Article

Neutron-Mirror Neutron Oscillations in Absorbing Matter

Yuri Kamyshkov [1,*] **, James Ternullo** [1] **, Louis Varriano** [2] **and Zurab Berezhiani** [3,4]

[1] Department of Physics and Astronomy, The University of Tennessee, Knoxville, TN 37996, USA; jternull@vols.utk.edu

[2] Department of Physics, University of Chicago, Chicago, IL 60637, USA; varriano@uchicago.edu

[3] Dipartimento di Fisica e Chimica, Universitá di L'Aquila, 67100 L'Aquila, Italy; zurab.berezhiani@aquila.infn.it or zurab.berezhiani@lngs.infn.it

[4] Laboratori Nazionali del Gran Sasso, INFN, 67100 L'Aquila, Italy

*   Correspondence: kamyshkov@utk.edu

Abstract: The possibility that a neutron can be transformed to a hidden sector particle remains intriguingly open. Proposed theoretical models conjecture that the hidden sector can be represented by a mirror sector, and the neutron n can oscillate into its sterile mirror twin n', exactly or nearly degenerate in mass with n. Oscillations $n - n'$ can take place in vacuum or in an environment containing regular matter and a magnetic field, in which only the neutron will be subject to interactions with the environment. We describe the propagation of the oscillating $n - n'$ system in a cold neutron beam passing through dense absorbing materials in connection to the possible regeneration type of experiments, where the effect of $n \to n' \to n$ transformation can be observed.

Keywords: neutron; mirror neutron; oscillation; density matrix

1. Introduction

Current interest in the hidden sector in particle physics is motivated mostly by the apparent existence of Dark Matter (DM), which is not contained in the Standard Model (SM) and the nature of which is not yet determined. One of the interesting possibilities is that DM is related to particles of some hidden sectors which also appear in string theory models. In these models, hidden sectors can include a new gauge group that is independent from the gauge group of the Standard Model or some of its extensions such as Grand Unification. Thus, the hidden sector particles do not interact with ordinary matter particles via Standard Model forces; however, they have gravitational interactions in common with ordinary matter. In the Mirror Matter model conjectured in [1,2], the hidden gauge sector is a replica of the ordinary sector including the same particle content so that two sectors are described by the Standard Model (SM) and its mirror copy SM′. They can have identical Lagrangians due to mirror Z_2 symmetry under the exchange of the particles between the two sectors (for reviews, see [3–5], and for a historical overview, see [6]). Mirror matter can be a viable candidate for DM, with specific cosmological implications, provided that the temperature of the mirror sector is smaller than that of the ordinary sector [7–10].

If mirror symmetry Z_2 is an exact symmetry, i.e., the Higgs doublets of the SM and SM′ have exactly the same vacuum expectation values (VEV), $\langle\phi\rangle = \langle\phi'\rangle$, then the ordinary and mirror sector should have identical particle spectra, so that all ordinary particles (electron e, proton p, neutron n, etc.) have mass degenerate mirror twins (e', p', n', etc.) which are sterile with respect to the SM interactions but have their own SM′ interactions. However, Z_2 symmetry can be spontaneously broken with two Higgses having different VEVs $\langle\phi\rangle \neq \langle\phi'\rangle$, in which case the mirror particle will have masses different from that of their ordinary partners [11,12].

Other than gravity, there can exist other interactions between the ordinary and mirror particles, possibly giving rise to observable effects. The cross-interactions which violate the lepton and/or baryon numbers of both sectors are of particular interest. From one side,

since these interactions violate both $B - L$ and $B' - L'$ symmetries, they can induce baryon asymmetries in both sectors [13,14], and such co-genesis mechanisms can explain the dark matter fraction in the Universe [15,16]. On the other side, they can induce the mixing and oscillation phenomena between the neutral particles from both sectors, e.g., the neutrino mixing $\nu - \nu'$ between two sectors, which makes mirror neutrinos the natural candidates for sterile neutrinos [17–20]. In addition, the neutron n can be mixed with a sterile neutron n', its partner from the mirror sector, $\epsilon \bar{n} n' +$ h.c. Interestingly, the oscillation $n - n'$ can be a rather fast process, with the characteristic oscillation time $\tau_{nn'} = \epsilon^{-1}$ as small as a few seconds; this possibility does not contradict the existing astrophysical limits, and it does not lead to nuclear instability [21]. That is different from the neutron–antineutron oscillation [22,23], for which the characteristic time should be $\tau_{n\bar{n}} > 10^8$ s as restricted by direct experimental limit as well by nuclear stability bounds [24]. In fact, both $n - n'$ and $n - \bar{n}$ mixing phenomena could be originated from the same theoretical framework [21,25]). This could also have interesting astrophysical implications, e.g., for extreme energy cosmic rays [26,27], for solar neutrons [28], and for neutron stars [29–31].

The $n - n'$ transition is affected by medium effects such as the presence of matter or magnetic fields [21,32] such that a transition faster than the neutron decay may not be immediately observed. However, this transition can be observed via neutron disappearance $n \to n'$ or regeneration $n \to n' \to n$ [21] in experiments with properly controlled background and environmental conditions. These experiments are convenient for observations due to the long lifetime of the neutron, the detection mechanism determined by strong interactions, and the large neutron fluxes available from reactors or spallation sources.

In the previous works, [21,33–35], various possible experiments for observing $n - n'$ oscillation effects were considered, including essentially the two detection methods: the disappearance experiments due to the neutron oscillation $n \to n'$ into a sterile neutron n', and the appearance (walking through the wall) experiments due to neutron regeneration from sterile state n', i.e., $n \to n' \to n$.

Several dedicated experiments have already been performed to search for $n \to n'$ oscillations via neutron disappearance in ultra-cold neutron (UCN) traps [36–42]. These experiments still do not exclude the possibility of $n - n'$ oscillation time to be much less than the neutron decay time, and some of them even show anomalous deviations from the null-hypothesis [43]. A new search is underway for testing these anomalies at the UCN facility of the Paul Sherrer Institute (PSI) [44].

As an alternative to UCN experiments, both the neutron disappearance $n \to n'$ and regeneration $n \to n' \to n$ can be experimentally tested with cold neutrons [34]. The latter search can be realized, e.g., in an experiment with an intense cold neutron beam where the transformation $n \to n'$ in the beam can be enhanced by applying specific environmental conditions. Then, the neutron beam can be removed by a strong absorber, leaving only n' states passing freely through. After passing the absorber, n' can effectively oscillate back to the n states under the same environmental conditions and will be detectable. Such experiments are underway at neutron sources in Oak Ridge National Laboratory (ORNL) [45–47] and at the newly constructed European Spallation Source (ESS) [48]. In the disappearance experiment, a small reduction in the neutron flux should be detected under applying a certain constant magnetic field over the neutron flight distance. In the regeneration mode, a small appearance effect can be directly measured possibly against a small background. For both methods, a resonant constant magnetic field should allow switching the effect on/off. There is also another type of experiment that can be related to the neutron regeneration to the antineutron, $n \to (n', \bar{n}') \to \bar{n}$, which is possible if the neutron n has mixings with both the mirror neutron n' and mirror antineutron $\bar{n}'$ [49].

In the 1960s, the regeneration method was used by O. Piccioni and his colleagues for a demonstration of the $K_L \to K_S$ transformation [50]. The mixing of neutral beam components K^0 and $\overline{K^0}$ forms a two-level quantum system with weak decay eigenstates K_L and K_S. Since K^0 and $\overline{K^0}$ components are interacting differently with the matter, the long-lived state K_L is transformed to a short-lived state K_S after passing solid iron or lead plates.

