University of L'Aquila

UNIVERSITY OF STRASBOURG (France)

And

UNIVERSITY degli Studi dell'Aquila (Italy)

Year 2025 N° of order

Thesis

To obtain the rank of

Doctor in mechanics of the University of Strasbourg

Doctoral School: Mathematics Information and Engineering Sciences-ED 269

And

Doctor in mathematics of university degli Studi dell'Aquila

Presented by:

David Izabel

Selected Geometric Aspects of General Relativity from the Standpoint of Continuum Mechanics: Implications for Measurements by VIRGO- and LISA-Type Interferometers

---

Submitted for the defense on 28 November 2025, before the jury composed of:

Thesis director: Yves Rémond, Emeritus Professor at the University of Strasbourg, School of Chemistry, Polymers and Materials (ECPM) - France
Co Thesis director: Francesco dell'Isola, Professor at the University of L'Aquila - Italy
Rapporteur: Nicolas Florsch, Professor at Sorbonne University - France
Rapporteur: Giovanni Ortenzi, Professor at the University of Turin - Italy
Examiner: Rachele Allena, Professor at Côte d'Azur University - France
Examiner: Yannick Hoarau, Professor at the University of Strasbourg - France
Examiner: Matteo Luca Ruggiero, Professor at the University of Turin - Italy
Examiner: Luca Placidi, Professor at Pegaso University, Napoli - Italy

David Izabel   Yves Remond   Francesco dell'Isola 1

Summary

1. Introduction - Why this thesis? What questions are being asked.

Summary

This chapter lays the foundations and framework of the research targeted by this thesis by analyzing and linking general relativity in weak field, space-time and the quantum vacuum through the prism of continuum mechanics. It explains that spacetime in a field of low gravity can be seen as an elastic medium. The approach is based on mechanical analogies to interpret gravitation. The aim is to propose an exploration between fundamental physics and structural mechanics. The method chosen is comparative, between physical and mechanical models. The originality comes from the use of materials resistance tools widely used in structural theory and civil engineering (the author's original training) to rethink cosmology and the functioning of the universe in terms of deformations and constraints measured for more than a century and therefore in generalized Hooke's law. This chapter emphasizes the importance of physical constants as mechanical parameters. It introduces the possibility that dark matter and dark energy have a mechanical interpretation. Finally, it sets the perspective of a global didactic understanding of general relativity in weak gravitational fields through engineering. It also lists the key questions addressed by this thesis.

Current physics is based on 3 main pillars. Special relativity, general relativity and quantum mechanics developed in quantum field theory. All 3 have been widely verified for more than a century. One concerns the infinitely large, general relativity, and the other, the infinitely small, quantum mechanics. One is determinism and operates in a continuous world, while the other operates because of probabilities of the presence of infinitely small energetic objects ($E=h\nu$) called quanta that behave under some experiments as waves or under others as particles. In short, the two theories seem irreconcilable. Then, general relativity applied to the entire universe gave rise to scientific cosmology and makes it possible to propose a model explaining the evolution of the universe from the Big Bang to the present day. However, this model is based on 3 entities, visible matter which represents 4.9% of the total, but also two huge unknowns. First, dark matter (26.8% of the total) which is necessary to explain gravitational effects that are greater than what the visible mass alone allows us to envisage but whose nature is totally unknown to us. Dark energy (68.3% of the total) which, according to the latest measurements, seems to have evolved over time and to be at the source of the accelerated expansion of the universe. Their deep natures are totally unknown to us. Moreover, the speed of light seems to be a constant specific to the very structure of space-time, of unsurpassable value without really knowing why. If light emits light, the resulting light will go at the speed of light, not twice the speed of light. The light velocities do not cumulate. Finally, all this is based on a 4-dimensional space-time where Einstein showed with special relativity that the closer we get to the speed of light, the more space contracts and the more time expands. Einstein also showed that gravity bends space-time and therefore that time varies with gravitation **[1]**, **[2]**. It slows down in the presence of a gravitational field, leaving us perplexed about its true nature. Worse still, Hawking, who is one of those who studied the situations where it is necessary to unify quantum mechanics with general relativity (the big bang and black holes), showed that black holes evaporate over time. The radiation which bears his name unifies quantum field theory with general relativity in strong gravitational fields and involves the use of Boltzmann's constant k_B, the light-speed constant c, Planck's quantum constant h and Newton's constant G. Gravitation is therefore also influenced by temperature T and time t. The first to theorize that general relativity could be linked to the theory of elasticity was A. Sakharov in 1968 **[3]**. Other theoretical physicists such as T. Damour, in his popular book **[4],** proposes the image of an elastic medium, of a jelly that deforms under the effect of the cosmic web that is within it and that charges it, and which follow a Hooke's law of the form D(g) = K T with D(g) the deformation, K the flexibility and T the tension. He therefore proposes a way to visualize the behavior of space-time. Others, such as R. Weiss **[5]**, propose a value of a Young's modulus of 10^{20} x $Y_{steel} = 2.1\ 10^{31}$ Pa to show the great but not infinite rigidity of spacetime. When we look at the state of the art, we realize that finally a large number of mechanical physicists and relativistic theoretical physicists have been interested in this analogy of space-time as an elastic medium **[3,4,5]**, **[8]**, **[11,12,13,14]**, **[17]**, **[20,21,22,23,24,25,26]**, **[28,29,30,31]**, **[80,81,82]**, **[86]**, **[89]**, **[105,106]**. This is why, faced with this scattered state of the art, I wanted to synthesize these different approaches and see if it is not possible to see in them a coherent and global mirror of the functioning of space-time and thus to use the mechanics of continuous media as a tool for an in-depth examination of general relativity. Indeed, in the case of general relativity, Einstein showed that space and time should be unified into a single entity, space-time, and that this space-time is not infinitely rigid as Newton thought, but can be distorted, bent by the presence of the masses energies that are within it. Thus, space-time is subject to deformations called as such in mechanics but called perturbations of the metric $h_{\mu\nu}$ in notations of general relativity. **But what is really deforming?** This is the central question to this whole thesis. It seemed appropriate to me, as others have done before me, to consider space-time by analogy as an elastic medium deformed by mass-energy densities present within it. In short, to transpose general relativity with a mechanistic vision derived from the mechanics of continuous media. To do this, I first analyzed what the different experiments that have made it possible to test general relativity have given in terms of perturbation of the metric (assimilated, and we will come back to this, to components of a deformation tensor of the intrinsic structure of space-time). Then I focused on those associated with gravitational waves that have been widely measured since 2015 and that have demonstrated forever the existence of black holes and their coalescences generating events so violent that they can distort the extremely rigid structure of space-time. Then I looked for models capable of reproducing these distortions of space-time resulting from gravitational waves but also those related to the other components of the perturbation tensor of the metric (Lense-Thirring effects by rotation of massive objects, terrestrial or solar gravitation, etc.). I also looked at how the power spectra of the cosmic microwave background in temperature and polarization could be seen by looking at it as an inverted X-ray diffractogram of the structure of the universe at these beginnings (in an X-ray diffractogram we project rays of X-ray light through a crystal and depending on the angle of these rays hitting or not atoms of its structure we see its structure, in the power spectra of the cosmic microwave background, light, and therefore photons, come out of the "space-time crystal" and depending on the angle at which they exit, we can see the "structure of space-time" 380000 years after the big bang). All these approaches by mechanical analogy with an elastic crystal modeling the behavior of space-time in connection with the mechanics of continuous media and the theory of defects **[92]** which can be associated by analogy in modified general relativity with a geometric torsion of space-time formalized for example in the form of Einstein Cartan's theory (we will come back to this later), therefore have the ultimate aim of understanding and reproducing by calculations the deformations of space-time but also to answer the following questions: Can we reproduce these deformations from models from the mechanics of continuous media? Why is the speed of light what it is and seems so intrinsically linked to the very nature of space-time? Why are there a priori only two polarizations associated with gravitational waves in the theory of linearized general relativity, are there not others? Is the universe isotropic? Is it not anisotropic under certain conditions? What models should be considered for the structure of isotropic space-time? Rigid, anisotropic, transverse isotropic, fluid, both depending on the loads? What could be the link between general relativity and quantum mechanics at the level of its internal structure of space-time and the vacuum that would no longer be empty? Would quantum beam models (physical object of engineering structure but of quantum scope and/or thickness **[111]**) allow to model the fine structure of space-time? In short, couldn't the mechanics of continuous media constitute another angle of view to unravel the mysteries of the structure of the universe?

We therefore finally focused on the following questions:

Q1: How to introduce mechanical behavior into Einstein's gravitational field equation by analogy and parallelism with elasticity theory? What are the mechanical characteristics of the analog elastic medium representing space-time?

Q2: Are there components of the spatiotemporal strain tensor associated with gravitational waves that are not measured by LIGO/VIRGO in the direction and perpendicular to the direction of propagation of the gravitational wave?

Q3: Can this whole analogy of the elastic medium give us an interesting new insight into the structure of the framework and the mechanical (isotropic? anisotropic) nature of space-time?

Q4: Is the twisting of space by two rotating black holes, and the associated deformations of gravitational waves measured by LIGO/VIRGO, and soon LISA, a way to determine these elastic parameters of space-time? In this case, can the geometric torsion seen from the Einstein-Cartan theory and its analogy with the theory of defects be reconcilable with the pure geometric torsion of a cylinder of elastic space? If so, how can it be physically interpreted in terms of the propagation of the constraint in space?

Q5: Does taking torsion into account generate complementary polarizations of gravitational waves? Do these have their images in terms of complementary deformations of the elastic medium?

Q6: Can space-time be considered only as a solid medium or as the analogy of the hydrodynamic medium, and does acoustic theory (Analog Gravity) also allow us to study the deformation of space during the rotation of binary acoustic sources equivalent to that of the coalescences of black holes in a liquid medium?

Q7: Can we find an experiment that will allow us to study the intrinsic framework of space by parallelism, and analogy with the X-ray diffractogram made for the current matter on Earth? Is there a current environment capable of reproducing the behaviour of space?

Q8: Can we find an elastic model capable of reproducing with sufficient precision the deformation measurements made in the case of gravitational waves, a Lense-Thirring effect or classical gravitation?

Q9: What are the realities and limitations of the analogy of the elastic behavior of space?

Finally, the ultimate objective of this thesis is to study this question on the analogy between the mechanics of continuous media and the general relativity of weak fields:

Q10: What are the didactic and predictive consequences of elastic analogy?

2. State of the art: Functioning of our Universe:

Overview of space-time distortions observed through general relativity in weak gravitational fields for more than 100 years. Example of angular deviations by Lense-Thirring effects **[6] [9]** – elongation-shortening effect in the case of gravitational waves GW150914 **[7]** – Some complementary deformations clearly remain to be discovered and measured.

Summary

This chapter summarizes the different distortions of space-time measured in weak gravitational fields (Newtonian gravitation, Lense-Thirring effects, gravitational lensing, gravitational wave). It shows that components remain to be studied (connected with the direction of gravitational wave propagation). It recalls the orders of magnitude of the measured deformations, which are extremely small both for elongation, shortening in the LIGO/VIRGO interferometers and for angular deformations (gravity PROBE B). It shows that in the case of gravitational waves, the gap between general relativity and measurements is very small. It concludes on the deep bridge between the perturbation tensor of the metric and the strain tensor in the mechanics of continuous media in 4 Dimensions. It also briefly recalls the key equations of general relativity in a weak field in the case of gravitational waves.

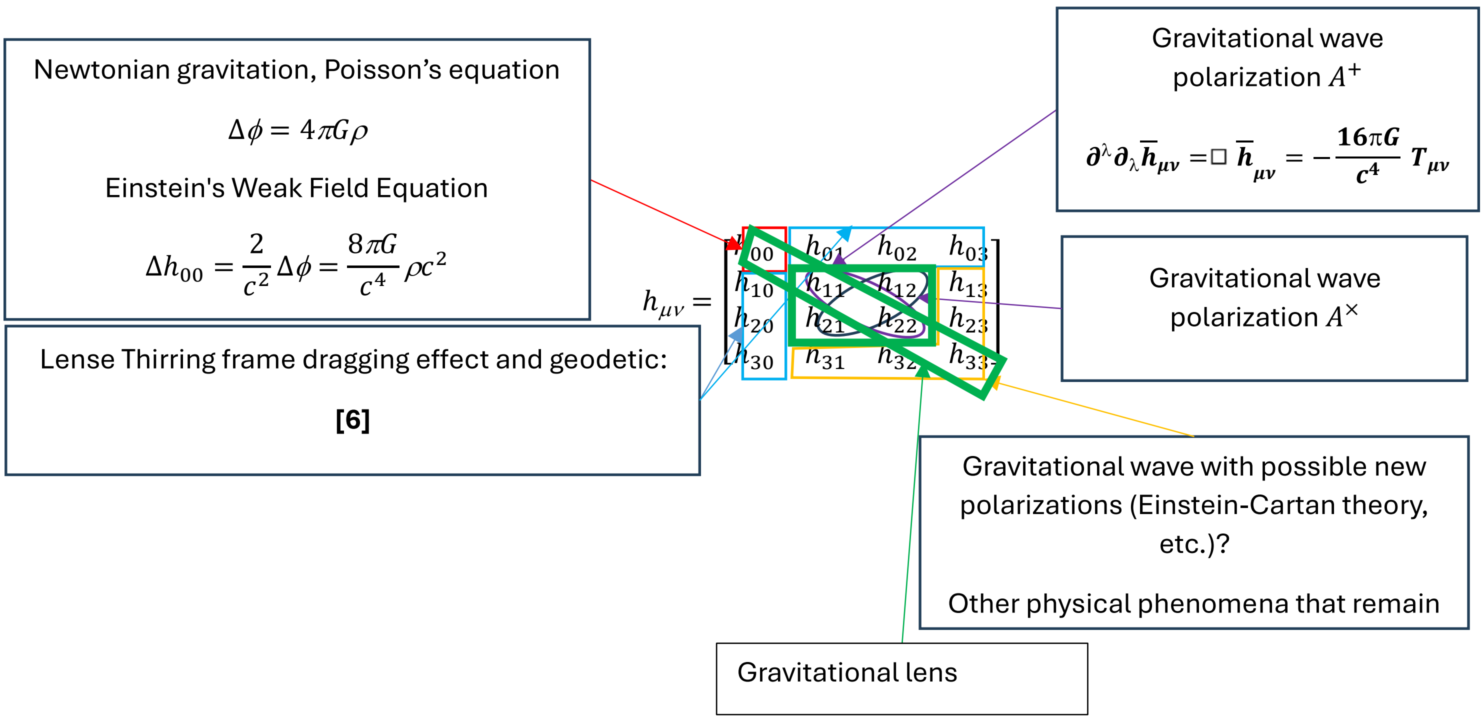


Figure 1: Overview of weak field spacetime deformations measured for more than 100 years and relationship to the perturbation tensor of the metric $h_{\mu\nu}$ **[8]**

The different perturbations of the metric in weak gravitational fields are gravitation in the solar system for example, gravitational lensing, Lense-Thirring effects and gravitational waves away from the producing source. All these phenomena are associated in different experimental frameworks with one or more components of the perturbation tensor of the metric $h_{\mu\nu}$. Figure 1 below, taken from the publication **[8]**, expresses this link.

