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Analysis of the behavior of the masonry Medici tower resorting on a hybrid discrete-kinematic methodology

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Abstract

This study presents a novel integrated discrete-analytical approach for analyzing the collapse behavior of the masonry Medici tower (L'Aquila, Italy). Due to their slenderness, masonry towers are characterized by high susceptibility to seismic actions and several approaches can be adopted to analyze their seismic vulnerability. Generally, engineers-practitioners and researchers study the local and global collapse mechanisms based on simplified kinematic analysis, as prescribed by national and international construction codes or, alternatively, more sophisticated approaches such as nonlinear finite element methods have been adopted to simulate the response of masonry towers. Although successful in some applications, these methods are limited in accurately capturing crack distributions and fracture mechanisms. In fact, they completely ignore the damage propagation phenomenon, starting from the trigger of the fracture up to the complete structural failure condition, that is instead fundamental aiming to analyze intermediate damage states for the check of serviceability limit states or to individuate a more realistic structural crack distribution in ultimate conditions. This work proposes a hybrid discrete-kinematic approach: first, the Lattice Discrete Particle Model (LDPM), that simulates masonry at meso-scale, is used to individuate the actual collapse mechanism; next, the individuated cracked configuration is used in the kinematic analysis for the analysis in ultimate conditions. The results show that the collapse of the Medici tower due to the 2009 L'Aquila earthquake is well predicted by LDPM and the corresponding limit analyses demonstrate the efficiency of the proposed hybrid approach applied to this case study. Additional results point out that different load configurations, more specifically variations in the direction of the seismic action, provoke in certain cases a more diffused damage and a clear failure pattern can not be identified for kinematic analyses. In these cases, relying mainly on comprehensive numerical models, such as LDPM, is fundamental to study the fracturing process from the cracks trigger up to the ultimate complex collapse mechanism.

Keywords: Seismic action; earthquake; Masonry tower; Discrete Modeling; Fracture Patterns; Kinematic analysis; Existing structures

1. Introduction

Slender masonry elements, such as towers and belfries, are one of the most recurrent architectural categories characterizing medieval historical towns (Mendes et al. (2005), Pieraccini et al. (2014), Bartoli et al. (2013)). These high elements are intrinsically vulnerable to seismic actions and they are prone to show extensive damage conditions

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and collapse when subjected to horizontal forces (Valluzzi et al. (2007), Lagomarsino (2012)). Two are the main reasons for the high susceptibility of these structures, i.e. their slenderness and the heterogeneous nature of their constituent materials (units and mortar joints) (Xu et al. (2012), Shadlou (2020), Gregori et al. (2022)). During the international debate on Cultural Heritage preservation, it has been recurrently underlined the importance of assessing the seismic vulnerability of masonry slender structures, aiming to preserve their usability for future generations (Committee I et al. (2005), Vailati et al. (2021)). Aiming to reach this goal it is fundamental to perform effective and extensive experimental campaigns on full-scale elements or structural sub-portions (Chourasia et al. (2016), Corradi et al. (2003)). However, the realization of experiments is very often an expensive activity and, nowadays, a growing interest is given from researchers to innovative emerging numerical tools. The most widely used numerical approach is the Finite Element Method (FEM) (Baraldi et al. (2018), Gregori et al. (2020) and Gregori et al. (2021)). Within this method, one very promising numerical strategy provides to model each component of the masonry element, i.e. brick and mortar joints, by means of a micro-scale approach (Drougkas et al. (2014)). Although FEM is very effective for regular masonry structures, it appears to be limited in simulating irregular masonries (Mercuri et al. (2021)). In fact, because of the heterogeneous nature of the material and the necessity of capturing complex crack distributions and fracture mechanisms, adopting a dedicated modeling tool is necessary.