Similarly, the mixed oscillating system of the neutron n and the mirror neutron n' with eigenfunctions of the propagation Hamiltonian n_1 and n_2 has the components interacting differently with the matter such that n can be absorbed and n' will be non-interacting with the material. In difference with mixed kaon system, the $(n, n')^T$ system can be a subject of additional interaction with the environment (e.g., magnetic field) that can lead to the suppression or enhancement of the transformation.

Regeneration experiments are particularly promising for testing $n - n'$ oscillations in the case when the two states n and n' have some small mass splitting of $\Delta m = m_n - m'_n \sim$ 100 neV or so. In particular, this situation was used in [51] for explaining the neutron lifetime discrepancy between the trap and beam experiments. Such a small splitting between the ordinary and mirror particles can be obtained if mirror symmetry Z_2 is very mildly broken, with the VEVs of two Higgses having a small difference, $\langle \phi' \rangle \approx \langle \phi \rangle$ [52,53].

The model with mass splitting between n and n' is being tested with the cold neutrons in a strong magnetic field by the NN' Collaboration at the Spallation Neutron Source in Oak Ridge National Laboratory using the regeneration method [47]. An essential element of the regeneration method is the absorber, where a two-component oscillating (n, n') system will propagate with only one component n strongly interacting with the material environment and the other component, n', is sterile. When the (n, n') system is passing the absorber, due to $n \leftrightarrow n'$ oscillations, both components n and n' are the subject of attenuation. To our knowledge, no quantum mechanical consideration of the evolution of such a system, through strongly absorbing materials, have been reported. We should note that the reflection/absorption of the mass-degenerate (n, n') system in the walls of the UCN traps was considered in Refs. [32,54], and the interactions of the two-level oscillating $(n, \bar{n})$ system with a magnetic and gaseous environment has been considered in Ref. [55]. In view of the aforementioned cold neutron experiments, including the regeneration search, we performed this study. We also found that a recent paper of the STEREO collaboration [56] describes the search of hidden (sterile) neutrons via regeneration in a shielded detector installed close to the ILL reactor without detailed treatment of absorption of n and n' inside the reactor shielding. We think that the STEREO collaboration can benefit from our approach, particularly in the region of larger $(\epsilon / \Delta E)$ not shown in the paper [56] that might be relevant [51] for the neutron lifetime anomaly [57,58].

2. Description of Approach

We shall describe the propagation of the oscillating $(n - n')$ system in a cold beam of neutrons, i.e., neutrons with a spectrum of velocities ranging from 200 m/s to 2000 m/s, through dense materials in view of the possible regeneration type experiments [34] where the effect of $n \rightarrow n' \rightarrow n$ transformation can be measured. The regeneration experiment can be described in the following way. The beam of free cold neutrons that can oscillate between n and n' states propagates in a vacuum. Before entering the absorber, it has probability P_n to be detected as n neutron (if a neutron detector were to be present at this location) and probability $P_{n'} = 1 - P_n$ to be in n' state (since n' is not detectable as such). The thickness of the absorber can be sufficiently large to remove almost all neutrons from the beam. The n' component should pass through the absorber without interaction. After exiting the absorber, n' can continue free oscillations, enriching the beam with n component. The latter traveling through some distance to the detector will be counted there. Thus, neutrons can be found to have passed through the absorbing wall.

The evolution of $n - n'$ system is described by the Schrödinger equation, $id|\Psi\rangle/dt = H|\Psi\rangle$, where:

$$|\Psi(t)\rangle = \begin{pmatrix} \psi_n(t) \\ \psi_{n'}(t) \end{pmatrix} \tag{1}$$

with each of the components $\psi_n = (\psi_n^+, \psi_n^-)^T$ and $\psi_{n'} = (\psi_{n'}^+, \psi_{n'}^-)^T$ being two-component spinors describing the two spin states of the ordinary and mirror neutron, respectively. Thus, $P_{nn}(t) = |\psi_n(t)|^2 = |\psi_n^+(t)|^2 + |\psi_n^-(t)|^2$ corresponds to the probability of detecting

the neutron, and $P_{nn'}(t) = |\psi_{n'}(t)|^2 = |\psi_{n'}^+(t)|^2 + |\psi_{n'}^-(t)|^2$ is the probability of $n - n'$ oscillation at time t.

A generic non-relativistic Hamiltonian in a medium has the form:

$$H = \begin{pmatrix} H_n & \epsilon \\ \epsilon & H_{n'} \end{pmatrix} \tag{2}$$

where non-diagonal term ϵ is the $n - n'$ mixing mass, and diagonal entries H_n and $H_{n'}$ correspond to n and n' states, respectively. (In this paper we use natural units $c = 1$ and $\hbar = 1$.) In particular, one has:

$$H_n = m + \frac{p^2}{2m} + \mu(\vec{\sigma} \cdot \vec{B}) + V - i\left(W + \frac{\Gamma}{2}\right) \tag{3}$$

where m is the neutron mass and μ is its magnetic moment; $\vec{B}$ is the magnetic field and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices; V is the neutron optical potential; W is the neutron absorption rate in matter; and $\Gamma = \tau_{\rm dec}^{-1}$ is the neutron decay rate. Similarly, $H_{n'}$ can be expressed in terms of a contribution from a mirror magnetic field and mirror matter density, whenever the latter can be present in the experiment.

We assume that there is a small mass splitting between ordinary and mirror neutrons, $\Delta m = m' - m$. For the sake of definiteness, we take $\Delta m \sim 10^2 \div 10^3$ neV, the values suggested in Ref. [51] for solving the neutron lifetime problem. The identical real contributions in H_n and $H_{n'}$ are irrelevant for the evolution of the system, and we can omit them. In particular, for cold neutrons, $v \sim 1$ km/s or so, the difference of the kinetic energies is negligibly small since $p^2/m - p^2/m' \simeq v^2 \Delta m \sim 10^{-11} \Delta m$. We also assume that contributions of mirror matter and mirror magnetic fields are negligible, and set V', W' and B' to zero. In this case, it is convenient to take the spin quantization axis as the direction of the magnetic field $\vec{B} = (0, 0, B)$. Therefore, the magnetic field contribution for two polarization states will be $\pm \mu B$. In addition, the last term in Equation (3) is the neutron decay width, which should be practically the same for the mirror neutron. Since the neutron decay time, $\tau_{\rm dec} \approx 880$ s, is very large as compared to the cold neutron observation time, typically $t \sim 0.1$ s, we can neglect this term in Equation (3).