However, these deformations are extremely small. Here are two examples:

In the case of the Lense-Thirring effects **[6]**, Table 1 below shows the difference measured between the predictions of general relativity and the measurements by the Gravity PROBE B experiment **[9]**. In this experiment, distortion angles by rotation of a massive object with a rotational/torsional effect of space-time were measured.

Figure 2 shows the experiment itself.

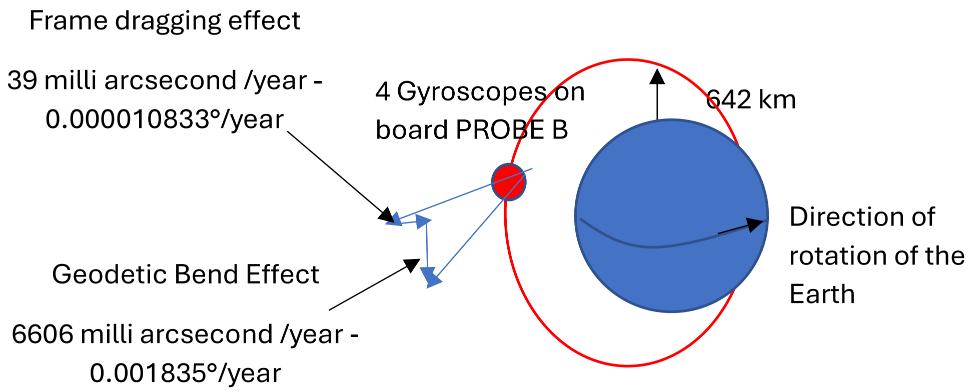


Figure 2: Gravity PROBE B experiment for measuring angular distortions of space-time **[9]**

Effect considered	**Prediction of general relativity in milliarc second per year (*)**	**Measurements made by the Gravity PROBE B satellite in milliarc second per year (*)**	**Error %**
Geodetic effect (vertical angular deviation)	-6606.1	-6601.8+/-18.3	0.28
Frame dragging effect (horizontal angular deflection)	-39.2	-37.2+/-7.2	19
1 milliarc second = 4.848x10^{-9} rad			

Table 1: Comparison of the results obtained according to general relativity with those measured by the Gravity PROBE B satellite **[9]**

The second example is the one chosen in the framework of the thesis, gravitational waves which were measured for the first time in 2014 during the coalescence event of two black holes GW150914 **[7]**. In this case, simultaneous lengthening and shortening of space at 90° from each other was measured by the 2 LIGO interferometer arms in the United States.

Figure 3 illustrates the δL-displacements (and deformations $h = \frac{\delta L}{L}$) measured by the LIGO-VIRGO interferometers during the passage of gravitational waves perpendicular to their plane xy.

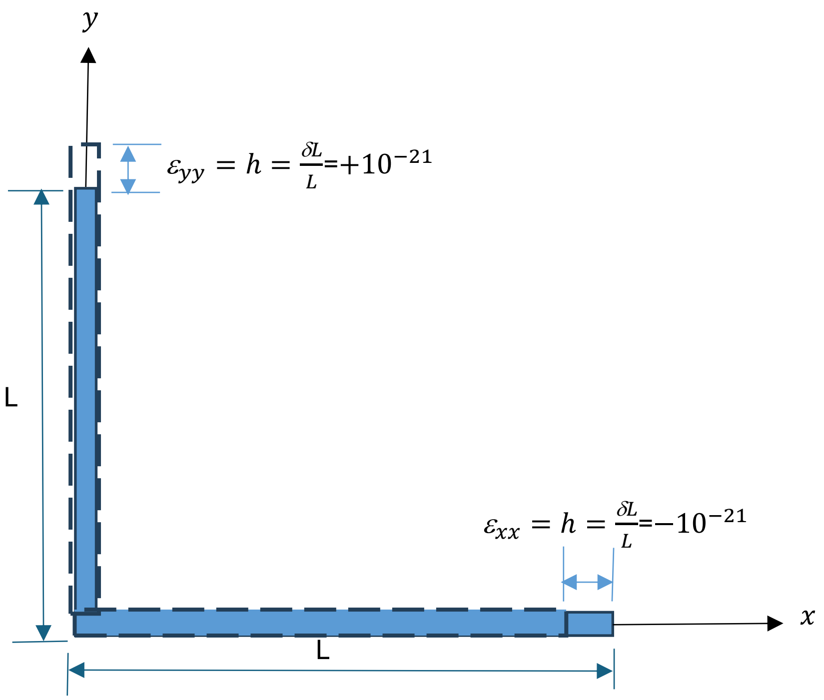


Figure 3: Typical displacements and deformations h of the arms of LIGO/VIRGO interferometers during the passage of gravitational waves perpendicular to their plane **[12]**

Figure 4 below shows the very good correlation between the predictions of general relativity and the measurements. The gap is again some percents.

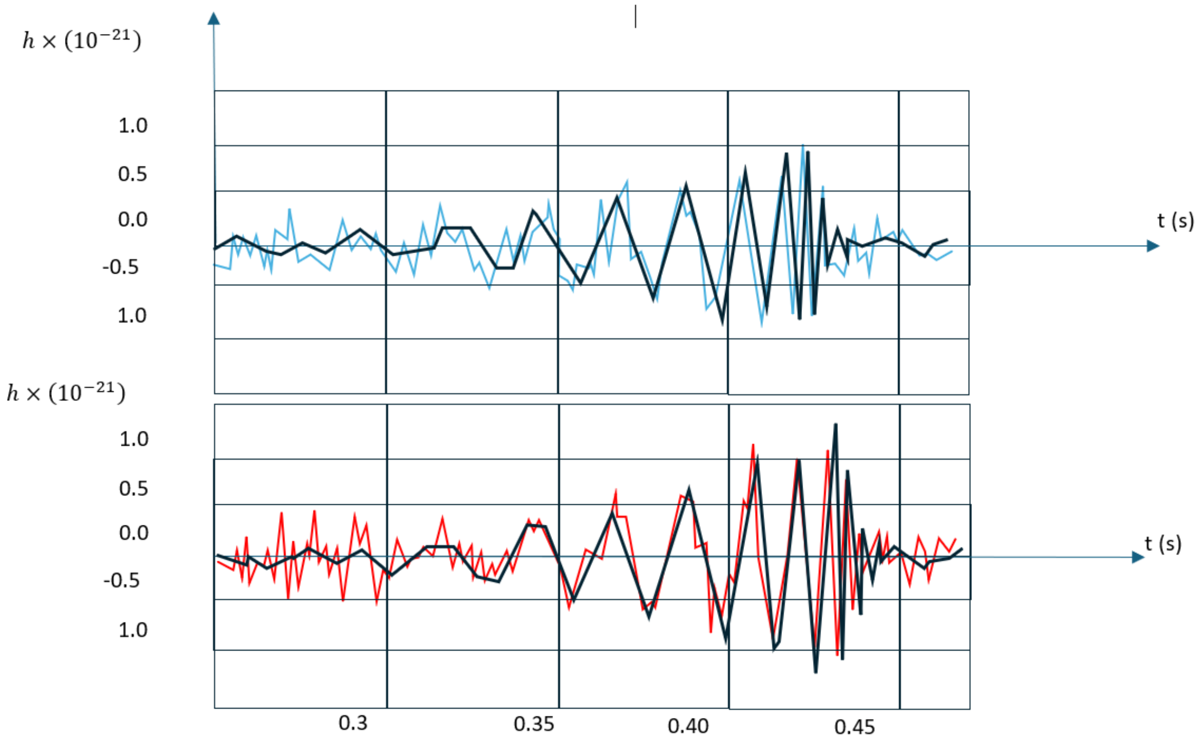


Figure 4: Deformations of the arms of the 2 interferometers measured during the passage of the gravitational wave GW150914 and visualization of the theoretical prediction according to general relativity **[7] [12] [14]**

We can see that here again the deformations are very small of the order of 10^{-21}. They are alternated in compression / tension.

Figure 5 shows for the signal considered (the longitudinal deformations $h = \frac{\Delta L}{L}$ of the arms of interferometers of length L = 4km) the difference between the theory and the measurements made in 2014 **[7]**.

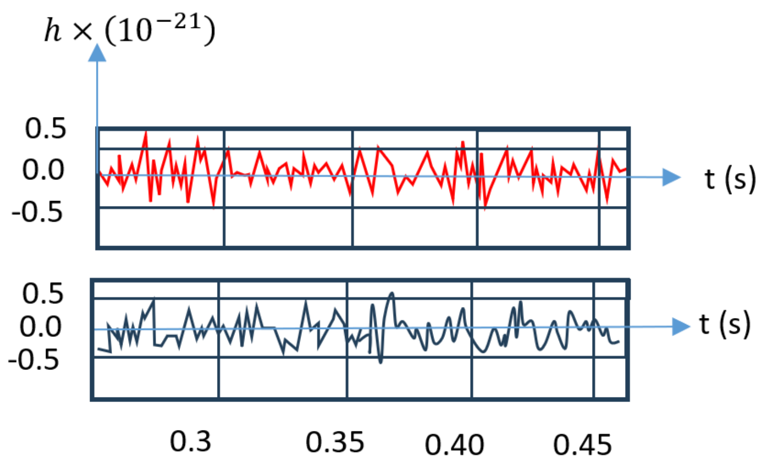


Figure 5: Discrepancy between the predictions of general relativity and the measurements made by LIGO interferometers in 2014 on the GW150914 event **[7] [14]**

Figure 6 below illustrates how space deforms when a gravitational wave propagates in the direction z. The deformations occur in planes transverse xy to this direction of propagation z. The deformations of these planes of space seem to be disconnected from each other, at least according to the predictions of classical general relativity.

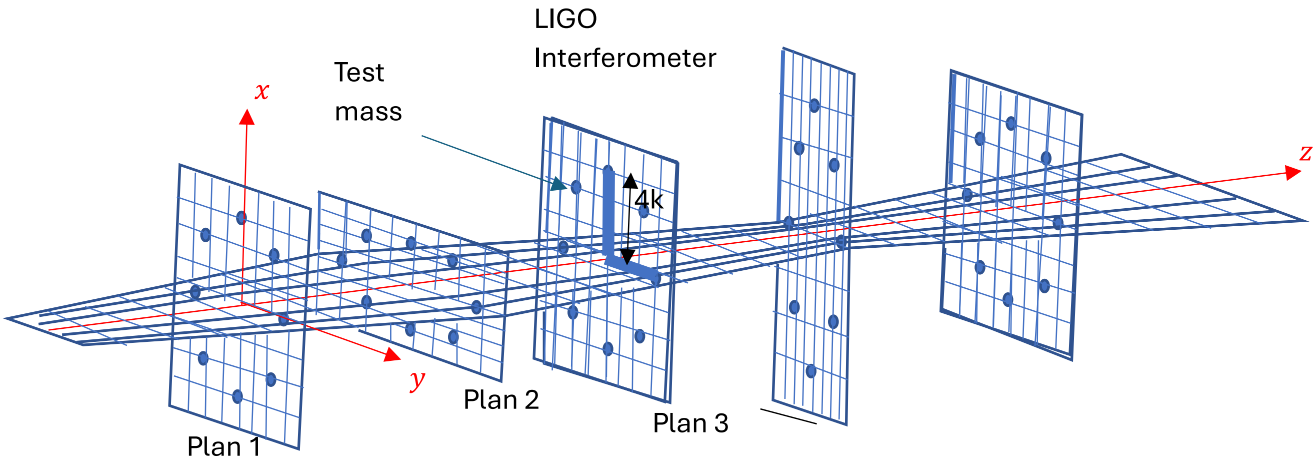


Figure 6: Visualization of the transverse deformations of space-time sheets during the passage of a gravitational wave **[14]**

The equations of general relativity make it possible to study these gravitational waves and deformations of associated space and are well known. According to Einstein's theory of general relativity, gravitation is the geometry of spacetime: specifically, spacetime is a four-dimensional pseudo-Riemannian manifold M, which is a pair $(\boldsymbol{M}, \boldsymbol{g}_{\mu\nu})$, where M is a connected 4-dimensional Haussdorf manifold and $\boldsymbol{g}_{\mu\nu}$ is the metric tensor[1]. Due to its Riemannian structure, spacetime is endowed with an affine connection compatible with the metric, known as the Levi-Civita connection. We are interested in gravitational waves, which are vacuum solutions of Einstein's equations:

$$G^{\mu\nu} = -\frac{8\pi G}{c^4} T^{\mu\nu}$$

where $\boldsymbol{G}^{\mu\nu}$ is the Einstein tensor defined in terms of the Ricci tensor $\boldsymbol{R}^{\mu\nu}$ and scalar curvature R, as $\boldsymbol{G}^{\mu\nu} = \boldsymbol{R}^{\mu\nu} - \frac{1}{2}\boldsymbol{g}^{\mu\nu}\mathbf{R}$ and $\boldsymbol{T}^{\mu\nu}$ is the energy-momentum tensor. To obtain the wave equations, we suppose that the metric tensor can be written in the form:

$$\boldsymbol{g}_{\mu\nu} = \boldsymbol{\eta}_{\mu\nu} + \boldsymbol{h}_{\mu\nu}$$

[1] Greek indices run from 0 to 3, while Latin indices run from 1 to 3.

where $\boldsymbol{h_{\mu\nu}}$ is a small perturbation of the Minkowski tensor $\boldsymbol{\eta_{\mu\nu}}$ of flat spacetime ($\mathbf{h}^{\mu\nu} << \eta^{\mu\nu}$). By setting $\boldsymbol{\overline{h}_{\mu\nu}} = \mathbf{h_{\mu\nu}} - \frac{1}{2}\ \boldsymbol{\eta_{\mu\nu}}\mathbf{h}$, where $\boldsymbol{h} = \boldsymbol{h^{\mu}_{\mu}}$, Einstein's equation can be written in the form:

$$\Box\, \overline{\boldsymbol{h}}_{\mu\nu} = -\frac{\mathbf{16\pi}\boldsymbol{G}}{\boldsymbol{c^4}}\boldsymbol{T}_{\mu\nu}$$

Where $\Box = \boldsymbol{\partial_\mu \partial^\mu} = \boldsymbol{\nabla^2} - \frac{\mathbf{1}}{\boldsymbol{c^2}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial t^2}}$. In a vacuum ($\boldsymbol{T_{\mu\nu}} = \mathbf{0}$) the above equation becomes $\Box\, \overline{\boldsymbol{h}}_{\mu\nu} = \mathbf{0}$, whose solutions are gravitational waves propagating in empty space, which can be written in the form:

$$\overline{\boldsymbol{h}}_{\mu\nu} = \boldsymbol{A_{\mu\nu}} \cos(\boldsymbol{k_\sigma x^\sigma})$$

$$\boldsymbol{A_{\mu\nu}} = \boldsymbol{A_+}\begin{bmatrix} 0 & -0 & -0 & -0 \\ 0 & +1 & -0 & -0 \\ 0 & -0 & -1 & -0 \\ 0 & -0 & -0 & +0 \end{bmatrix} + \boldsymbol{A_\times}\begin{bmatrix} 0 & -0 & -0 & -0 \\ 0 & -0 & +1 & -0 \\ 0 & +1 & -0 & -0 \\ 0 & -0 & -0 & +0 \end{bmatrix}$$

Where $\boldsymbol{A_+}$ and $\boldsymbol{A_\times}$ are the amplitude of the wave in the two polarization states, and $\boldsymbol{k^\sigma}$is the four-plane wave vector $\boldsymbol{k^\sigma} = \left(\frac{\boldsymbol{\omega}}{\boldsymbol{c}}; \vec{\boldsymbol{k}}\right)$, where $\boldsymbol{\omega}$ is the frequency and $\boldsymbol{k} = \left|\left|\vec{\boldsymbol{k}}\right|\right| = \frac{\boldsymbol{\omega}}{\boldsymbol{c}}$ is the wave number, with $\boldsymbol{k^\sigma k_\sigma} = \mathbf{0}$. This solution is given in the so-called TT gauge: the deformation caused by the wave is transverse to the propagation direction, and the amplitude tensor $\boldsymbol{A_{\mu\nu}}$ is traceless. For instance, if we consider test masses positioned on a circle of radius R before the passage of the wave, the deformation provoked by the $\boldsymbol{A_+}$ polarization is given by:

$$\Delta \boldsymbol{S} = \boldsymbol{R}\left[\mathbf{1} + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{A_+ cos}\omega\boldsymbol{t}\ \boldsymbol{cos}2\theta\right]$$

and it is depicted in Figure 6, for a wave propagating along the z direction. A similar deformation is obtained for polarization $\boldsymbol{A^\times}$, which corresponds to a rotation of Figure 6 by 45 degrees.