To analyze the seismic behavior of masonry elements, the Italian construction code prescribes to perform the analysis of local and global collapse mechanisms. Identifying collapse mechanisms is not a trivial activity, it involves a subjective judgment on the modalities and possibilities of structural activation under seismic excitation and it requires a preliminary thorough study of the unreinforced masonry structure. The knowledge of the construction should be reached through an exhaustive survey that allows an understanding of the different historical phases affecting the building stratifications over the course of the years, and the full characterization of the structural geometry and material (Brandi (2022), Mercuri et al (2020)). Afterward, it is possible to apply a consolidated methodology for the assessment of the vulnerability of unreinforced masonry structures (Giuffré (1993)), that consists in identifying a priori the collapse mechanisms of the structure, by considering the involved portions of the building as a number of rigid blocks connected by unilateral hinges or sliding joints, in order to obtain a kinematic chain. The assumptions of this approach provide each rigid macro-elements to have unlimited compressive strength and their reciprocal interfaces to be characterized by the absence of tensile strength. Finally, for each rigid block, the linear and non-linear kinematic analyses can be performed and, therefore, the mechanism most likely to occur of all the possible local mechanisms can be identified. This methodology is satisfying to some extent, but has three main limitations: (i) the a priori choice of the collapse mechanisms brings an intrinsic level of uncertainty, that is related to the subjective experience of the analyst, (ii) the procedure is very tedious to be performed as both linear and non-linear kinematic analyses have to be applied to each rigid block, (iii) for complex masonry geometries, the identification of simplified collapse modalities is too simplistic and, thus, the results may be inaccurate.

It is possible to overcome the aforementioned drawbacks directly simulating the fracturing behavior of masonry towers. In this case, resorting on fracture mechanics theory is fundamental (Bažant (2002), Mercuri (2022)). Several numerical methods can be used, as they capture the mechanical behavior of quasi-brittle materials with different degrees of accuracy (Cusatis and Cedolin (2007)). The so-called Lattice Discrete Particle Model (LDPM) is adopted in this study to model the masonry fracturing behavior at meso-scale (the reader is referred to the Appendix for a deep review of the model). LDPM allows to correctly localize the crack pattern that triggers the collapse mechanism. In this way, the pre-definition of multiple collapse mechanisms can be avoided and just one kinematic analysis can be directly applied to the numerically calibrated fractured structural configuration. In alternative, since LDPM captures the complete damage evolution phenomena, starting from cracks localization, propagation and up to the overall collapse, it can be used alternatively to the kinematic analysis. In this study, the LDPM is used as a complementary tool coupled with the kinematic analysis to describe the fracturing behavior of the Medici tower subjected to the 2009 L'Aquila earthquake. In particular, the cracked configuration is first identified from the numerical results and, after, the kinematic analysis is performed. Finally, a comparison between the kinematic analysis and the lattice discrete modeling is carried out, underlying the advantages and drawbacks of the proposed integrated approach.

2. Lattice discrete modeling of the masonry tower

2.1. Architectural and historical features

Medici tower is one of the most relevant architectures of central Italy, as it is located within a spectacular natural landscape and in the core of a medieval town (Santo Stefano di Sessanio, L'Aquila, Italy), as shown in Fig. 1. The actual shape of the tower is the result of many stratifications that occurred over the course of the years. During the 12th century, the tower was erected as a cylindrical body (internal diameter of the cross-section equal to 3.86 m and thickness equal to 1.50 m) without the top crowning, which was instead built during the Angioin's domination period. During the Second World War, a concrete slab was added on top of the structure, as the tower's function changed, being used as an antiaircraft station. During the 2009 L'Aquila earthquake, the Medici tower collapsed almost completely and, after, was rebuilt keeping intact the geometrical and architectural features.

Fig. 1. (a) Location of the Medici tower in the landscape and within the town core; (b) architectonic configuration of the tower before the 2009 L'Aquila earthquake; (c) reproduction of the tower's original profile through scaffolding placed after the collapse in 2009.