Therefore, our Hamiltonian (2) can be reduced to the following effective form:

$$\mathcal{H} = \begin{pmatrix} U - iW & \epsilon \\ \epsilon & 0 \end{pmatrix} = \begin{pmatrix} V - \Delta m \pm \mu B - iW & \epsilon \\ \epsilon & 0 \end{pmatrix} \tag{4}$$

which is non-hermitian in the presence of the absorptive contribution. The diagonal real quantities in Equation (4) are combined into $U = V - \Delta m \mp |\mu B|$. This Hamiltonian can also be split in two parts, hermitian and anti-hermitian (absorptive):

$$\mathcal{H} = \mathcal{H}_{\rm osc} + \mathcal{H}_{\rm abs}, \quad \mathcal{H}_{\rm osc} = \begin{pmatrix} U & \epsilon \\ \epsilon & 0 \end{pmatrix} \quad \mathcal{H}_{\rm abs} = -i\begin{pmatrix} W & 0 \\ 0 & 0 \end{pmatrix} \tag{5}$$

Hamiltonian $\mathcal{H}$ is non-hermitian if $W \neq 0$. However, it can be diagonalized by a canonical transformation:

$$S\,\mathcal{H}S^{-1} = \mathcal{H}_{\rm diag} = {\rm diag}(H_1, H_2) \tag{6}$$

or vice versa, $\mathcal{H} = S^{-1}\mathcal{H}_{\rm diag}S$. Without loss of generality, the matrix S can be taken as unimodular, with Det $S = 1$:

$$S = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}, \quad c = \cos \zeta = \frac{1}{2}\left(e^{i\zeta} + e^{-i\zeta}\right), \quad s = \sin \zeta = \frac{1}{2i}\left(e^{i\zeta} - e^{-i\zeta}\right) \tag{7}$$

with

$$\tan 2\zeta = \frac{2\epsilon}{U - iW} = \frac{2\epsilon(U + iW)}{U^2 + W^2} \tag{8}$$

where the parameter $\zeta = \theta + i\omega$ as well as the eigenvalues $H_{1,2}$ are generally complex. Namely, since the transformation (6) conserves the trace and determinant of $\mathcal{H}$, we have $H_1 + H_2 = U - iW$ and $H_1 H_2 = -\epsilon^2$. In this way, we obtain:

$$H_{1,2} = U_{1,2} - iW_{1,2} = \frac{1}{2}\left(U - iW \pm \sqrt{(U - iW)^2 + 4\epsilon^2}\right) \tag{9}$$

Then, the Schrödinger equation $i\frac{d}{dt}|\Psi\rangle = \mathcal{H}|\Psi\rangle$ in a constant medium is formally solved as:

$$|\Psi(t)\rangle = \mathcal{S}(t)|\Psi(0)\rangle, \quad \mathcal{S}(t) = e^{-i\mathcal{H}t} = e^{-iS^{-1}\mathcal{H}_{\text{diag}}St} = S^{-1}\text{diag}(e^{-iH_1 t}, e^{-iH_2 t})\,S \tag{10}$$

and so the transition probabilities are described by the evolution matrix:

$$\mathcal{S}(t) = \begin{pmatrix} \mathcal{S}_{nn}(t) & \mathcal{S}_{nn'}(t) \\ \mathcal{S}_{n'n}(t) & \mathcal{S}_{n'n'}(t) \end{pmatrix} = \begin{pmatrix} c^2 e^{-iH_1 t} + s^2 e^{-iH_2 t} & cs(e^{-iH_1 t} - e^{-iH_2 t}) \\ cs(e^{-iH_1 t} - e^{-iH_2 t}) & s^2 e^{-iH_1 t} + c^2 e^{-iH_2 t} \end{pmatrix} \tag{11}$$

Namely, starting at $t = 0$ from a neutron state $|\Psi(0)\rangle = |n\rangle = (1, 0)^T$, we obtain:

$$P_{nn}(t) = |\mathcal{S}_{nn}(t)|^2 = |c|^4 e^{-2W_1 t} + |s|^4 e^{-2W_2 t} + 2\text{Re}\left(c^{*2}s^2 e^{i\Delta E t}\right)e^{-Wt}$$

$$P_{nn'}(t) = |\mathcal{S}_{n'n}(t)|^2 = \frac{1}{4}|\sin 2\zeta|^2\left[e^{-2W_1 t} + e^{-2W_2 t} - 2\cos(\Delta E t)e^{-Wt}\right] \tag{12}$$

where $\Delta E = U_1 - U_2$.

Once again, if the absorptive part W is vanishing, then the matrix S (7) becomes orthogonal with $\zeta = \theta$ real, $\tan 2\theta = 2\epsilon/U$, and the eigenvalues $H_{1,2}$ are also real, so the evolution matrix $\mathcal{S}(t)$ becomes unitary. In this case we have: $H_1 - H_2 = \Delta E = \sqrt{U^2 + 4\epsilon^2}$, $4c^2 s^2 = \sin^2 2\zeta = 4\epsilon^2/(U^2 + 4\epsilon^2)$, and Equation (12) reduces to a standard expression for the probability of $n - n'$ oscillation:

$$P_{nn'}(t) = \sin^2 2\theta\left[1 - \cos(\Delta E t)\right] = \frac{4\epsilon^2}{\Delta E^2}\sin^2\left(\frac{\Delta E}{2}t\right) \tag{13}$$

and $P_{nn}(t) = 1 - P_{nn'}(t)$. In the vacuum conditions, $V = 0$, and in the absence of magnetic field, $B = 0$, we have $U = -\Delta m$, and the $n - n'$ oscillation amplitude is $\sin^2 2\theta_0 = 4\epsilon^2/(\Delta m^2 + 4\epsilon^2)$. The presence of medium modifies the oscillation probability. Namely, if $|U| < |\Delta m|$, then $n - n'$ oscillation probability is enhanced, $\sin^2 2\theta > \sin^2 2\theta_0$, and for $U = 0$, i.e., in the case of full cancellation between Δm and medium contributions, we have maximal oscillations with $\theta = \pi/2$ and $\Delta E = 2\epsilon$ so that the oscillation probability becomes $P_{nn'}(t) = \sin^2(\epsilon t)$.

Another interesting case is when $W \neq 0$, but instead, U is vanishing, assuming, e.g., an exact cancellation between Δm and medium contributions (notice, however, that in the presence of magnetic field this is possible only for one polarization of the neutron, $+$ or $-$). Then, in the absence of $n - n'$ mixing, $\epsilon = 0$, we would have the exponential extinction of the neutron flux, $P_{nn}(t) = e^{-2Wt}$ and $P_{nn'}(t) = 0$.

If $\epsilon < W/2$, both eigenvalues are imaginary, $H_{1,2} = -iW_{1,2}$ (so that $\Delta E = 0$), where both W_1 and W_2 are positive, with $W_1 + W_2 = W$ and $W_1 - W_2 = \Delta W = \sqrt{W^2 - 4\epsilon^2}$. The mixing parameter is imaginary, $\zeta = i\omega$, with $|\sin 2\zeta|^2 = 4\epsilon^2/\Delta W^2$. Hence, from Equation (12), we obtain the following for the transition probabilities:

$$P_{nn}(t) = \left[\cosh(\Delta Wt/2) - \frac{W}{\Delta W}\sinh(\Delta Wt/2)\right]^2 e^{-Wt}$$

$$P_{nn'}(t) = \left[\frac{2\epsilon}{\Delta W}\sinh(\Delta Wt/2)\right]^2 e^{-Wt} \tag{14}$$

In particular, in the limit $\Delta W \to 0$, i.e., $W_1 = W_2 = W/2 = \epsilon$, one obtains $P_{nn}(t) = (1 - \epsilon t)^2 e^{-2\epsilon t}$ and $P_{nn'}(t) = (\epsilon t)^2 e^{-2\epsilon t}$. Thus, for short flight times, $\epsilon t \ll 1$, we obtain $P_{nn'}(t) \approx (\epsilon t)^2(1 - 2\epsilon t)$ and $P_{nn}(t) \approx 1 - 4\epsilon t$, while for large times, $\epsilon t \gg 1$, both probabilities P_{nn} and $P_{nn'}$ are exponentially suppressed. In the case $\epsilon \ll W$, we have $\Delta W \approx W - 2\epsilon^2/W$, so that $W_2 \approx \epsilon^2/W \ll W_1 \approx W - \epsilon^2/W$. Thus, for large flight times, $Wt \gg 1$, we obtain $P_{nn'}(t) \approx (2\epsilon^2/W)^2 e^{-(2\epsilon^2/W)t}$.