According to this description, the deformations provoked by the wave are in the plane orthogonal to the propagation direction: strictly speaking, this is true if we confine ourselves to the first order with respect to the reference position.

At this stage we can conclude as follows:

1) Space-time is not rigid as Newton thought **[10]**, it is deformed. Einstein is right with his theory of general relativity **[1] [2]**.

2) Space-time undergoes very small deformations that are either elongations or shortening (gravitational waves) **[7]** or angular distortions (Lense-Thirring effects) **[6]**.

3) The perturbations of the metric $h_{\mu\nu}$ called as such in general relativity are close to the notion of deformations $\varepsilon_{\mu\nu}$ in the mechanics of continuous media.

Various publications propose that $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$ **[8]**, **[11]** to **[14]** with γ the angular distorsion of the medium issued of the Hooke law τ = Gγ and ε the lengthening and shortening of the Hooke law : σ = Eε.

$$\begin{bmatrix} \boldsymbol{h_{tt}} & \boldsymbol{h_{tx}} & \boldsymbol{h_{ty}} & \boldsymbol{h_{tz}} \\ \boldsymbol{h_{xt}} & \boldsymbol{h_{xx}} & \boldsymbol{h_{xy}} & \boldsymbol{h_{xz}} \\ \boldsymbol{h_{yt}} & \boldsymbol{h_{yx}} & \boldsymbol{h_{yy}} & \boldsymbol{h_{yz}} \\ \boldsymbol{h_{zt}} & \boldsymbol{h_{zx}} & \boldsymbol{h_{zy}} & \boldsymbol{h_{zz}} \end{bmatrix} \rightarrow 2 \times \begin{bmatrix} \varepsilon_{tt} & \frac{1}{2}\gamma_{tx} & \frac{1}{2}\gamma_{ty} & \frac{1}{2}\gamma_{tz} \\ \frac{1}{2}\gamma_{xt} & \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yt} & \frac{1}{2}\gamma_{yx} & \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zt} & \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_{zz} \end{bmatrix}$$

4) The deformations are elastic (they disappear, and space-time returns to its original shape once the event is over.

So, to consider that space-time behaves by analogy as a deformable elastic medium is a reasonable analogy at first sight. Sakharov was the first to write it in 1968 **[3]**. But in this case, how to use the equations of the mechanics of continuous media originally developed in 3 dimensions to space-time in 4 dimensions? Which elastic medium should be considered? Isotropic or anisotropic? This is what we will see in the next chapter.

3. State of the art on isotropic rigid elastic models: Use of the mechanics of continuous media, to reproduce the deformations of gravitational waves

Summary

This chapter examines mathematically and mechanically the analogy between classical elastic media and space-time. Isotropic models are recalled: homogeneity, rigidity and symmetries. Young's modulus, the Poisson's ratio and Hooke's laws are described. Spacetime is compared to a rigid elastic medium in some theories. The interest is to express gravitational curvature as mechanical deformation. The limits of isotropy appear to describe the anisotropy of the universe in the case of gravitational waves. Previous attempts at space-time/elastic solid assimilation are recalled. This chapter highlights the need to have a complementary tensor related to the very structure of the vacuum and therefore its quantum energy to account for the energy momentum tensor to be zero in a vacuum. The isotropic approach accounts for large structures, but not for local anomalies (transverse deformations in the case of gravitational waves). The parallel with standard cosmology is discussed. The chapter prepares the introduction of an anisotropic model.

We saw in the previous chapter that space-time behaves like a deformable elastic medium with deformations that can be visualized and characterized by an equivalent deformation tensor defined by $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$ **[8]**, **[11]** to **[14]**. We have also seen that we can by analogy consider space-time as an elastic medium, i.e. a medium that follows Hooke's law relating deformations to the stresses that generate them via a constant that depends on the material constituting the elastic medium. This Hooke's law is classically written with the stress tensor σ_{ij} and the strain tensor ε_{kl}:

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$$

With: $c_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$

λ and μ being the Lamé coefficients as a function of the elastic medium characterized by Young's modulus E=Y and Poisson's modulus ν. We have by introducing c_{ijkl} into the expression of the stress tensor σ_{ij}:

$$\sigma_{ij} = \frac{E}{(1+\nu)}\left(\varepsilon_{ij} + \frac{\nu}{(1-2\nu)}\varepsilon_{kk}\delta_{ij}\right)$$

With δ_{ij} the Kronecker symbol. We can then write U_{ij} the energy of deformation of the elastic medium under stress:

$$U_{ij} = \frac{1}{2}\sigma_{ij}\varepsilon^{ij}$$

We therefore obtain for the energy by replacing σ_{ij} by its expression above:

$$U_{ij} = \frac{E}{2(1+\nu)}\left(\varepsilon_{ij} + \frac{\nu}{(1-2\nu)}\varepsilon_{kk}\delta_{ij}\right)\varepsilon^{ij}$$

Or to be in a form with parallelism with the expression of general relativity:

$$\left(\varepsilon_{ij} + \frac{\nu}{(1-2\nu)}\varepsilon_{kk}\delta_{ij}\right)\varepsilon^{ij} = \frac{2(1+\nu)}{E}U_{ij}$$

In this expression $\frac{2(1+\nu)}{E}$ represents the mechanical flexibility of the deformed elastic medium. In the case of gravitational waves, the deformations manifesting themselves in transverse planes independent of each other (see figure 6), we can then at first approach apply the mechanics of continuous media in 2 dimensions in these planes. This is what I have done in the publications **[12]** and **[13]** reproduced in **[14]**. Other authors have chosen to extend general relativity in 4 dimensions by studying Hyper-plates **[11]** **[17]** or have developed a theory of elasticity in 4 dimensions **[20]**. A complete state of the art is given in **[3]** **[8]** **[11-14]** **[17]** and **[20]** to **[33]**. It shows that there are in fact two ways of doing things. The first consists of starting from the mechanics of continuous media and seeking to develop it in 4 dimensions. The second is to start from general relativity and transform it, to "mechanize" it by finding bridges between relativistic parameters (G, c, h, R...) and mechanical parameters (E=Y, ν, G=μ, f, ρ) to get closer to the mechanics of continuous media. It is this second approach that we have followed in this thesis and in the associated publications **[12]** to **[14]**. Why such a choice? General relativity predicts distortions of space-time as a function of its load. These deformations have been measured with precision for 100 years. From these deformations, it seems natural and safer to look for the mechanical theory that can be applied to it to reproduce this functioning and then to see the possible consequences by mirror effects on general relativity from both a didactic and a predictive point of view. We can then propose covariant tensor mathematical amendments to this general relativity to match as closely as possible the precision of the measurements of these observed deformations. Thus, if we take the example of a plate in continuum mechanics (space is considered to have almost zero curvature according to the Planck mission surveys **[60]** to **[62]**), this expression becomes with $\frac{1}{R_{ii}}$ the curvatures of the plate, t the thickness of the plate and U the strain energy according to S. Timoshenko **[16]**.

$$\left[\left(\frac{1}{R_x}\right)^2 + \left(\frac{1}{R_y}\right)^2 + 2(1-\nu)\left(\frac{1}{R_{xy}}\right)^2 + 2\nu\left(\frac{1}{R_x R_y}\right)\right] = \frac{24(1-\nu^2)}{Et^3} \times \frac{\Delta U}{\Delta x \Delta y}$$

To move from space to space-time, T. Tenev and M. F. Horstemeyer took this 3-dimensional plate approach and generalized it in **[11]** and **[17]** into a 4-dimensional super-plate. Starting from the theory of gravitational waves recalled in the previous chapter, where h is the trace of $\boldsymbol{h}_{\mu\nu}$, that is connected to the disruption of the metric based on the variable change $\boldsymbol{\overline{h}_{\mu\nu}}$ $\boldsymbol{\overline{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{h}}$, we can highlight the parallelism of Einstein's gravitational field equation with the analogy of the elastic medium according to a Hooke's law:

$$\boldsymbol{\partial^{\lambda}\partial_{\lambda}\overline{h}_{\mu\nu} = \Box\, \overline{h}_{\mu\nu} = -\frac{16\pi G}{c^4}\, T_{\mu\nu} = 2\kappa T_{\mu\nu}}$$

$$\boldsymbol{\Box\left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\overline{\varepsilon}\right) = -\frac{16\pi G}{c^4} T_{\mu\nu}}$$

Which we can compare with Hooke's law:

$$\left(\varepsilon_{ij} + \frac{\nu}{(1-2\nu)}\varepsilon_{kk}\delta_{ij}\right)\varepsilon^{ij} = \frac{2(1+\nu)}{E}U_{ij}$$

The "mechanization" of general relativity is then based on the following bridges:

- The perturbation tensor of the metric approximates the strain tensor by the expression $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$ **[8]**, **[11]** to **[14]**
- Einstein's constant κ is a close to the flexibility of space-time, the $\frac{24(1-\nu^2)}{Et^3}$in plate theory) this is what we did in Pramana's publication **[12]**.

- The tensor energy impulse in French called in English stress-energy is to be compared to the energy of deformation of the elastic medium U_{ij} . This tensor nevertheless has a double possible interpretation in mechanics. An energy density or a stress. Indeed, we can thus show **[12]** that the stress energy tensor in English is aptly named since for space coordinates it is equivalent to the stress tensor $\sigma_{ij} = \rho v_i v_j$ and that for general relativity, for time components, it represents energy densities and is written $T_{\mu\nu} = \rho u_{\mu} u_{\nu}$with u_{μ} and u_{ν} four velocities and v_i and v_j velocities and ρ a density of the medium. Thus, the component $T_{00} = \rho c^2$ has the unit of an energy density N.m/m³ or a stress in N/m² = Pa.

However, at this stage, a first difficulty appears. Indeed, gravitational waves propagate in a vacuum and, according to Einstein **[88]**.

$$\boldsymbol{\partial^{\lambda}\partial_{\lambda}\overline{h}_{\mu\nu} = \Box\, \overline{h}_{\mu\nu} = 0}$$

Similarly, the Lense-Thirring effects are measured in a vacuum. As a result, if we look at the deformation of space-time in a vacuum, the classical stress energy tensor becomes zero: $T_{\mu\nu} = 0$.

In fact, $T_{\mu\nu}$ in Einstein's mind, represents the loading energy density of spacetime within it, not the energy density of spacetime itself. This is why, during the propagation of the gravitational wave in a vacuum (without space-time loading), there are measured deformations of something even though apparently no stress or charge is present! which seems very strange and contrary to Hooke's law which involves both stresses and strains. The question is: what do these strains come from? Is it the deformation of what object physically? Are there no intrinsic stresses in the structure of space-time itself? And from which deformation structure comes the deformation into elastic energy of the gravitational wave in a vacuum in this case? So, we have no choice; It must be space-time itself, considered by analogy, as an equivalent elastic medium. This is the basis of this analogy and of our thesis. Another way to look at this point is to say, as seen above, that general relativity is a 4-dimensional Hooke's law **[20]**. So, in this case, $h_{\mu\nu}$is the tensor that groups all the deformations, κ is the flexibility, $T_{\mu\nu}$is the set of stresses that are absent if we are in a vacuum. Thus, to retain a Hooke's law associating deformations and stresses via a constant of proportionality, the only solution is to complete associated with the charge of space-time with a complementary tensor related to its proper elastic deformation: the energy source of this tensor is that of the vacuum. We know from quantum field theory, the Casimir force or the Higgs boson that the vacuum is empty on average but that in fact it does not have it because virtual particles are constantly created and annihilated. This energy of potential distortions of space-time itself is somehow found in its elastic texture itself. If we measure deformations via distortion angles, elongations and shortenings, there is indeed something physical that is physically distorted. In structural mechanics and engineering, an image would be to associate, for example, with the self-weight of a beam or a slab this $t_{\mu\nu}$ while the operating loads would be associated with the stress energy tensor defined by Einstein $T_{\mu\nu}$. Of course, it's only an image because it acts within space-time itself and in addition, we are 4 dimensions while the plate is in 3 dimensions. So, and this is a fundamental pillar of this thesis, there is no choice, we must add to Einstein's stress energy tensor an elastic deformation tensor of space-time, itself noted $\boldsymbol{t}_{\mu\nu}$ or $\boldsymbol{t}_{\mu\nu,el}\ or \sigma_{\mu\nu}$ or according to the publications and which remains present when $\boldsymbol{T}_{\mu\nu} = 0$ in a vacuum. Thus, Hooke's law is re-established, and the analogy can work again. This is what is proposed in the publications **[21]** to **[24]**.

