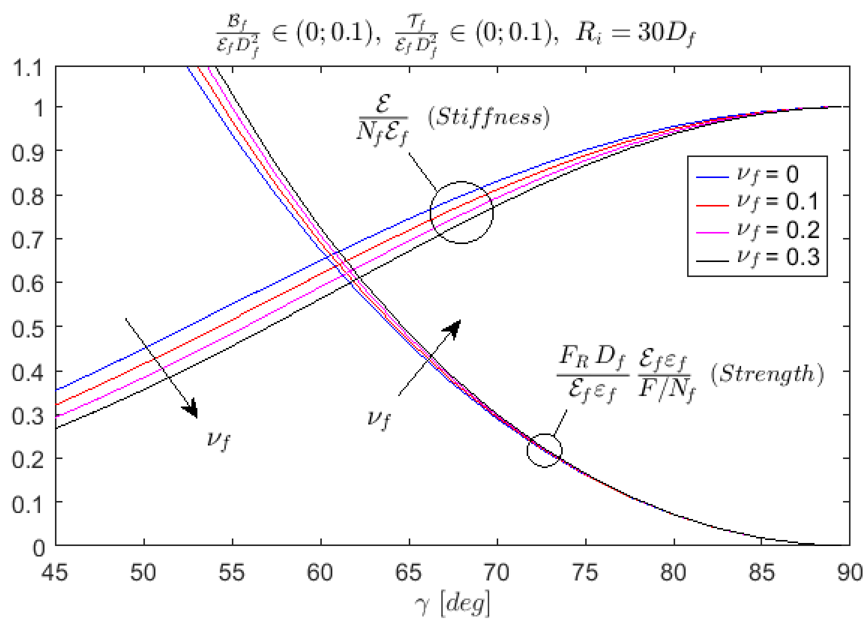

Figure 5: Cable stiffness and strength as a function of fiber stiffness, Poisson's ratio, and slope γ.

Remarkably, the cable axial *stiffness* increases with the slope γ, as opposed to the parameter related to the cable *strength*. Therefore, a value of $\gamma = 90°$, which optimizes the stiffness, may not optimize the strength. This suggests the existence of an optimal compromise between strength and stiffness of the cable, which needs to be further investigated and verified experimentally.

Conclusions

Thanks to their exceptional mechanical properties, CNTFs appear particularly suitable for being the basic constituents of cables for advanced structural applications. An ad-hoc theoretical model has been proposed to describe the mechanical properties of cables made of wound CNTFs.

The approach is variational and considers the minimization of the strain energy that additively accounts for the contribution of the individual CNTFs in the cable. Using symmetry, the problem is reduced to the paradigmatic case of an isolated fiber wound onto a cylinder, whose equilibrium states are found to be helixes with constant pitch. Analytical expressions have been found for the stiffness parameters of a cable composed by several concentric layers of helical CNTFs, as well as for the internal stresses in such CNTFs. A parametric analysis has been performed to evaluate the dependence of the cable mechanical response on the stiffness, Poisson's ratio, and helical pitch of the constituent CNTFs. The analysis has demonstrated the important influence of the fiber axial stiffness and helical pitch, the limited effect of the fiber Poisson's ratio for large helical pitch, and the negligible contribution of the fiber bending stiffness and torsional stiffness when dealing with the calculation of the stiffness and strength parameters of the cable.

The theoretical model has disclosed important aspects to be corroborated by experiments. If the characterization of the axial stiffness and bending stiffness of the CNTFs has been the subject of intense experimental tests, little has been done on Poisson's ratio and torsional stiffness. Another important issue concerns the radial contact forces between helically wound CNTFs, which result from the stretching or twisting of the cable. Such forces can increase the bond between the CNTs that form the CNTFs, enhancing their effective strength, and produce a friction constraint which improves the cable resilience in the event of rupture of one CNTF. This study has shown that the radial contact forces increase by reducing the helical pitch of the cable CNTFs, but the cable axial stiffness shows an opposite trend. This suggests the existence of an optimal compromise between the strength and stiffness of the cable. An extended experimental campaign has been planned to corroborate the theoretical findings.

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