As for the regime $\epsilon > W/2$, $H_{1,2}$ are not purely imaginary since $\sqrt{4\epsilon^2 - W^2} = \Delta E$ becomes real. In this case, we obtain $H_{1,2} = \frac{1}{2}(-iW \pm \Delta E)$, $\sin 2\zeta = 2\epsilon/\Delta E$ and we obtain:

$$P_{nn}(t) = \left[\cos(\Delta Et/2) + \frac{W}{\Delta E}\sin(\Delta Et/2)\right]^2 e^{-Wt}$$

$$P_{nn'}(t) = \left(\frac{2\epsilon}{\Delta E}\right)^2 \sin^2(\Delta Et/2)\, e^{-Wt} \tag{15}$$

Therefore, for short times, $Wt \ll 1$, $P_{nn'}$ reproduces the standard result of Equation (13), while for large times, $Wt \gg 1$, both P_{nn} and $P_{nn'}$ are exponentially suppressed.

In this paper, we follow the approach of our previous paper [35]. For our purposes, it will be convenient to use the density matrix formalism and describe the evolution of (n, n') system evolution via the Liouville–von Neumann equation (see, for example, [59]):

$$\dot{\rho} = -i(\mathcal{H}\rho - \rho\mathcal{H}^\dagger) = -i[\mathcal{H}_{\text{osc}}, \rho] - i\{\mathcal{H}_{\text{abs}}, \rho\} \tag{16}$$

where $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$ is a 4×4 Hermitian density matrix: $\rho_{ij}(t) = \psi_i(t)\psi_j^*(t)$, where $i, j = n, n'$ of two polarizations. The diagonal terms $\rho_{nn}(t)$ and $\rho_{n'n'}(t)$ represent the probability to detect the neutron and sterile neutron correspondingly at time t. The first term in Equation (16) is related to the hermitian part of the Hamiltonian and contains a commutator. The second term related to its absorptive part contains an anti-commutator. We set $\rho_{nn}(0) = 1$ at initial moment $t = 0$, corresponding to a pure n state.

We note that the Hamiltonian Equation (2) describing the interaction of (n, n') system with an absorber is incomplete. It omits effects of scattering of the neutrons on the nuclei of the absorber material. Since mirror neutrons do not scatter off nuclei, the scattering at any angle different than zero will lead to decoherence of the oscillating (n, n') system. This decoherence should be properly treated with the more complicated Lindblad Master equation [60,61]. Elastic scattering treatment similar to Lindblad equation for the oscillating muonium–antimuonium system in a gas environment was considered in an early paper [62]. For neutron scattering in condensed matter, the classical Van Hove theory with inclusion of Lindblad treatment was presented in the paper [63]. For thick absorbers and when the length of neutron scattering is smaller than absorption length, such as in the STEREO experiment [56], the approach of this paper can be useful.

For the regeneration effect that we consider in this paper, the elastic scattering of neutrons at angles larger than zero will additionally reduce the number of neutrons at coordinate z along the beam paths that are removed by absorption. For the mirror neutron component entering the absorber, transitional oscillations are damped at the neutron absorption length, which is much smaller than the length for elastic scattering. Thus, if, e.g., cadmium is used as an absorber, the absorption length is ~ 0.04 mm, while the elastic scattering length is ~ 20 mm.

In the next section, we will start with a discussion of the preparation of the initial state of the density matrix in a vacuum and the calculation of an average density matrix as the initial state. Next, we will discuss the case when a magnetic field is absent, i.e.,

$U = -\Delta m + V_{op}$, and will calculate numerically the evolution of the density matrix for an idealized regeneration experiment where a neutron is passing the vacuum–absorber–vacuum–detector environment sequence. If the regeneration experiment occurs in a constant magnetic field, then this can be described in the same way as a zero magnetic field but with modification of the magnitude of U and by considering two possible beam polarizations with $-\mu B$ and $+\mu B$. Finally, we will discuss the calculations with constant U for the case of weak and strong absorbers. The case with $U(z)$ including Δm and complex V_{opt} in non-uniform magnetic field $B(z)$ was used in calculations for the regeneration experiment [47] and not presented in this paper.

3. Density Matrix in Vacuum

Now, following the paper [51], we can consider an oscillating (n, n') system with $U = -\Delta m$ propagating in a vacuum, with $B = 0$, $V = 0$, and $W = 0$:

$$\mathcal{H} = \begin{pmatrix} -\Delta m & \epsilon \\ \epsilon & 0 \end{pmatrix} \tag{17}$$

This Hamiltonian will be used in Equation (16) together with the density matrix:

$$\rho(t) = \begin{pmatrix} \rho_{nn}(t) & \rho_{nn'}(t) \\ \rho_{n'n}(t) & \rho_{n'n'}(t) \end{pmatrix} \tag{18}$$

At the point where the neutron is produced in the nuclear reactor or in spallation process, or as a result of neutron scattering or decay of other particles, the initial state of the density matrix for (n, n') system at $t = 0$ can be described as:

$$\rho(0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{19}$$

The time evolution of the density matrix Equation (18) will depend on the oscillation frequency ω and the mixing angle θ_0 defining the amplitude of oscillations:

$$\Delta E / 2 = \omega = \sqrt{(\Delta m / 2)^2 + \epsilon^2} \tag{20}$$

$$\tan 2\theta_0 = -2\epsilon / \Delta m \tag{21}$$

The solution of Equation (16) under the initial conditions of Equation (19) can be explicitly found by:

$$\rho_{nn}(t) = 1 - \sin^2 2\theta_0 \cdot \sin^2(\omega t), \quad \rho_{n'n'}(t) = \sin^2 2\theta_0 \cdot \sin^2(\omega t)$$
$$\rho_{nn'}(t) = -\frac{1}{2}\sin 4\theta_0 \cdot \sin^2(\omega t) - \frac{i}{2}\sin 2\theta_0 \cdot \sin(2\omega t), \quad \rho_{n'n}(t) = \rho_{nn'}^*(t) \tag{22}$$

If $\Delta m > 10~neV$, then for an arbitrarily small ϵ, the frequency ω will be large enough such that a neutron in the Lab with a velocity ~ 1000 m/s will have an oscillation length smaller than 1 mm. If the size of the experiment is much larger than 1 mm, then the time-dependent probabilities of Equation (22) can be replaced by time-averaged values. Thus, we come to the time-averaged density matrix (TADM). In the TADM, the oscillation phases induced by variable initial phases and velocities of the beam neutrons will be averaged in the following way (we also use here the smallness of the angle θ_0):

$$\bar{\rho} = \begin{pmatrix} 1 - 2\theta_0^2 & -\theta_0 \\ -\theta_0 & 2\theta_0^2 \end{pmatrix} \tag{23}$$

This matrix when taken as the initial condition for Equation (16) with the Hamiltonian Equation (17) reproduces itself in evolution. This density matrix Equation (23) provides phase averaging and can be used as an initial condition for evolution of the (n, n') system

coming from vacuum through the absorbing material. This density matrix can be understood as a final state of a single (n, n') system having probabilities similar to that of the average of the large ensemble of neutrons in the beam.

4. Evolution through Weakly Absorbing Material

In practice, sometimes it is necessary to transport a cold neutron beam through air. This is an example of a weakly absorbing environment for the propagation of the (n, n') system. We will try to construct an average density matrix that will be a result of evolution through homogeneous weakly absorbing material described by the Hamiltonian Equation (4) without a magnetic field, such that $U = -\Delta m + V$ and $W \ll U$.