2.2. Numerical analysis

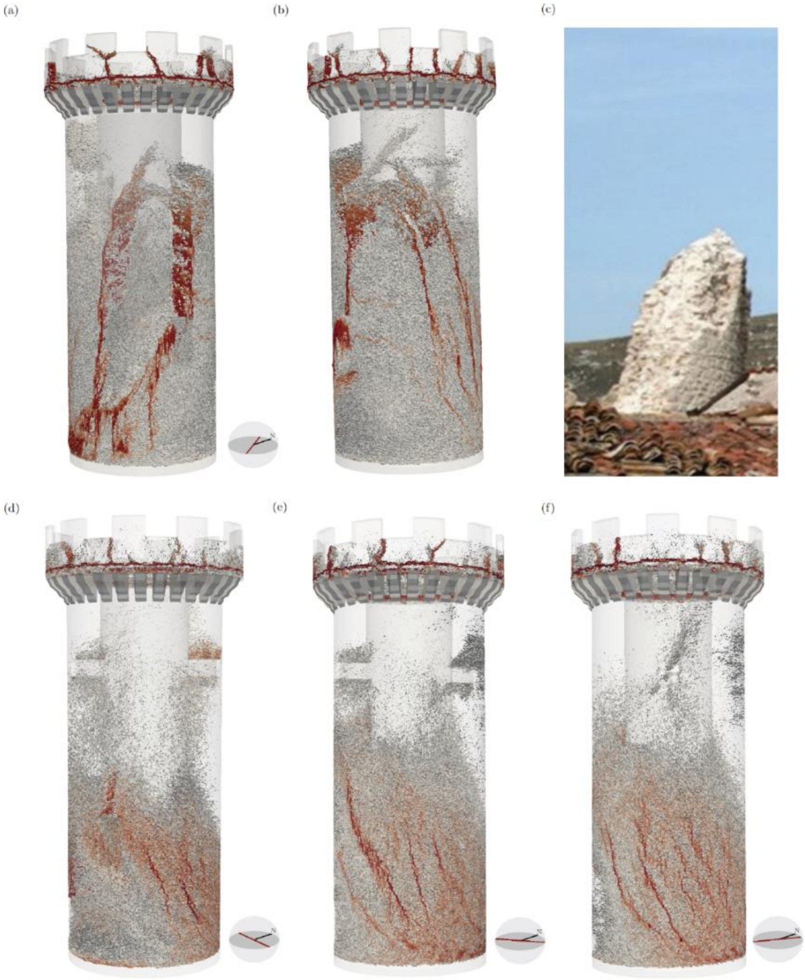
This section presents the numerical analysis of the Medici tower subjected to the 2009 L'Aquila earthquake. The Lattice Discrete Particle model is adopted for this purpose, being able to characterize the irregular masonry at meso-scale level, simulating the interaction between the stones and the mortar joints. The Lattice Discrete Particle Model was proposed by Cusatis et al. (2011a,b) to simulate the mechanical behavior of granular quasi-brittle materials such as concrete, fiber-reinforced concrete, mortar, and rocks and it was later coupled with multi-physics models to simulate complex phenomena, such as hygro-thermal-chemical processes. Fro a deeper understanding of the LDPM and for all the details related to its implementation, the reader is referred to the works of Cusatis et al. (2011a, 2011b). The meso-scale structure of LDPM is built by randomly placing spherical particles into a volume of material, from the largest to the smallest size. The particle size distribution follows a Fuller sieve curve. Next, a Delaunay tetrahedralization is first performed with the centers of the spherical particles to generate a lattice system. A domain tessellation is then performed to specify possible failure locations between adjacent particles. These latter two steps result in the generation of a system of polyhedral cells, each of them enclosing a spherical particle. The surface of each polyhedral cell is composed of triangular facets in which the LDPM constitutive equations are defined. The geometrical parameters adopted in this study were previously calibrated for the in-plane and out-of-plane behavior of unreinforced masonry (Angiolilli et al. (2020), Mercuri et al. (2020), Angiolilli et al. (2021)). More specifically, the stone-to-mortar ratio a/c = 3.4 corresponding to the ratio between the volume of stones and the volume of cement-mortar, and the water-to-mortar ratio w/c = 0.5 were assumed based on the actual masonry composition. The cement-mortar content parameter c=427.5 kg/m^3 was calculated such that the total mass density ρ was equal to 1,800.0 kg/m^3 using the formula: $\rho=c(1+w/c+a/c)$. The stones were assumed to have characteristic size within the range d_0=33 mm and d_a=200 mm in order to simulate the coarse gravel. It is worth specifying the stone dimensions was chosen to be consistent with experimental data performed on unreinforced masonry in both in plane and out of plane conditions (Modena (1999), Corradi et al. (2003), Degli Abbati et al. (2014)). The Fuller coefficient n_f=0.5 was assumed as the default parameter since no specific sieve curve was assumed for the preparation of the specimens in the experimental campaign. The identified model parameters are as follows: normal elastic modulus E_0=1,200 MPa, shear-normal coupling parameter α=0.065, tensile strength σ_t=0.3 MPa, tensile fracture energy G_t=11 N/m, shear-to-tensile strength ratio σ_s/σ_t=1.25, softening exponent n_t=0.2, yielding compressive stress σ_{c0}=125 MPa, initial hardening modulus H_{c0}/E_0=0.4, final hardening modulus H_{c1}=1, transitional strain ratio k_{c0}=1.75, deviatoric strain threshold ratio k_{c1}=1, deviatoric damage parameter k_{c2}=5, initial friction coefficient μ_0=0.2, asymptotic friction coefficient μ_∞=0, transitional stress σ_{N0}=42 MPa, densification ratio E_d/E_0=1 and r_s=0.