With the elastic analogy, there is no choice. The unresolved question is whether this tensor really exists and from where it can emerge mathematically speaking. But for our analogy, we absolutely need it. If we take the hypothesis and postulate that yes, in this case, the modified Einstein equation becomes following **[8]**:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} + t_{\mu\nu}\right)$$

In a vacuum, we have it because of the elasticity of the vacuum itself:

$$\boldsymbol{T}_{\mu\nu} = \mathbf{0} + \boldsymbol{t}_{\mu\nu,\boldsymbol{elastic\ of\ vacuum}}$$

In **[22]** and **[23]**, the author defined a modified formula for general relativity as follows:

$$G_{\mu\nu} = \kappa T_{\mu\nu} + T_{e\mu\nu}$$

With:

$$T_{e\mu\nu} = \lambda\varepsilon\varepsilon_{\mu\nu} + 2\mu\varepsilon_{\ \mu\nu}$$

In **[24]**, the author followed the same approach:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} + \sigma_{\mu\nu}\right)$$

With:

$$\varepsilon_{\mu\nu} = \frac{1}{2}\left(D_\mu G_\nu + D_\nu G_\mu\right)$$

$$\sigma_{\mu\nu} = C_{\mu\nu\delta\gamma}\varepsilon^{\delta\gamma}$$

Moreover, this approach to the additional tensor is not totally new because it has already been added to calculate the energy of the gravitational wave in a vacuum. Indeed, for the determination of the gravitational wave, Einstein's equation linearized in a vacuum is written without any $T_{\mu\nu}$. It has therefore become difficult to find a vacuum energy, since space-time is not charged. As we saw in the previous chapter:

$$\boldsymbol{\partial}^\lambda \boldsymbol{\partial}_\lambda \overline{\boldsymbol{h}}_{\boldsymbol{\mu\nu}} = \Box\, \overline{\boldsymbol{h}}_{\boldsymbol{\mu\nu}} = \mathbf{0}$$

To solve this dilemma, relativistic physicists have used a similar approach to this additional tensor to the classical tensor of stress energy. This approach is given by several authors **[18]** and **[25]** to **[27]**. So, when we want to establish the energy of gravitational waves in a vacuum, the Einstein field equation becomes:

$$\boldsymbol{G}^{(\mathbf{1})}_{\mu\nu} = -\frac{\mathbf{8}\pi \boldsymbol{G}}{\boldsymbol{c}^{\mathbf{4}}}\left(\boldsymbol{T}_{\mu\nu} + \boldsymbol{t}_{\mu\nu}\right)$$

$\boldsymbol{G}^{(\mathbf{1})}_{\mu\nu}$ is constructed from terms $\boldsymbol{G}_{\mu\nu}$of which are linear in $\boldsymbol{h}_{\mu\nu}$ (see formula 4 of **[26]**) and $t_{\mu\nu}$ is defined as follows:

$$\boldsymbol{t}_{\mu\nu} = \boldsymbol{T}^{\boldsymbol{GW}}_{\mu\nu} = \frac{\boldsymbol{c}^{\mathbf{4}}}{\mathbf{8}\pi \boldsymbol{G}}\left[\boldsymbol{G}^{(\mathbf{2})}_{\mu\nu} + \cdots\right]$$

$\boldsymbol{G}_{\mu\nu}^{(2)}$ is constructed from the quadratic terms of $\boldsymbol{h}_{\mu\nu}$ (see formula 5 of **[26]**):

Conclusion

In many publications, an additional elastic stress energy tensor is added to simulate the stresses inside the elastic medium that intrinsically model the deformations of spacetime within it in the denoted vacuum ($T_{el.}^{ik}$ or $T_{e\mu\nu}$or $\sigma_{\mu\nu}$ etc). Thus, in a vacuum, Hooke's law is conserved (curvature via $\boldsymbol{R}_{\mu\nu}$ and R as a function of deformations, flexibility of spacetime via as κ a function of 1/Y = 1/E, and without energy density or mass external to spacetime, the internal constraints of spacetime via this additional elastic stress energy tensor $T_{el.}^{ik}$ or $T_{e\mu\nu}$ or $\sigma_{\mu\nu}$). This additional elastic tensor, used both in general relativity to calculate the strain energy of the gravitational wave in a vacuum **[18]** and **[25]** to **[27]** or introduced by an author **[22]** to **[24]**, **[20]** to model the elasticity of spacetime itself, related to the intrinsic structure of spacetime itself, is the foundation of our analogy in this thesis.

Thus, based on this hypothesis of an elastic strain density tensor of space-time itself and therefore of the vacuum, space-time is studied for the first time in the state of the art as an isotropic medium. In **[12]** I show that we can establish different expressions for κ as a function of the Young's modulus E = Y of spacetime and the Poisson ratio ν. I show this by considering the interferometer tubes as tensioned and compressed bars (see Figure 7) and in the case of an isotropic space-time cylinder stressed in torsion (See Figure 8).

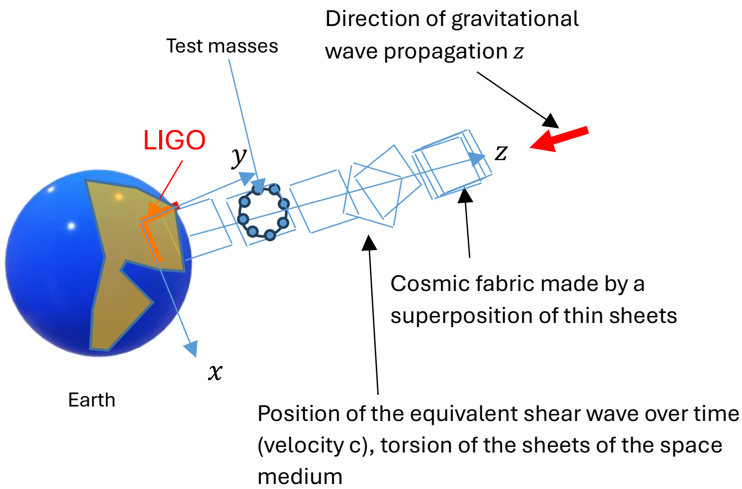


Figure 7: Space tube deformations in interferometer arms during the passage of a gravitational wave **[12]**

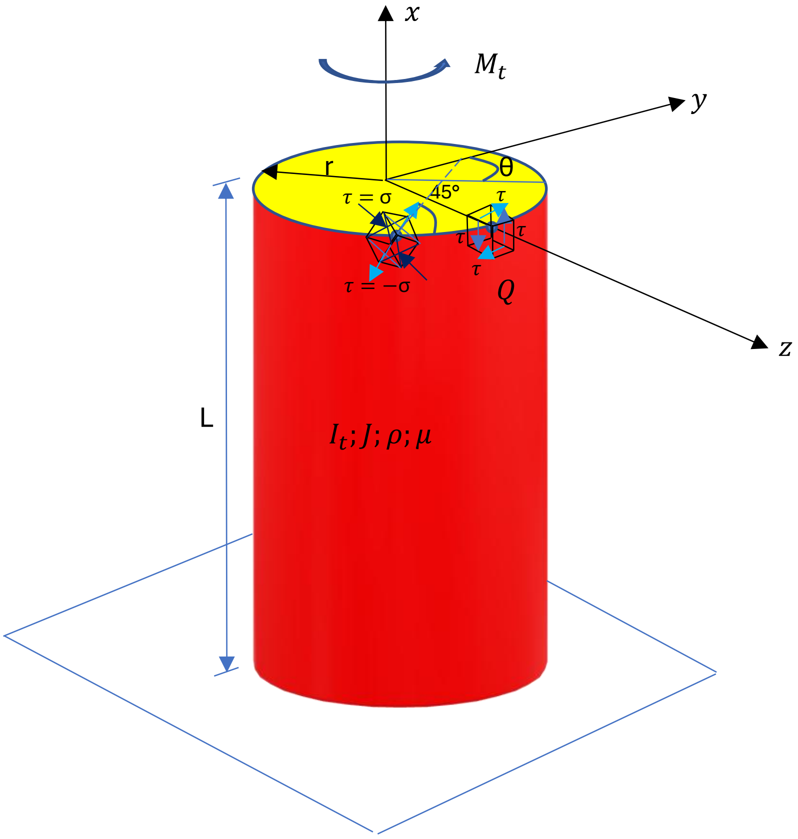


Figure 8: Cylinder of Torsionally Stressed Space **[12]**

From these approaches in isotropic elastic medium, it follows:

That it is possible in the plane of interferometers to apply the mechanics of traditional continuous media and to find a form of Einstein's equation provided that the Poisson ratio ν= 1 **[11]** to **[14]** is taken for the Poisson ratio.

a) In the case of Figure 7 of the interferometer arms, we thus obtain **[12]** to **[14]**

$$\frac{1}{L^2}(\varepsilon_{xx})^2 = 4(1+\nu) \times \pi \times \frac{\pi f^2}{\rho} \times \frac{1}{c^4} \times \frac{U}{V}$$

$$\frac{1}{L^2}\left(\varepsilon_{yy}\right)^2 = 4(1+\nu) \times \pi \times \frac{\pi f^2}{\rho} \times \frac{1}{c^4} \times \frac{U}{V}$$

Which becomes:

$$\begin{bmatrix} \frac{1}{L^2} & 0 \\ 0 & \frac{1}{L^2} \end{bmatrix} \begin{bmatrix} (\varepsilon_{xx})^2 & 0 \\ 0 & \left(\varepsilon_{yy}\right)^2 \end{bmatrix} = \frac{8\pi G}{c^4} \begin{bmatrix} T_{xx} & 0 \\ 0 & T_{yy} \end{bmatrix}$$

Tensor form written in the isotropic transverse plane of the interferimeter arms that is very reminiscent of Einstein's form, this time written in 4 dimensions of space-time.

b) In the case of the torsionally stressed space cylinder **[12]** to **[14]**:

$$\frac{\gamma^2}{L^2} = 16\pi\frac{\pi f^2}{\rho}\frac{1}{c^4} \times \frac{U}{V}$$

That it is then possible to "mechanize" the gravitational constant G by expressing it from the characteristics of the elastic medium. In **[12]** I obtain with f the frequency of vibration of space-time and ρ the density of the vacuum:

$$G = \frac{\pi f^2}{\rho}$$

This gives with $c = \sqrt{\frac{Y}{\rho}}$ and $\omega = 2\pi f$ considering the compression waves (Figure 7):

$$\kappa = 2\rho\left(\frac{\omega}{E}\right)^2$$

Or considering shear wave with $c = \sqrt{\frac{\mu}{\rho}}$ and $\omega = 2\pi f$ (Figure 8):

$$\kappa = 2\rho\left(\frac{\omega}{\mu}\right)^2$$

These expressions of G are found in **[28]** and **[29]**.

T. Tenev and M. F. Horstemeyer **[11]** and **[17]** obtain an expression of G of the form with them also ν = 1:

$$Y = \frac{6c^7}{2\pi\hbar G^2}$$

It should be noted that as with Hawking with his black hole radiation, the elastic continuous media approach allows relativistic and quantum constants to intervene simultaneously.

That the two polarizations of gravitational waves can be seen as the two expressions of a strain tensor associated with a pure torsion of the medium **[12]** to **[14]** (which seems logical given the mutual rotation of the 2 black holes between them during coalescence). Depending on the facets considered as a function of the angle (see Figure 8), either elongations, shortening associated with σ or angular mirror distortions γ associated with the shear τ (τ = σ in intensity) of the two polarizations of gravitational waves appear. These distortions are a novelty and a first contribution of mechanical analogy. Lateral movements of the interferometer arms should exist (see Figure 9).

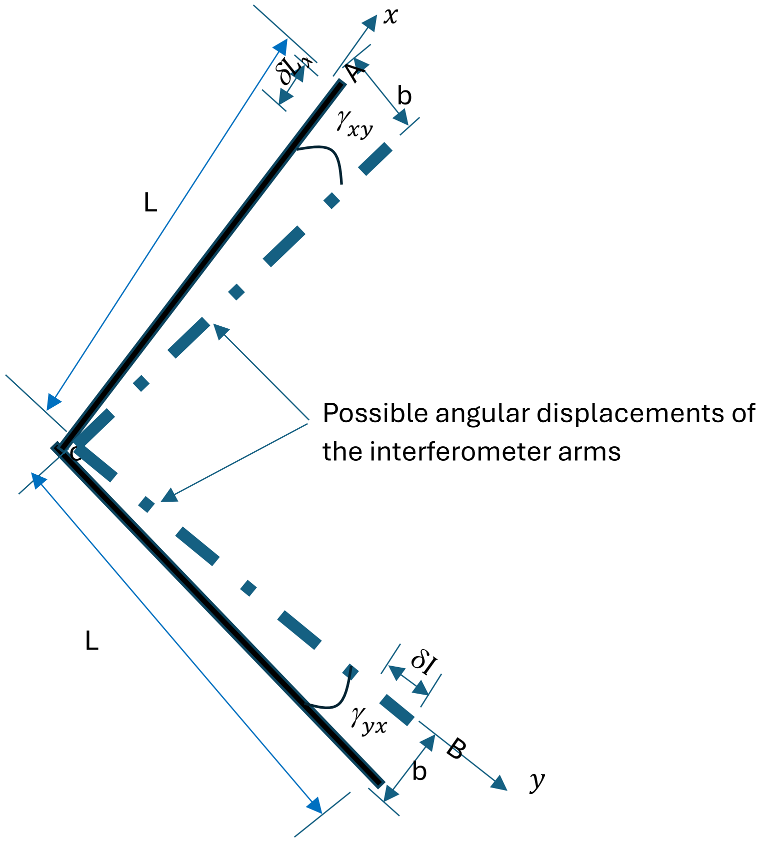


Figure 9: Lateral displacement of interferometer arms if the component is considered as a source of shear deformations by torsion of space sheets $A^{\times}$**[12]** to **[14]**

Thus, using the elastic bridge between the perturbation tensor of the metric and the strain tensor $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$ **[8]**, **[11]** to **[14]**, the 2 classical polarizations A^{+} and $A^{\times}$ gravitational waves can be read as follows **[12]** to **[14]**:

$$h_{\mu\nu} = A_{+}\cos\left(\frac{\omega}{c}(ct-z)\right)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \varepsilon_{xy\,(A_{+})} = \frac{1}{2}A_{+}\cos\left(\frac{\omega}{c}(ct-z)\right)\begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

These two components of deformations correspond to elongations and shortenings, and:

$$h_{\mu\nu} = A_{\times}\cos\left(\frac{\omega}{c}(ct-z)\right)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \varepsilon_{xy\,(A_{\times})} = \frac{1}{2}A_{\times}\cos\left(\frac{\omega}{c}(ct-z)\right)\begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

These two components of deformations correspond to distortions γ_{xy}, γ_{yx}, which in elasticity correspond to pure torsion.

1) Let the speed of light become associated with Young's modulus and the density of the elastic medium under compression waves by the expression $c = \sqrt{\frac{Y}{\rho}}$ or shear modulus G=μ and density by the expression $c = \sqrt{\frac{\mu}{\rho}}$ if we are interested only in transverse deformations **[12]** to **[14]**:

2) That it is possible to model space-time and its loading with engineering elements, a kind of relativistic Eurocode approach. The authors **[11]**, **[17]**, **[24]**, **[29]**, **[30]**, **[79]** to **[81]** modeled spacetime as a membrane, others from plate or hyperplate **[11]**, **[17]**, **[31]** or beam **[12]** to **[14]**. Finally, others used cylinders **[12]**. This is what the state-of-the-art shows. We have also developed this via beam lattices or quantum thickness membranes as well as with pressure shells. Moreover, to support the quantum interpretation, reference **[111]** demonstrates that the dynamic self-vibration of a beam resting on two supports serves as an analog to a quantum particle confined in an infinite potential well.

However, this initial approach, a rough roughing up of the mechanization of space-time, presents a small flaw on closer inspection! the Poisson's ratio ν must take a value of 1 (the compression deformations following one of the arms of the interferometer are simultaneously of tension and of the same intensity on the other arm at 90°) **[11]** to **[14]** and **[17]** (Figure 3). This value is uncommon with conventional materials on Earth. Worse still, this Poisson coefficient seems zero (or very, very low, because there is a lack of information with general relativity about what happens in the direction of propagation of gravitational waves in the z direction see figure 6) between the sheets of space-time that deform apparently independently of each other. These values of Poisson coefficients render obsolete the hypothesis taken in physics of a homogeneous and isotropic universe.