For the propagation of cold neutrons in air, at NTP, we use the following values for $V = 5.668 \times 10^{-11}$ eV and $W = (5.773 \times 10^{-15} + v \cdot 8.328 \times 10^{-18})$ eV, where v is neutron velocity in m/s. By direct numerical computation of $\rho(t)$ in the evolution of Equation (16), we obtain the following density matrix for the (n, n') system following a path in air: $\Delta z = v \cdot \Delta t$. As we mentioned before, we do not consider in the evolution the elastic scattering of cold neutrons off the nuclei in the gas, since elastic scattering will result in a dropout of neutrons from a highly collimated beam and effectively reduce the beam intensity but will not fundamentally affect the propagation of (n, n') system:

$$
\begin{aligned}
\bar{\rho}_{nn} &= \left(1 - \tfrac{1}{2} \cdot \sin^2 2\theta_0\right) \cdot e^{-\Delta z/L_{air}} \\
Re\bar{\rho}_{nn'} = Re\bar{\rho}_{n'n} &= -\tfrac{1}{4} \cdot \sin 4\theta_0 \cdot e^{-\Delta z/L_{air}} \\
Im\bar{\rho}_{nn'} = Im\bar{\rho}_{n'n} &= 0 \\
\bar{\rho}_{n'n'} &= \tfrac{1}{2} \cdot \sin^2 2\theta_0 \cdot e^{-\Delta z/2L_{air}}
\end{aligned}
\tag{24}
$$

where $\tan 2\theta_0 = -2\epsilon/(\Delta m - V_{air})$, and L_{air} is the absorption length in air calculated as $L_{air} = \hbar v/(2W)$. For example, for a neutron velocity $v = 1000$ m/s, this gives an absorption length $L_{air} = 23.338$ m. The averaged density matrix in Equation (24) can be used as the initial state of the evolution for the (n, n') system entering a strongly absorbing material after passing the distance Δz in air or in another weakly absorbing medium. These equations correspond to the case considered in Equation (15) and follow from Equation (12)'s general solution.

5. Evolution in the Strongly Absorbing Material

The evolution of the (n, n') system inside the absorber can be described by the same Hamiltonian Equation (4) without a magnetic field with different values for V and W for the particular absorbing material. As an example calculation for practical reasons, we have chosen two absorber materials: a 3.5 mm thick cadmium (Cd) and a 32 mm boron carbide (B$_4$C) with natural isotope abundance. For Cd, $V = 5.877 \times 10^{-8}$ eV and $W = (8.4558 \times 10^{-9} + v \cdot 9.914 \times 10^{-15})$ eV. For B$_4$C, $V = 1.992 \times 10^{-7}$ eV and $W = (6.102 \times 10^{-9} + v \cdot 2.397 \times 10^{-14})$ eV. An example of the evolution calculations for Cd is shown in Figure 1 for parameters $\Delta m = 300$ neV and $\theta_0 = 1 \times 10^{-2}$ and 1×10^{-3}.

As expected, the mirror neutron component $\rho_{n'n'}$ remains practically constant throughout the length of the absorber, but at the entrance it experiences some damped oscillations due to the re-arrangement of the energy eigenvalues of the system. The neutron component ρ_{nn} shows fast absorption with the conventional absorption length known from the neutron cross sections until it reaches the level that is determined by oscillation feedback of mirror neutrons to neutrons. Since the probability of n' remains almost constant, it provides an equilibrium level of neutron probability. The oscillation probability is slightly dampened in this equilibrium, though this is at a very small level, invisible in the figure. In equilibrium, the attenuation of both components is near equal. Interestingly, the probability for n' levels is approximately θ_0^2, while the constant level of probability for neutrons is $\sim\theta_0^4$, thus demonstrating a regeneration effect already inside the absorber. Since oscillations are suppressed inside the absorber, the system does not obtain the phase shift factor that would

lead to the significant variation in the probability at the exit of the absorber, thus providing the state of the density matrix that is averaged and can serve as an initial condition for the propagation in vacuum (or in air, or in magnetic field) behind the absorber.
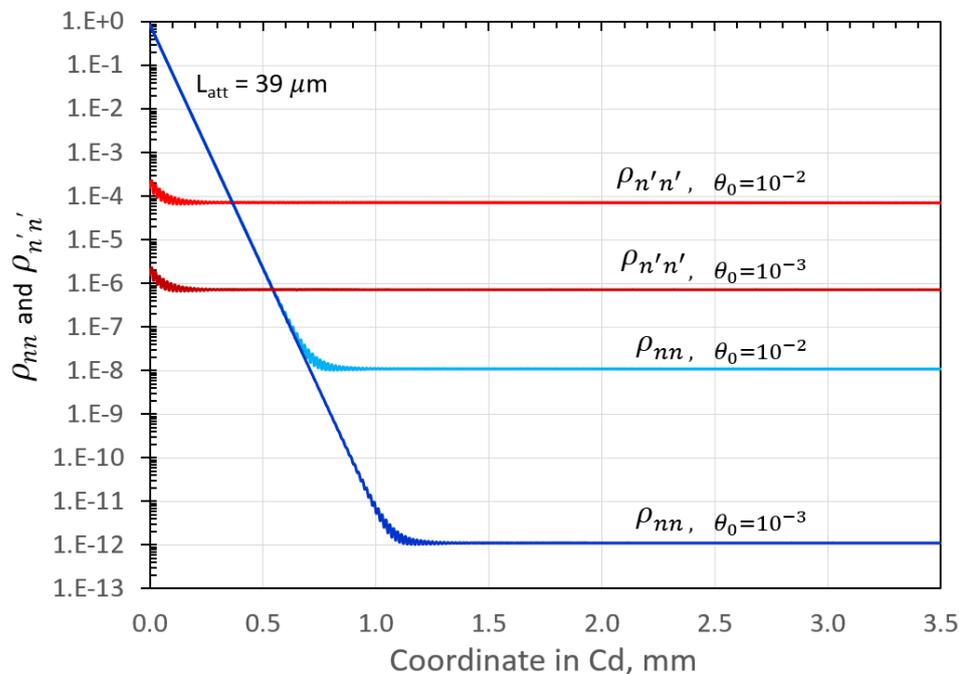

Figure 1. Evolution of (n, n') system in a 3.5 mm Cd absorber for $\Delta m = 300$ neV, $\theta_0 = 10^{-2}$ and 10^{-3}, and for velocity $v = 1000$ m/s. The averaged density matrix at the entrance of the Cd absorber after passing 3 m in air is used as an initial condition. Magnetic field is $B = 0$. Neutron components ρ_{nn} are shown in light/dark blue and mirror neutron component $\rho_{n'n'}$ in light/dark red colors.

This interesting behavior of the (n, n') system inside the absorber opens up a new, very simple way of observation for the presence of mirror neutrons in the intense beam of cold neutrons. For that, it should be sufficient to measure the attenuation of the neutron beam intensity as a function of the absorber thickness. At some thickness, the attenuation regime should be stopped and replaced by a constant irreducible intensity. The latter should be above the level of the background in the neutron detector. Since the constant level of neutron probability in essentially determined by the $\sim\theta_0^4$, such measurements will not be difficult to perform for larger values of θ_0. The parameter $\theta_0 = \epsilon/\Delta m$ can be limited by measurements for all values of Δm.