The modeling strategy consisted in reproducing the structural and functional stratifications that modified the tower during the centuries. The original hollow cylindrical body of the tower and the battlement composed by 10 merlons were modeled together using a single LDPM mesh and the openings were included within the main body of the tower to reproduce the presence of windows and doors. The 30 corbels were singularly included as elastic tetrahedral finite elements and also the concrete was modeled using elastic finite elements. The nodes belonging to the bottom surface of the tower are restrained by fixing all the rotations and the translations perpendicular to the direction of the seismic action. Three different LDPM meshes corresponding to three random stone distributions within the volume of the body tower were used for each set of simulations. The presence of the gravity load was accounted for with the preliminar application of the self-weight. The boundary conditions prescribed to the tower consisted in constraining all vertical translations related to the particles belonging to the bottom surface. The load was then prescribed as a velocity to all the particles contained in the lower volume of the tower of 50 cm high. The characteristics of the velocity-load were chosen to be consistent with the ones related to the 2009 L’Aquila earthquake (Decanini et al. (2012)) and, in particular, the direction was set to 147° of Strike and a magnitude of 590 mm/s, being this latter value the sum of two consecutive maximum Peak Ground Velocity, i.e. 330 mm/s and 260 mm/s, taken in absolute value (Decanini et al (2009)). In addition to the just described benchmark case, other five cases were investigated, making varying either the direction or the magnitude of the velocity-load: two additional cases consisted in fixing the direction of the velocity to 147° with respect to the North-South axis and setting the magnitude of the velocity to -330 mm/s and 260 mm/s, respectively; additionally, the other three set of simulations consisted in fixing the magnitude of the velocity to 590 mm/s and to make varying the direction of the velocity, i.e. setting it equal to 15°, 45°, 90° with respect to the North-South axis.

2.3. Results and discussion

The results of numerical simulations are shown in Fig. 2. For the benchmark case, Fig. 2a and Fig. 2b show the predictivity of LDPM in simulating the behavior of the tower subjected to the 2009 L’Aquila earthquake (the reader is also referred to Fig. 2c for a qualitative comparison with the real collapse configuration). It is worth pointing out that LDPM describes the progressivity of the fracturing phenomena: the damage first occurred in the top portion of the structure, in correspondence with the location of the concrete slab (that turned out to fall down almost untouched as a consequence of the April 2009 earthquake) and, after, the main crack triggered from the bottom opening and propagated as a diagonal macro-fracture up to the top narrow window, thus suggesting a correlation between the type of rupture with the presence of the opening within the main body of the tower.

Additional results, shown in Fig. 2d, Fig. 2e, and Fig. 2f, point out that the fracturing behavior of the tower mostly depends on the direction of the seismic action rather than the magnitude of the velocity. In fact, the qualitative results related to the two analyzed cases with Strike equal to 147° and magnitude of 330 m/s and 260 m/s almost coincide with the benchmark case (shown in Fig 2a and Fig 2b) and, therefore, they are not reported in this manuscript. As opposed to the previous observations, the three different failure mechanisms of the tower related to cases with a fixed magnitude of the velocity and different directions of the seismic action show different crack configurations. In particular, the fractures become more diffused and almost vertical for cases in which the seismic direction rotates with respect to the North-South direction of 90°, 45°, and 15°, respectively. This result may be due to the variable relative position of the openings with respect to the direction of the seismic action. As a general observation, LDPM is characterized by a very wide capability of predicting fracturing phenomena from the trigger of the fracture up to the complete structural failure, for simple and complex geometries subjected to a variety of



loading conditions. This capability is fundamental for complex geometries, for which the application of simplified methods is most of the times a crude approximation.

Fig. 2. Cracked configuration of the tower simulating the 2009 L'Aquila earthquake (a) in the South-Est facade; (b) in the North-West facade; (c) Tower stump after the 2009 L'Aquila earthquake; meso-scale crack openings assuming a direction of the seismic action rotated with respect to the North-South direction of: (d) 90°; (e) 45°; (f) 15°.