We therefore come across an anisotropic behavior of space-time contrary to the hypothesis of a homogeneous and isotropic universe. We are thus reaching the limits of elastic analogy, or we must revise the hypothesis of homogeneity and isotropy of space-time. This is therefore the starting point of the research work of the thesis, the main lines of which are published in **[14]**. Having studied the state of the art on the analogy of space-time as an isotropic elastic medium, it remains to study the conditions of anisotropy of space-time and the potential consequences on its behavior. So, either the universe is anisotropic, and the hypothesis of homogeneous and isotropic physics falls. Either the universe is isotropic, and, in this case, general relativity must be modified to generate other polarizations and therefore by mirror effect of other deformation to reassemble the disjoint space-time sheets in the case of polarization predicted by general relativity only. It would also have to simultaneously study the behavior of space-time under the different components of the perturbation tensor of the metric $h_{\mu\nu}$ **[8]**. We will therefore see in the next chapter how to study space-time in a transverse isotopic way and see all the consequences of this hypothesis on the structure of space-time and on general relativity itself.

4. New anisotropic model in continuum mechanics to better represent gravitational wave deformations within space-time: Basic hypothesis of the thesis and proposed new model

Summary

Based on the analysis of space-time deformations in the case of gravitational waves, this chapter proposes an innovative anisotropic model. The space in the framework of the equivalent elastic medium seems to consist of sheets disconnected from each other in terms of deformations in planes transverse to the direction of propagation of gravitational waves on the one hand, and the Young's moduli and Poisson coefficients appear different according to the directions considered on the other hand. It is recalled that the theory of defects and the relativistic geometric torsion present a strong analogy allowing these leaves to be reconnected to each other via the Burger vector. This type of operation could make it possible to explain in the transverse isotropic model the successive slippages from sheet to sheet reconnected by geometric torsion in a way.

We have seen in the previous paragraphs, that the disjunct leaf behavior of space during the passage of a gravitational wave and the associated mechanical modeling implies a Poisson ratio ν of 1 in the plane of these leaves and close to 0 perpendicular to them, bringing down the hypothesis of an isotropic hormonal elastic medium associated by analogy with space-time.

This Poisson coefficient of 1 **[11]**, **[17]** and **[12]** to **[14]** is found in the very shape of the polarization $A^{+}$and deformations $\varepsilon_{xx} = -\varepsilon_{yy}$ associated in the plane xy of elastic medium. Figure 10 illustrates these deformations well in the case of test masses deposited in a vacuum far from any gravitational field.

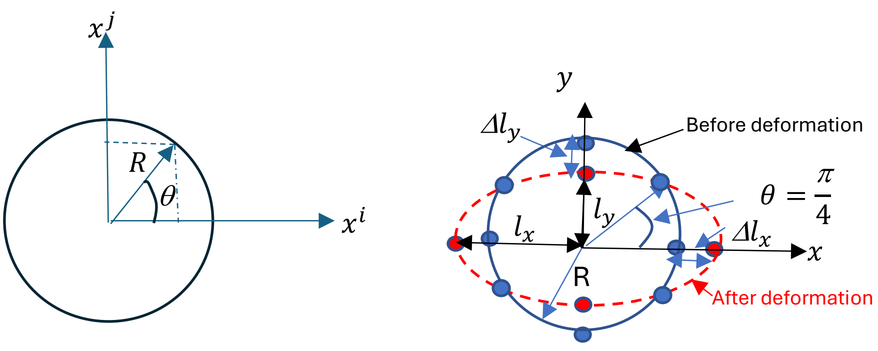


Figure 10: Displacement of test masses in the plane xy (polarization A+) when a gravitational wave propagates in the direction z- **[12] [14]**

Thus, in the case of a transverse isotropic medium, Hooke's law is written:

$$\begin{Bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{Bmatrix} = [K^{-1}] \begin{Bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{Bmatrix}$$

With for flexibility:

$$\begin{bmatrix} \frac{1}{E_L} & \frac{-1}{E_L} & 0 & 0 & 0 & 0 \\ \frac{-1}{E_L} & \frac{1}{E_L} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{E_N} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{4}{E_L} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{LN}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{LN}} \end{bmatrix}_{M\,(N,L,T)}$$

At this point, we therefore have a behavior of space-time during the passage of a gravitational wave which behaves in a way as shown in figure 11 below:

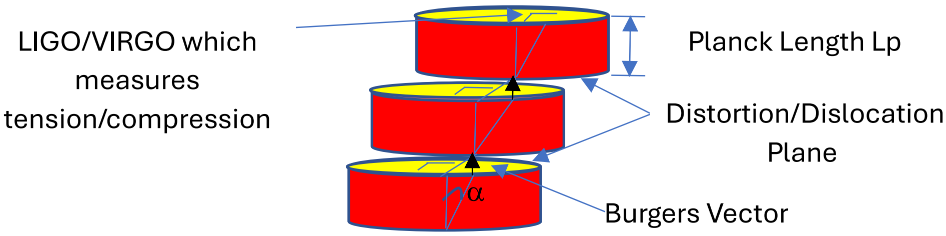


Figure 11: Behavior of space sheets during the passage of a gravitational wave **[14]**

This type of space-time behavior is reminiscent of that of a crystal with local imperfections or defects creating breaks in the overall behavior **[49]**, **[50]**. These defects are dislocations in wedges or screws as shown in Figure 12 below.

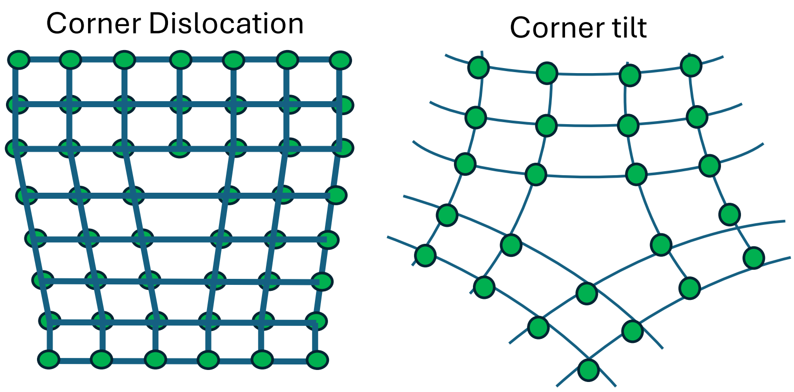


Figure 12: Example of wedge dislocation and wedge disinclination in crystal lattices of atoms **[14]**

Figure 13 below shows how the Burger vector $\vec{b}$ **[14] [32]** makes it possible to reconstruct a certain continuity between two planes disjointed by a defect when a parallel transport of a vector along a crystal lattice perturbed by a defect is carried out along a path.

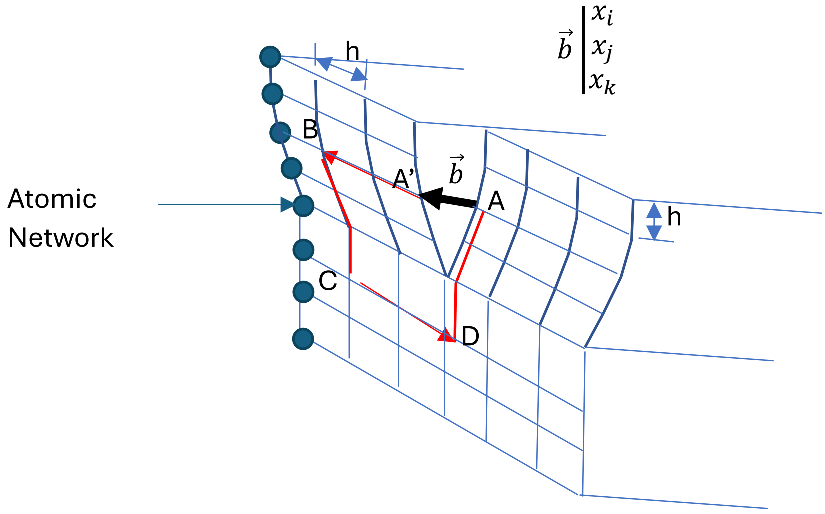


Figure 13: Parallel transport of a vector along a path ABCD in a crystal lattice disturbed by a defect A'A **[14]**

It can be seen that this kind of defect, which occurs during a local plasticization of a crystal, is reminiscent of those observed in the case of gravitational waves (see Figure 11) since they are transverse waves where the deformations are located in planes xy perpendicular to the direction of propagation z of these waves, a plane that seems to be at odds with each other as shown in Figure 12 above.

Mathematically, these defects and this Burger vector are in the following form:

Writing the path in Figure 13, we can see that mathematically the equivalent Burgers vector is expressed as follows $ABCD$**:[14]**, **[32]**, **[33]**, **[34]**, **[35]**.

$$\boldsymbol{db^{\mu} = -\Gamma_{\nu\lambda}^{\mu} dA^{\nu\lambda}}$$

$dA^{\nu\lambda}$ Being antisymmetric, the symmetric part of the affine bond is excluded, and only the antisymmetric part $\Gamma_{[\nu\lambda]}^{\mu}$ exists. Thus, the equivalent Burgers vector is written:

$$db^{\mu} = -\Gamma_{[\nu\lambda]}^{\mu} dA^{\nu\lambda}$$

The bridge is then made as follows with the geometric torsion tensor:

$$-T_{\mu\nu}^{\lambda} = -2\Gamma_{[\mu\nu]}^{\lambda}$$

By defining:

$$-[\nu\lambda] = \frac{1}{2}(\lambda\nu - \nu\lambda)$$

Thus, our transverse isotropic model by analogy has oriented us in the case of gravitational waves towards a crystalline structure of space-time **[49] [50]** having within it defects subject to local plasticizations compatible with the energy of deformations of gravitational waves that causes them locally during their passage. This local plasticization does not propagate; it is the wave during its passage that sets them in motion locally. Several authors have been in this direction and towards this hypothesis of Crystal space-time **[33]** to **[36]** and **[49] [50]**.

We shall see, extraordinarily, that this crystal-like approach to space-time can be transposed to general relativity **[49] [50]** in the form of a modified theory of general relativity, and that this implies modifications of the current version of the theory into a larger one but also implies measurable complementary physical phenomena.

5. Consequences of the new rigid anisotropic elastic model on the structure of space-time: link of crystallography and defect theory and geometric torsion of the Einstein-Cartan type

Summary

This chapter reminds us that defect theory related to crystallography and geometric torsion are connected in evolved versions of general relativity of the Einstein-Cartan type. The Einstein-Cartan equations with numbers of two involve the Einstein equation but also a spin equation useful for getting closer to quantum field theory. This reflects the dissymmetry of the medium that is found in the leaf behavior in the case of gravitational waves (the affine connections do not cancel each other in the constitution of the torsion tensor). The analogy is made with the strength of the materials where a beam on two supports under centered point load is symmetrical and where only an equilibrium equation is sufficient to find the forces at the supports, while under dissymmetric loading (applied force not centered at mid-span) two equilibrium equations are necessary (in force and momentum) to find the support reactions.

We have seen in the previous chapter that gravitational waves imply in the case of our analogy of a transverse isotropic medium a Poisson ratio of 1 in the transverse planes **[11]** **[17]** and **[12]** to **[14]** and of 0 between these planes in the direction of propagation of these waves. We have seen that, according to traditional general relativity, this implies deformations in planes identified thanks to test masses distributed in a circle (Figure 10) and that these planes are like so many sheets dislocated in relation to each other during the passage of the wave, recalling by analogy and mirror effect the theory of defects in crystallography. Finally, we have seen that, mathematically, these defects correspond to a geometric torsion of the medium involving a Burgers vector when we make a parallel transport of a vector along a path in the crystal lattice passing through this defect that appeared by local plasticization.

The publication **[14]** allows us to make the link with a completed version of general relativity (addition of the elastic tensor $t_{\mu\nu}$) and our analogy of anisotropic medium and crystal with defects. Indeed, to unify general relativity with quantum field theory, an Einstein-Cartan relativity has been developed **[33]** and **[35]**. In this theory, there are no longer one but two equations to translate the dynamics of distorted space-time. The first is the classical one mentioned at the beginning of this summary of the thesis, the second is related to the spin equivalent of particles. They rely on each other, and this is where the miracle and the coherence with our previous chapter with the geometric torsion of space-time occurs. In this approach of Einstein Cartan, the Riemann tensor that Einstein transformed into the Ricci tensor by tensor contraction of an index, is more complicated and incorporates a complementary term which is the same term as the one we have shown with the geometric torsion associated with the Burgers vector. Our transverse anisotropic model therefore converges towards a modified Einstein-Cartan-type general relativity.

If we write according to **[37]** the parallel transport of the vector V^ρ along the two paths defined in Figure 14 below, we obtain the Riemann tensor completed by a term which is the torsion tensor.

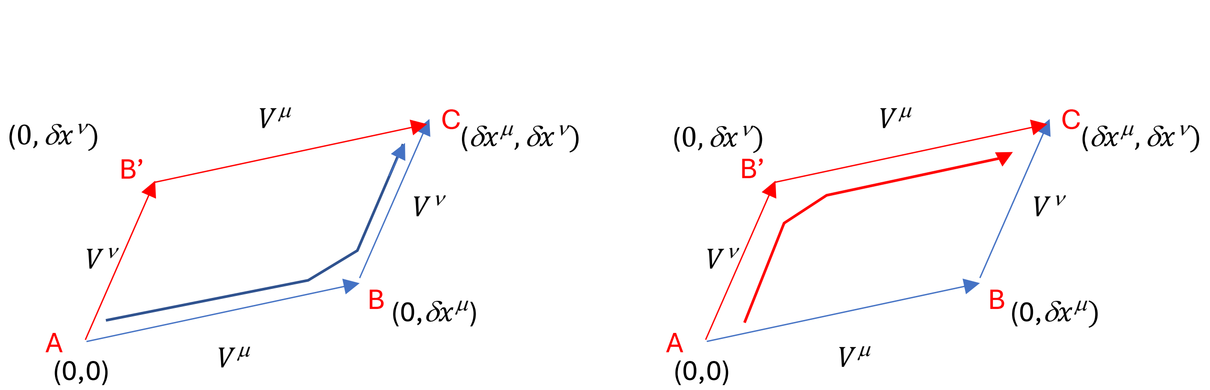


Figure 14: Trajectory of the vector V^ρ transported parallel along the path AB (vector V^μ) then along the path BC (vector V^ν)then by AB' then B'C – **[37]**

All calculations made, the different expressions of the Riemann tensor completed by the torsion tensor expressed according to the different notations found in the literature are given below with D_μ the covariant derivative and $[D_\mu, D_\nu] V^\rho$ the second covariant derivative of vector V^ρ.

$$[D_\mu, D_\nu]V^\rho = D_\mu D_\nu V^\rho - D_\nu D_\mu V^\rho = R^\rho_{\sigma\mu\nu}V^\sigma - T^\lambda_{\mu\nu}D_\lambda V^\rho$$
$$[D_\mu, D_\nu]V^\rho = D_\mu D_\nu V^\rho - D_\nu D_\mu V^\rho = R^\rho_{\sigma\mu\nu}V^\sigma - 2\Gamma^\lambda_{[\mu\nu]}D_\lambda V^\rho$$
$$[D_\mu, D_\nu]V^\rho = D_\mu D_\nu V^\rho - D_\nu D_\mu V^\rho = (\partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma})V^\sigma + (\Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\mu\nu})D_\lambda V^\rho$$
$$[D_\mu, D_\nu]V^\rho = D_\mu D_\nu V^\rho - D_\nu D_\mu V^\rho = (\partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma})V^\sigma + (\Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\mu\nu})(\partial_\lambda V^\rho + \Gamma^\rho_{\lambda\sigma}V^\sigma)$$

We can see that in the second expression we find the one used for the Burgers vector in Crystallography, namely $-T^\lambda_{\mu\nu} = -2\Gamma^\lambda_{[\mu\nu]}$

Einstein Cartan's general relativity takes the following form when we include geometric torsion. Having introduced a new degree of freedom in spin, Einstein's starting equation is now decomposed into 2 equations described in the Einstein-Cartan formalism (see **[33]** formula (34) and (35)):

$$\begin{cases} G^{\mu\nu} - \frac{1}{2}D^*_\gamma(P^{\mu\nu\gamma} - P^{\nu\gamma\mu} + P^{\gamma\mu\nu}) = \kappa T^{\mu\nu} \\ P^\gamma_{\mu\nu} = \kappa \sum^\gamma_{\mu\nu} \end{cases}$$

With:

D_γ The covariant derivative

$D^*_\gamma = D_\gamma + 2S_\gamma = D_\gamma + 2S^\lambda{}_{\gamma\lambda}$

$P^\gamma_{\mu\nu}$ is the Palatini tensor: $\frac{1}{2}P^\gamma{}_{\mu\nu} = S^\gamma_{\mu\nu} + \delta^\gamma_\mu S_\nu - \delta^\gamma_\nu S_\mu$

$S^h_{ml} = \alpha^h_{ml}$ for the torsion tensor

At this stage of our reflection, three questions then arise:

- What are the polarizations of space-time if we introduce geometric torsion into general relativity according to the approach of Einstein Cartan, for example?