One can notice that in Equation (4), without a magnetic field, $U = -\Delta m + V$, and there is a possibility that $U = 0$ and $\epsilon > W/2$ with a modified oscillation frequency of:

$$\Delta E/2 = \omega = \sqrt{\epsilon^2 - (W/2)^2}, \qquad \epsilon = (\Delta m/2) \times \tan 2\theta_0 \qquad (25)$$

This case also can be calculated with the evolution Equation (16) and with the Hamiltonian in Equation (4). We show the results of such calculations as the values of ρ_{nn} and $\rho_{n'n'}$ in Figure 2 for a B$_4$C 32 mm absorber and in Figure 3 for a Cd 3.5 mm absorber. For both figures, we calculate the density matrix components at the exit of the absorber as a function of Δm for two values of $\theta_0 = 0.01$ and 0.001.

The Fermi potential of B$_4$C is $V = 1.992 \times 10^{-7}$ eV corresponding to a pronounced structure of probability in Figure 2 around $\Delta m = 200$ neV. For a small angle θ_0, the resonance enhances the regeneration of neutrons ρ_{nn} and increases the yield of mirror neutrons $\rho_{n'n'}$. For larger angles at a stronger mixing parameter, the absorption of neutron components starts to dominate in the resonance region.

As we mentioned above, the presence of a constant magnetic field can play the same role in U as Δm or the real part of the optical potential V_{op}. Therefore, the position of the resonance can be controlled for some region of Δm and the magnetic field. For the *Cd* absorber where $V_{op} = 58.8$ neV, similar resonance behavior in Figure 3 is not as pronounced as with B_4C for the same values of mixing angle θ_0.
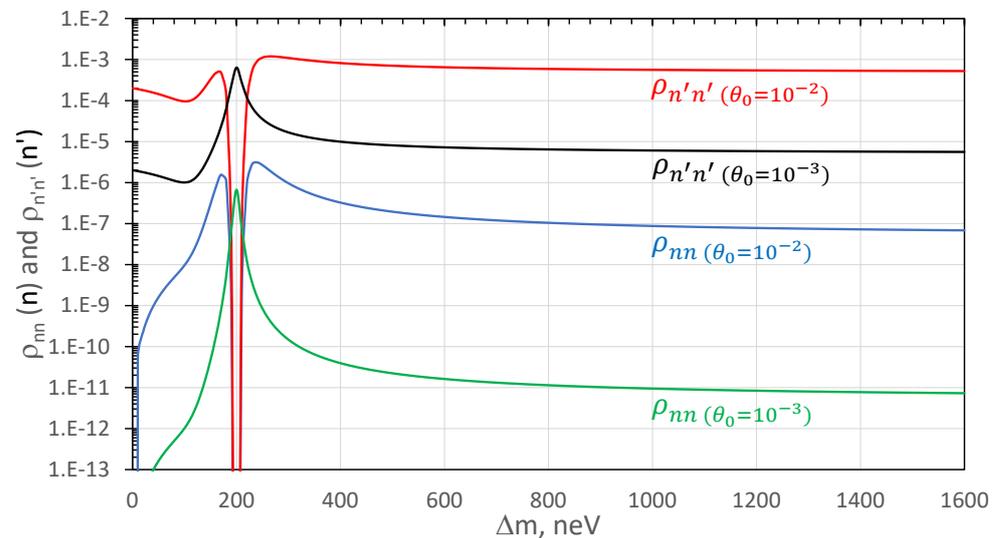

Figure 2. Δm dependence of probability ρ_{nn} and $\rho_{n'n'}$ for two values of angles $\theta_0 = 0.01$ and 0.001 after the neutron passes through the 32 mm B_4C absorber starting from the average density matrix in a vacuum. The value for the magnetic field $B = 0$.
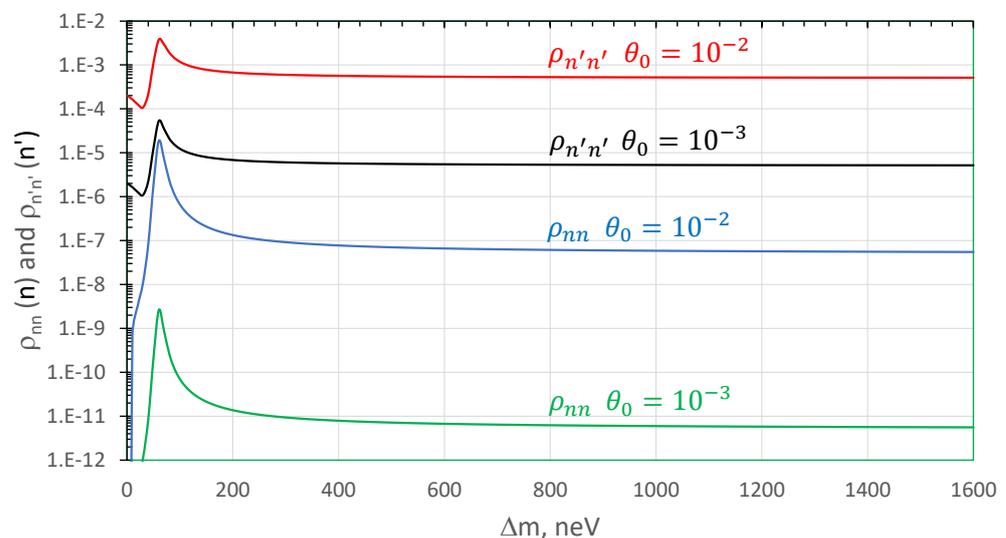

Figure 3. Δm dependence of probability ρ_{nn} and $\rho_{n'n'}$ for two values of angles $\theta_0 = 0.01$ and 0.001 after neutron passes through the 3.5 mm *Cd* absorber starting from an averaged density matrix in vacuum. The value for the magnetic field $B = 0$.

6. Summary

By computing the time evolution of the density matrix of the two-level system (n, n') passing the environment where one of the components in the Hamiltonian is strongly interacting with an environment and another component is sterile, we provided the method for understanding the process of regeneration that can be used in experiments with cold neutron beams, e.g., in [47]. The real part of the potential of the Hamiltonian describing the two-level (n, n') oscillating system interacting with the environment can include the effect of $\Delta m_{nn'}$ that can be positive and/or negative, the positive optical potential of the

material, and the magnetic field whose contribution will depend on the polarization of the neutron. If a magnetic field varies along the path of the neutron beam, it may compensate the overall real potential to zero and will lead to the resonance behavior inside the absorber that might essentially modify the regeneration process. This method can also be applied to the experiments with lower magnetic fields, where n and n' are degenerate in mass but the mirror magnetic field B' and/or eventual mirror matter gas can contribute to the energy splitting. In this case, the latter contribution should not be neglected in the Hamiltonian (2).