3. Linear kinematic analysis

After having correctly individuated the fractured configuration that triggers the collapse mechanism by means of LDPM, the kinematic analysis is performed. All the further discussion will be related to the cracked configuration shown in Fig. 2a and Fig. 2b related to the benchmark case of Sec. 2. The main contour of the fracture is diagonal and goes from the bottom opening (located at 3.80 m from the ground) up to the top narrow window (at about 13.00 m from the ground). The performance of the linear kinematic analysis allows the check of the Life Safety Limit State (SLV), carrying out a comparison between the activating acceleration a_0*, representing the capacity of the structure, with the maximum spectral acceleration for the structure in ultimate conditions $a_{exp,SLV}$, that indicates the demand: $a_0{}^* \geq a_{exp,SLV}$, or, alternatively, one can just verify that the acceleration factor $f_{a,SLV}$ is grater than 1 (where $f_{a,SLV} = a_0{}^*/a_{exp,SLV}$). Both the activating acceleration $a_0{}^*$ and the maximum spectral acceleration for the structure in ultimate conditions $a_{exp,SLV}$, as well as $f_{a,SLV}$ are computed according to Code NTC (2018) and they are reported in the following Table 1 (case b-B).

4. Discussion

From the numerical results related to the benchmark case (in Fig. 2a and Fig. 2b) it can be inferred that the main crack created by the 2009 L'Aquila earthquake was triggered from the bottom opening, and propagated diagonally reaching the top narrow window. However, since the rubble stone masonry is characterized by heterogeneous nature, the mutual position between the fracture contour and the openings is not clearly defined in the numerical results. On the other hand, while performing the kinematic analysis performed in Sec. 3, it has been assumed the diagonal crack to start from the bottom of the lower opening, at about 3.80 m from the ground, reaching the bottom of the top window, at about 13.00 m from the ground (Case b-B in Fig. 4). This assumption could not be on the safe side, as the reciprocal position between the diagonal fracture and the location of the openings could play a role while bridging LDPM with kinematic analysis and assessing the SLV check. In this perspective, five additional cases are analyzed making varying the mutual position between the diagonal fracture and the location of the openings.

Figure 3 shows the fractured configurations: the first set of three cases (cases b-B, m-B and h-B, corresponding to Fig. 3a, Fig. 3b and Fig. 3c, respectively) provides the diagonal crack to reach the bottom of the top window, at about 13.00 m from the ground, and to trigger from the bottom, the middle and the top of the lower opening, at 3.80 m, 4.90 m and 6.0 m from the ground, respectively; the second set of three cases (cases b-H, m-H and h-H, corresponding to Fig. 3d, Fig. 3e and Fig. 3f, respectively) assumes the diagonal crack to reach the top of the highest window, at about 13.50 m from the ground, and to trigger from the bottom, the middle and top of the lower opening, at 3.80 m, 4.90 m and 6.0 m from the ground, respectively. The results of the kinematic analysis are reported in Fig. 4 and Tab. 1. As expected, the SLV check is not verified for all analyzed cases and the collapse mechanisms are activated, being $a_0{}^* < a_{exp,SLV}$ (the reader is referred to Fig. 4b), but it is worth underlying that each case shows a different value of the acceleration factor $f_{a,SLV}$. This factor ranges from 0.789 and 0.970 and it is mportant observing that the assumption made for the benchmark case, i.e. crack starting from the bottom of the bottom window to the bottom of the top window, is in favor of safety as the acceleration factor is one of the lowest among the six cases ($f_{a,SLV} = 0.809$). Additionally, a trend related to the acceleration factor can be observed: $f_{a,SLV}$ increases as the crack in the vicinity of the bottom opening is assumed to be located at the bottom, the middle, or at the top (see Fig 4c). As a general result, it can be noticed that the method proposed in this paper, i.e. identifying the main failure pattern from the lattice discrete modeling, and then performing and refining the simplified kinematic analysis, allows to obtain realistic ultimate conditions for a given geometry. However, in other situations, it appears difficult or impossible to proceed this way, as the damage cannot be described by a single crack. For instance, Fig 2c, Fig 2d and Fig. 2e showed that the fracture becomes less localized and more distributed as the seismic direction changes. In the most general case, it is necessary to analyze the fracturing and the collapse behavior of masonry structures with elaborate more numerical models, such as LDPM, in addition or in alternative to simplified kinematic analysis. These advanced numerical models showed that the failure criteria and, more generally, the structural behavior depend on: (i) the presence of openings within walls, (ii) the correct application of boundary conditions and (iii) the proper consideration of the seismic direction (Mercuri et al. (2021b)).