- Since polarizations are associated with the strain tensor of space-time, what deformations appear when considering geometric torsion in general relativity given the link with crystallography and defect theory **[33]**?

- Is the elastic mirror effect between the perturbation tensor of the metric and the strain tensor $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$ preserved? A priori no, because in the case of defects, we are in plasticity. In which case, what is the rule of passage between the perturbation tensor of the metric and the strain tensor in plasticity?

This is what we will see in the next chapter.

6. Consequences of the analogy of the anisotropic elastic medium on general relativity:

Refinement of the anisotropic model by analogies with crystallography and space-time **[49] [50]**, defect theory **[33]**, geometric torsion in the Riemann tensor - complementary polarizations of gravitational waves in the direction of propagation **[38]** and connections with recent results in the literature related to a plasticization of space-time.

Summary

This chapter shows that if we consider the twist in modified general relativity of Einstein Cartan, complementary polarizations appear. He also shows that the bridge between polarizations and deformations is no longer linear due to the local plasticizations of the equivalent cosmological crystal. It then appears that the complementary polarizations and their associated deformations make it possible to complete the deformation tensor to glue the space sheets back together. All the new components of the polarizations are placed where they are needed in the strain tensor. This chapter also reminds us that there are many other types of general relativity modified with geometric torsion, all or almost all these good polarizations. This chapter shows that in the state of the art, a second-order study of general relativity in weak fields by gravito-electromagnetism reveals deformations in the direction of propagation of gravitational waves.

The answers to the three questions raised in our previous chapter have been resolved by **[38]** and **[39]**.

In the Einstein-Cartan theory, geometrical torsion is therefore present in addition to curvature **[33]** and **[35]**. More precisely, the torsion is related to the spin of the sources of the gravitational field **[33]**. For our purposes, it is interesting to note that this theory introduces additional polarizations for gravitational waves **[38]**. Our complementary polarizations appear as shown in Figure 15 and are associated with the following polarization matrices:

$$P_{\mu\nu} = p_{(+)}P_{\mu\nu}^{(+)} + p_{(\times)}P_{\mu\nu}^{(\times)} + p_{(b)}P_{\mu\nu}^{(b)} + p_{(l)}P_{\mu\nu}^{(l)} + p_{(xz)}P_{\mu\nu}^{(xz)} + p_{(yz)}P_{\mu\nu}^{(yz)}$$

With the following closing condition according to **[38]**:

$$\bar{p}_{(+)}p_{(+)} + \bar{p}_{(\times)}p_{(\times)} + \bar{p}_{(b)}p_{(b)} + \bar{p}_{(l)}p_{(l)} + \bar{p}_{(xz)}p_{(xz)} + \bar{p}_{(yz)}p_{(yz)} = 1$$

$$P_{\mu\nu}^{(+)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad P_{\mu\nu}^{(\times)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_{\mu\nu}^{(b)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad P_{\mu\nu}^{(l)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P_{\mu\nu}^{(xz)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad P_{\mu\nu}^{(yz)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Their graphical representation is given in Figure 15 below.

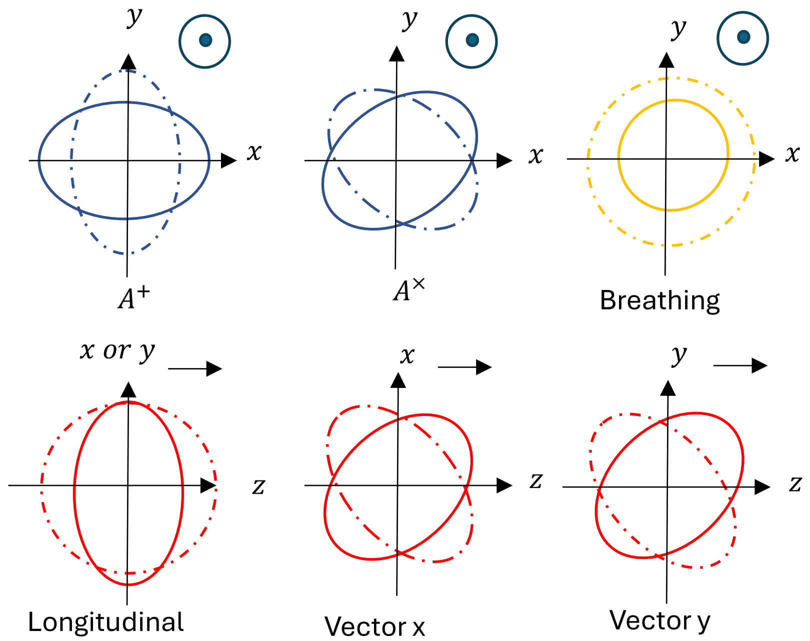


Figure 15: Complementary polarizations that appear in the case of torsionally modified general relativity in the case of the Einstein-Cartan-Sciamma-Kibble theory **[58]**

The bridge between the components of the gravitational wave polarization tensors given above in the case of modified general relativity (Einstein-Cartan) **[33]** and **[35]** and, by analogy, the deformations related to an associated elasto-plastic medium was made in **[39]**. It is no longer linear. Indeed, the direct linear transposition that we had done via equation $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$**[11]** to **[14]** is no longer possible because of this local plasticization associated by analogy with these defects in the equivalent space-time crystal.

The authors therefore considered a gravitational wave as a defect (propagation of a Burgers vector) propagating in an equivalent solid medium. The result of their study is again a compression component H in space and shear and distortion components$\pm\sqrt{2}a_i$, as described in the expression below in 4 dimensions and in the following expression below in 3 dimensions **[39]**.

$$\varepsilon_{\mu\nu} = \frac{1}{2}\begin{pmatrix} H & -\sqrt{2}a_1 & -\sqrt{2}a_2 & H \\ -\sqrt{2}a_1 & 0 & 0 & -\sqrt{2}a_1 \\ -\sqrt{2}a_2 & 0 & 0 & -\sqrt{2}a_2 \\ H & -\sqrt{2}a_1 & -\sqrt{2}a_2 & H \end{pmatrix}$$

They also cite the existence of a torsion wave in **[12]**, which they call a J-dependent giratonic wave (a spin of space), in which the J function is related to the rotating nature of gyratons. **[39]**:

$$\varepsilon^{(i)(j)} = \frac{1}{2}\begin{pmatrix} 0 & 0 & \frac{\sqrt{2}J}{\rho A}sin\Phi \\ 0 & 0 & -\frac{\sqrt{2}J}{\rho A}cos\Phi \\ \frac{\sqrt{2}J}{\rho A}sin\Phi & -\frac{\sqrt{2}J}{\rho A}cos\Phi & H/A \end{pmatrix}$$

It should be noted that in this study **[39]**, the authors placed themselves in the framework of nonlinear plane gravitational waves, or frontal plane waves propagating parallel (P-P waves) **[40]**. Indeed, while this study is interested in the parallelism between crystalline plasticity, the implementation of which characterizes a plasticization of the medium by rearrangement of atoms (and therefore a non-linearity between deformations and stresses) **[34]**, and the modified nonlinear general relativity of Einstein Cartan associated with this theory **[34]** and **[41]** **[42]**, it makes sense to move away from the domain of traditional elastic waves to consider nonlinear plane waves called PP.

Note also that the result they obtain (expressions above) for a gravitational wave propagating in the direction z is related to polarizations A^{+} and $A^{\times}$ gravitational waves in classical general relativity via the expressions below of **[40]**:

$$H = A_{+}(u)(x^2 - y^2)$$

$$H = A_{\times}(u)xy$$

Recently, in **[43]** to **[46]**, the authors consider a shape memory of space (i.e., a certain residual plasticity of this medium).

The zero values in the above two strain tensors result from the fact that the authors focused on the geometric torsion part associated with the Burgers vector itself, which is associated with crystalline plasticity **[34]**, i.e. the second equation of the Einstein–Cartan theory that presents an analogy with this theory. Thus, as in **[38]** where complementary polarizations appear because of this geometric torsion in the components (zz), (zx), (zy) and (zt) in 4 dimensions, in **[39]** complementary deformations appear as a function of these same components and only for them. However, this publication **[39]** shows that, contrary to the classical equation of general relativity where the bridge between polarization and deformation is direct by expression $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$, this time, for the torsion component, the correspondence between the components of the polarization tensors and the 4 and 3 dimensional strain tensors is no longer direct of the form. The publication **[39]** explains mathematically how to make this transition.

In conclusion, now with the Einstein-Cartan approach, polarizations in the direction of propagation of gravitational waves appear on the one hand, and the sheets of space again present interactions between them, thus reconstituting a 3-dimensional space. A certain homogeneity and coherence of the space is thus regained. Finally, it should be noted that there are different forms of general relativity with torsion, as shown in Table 2 below, all of which have complementary polarizations.

Theory	+ (*)	× (*)	x (*)	y	Breathing (*)	Longitudinal (*)
General relatvity	Yes	Yes	No	No	No	No
GR in noncompactified 4/6D Minkowski	Yes	Yes	Yes	Yes	Yes	Yes
Einstein/Aether	Yes	Yes	Yes	Yes	Yes	Yes
5D Kaluza-Klein	Yes	Yes	Yes	Yes	Yes	No
Randall-Sundrum braneworls	Yes	Yes	No	No	No	No
Dvali-Gabadadze-Porrati braneword	Yes	Yes	Dep	Dep	Dep	Dep
Brans-Dicke Massive	Yes	Yes	No	No	Yes	Yes
Brans-Dicke Mass-less	Yes	Yes	No	No	Yes	No
F(R) Metric gravity	Yes	Yes	No	No	Yes	Yes
Bimetric Theory	Yes	Yes	Yes	Yes	Yes	Yes
Palatini Gravity	Yes	Yes	No	No	No	No
Scalar tensor theory	Yes	Yes	No	No	Yes	Yes
(*) See Figure 15 for the definition of polarization × + respiration and logitudinal typexy Dep : depend						

Table 2: Different theories of modified general relativity with associated polarisations **[47] [48]**

Finally, in studies (D. Baskaran and L. P. Grishchuk **[51]** and M.L. Ruggiero in 2022 **[52]**), aimed at studying the second order by analogy in gravitational magnetism, the authors have shown that complementary polarizations also appear in the direction of gravitational wave propagations. (See Figure 16).

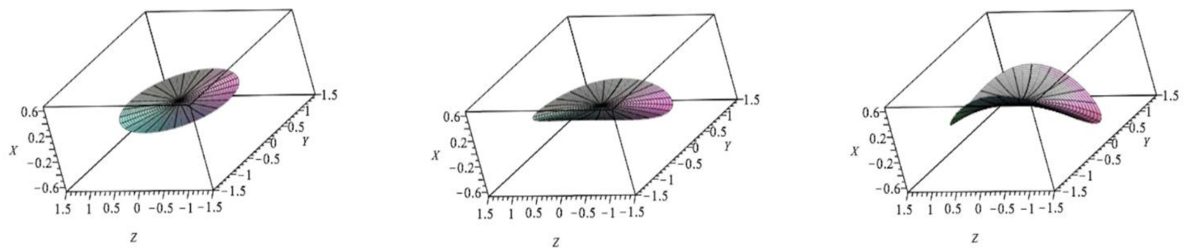


Figure 16: Temporal evolution of the test masses because of A+ polarization up to the second order **[52]**

Thus, several different approaches to modified general relativity all converge towards its complementary polarizations of space under gravitational waves, which is consistent with our model of transverse isotropic space, which requires deformations acting between sheets of space rather than a rigid space, consisting of isotropic sheets that are completely independent of each other.

To confirm that this cosmological crystal hypothesis is credible, we would like to quote the publication **[109]** "On Nanocones as Gravitational Analog Systems" published in 2025. Indeed, in this publication **[109]** the authors demonstrate that graphene exhibits mechanical behavior with noteworthy parallels to spacetime, estimating graphene's Young's modulus at 10^{12} Pa. They further propose that certain graphene structures may provide insight into the microstructure of spacetime itself, within the Teleparallel Equivalent of General Relativity (TEGR) framework originally suggested by Einstein in his attempts to unify electromagnetism with gravitation. The authors demonstrate that the coupling constant κ converges toward its value in general relativity, thereby illuminating both the possible microstructure of spacetime and the origin of its intrinsic stiffness. Their approach explicitly incorporates geometric torsion - which, as we showed in **[14]**, is essential for maintaining a coherent stacking of spacetime 'sheets. Their results are as follows: I quote:

"If the preceding hypothesis is accurate, as substantiated by these findings, and we conceptualize spacetime as a cellular structure comprised of minuscule particles, we can deduce that the force between these particles corresponds to the coupling constant. Expressed in SI units, this equation results in $c^4/16\pi G = 2.41596\times10^{42}$ N. The immense magnitude of this force indicates that spacetime possesses an extremely high resistance to deformation (about 10^{30} times greater than that of a one-square-meter graphene sheet). Consequently, gravity, which arises from the deformation of spacetime, is a remarkably feeble phenomenon. If the force of deformation is insufficient, meaning that smaller amounts of energy cannot cause considerable deformation of spacetime. This aligns with our understanding of gravity."

The authors therefore arrive at an order of magnitude of the rigidity of the graphene structure of the same order of magnitude than that of space-time...