Author Contributions: All authors contributed equally to the manuscript preparation. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Kobzarev, I.Y.; Okun, L.B.; Pomeranchuk, I.Y. On the possibility of experimental observation of mirror particles. *Sov. J. Nucl. Phys.* **1966**, *3*, 837–841.
2. Foot, R.; Lew, H.; Volkas, R.R. A Model with fundamental improper space-time symmetries. *Phys. Lett. B* **1991**, *272*, 67–70. [CrossRef]
3. Berezhiani, Z. Mirror world and its cosmological consequences. *Int. J. Mod. Phys. A* **2004**, *19*, 3775–3806. [CrossRef]
4. Berezhiani, Z. Through the looking-glass: Alice's adventures in mirror world. In *From Fields to Strings: Circumnavigating Theoretical Physics: A Tribute to Ian Kogan*; World Scientific: Singapore, 2005; pp. 2147–2195. [CrossRef]
5. Foot, R. Mirror dark matter: Cosmology, galaxy structure and direct detection. *Int. J. Mod. Phys. A* **2014**, *29*, 1430013. [CrossRef]
6. Okun, L.B. Mirror particles and mirror matter: 50 years of speculations and search. *Phys. Usp.* **2007**, *50*, 380–389. [CrossRef]
7. Berezhiani, Z.; Comelli, D.; Villante, F.L. The Early mirror universe: Inflation, baryogenesis, nucleosynthesis and dark matter. *Phys. Lett.* **2001**, *B503*, 362–375. [CrossRef]
8. Ignatiev, A.Y.; Volkas, R.R. Mirror dark matter and large scale structure. *Phys. Rev. D* **2003**, *68*, 023518. [CrossRef]
9. Berezhiani, Z.; Ciarcelluti, P.; Comelli, D.; Villante, F.L. Structure formation with mirror dark matter: CMB and LSS. *Int. J. Mod. Phys.* **2005**, *D14*, 107–120. [CrossRef]
10. Berezhiani, Z.; Cassisi, S.; Ciarcelluti, P.; Pietrinferni, A. Evolutionary and structural properties of mirror star MACHOs. *Astropart. Phys.* **2006**, *24*, 495–510. [CrossRef]
11. Berezhiani, Z.G.; Dolgov, A.D.; Mohapatra, R.N. Asymmetric inflationary reheating and the nature of mirror universe. *Phys. Lett. B* **1996**, *375*, 26–36. [CrossRef]
12. Berezhiani, Z.G. Astrophysical implications of the mirror world with broken mirror parity. *Acta Phys. Polon. B* **1996**, *27*, 1503–1516.
13. Bento, L.; Berezhiani, Z. Leptogenesis via collisions: The Lepton number leaking to the hidden sector. *Phys. Rev. Lett.* **2001**, *87*, 231304. [CrossRef] [PubMed]
14. Bento, L.; Berezhiani, Z. Baryon asymmetry, dark matter and the hidden sector. *Fortsch. Phys.* **2002**, *50*, 489–495. [CrossRef]
15. Berezhiani, Z. Unified picture of ordinary and dark matter genesis. *Eur. Phys. J. Spec. Top.* **2008**, *163*, 271–289. [CrossRef]
16. Berezhiani, Z. Matter, dark matter, and antimatter in our Universe. *Int. J. Mod. Phys.* **2018**, *A33*, 1844034. [CrossRef]
17. Akhmedov, E.K.; Berezhiani, Z.G.; Senjanovic, G. Planck scale physics and neutrino masses. *Phys. Rev. Lett.* **1992**, *69*, 3013–3016. [CrossRef] [PubMed]
18. Foot, R.; Volkas, R.R. Neutrino physics and the mirror world: How exact parity symmetry explains the solar neutrino deficit, the atmospheric neutrino anomaly and the LSND experiment. *Phys. Rev. D* **1995**, *52*, 6595–6606. [CrossRef] [PubMed]
19. Berezhiani, Z.G.; Mohapatra, R.N. Reconciling present neutrino puzzles: Sterile neutrinos as mirror neutrinos. *Phys. Rev.* **1995**, *D52*, 6607–6611. [CrossRef]
20. Silagadze, Z.K. Neutrino mass and the mirror universe. *Phys. Atom. Nucl.* **1997**, *60*, 272–275.
21. Berezhiani, Z.; Bento, L. Neutron—Mirror neutron oscillations: How fast might they be? *Phys. Rev. Lett.* **2006**, *96*, 081801. [CrossRef]
22. Kuzmin, V.A. CP violation and baryon asymmetry of the universe. *Pisma Zh. Eksp. Teor. Fiz.* **1970**, *12*, 335–337.
23. Mohapatra, R.N.; Marshak, R.E. Phenomenology of neutron oscillations. *Phys. Lett.* **1980**, *94B*, 183. [CrossRef]
24. Phillips, D.G., II.; Snow, W.M.; Babu, K.; Banerjee, S.; Baxter, D.V.; Berezhiani, Z.; Bergevin, M.; Bhattacharya, S.; Brooijmans, G.; Castellanos, L.; et al. Neutron-Antineutron Oscillations: Theoretical Status and Experimental Prospects. *Phys. Rept.* **2016**, *612*, 1–45. [CrossRef]

25. Berezhiani, Z. Neutron–antineutron oscillation and baryonic majoron: Low scale spontaneous baryon violation. *Eur. Phys. J. C* **2016**, *76*, 705. [CrossRef]

26. Berezhiani, Z.; Bento, L. Fast neutron—Mirror neutron oscillation and ultra high energy cosmic rays. *Phys. Lett. B* **2006**, *635*, 253–259. [CrossRef]

27. Berezhiani, Z.; Gazizov, A. Neutron Oscillations to Parallel World: Earlier End to the Cosmic Ray Spectrum? *Eur. Phys. J. C* **2012**, *72*, 2111. [CrossRef]

28. Mohapatra, R. N.; Nasri, S.; and Nussinov, S Some implications of neutron mirror neutron oscillation *Phys. Lett. B* **2005**, *672*, 124–130. [CrossRef]

29. Goldman, I.; Mohapatra, R.N.; Nussinov, S. Bounds on neutron-mirror neutron mixing from pulsar timing. *Phys. Rev. D* **2019**, *100*, 123021. [CrossRef]

30. Berezhiani, Z.; Biondi, R.; Mannarelli, M.; Tonelli, F. Neutron-mirror neutron mixing and neutron stars. *Eur. Phys. J. C* **2021**, *81*, 1036. [CrossRef]

31. Berezhiani, Z. Antistars or antimatter cores in mirror neutron stars? *arXiv* **2021**, arXiv:2106.11203.

32. Berezhiani, Z. More about neutron—Mirror neutron oscillation. *Eur. Phys. J.* **2009**, *C64*, 421–431. [CrossRef]

33. Pokotilovski, Y.N. On the experimental search for neutron —> mirror neutron oscillations. *Phys. Lett.* **2006**, *B639*, 214–217. [CrossRef]

34. Berezhiani, Z.; Frost, M.; Kamyshkov, Y.; Rybolt, B.; Varriano, L. Neutron Disappearance and Regeneration from Mirror State. *Phys. Rev. D* **2017**, *96*, 035039. [CrossRef]

35. Berezhiani, Z.; Biondi, R.; Kamyshkov, Y.; Varriano, L. On the Neutron Transition Magnetic Moment. *Physics* **2019**, *1*, 271–289. [CrossRef]

36. Ban, G.; Bodek, K.; Daum, M.; Henneck, R.; Heule, S.; Kasprzak, M.; Khomutov, N.; Kirch, K.; Kistryn, S.; Knecht, A.; et al. A Direct experimental limit on neutron: Mirror neutron oscillations. *Phys. Rev. Lett.* **2007**, *99*, 161603. [CrossRef] [PubMed]

37. Serebrov, A. P. et al. Experimental search for neutron – mirror neutron oscillations using storage of ultracold neutrons. *Phys. Lett. B* **2008**, *663*, 181–185. [CrossRef]

38. Altarev, I.; Baker, C.A.; Ban, G.; Bodek, K.; Daum, M.; Fierlinger, P.; Geltenbort, P.; Green, K.; van Der Grinten, M.G.; Gutsmiedl, E.; et al. Neutron to Mirror-Neutron Oscillations in the Presence of Mirror Magnetic Fields. *Phys. Rev.* **2009**, *D80*, 032003. [CrossRef]