5. Conclusions

This study presents an advanced numerical model, the so-called Lattice Discrete Particle Model (LDPM), to investigate the fracturing behavior of slender masonry elements and to predict the most likely to occur collapse mechanism. In particular, the LDPM was adopted to predict the fracturing behavior of the Medici masonry tower subjected to the 2009 L'Aquila earthquake. Six individuated cracked configurations are then used to perform the kinematic analysis. From the obtained results, the following conclusions can be deduced:

- The progressive fracturing behavior of the Medici tower under seismic excitation is replicated by LDPM
- The crack contour depends on the direction of the seismic action rather then its magnitude
- Identifying the correct fracture configuration is fundamental prior to perform the kinematic analysis and to obtain reliable SLV check.

- Simplified analytical methods, as kinematic analysis, are inaccurate and imply crude approximations of the real behavior for complex geometries and when the fracturing pattern is not clear neat. In these cases, using an advanced numerical tool, such as LDPM, is necessary.

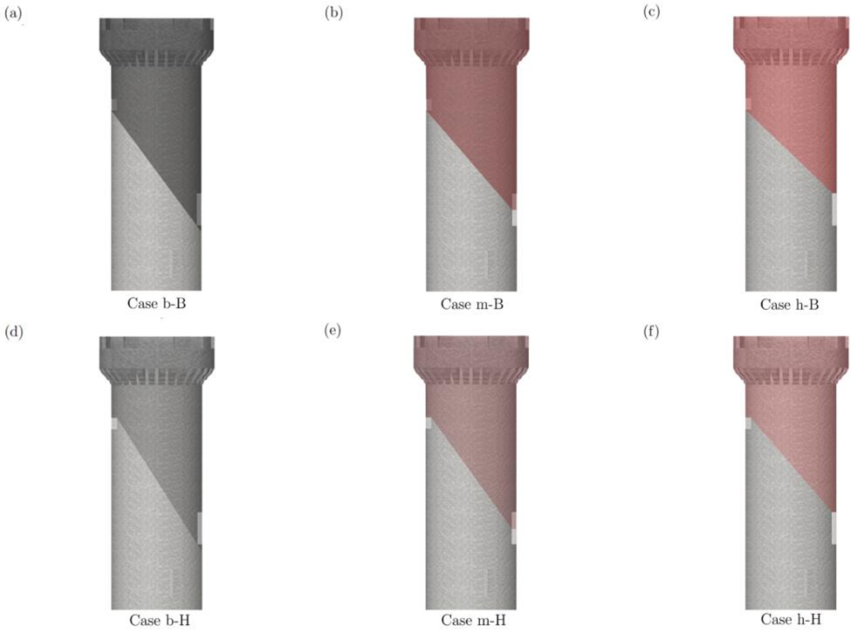


Fig. 3. Geometrical configurations for kinematic analysis performed assuming different diagonal fractures in function of the openings position.

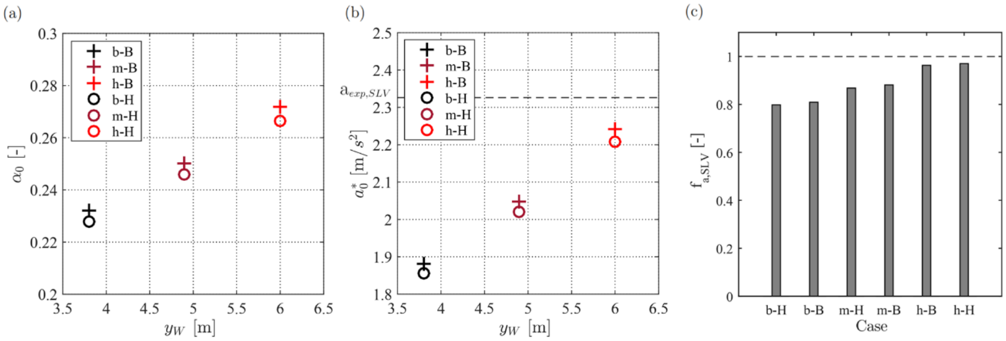


Fig. 4. (a) Horizontal multiplier α_0 as a function of the openings position y_W; (b) activating acceleration a_0* as a function of the openings position y_W.

Table 1. Parameters for the SLV check related to different relative position between crack and openings.

Case	a_0* [g]	$a_{exp,SLV}$ [m/s^2]	$f_{a,SLV}$ [-]	*SLV check*
b-B	1.882	2.326	0.809	x
m-B	2.049	2.325	0.881	x
h-B	2.243	2.329	0.963	x
b-H	1.857	2.327	0.798	x
m-H	2.020	2.327	0.868	x
h-H	2.209	2.200	0.970	x

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