Finally, in classical general relativity, formulated on Riemannian geometry, the Levi-Civita connection is symmetric in its lower indices. The consequence of this symmetry is that some antisymmetric contributions in the Riemann tensor cancel each other out, so that the dynamics of spacetime are entirely governed by the equilibrium of "geometric forces," i.e., curvature. Conversely, in the Einstein–Cartan–Sciama–Kibble (ECSK) theory, the affine connection contains an antisymmetric part, the torsion. This introduces additional terms into the generalized Riemann tensor that no longer cancel each other out, even in situations of global symmetry. In other words, geometric torsion keeps track of antisymmetric imbalances and explicitly encodes the spin contributions of matter.

The analogy with the strength of materials illustrates this distinction well. In a perfectly symmetrical system, such as a beam on two supports loaded in the middle, the solution by the equations of equilibrium of forces alone is sufficient: the internal moments compensate each other by symmetry. On the other hand, in an asymmetrical system, the equations of moments become indispensable to obtain support reactions and deformations. In the same way, torsion-free general relativity corresponds to a "symmetric" equilibrium where antisymmetric contributions disappear, while the addition of torsion, as in ECSK, corresponds to the explicit writing of the equations of moments in geometry: the antisymmetric terms of the torsion tensor complete the field of equations and prevent cancellations. It is then understood that geometric torsion plays a negligible role in perfectly symmetric systems, but becomes decisive in dissymmetric configurations, such as those encountered for example in binary systems of strongly unequal masses.

It is therefore understandable why two tensor field equations appear in ECSK. In addition, it would be interesting to analyze the coalescence library of black holes measured by LIGO/VIRGO/KAGRA for the past 10 years to see whether or not the gap between the predictions of general relativity in terms of the h(t) signal increases with asymmetric binary systems, a possible signature of a geometric torsional effect not taken into account in classical non-torsional general relativity.

At this point, it is interesting to note that we have concentrated under a rigid space-time behavior under the action of gravitational waves propagating at the speed of light. Now, the simple fact of observing in astronomy the motion of the planets around the sun shows that they move freely in a vacuum at a low-speed relative to the speed of light, and that at these small speeds space-time seems to behave like a fluid. So, let's see what the state of the art says about modeling space-time as a fluid. Thus, our elastic medium becomes anisotropic, transverse and extremely rigid when we move through it at the speed of light (this is the case with gravitational waves) and its texture transforms it into a fluid when we move at much lower speeds. The speed of light in our model is therefore intrinsically related to the structure (see texture by crystal analogy) of spacetime. Photons cannot travel faster than the speed of light in space because of its density ρ and Young's modulus in our elastic model. Recent studies show that with AI new materials have been created that have the same property of being rigid under fast dynamic actions and fluid under slow actions **[53]**. It is interesting to note that these new materials have structures where the crystalline atoms are replaced by complex polygonal shapes conducive to defects as seen above.

7. State of the art related to other flexible (fluid) models of space-time: Modeling in hydroacoustics, convergence with rigid models, and prediction of complementary polarizations in the direction of gravitational wave propagation

Summary

This chapter shows the state of the art in terms of modeling space-time as a fluid compared to the rigid model studied so far. This seems unavoidable given that the stars move freely within space-time by following geodesics. In addition, coalescence modeling in fluid mechanics shows that compression waves are possible, confirming the compression polarization discussed in the previous chapter.

In publications **[54]** and **[55]** the authors show that it is possible to construct a metric that presents analogies with that of general relativity. The assumptions are as follows (see **[54]**):

c is the speed of sound relative to the fluid,

v the velocity of the fluid in motion,

n the unit vector,

We can therefore write with respect to the laboratory that the sound ray propagates with respect to the fluid at a speed:

$$\frac{d\vec{x}}{dt} = c\vec{n} + \vec{v}$$

$$dx - vdt = cndt$$

Let us define $n^2 = 1$: normalization condition that defines a sound cone:

$$-c^2dt^2 + (dx - vdt)^2 = 0$$

Or by developing the expression below:

$$-c^2dt^2 + dx^2 - 2vdxdt + v^2dt^2 = 0$$

$$[\boldsymbol{v}^2 - \boldsymbol{c}^2]dt^2 + \mathbf{1}\, dx^2 - \mathbf{2}\boldsymbol{v}dxdt = 0$$

So, we have a metric of the following form with I the identity matrix:

$$\boldsymbol{g}_{\mu\nu(\boldsymbol{t},\boldsymbol{x})} = \frac{\rho}{\boldsymbol{c}}\begin{bmatrix} -[\boldsymbol{c}^2 - \boldsymbol{v}^2] & -\boldsymbol{v}^T \\ -\boldsymbol{v} & \boldsymbol{I} \end{bmatrix}$$

In this metric, the moving fluid represents space-time, and acoustic waves represent the speed of light. As an example, taken from the publication **[55]**, we take up the development of fluid dynamics equations which presents analogies with the expression of general relativity in weak fields. Let us consider a uniform medium of uniform density ρ_0 and uniform pressure p_0 solicited by no external action. The differential equations of fluid mechanics that apply to this medium are classically:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho v) = 0$$

$$\frac{\partial (v)}{\partial t} + v\nabla(v) = \frac{\nabla p}{\rho}$$

The following disturbances are introduced from this environment:

$$p(\vec{x},t) = p_0 + p'(\vec{x},t)$$

$$\rho(\vec{x},t) = \rho_0 + \rho'(\vec{x},t)$$

$$v(\vec{x},t) = v'(\vec{x},t)$$

The displacement of the fluid is assumed to be irrotational:

$$\nabla \times v = \nabla \times \mathrm{v}'$$

In the publication **[55]**, on the basis described above, the author arrives at a metric as stated above:

$$g_{\mu\nu}(\vec{x},t) = \frac{\rho}{c}\begin{bmatrix} -(c^2 - v'^2) & -v'^i \\ -v'^i & \delta_{ij} \end{bmatrix}$$

Note:

Sometimes v is denoted U.

The metric can also be broken down into 2 parts $g_{(0)\mu\nu}(\vec{x},t)$ and $g_{(1)\mu\nu}(\vec{x},t)$: either according to **[55]**:

$$g_{\mu\nu}(\vec{x},t) = g_{(0)\mu\nu} + g_{(1)\mu\nu}(\vec{x},t) = \frac{\rho_0}{c_0}\begin{bmatrix} -c_0^2\left(1 + \frac{(\gamma+1)}{2} \times \frac{\rho'_{(1)}(\vec{x},t)}{\rho_0}\right) & -v'^i_{(1)}(\vec{x},t) \\ -v'^i_{(1)}(\vec{x},t) & \delta_{ij}\left(1 + \frac{(3-\gamma)}{2} \times \frac{\rho'_{(1)}(\vec{x},t)}{\rho_0}\right) \end{bmatrix}$$

With in this expression:

$$p = K\rho^{\gamma}$$

γ is a specific heat coefficient,

K is a constant to have:

$$c_0^2 = \frac{p'_{(1)}(\vec{x},t)}{\rho'_{(1)}(\vec{x},t)} = \frac{\gamma p_0}{\rho_0}$$

With:

$$\frac{c_{(1)}(\vec{x},t)}{c_0} = \frac{(\gamma-1)}{2}\frac{\rho'_{(1)}(\vec{x},t)}{\rho_0}$$

$$\frac{c_{(1)}^2(\vec{x},t)}{c_0^2} = (\gamma-1)\frac{\rho'_{(1)}(\vec{x},t)}{\rho_0}$$

To make the comparison with the linearized metric in general relativity, the author proposes the same formalism:

$$\tilde{g}_{\mu\nu}(\vec{x},t) = \frac{c_0}{\rho_0} g_{\mu\nu}(\vec{x},t) = (\eta_A)_{\mu\nu} + h_{\mu\nu}(\vec{x},t)$$

$(\eta_A)_{\mu\nu}$ is the analogue of the Minkowski metric $(\mathrm{diag}[-c_0^2, +1, +1, +1])_{\mu\nu}$ and $h_{\mu\nu}$ is the linear perturbation of the medium acoustic metric described at the beginning of this chapter.

The author is interested in this example of an acoustic wave propagating as in true gravitational waves along the axis z.

As with gravitational waves, the equivalent of the Dalembertian in acoustics must be solved:

$$\Box_A h_{\mu\nu}(z,t) = 0$$

With:

$$\Box_A = -\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} + \nabla^2$$

The solution of this equation is of the form:

$$h_{\mu\nu}(z,t) = h_{\mu\nu}(z \pm ct)$$

The perturbation of the metric is a plane wave of the form:

$$h_{\mu\nu}(z,t) = e_{\mu\nu}\, e^{(i(kz-\omega t))}$$

Once all the calculations have been made, the author obtains for the perturbation of the metric:

$$h_{\mu\nu}(z,t) = \begin{bmatrix} -c_0^2\frac{(\gamma+1)}{2}\times\frac{\rho'_{(1)}(z,t)}{\rho_0} & 0 & 0 & -v'^3_{(1)}(z,t) \\ 0 & \frac{(3-\gamma)}{2}\times\frac{\rho'_{(1)}(z,t)}{\rho_0} & 0 & 0 \\ 0 & 0 & \frac{(3-\gamma)}{2}\times\frac{\rho'_{(1)}(z,t)}{\rho_0} & 0 \\ -v'^3_{(1)}(z,t) & 0 & 0 & \frac{(3-\gamma)}{2}\times\frac{\rho'_{(1)}(z,t)}{\rho_0} \end{bmatrix}$$

We notice that:

- There are two types of components:

$$h_{11} = h_{22} = h_{33} = -\frac{(3-\gamma)}{c_0^2(\gamma+1)}h_{00}$$

$$h_{30} = h_{03}$$

- Classical gravitational waves also have two independent non-trivial components. Acoustic analogy also has two non-trivial components:

$h_{00(z,t)}$ which is proportional to the linear disturbance of the density of the medium $\rho'_{(1)}(z,t)$.

$h_{03(z,t)}$ which is proportional to the linear disturbance of the velocity along the z-axis of the middle $v'_{(1)}(z,t)$.

- At the sign level, the analogue of the gravitational wave is the speed of sound.

When it is positive "+", it implies that the wave propagates in the negative direction z.

When negative, "-" implies that the wave propagates in the positive direction z.

Comment on this example

In this thesis, we study the analogy of the behavior of an elastic medium (solid or liquid in this chapter) with general relativity based on solid deformations measured by the LIGO/VIRGO interferometer.

We have shown following the state of the art and analyses of this thesis that $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$. Thus, we can read the perturbation of the metric tensor $h_{\mu\nu}$ as a strain tensor $\varepsilon_{\mu\nu}$. We therefore see in the framework of our analogy, that even if the acoustic waves are longitudinal (deformations in the direction of z-propagation of the plane wave) and homogeneous in all directions, the tensor has components not only diagonally (compression) but also in the corners of the matrix reconstituting a "spatial behavior".

This trend is also confirmed and amplified by the application of a rotation of an angle θ with respect to the axis y (see **[55]** results).

$$h' = \begin{bmatrix} -c_0^2 \frac{(\gamma+1)}{2} \frac{\rho'_{(1)}}{\rho_0} & v'^{3}_{(1)} sin\theta & 0 & -v'^{3}_{(1)} cos\theta \\ v'^{3}_{(1)} sin\theta & \frac{(3-\gamma)}{2} \frac{\rho'_{(1)}}{\rho_0} & 0 & 0 \\ 0 & 0 & \frac{(3-\gamma)}{2} \frac{\rho'_{(1)}}{\rho_0} & 0 \\ -v'^{3}_{(1)} cos\theta & 0 & 0 & \frac{(3-\gamma)}{2} \frac{\rho'_{(1)}}{\rho_0} \end{bmatrix}$$

Thus, the acoustic analogy in this example also complements the perturbation metric tensor. Just as the introduction of geometric torsion into general relativity or other unverified theories of modified general relativity completes the perturbation tensor of the metric with components following the direction of wave propagation, the acoustic analogy leads to the same result. The big difference in this analogy is that there are only longitudinal waves. No transverse waves as in gravitational waves due to the very nature of the fluid.

We will see in the next chapter that there are also acoustic analogies developing simultaneously transverse waves and longitudinal waves, in the case of acoustic binaries (viscosity).

The following approach is issued of **[56]**. We have shown that Einstein's linearized equation in weak fields is written:

$$\Box \bar{h}^{\mu\nu} = -\frac{16\pi G}{c^4} T^{\mu\nu}$$

Non-relativistic sources call for speeds much smaller than the speed of light. The 00 component of the stress energy tensor is:

$$T^{00} = \rho c^2$$

For this type of source, calibrating the above equation to the 00 component, Einstein's linear differential equations on the metric perturbation $h^{\mu\nu}$ boil down to h^{00} and T^{00} with the previous definition of $T^{00} = \rho c^2$:

$$\Box \bar{h}^{00} = -\frac{16\pi G}{c^4} T^{00} = -\frac{16\pi G}{c^2} \rho$$

This equation is a typical scalar wave equation in fluid acoustics. Indeed, if we compare the shape of the equation of the typical analog equation of the scalar wave with the relationship obtained in the previous chapter between the sound pressure p and the perturbation of the metric h_{00} on the one hand and with the equation of the sound pressure wave on the other hand:

$$\nabla^2 \bar{h}^{00} = -\frac{16\pi G}{{c_0}^2}\rho_0$$
$$\frac{{c_0}^2}{2}\nabla^2 \bar{h}^{00} = -8\pi G \rho_0$$

It can be shown that:

$$p = -\rho_0 \frac{d\phi}{dt} = \rho_0 \frac{c_0^2}{2} h_{00}$$

Or:

$$\frac{c_0^2}{2} h_{00} = \frac{p}{\rho_0} = v^2 = -\frac{d\phi}{dt}$$

With in this analogy, a mathematically defined potential acoustic scalar ϕ defined as follows:

$$\vec{v}^k = \frac{dx^k}{dt} = grad\vec{\phi}^k = \nabla \phi^k$$

With $\vec{v}^k$ the velocity vector of the particles in the moving fluid.

And in fluid mechanics, we have:

$$\nabla^2\left(\frac{p}{\rho_0}\right) - \frac{1}{c^2}\frac{\partial^2\left(\frac{p}{\rho_0}\right)}{\partial t^2} = 0$$

Replacing $\frac{p}{\rho_0}$ according to its expression of h_{00} we obtain:

$$\nabla^2\left(\frac{c_0^2}{2}h_{00}\right) - \frac{1}{c^2}\frac{\partial^2\left(\frac{c_0^2}{2}h_{00}\right)}{\partial t^2} = 0$$

Thus:

$$\frac{c_0^2}{2}\square^2\left(\bar{h}^{00}\right) = 0$$

In the incompressible acoustic field close to the mass source ρ , i.e. for $\frac{\omega r}{c_0} \ll 1$, where r is the distance to the source, the d'Alembertian reduces to the Laplacian:

$$\nabla^2 \bar{h}^{00} = -\frac{16\pi G}{c^2}\rho_0$$

Conclusion

The linearized equation of general relativity can be thought of as an acoustic wave equation in a medium that is a fluid in motion.

Based on the principles developed above, the author in **[56]** modeled the gravitational wave GW150914 and we give his results figure 17 below. Thus, space-time becomes a fluid in motion and the gravitational wave an acoustic wave propagating through it.