39. Bodek, K.; Kistryn, St.; Kuźniak, M.; Zejma, J.; Burghoff, M; Knappe-Grüneberg, S.; Sander-Thoemmes, T.; Schnabel, A.; Trahms, L.; Ban, G.; Lefort, T.; et al. Additional results from the first dedicated search for neutron-mirror neutron oscillations. *Nucl. Instrum. Meth.* **2009**, *A611*, 141–143. [CrossRef]

40. Serebrov, A.P.; Aleksandrov, E.B.; Dovator, N.A.; Dmitriev, S.P.; Fomin, A.K.; Geltenbort, P.; Kharitonov, A.G.; Krasnoschekova, I.A.; Lasakov, M.S.; Murashkin, A.N.; et al. Search for neutron–mirror neutron oscillations in a laboratory experiment with ultracold neutrons. *Nucl. Instrum. Meth. A* **2009**, *611*, 137–140. [CrossRef]

41. Berezhiani, Z.; Biondi, R.; Geltenbort, P.; Krasnoshchekova, I.A.; Varlamov, V.E.; Vassiljev, A.V.; Zherebtsov, O.M. New experimental limits on neutron - mirror neutron oscillations in the presence of mirror magnetic field. *Eur. Phys. J. C* **2018**, *78*, 717. [CrossRef]

42. Abel, C.; Ayres, N.J.; Ban, G.; Bison, G.; Bodek, K.; Bondar, V.; Chanel, E.; Chiu, P.J.; Crawford, C.; Daum, M.; et al. A search for neutron to mirror-neutron oscillations using the nEDM apparatus at PSI. *Phys. Lett. B* **2021**, *812*, 135993. [CrossRef]

43. Berezhiani, Z.; Nesti, F. Magnetic anomaly in UCN trapping: Signal for neutron oscillations to parallel world? *Eur. Phys. J.* **2012**, *C72*, 1974. [CrossRef]

44. Ayres, N.J.; Berezhiani, Z.; Biondi, R.; Bison, G.; Bodek, K.; Bondar, V.; Chiu, P.J.; Daum, M.; Dinani, R.T.; Doorenbos, C.B.; et al. Improved search for neutron to mirror-neutron oscillations in the presence of mirror magnetic fields with a dedicated apparatus at the PSI UCN source. *arXiv* **2021**, arXiv:2111.02794.

45. Broussard, L.J.; Bailey, K.M.; Bailey, W.B.; Barrow, J.L.; Chance, B.; Crawford, C.; Crow, L.; DeBeer-Schmitt, L.; Fomin, N.; Frost, M.; et al. New Search for Mirror Neutrons at HFIR. Meeting of the APS Division of Particles and Fields. *arXiv* **2017**, arXiv:1710.00767.

46. Broussard, L.J.; Bailey, K.M.; Bailey, W.B.; Barrow, J.L.; Berry, K.; Blose, A.; Crawford, C.; Debeer-Schmitt, L.; Frost, M.; Galindo-Uribarri, A.; et al. New search for mirror neutron regeneration. *EPJ Web Conf.* **2019**, *219*, 07002. [CrossRef]

47. Broussard, L.J.; Barrow, J.L.; DeBeer-Schmitt, L.; Dennis, T.; Fitzsimmons, M.R.; Frost, M.J.; Gilbert, C.E.; Gonzalez, F.M.; Heilbronn, L.; Iverson, E.B.; et al. Experimental Search for Neutron to Mirror Neutron Oscillations as an Explanation of the Neutron Lifetime Anomaly. *arXiv* **2021**, arXiv:2111.05543.

48. Addazi, A.; Anderson, K.; Ansell, S.; Babu, K.S.; Barrow, J.L.; Baxter, D.V.; Bentley, P.M.; Berezhiani, Z.; Bevilacqua, R.; Biondi, R.; et al. New high-sensitivity searches for neutrons converting into antineutrons and/or sterile neutrons at the HIBEAM/NNBAR experiment at the European Spallation Source. *J. Phys. G* **2021**, *48*, 070501. [CrossRef]

49. Berezhiani, Z. A possible shortcut for neutron–antineutron oscillation through mirror world. *Eur. Phys. J. C* **2021**, *81*, 33. [CrossRef]

50. Muller, F.; Birge, R.W.; Fowler, W.B.; Good, R.H.; Hirsch, W.; Matsen, R.P.; Oswald, L.; Powell, W.M.; White, H.S.; Piccioni, O. Regeneration and Mass Difference of Neutral *K* Mesons. *Phys. Rev. Lett.* **1960**, *4*, 418–421. [CrossRef]

51. Berezhiani, Z. Neutron lifetime puzzle and neutron–mirror neutron oscillation. *Eur. Phys. J. C* **2019**, *79*, 484. [CrossRef]

52. Mohapatra, R.N.; Nussinov, S. Constraints on Mirror Models of Dark Matter from Observable Neutron-Mirror Neutron Oscillation. *Phys. Lett. B* **2018**, *776*, 22–25. [CrossRef]

53. Berezhiani, Z. Neutron lifetime and dark decay of the neutron and hydrogen. *LHEP* **2019**, *2*, 118. [CrossRef]
54. Kerbikov, B.; Lychkovskiy, O. Neutron-Mirror-Neutron Oscillations in a Trap. *Phys. Rev. C* **2008**, *77*, 065504. [CrossRef]
55. Gudkov, V.; Nesvizhevsky, V.V.; Protasov, K.V.; Snow, W.M.; Voronin, A.Y. A new approach to search for free neutron-antineutron oscillations using coherent neutron propagation in gas. *Phys. Lett. B* **2020**, *808*, 135636. [CrossRef]
56. Almazán, H.; Bernard, L.; Blanchet, A.; Bonhomme, A.; Buck, C.; del Amo Sanchez, P.; Atmani, I.E.; Labit, L.; Lamblin, J.; Letourneau, A.; et al. Searching hidden neutrons with a reactor neutrino experiment: New constraint from the STEREO experiment. *arXiv* **2021**, arXiv:2111.01519.
57. Serebrov, A.P. Neutron Lifetime: Experimental Problem or Anomaly? Talk at NORDITA Workshop: Particle Physics with Neutrons at the ESS, Stockholm. December 2018. Workshop Website. Available online: https://agenda.albanova.se/getFile.py/access?contribId=259&sessionId=252&resId=251&materialId=slides&confId=6570 (accessed on 8 January 2022).
58. Wietfeldt, F.E. Measurements of the Neutron Lifetime. *Atoms* **2018**, *6*, 70. [CrossRef]
59. Breuer, H.; Petruccione, F. *The Theory of Open Quantum Systems*; Clarendon Press: Oxford, UK, 2002; p. 110, ISBN 978-0-19-852063-4.
60. Lindblad, G. On the Generators of Quantum Dynamical Semigroups. *Commun. Math. Phys.* **1976**, *48*, 119. [CrossRef]
61. Gorini, V.; Kossakowski, A.; Sudarshan, E.C.G. Completely Positive Dynamical Semigroups of N Level Systems. *J. Math. Phys.* **1976**, *17*, 821. [CrossRef]
62. Feinberg, G.; Weinberg, S. Conversion of Muonium into Antimuonium. *Phys. Rev.* **1961**, *123*, 1439–1443. [CrossRef]
63. Lanz, L.; Vacchini, B. Incoherent dynamics in neutron matter interaction. *Phys. Rev. A* **1997**, *56*, 4826–4838. [CrossRef]