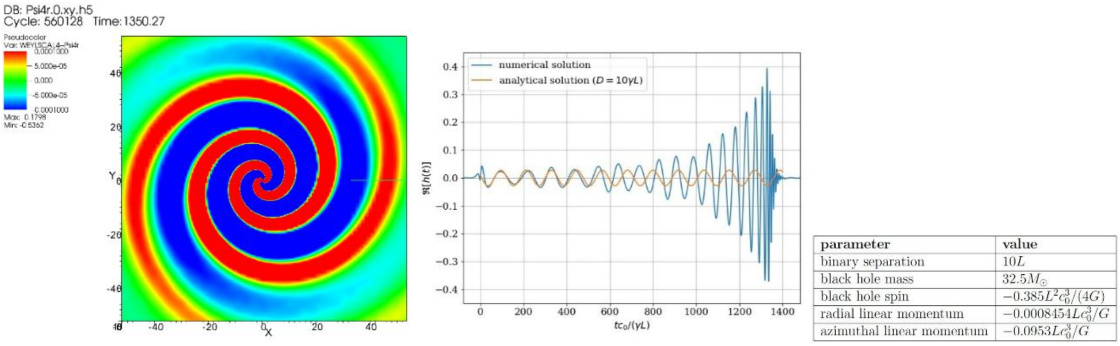


Figure 17: Gravitational wave modelling GW150914 source **[56]**

To clearly visualize the analogy of the mathematical formulations, we have reported in Table 3 below the formulations computed by **[56]** which are compared to the formulations of the classical theory of gravitational waves.

Parameter	**Classical theory of the gravitational wave [56]**	**Acoustic binary analogy in a moving fluid (aeroacoustic quadrupole) [56]**
Expression of the strain h generated by the binary on the medium (transverse staining)	$h_{ij}^{TT} = \bar{h}_{ij}^{TT} = \frac{2G}{Rc^4}\frac{d^2}{dt^2}I_{ij}^{TT}\left(t - \frac{R}{c}\right)$ Cross-cutting component	$h_{jk}^{TT} = \bar{h}_{jk}^{TT} = \frac{kG}{4\pi r {c_0}^4}\frac{d^2}{dt^2}I_{jk}^{TT}\left(t - \frac{r}{c_0}\right)$ Magnitude of metric perturbation see Formula 3.27 **[56]** with k=8π
Deformation in the longitudinal direction (direction of wave propagation)	Does not exist, we generally assume that space is an incompressible medium. No longitudinal component	If the compressible medium of density ρ_0 in a linear theory with a specific Newtonian gauge at the higher order of the compactness of the source, this implies that an additional compression wave is possible. $\lvert h_{00}^{lg}\rvert \frac{c_0}{\Omega_r} = \frac{2}{5}\left(\frac{\Omega D}{2c_0}\right)^2 \lvert \bar{h}_{jk}^{TT}\rvert$ Magnitude of metric disturbance see Formula 3.28 **[56]** With: D is the binary separation (distance between the two black holes/acoustic vortex), Ω is the angular frequency of their rotation (half the frequency of the waves generated, c_0 it is the celerity of the wave

Table 3: Comparison of formulations between the classical theory of gravitational waves and the analogy with the waves of an acoustic binary in a moving fluid

Remark

In the linear theory of compact and weak wave sources (of the first order), there is no longitudinal (compressive) gravitational wave. To possibly see a possible compression wave, it is necessary to consider a specific Newtonian gauge in a linear theory but at a higher order of the compactness of the source. This Newtonian gauge is possible because the acoustic observer is "classical," in the sense that he exists in Newtonian space and time, and not in relativistic space-time or acoustic space-time."

This correction of Newtonian gravity due to gravitational wave radiation corresponds to an acoustic near field that cannot be compressed in the vicinity of a compact sound source. This field decreases with distance from the source as $\frac{1}{r^2}$, rather than 1/r. The extension of this action of the source in the far field (to "infinity") corresponds to sound waves, but, again, only in the Newtonian gauge.

Thus, acoustic analogy does not exclude the existence of longitudinal gravitational waves either, for example for non-compact or strong sources, but only in a higher-order linear theory of the compactness of the source. and in the case of a specific Newtonian gauge.

In the case of a higher-order linear theory of source compactness, the longitudinal strain component is possible but very small compared to the transverse strains.

The author in **[56]** also gives the order of magnitude of each deformation (first transverse) (3.1 Formula 3.27 quadripolar aeroacoustic **[56]**), longitudinal second (Formula 3.28 **[56]**).

For the transverse perturbation components of the metric (equivalent gravitational wave) (j, k:≠ 0)

$$\left|h_{jk}^{TT}\right| \approx \left|\bar{h}_{jk}\right| \approx \left(\frac{\omega L}{c_0}\right)^2$$

For the longitudinal perturbation component of the metric (acoustics) (00):

$$|h_{00}| \approx \left|\frac{\vec{v}_{lg}}{c_0}\right| \approx \left(\frac{\omega L}{c_0}\right)^4$$

In these expressions:

c_0 is the speed of the acoustic wave,

L is D/2,

ω is a circular frequency.

We have therefore seen that by a solid or liquid medium approach, the modeling of space-time was possible and, in both cases, led to deformation waves. We have seen, in particular, in the case of a solid medium associated with gravitational waves, that space appears to be laminated and that it is necessary to resort to geometric torsion to reconstitute a medium in 3D, the fourth dimension corresponding in a way to a delayed arrival of deformations due to the texture of space-time which limits the propagation of deformations to a maximum that is the speed of light. But it remains to be verified whether this lamination is specific only to the response of space-time to the solicitations generated by gravitational waves or whether this laminated texture is more general and constitutes a kind of cosmological crystal. To do this, we must go back to creation, or sometime after the creation of space-time, more precisely 380,000 years after the big bang. Indeed, this is the first photo of it that we have and which, through the invalid variations in temperature measured by the Planck satellite, reflects the nature of its structure at that time, knowing that then the universe expanded and has only grown since that date. So, the question is: can we extract information about the texture of space-time from the cosmic microwave background? This texture is like a lamination as suggested by the analysis of gravitational waves.

This is what we will study in the next chapter.

8. Potential consequences of the transverse isotropic analogy on the possible texture of space-time:

Analogy between the X-ray diffractograms of crystals and the power spectra of the cosmic microwave background – hypothesis of plasticity, liquefaction and self-clogging of space-time – analogy with the behavior of certain lamellar clays

Summary

This chapter studies the layered structure of space and therefore of the cosmological crystal that results from our transverse isotropic analysis of gravitational waves and attempts to answer this question. Space appears to be laminated in the case of gravitational waves. But is this texture specific to this mode of solicitation or more general? Is anisotropy local or global? To do this, based on the observation that according to current knowledge, space and therefore the quantum vacuum has only swelled since the Big Bang, do we have an attempt that would allow us to measure this potential foliation of space on a large scale? To do this, in this chapter, we assume to consider the cosmic microwave background and the associated power spectra in temperature/energy and polarization as an inverted X-ray diffractogram of space (same angular abscissa and energy coordinate). It then appears that if this hypothesis is correct, then a certain foliation of the universe appears by repetition of the peaks seen in the power spectra of the cosmic microwave background on the one hand and the presence of the geometric torsion in the B-mode polarizations of this cosmic microwave background according to the current state of the art on the other hand. This chapter shows that Einstein's constant κ becomes a matrix and perhaps shows by its complexity the limits of analogy. A comparison by analogy with the behavior of large-scale transverse isotropic lamellar clays is also made showing similarities with the behavior of space on a large scale (self-clogging behind rotating black holes, local liquefaction, plasticity).

We have seen that our transverse isotropic rigid elastic model of spacetime requires the addition of geometric torsion to the Riemann tensor to reassemble the disjoint spacetime sheets as they appear in traditional general relativity applied to gravitational transverse waves. If this geometric twist is an intrinsic characteristic linked to the intimate structure of space-time, as it has only swelled since the big bang, we should find it or at least traces in the beginnings of its initial structure. It just so happens that we have an image of space-time 380,000 years after the big bang! the cosmic microwave background. In addition, it has been widely and precisely studied **[60]** to **[62]**. So, we must analyze the broad outlines of this cosmic microwave background to see whether or not we can distinguish geometric torsion in its internal structure. If this is the case, as this geometric twist has the theory of defects as an image in crystallography, then we should also find in this cosmic microwave background by mirror effect of our analogy, signs or properties similar to crystals, especially when they are observed and analyzed by X-ray diffractograms (Figure 8), thus informing us about the possible structure of space-time 380000 years after its birth. This is what we will summarize in this chapter. The study of this cosmic microwave background by the Planck satellite **[60]** to **[63]** has given rise to three main results.

- The first concerns the infinitesimal temperature variations as a function of the aperture of the angle of observation. This is the temperature power spectrum (see Figure 18). The curve obtained corresponds with great precision to the theoretical curve, thus validating the big bang model
- The second concerns the polarizations of photons as a function of deformations and therefore of the structure and nature of the original plasma 380000 years after the big bang according to two modes E and B (see Figure 19). Polarization spectra for each of these modes were also established. They also correspond with great precision to the theoretical spectra from the big bang theory.
- The third, the universe has almost zero curvature.

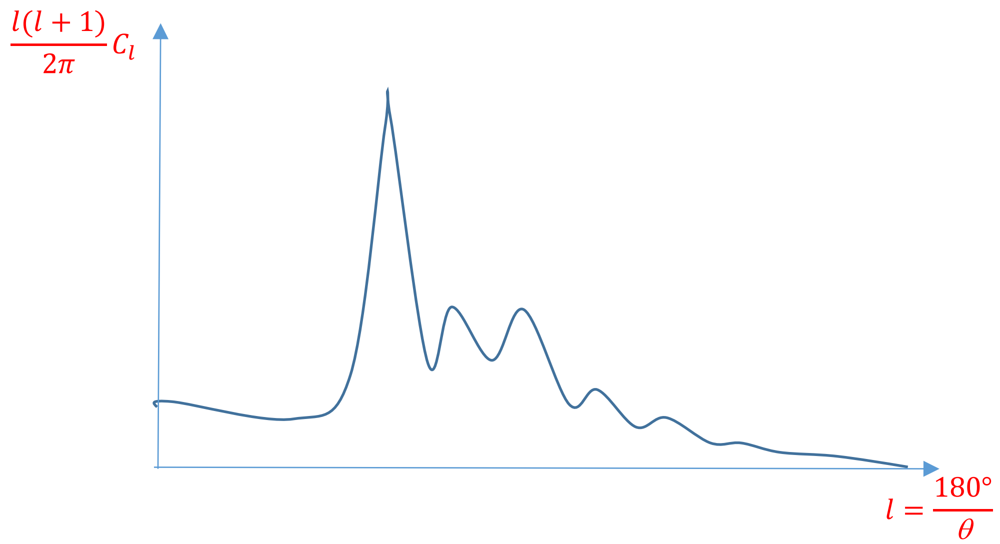


Figure 18: Cosmic microwave background power spectrum (wide angles left and small angles right) – $(\mu K)^2$ **[57] [58]**

$$C_\ell = \frac{1}{2\ell+1}\sum_{m=-\ell}^{m=+\ell}|a_{lm}|^2 = \langle a^*_{\ell m}a_{\ell m}\rangle = \langle |a_{lm}|^2\rangle_m$$

In the study of the Cosmic Microwave Background, the angular power spectrum C_ℓ quantifies the amount of temperature fluctuation on different angular scales across the sky. These fluctuations are decomposed using spherical harmonics, with coefficients $a_{\ell m}$representing the amplitude of each mode. Each multipole moment ℓcorresponds to a specific angular scale (higher ℓvalues probe smaller scales). The spectrum C_ℓ is defined as the average of the squared amplitudes $|a_{\ell m}|^2$ over all values of m, providing an isotropic statistical measure of the fluctuations. This spectrum is a cornerstone of modern cosmology, as it encodes key information about the geometry, composition, and evolution of the universe, including parameters related to dark matter, dark energy, and primordial inflation.

The expression of $a_{\ell m}$ for the integration of the solid angle over the whole sphere (4π) is written:

$$a_{\ell m} = \int_{4\pi} \frac{\Delta T_{(\theta,\varphi)}}{T} Y^*_{\ell m(\theta,\varphi)} d\Omega$$

And $\boldsymbol{Y^*_{\ell m}}_{(\theta,\varphi)}$ is deduced from the infinitesimal temperature variations T of the cosmic microwave background:

$$\frac{T_{(\theta,\varphi)} - T}{T} = \frac{\Delta T_{(\theta,\varphi)}}{T} = \sum_{l=0}^{m} \sum_{m=-l}^{l} a_{lm} Y_{lm(\theta,\varphi)} = \sum_{l=0}^{m} \sum_{m=-l}^{l} a_{lm} \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P^m_{l\,(cos\theta)} e^{\pm im\varphi}$$

And for polarizations, we have two modes:

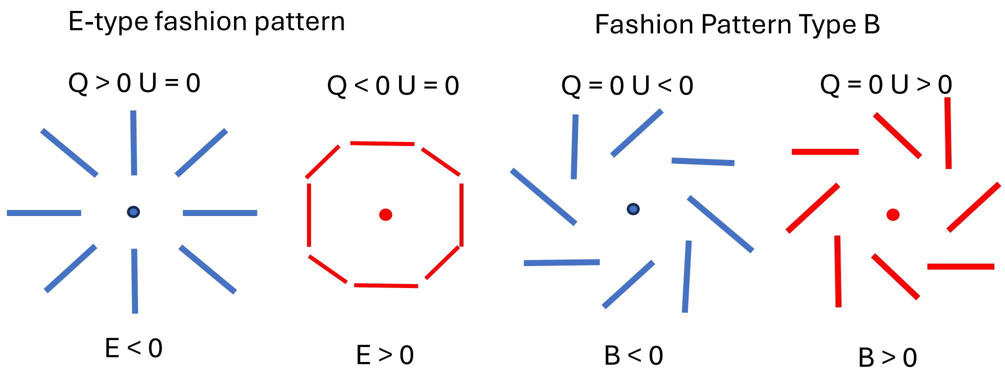


Figure 19: View of the different polarizations of the E and B modes of the source cosmic background **[57] [58]**

The link of the cosmic microwave background with the geometric torsion in the sense of Einstein-Cartan relativity is given in the publication **[59]**. Indeed, in the publication **[59]**, it is shown that the B mode of polarization of the cosmic microwave background is correlated with the torsion tensor shown in Einstein's Cartan theory. Thus, in **[59]** we can read, and I quote: "The coupling of torsion to electromagnetism in a gauge-invariant manner was first achieved by Novello **[64]**, De Sabbata and Gasperini **[65]** and Duncan, Kaloper and Olive **[66]**.

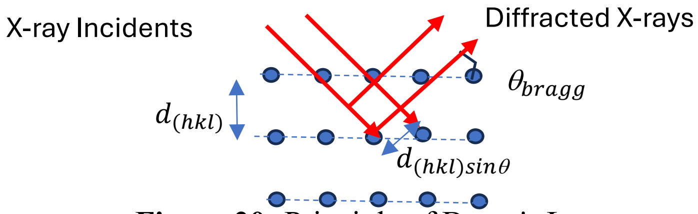


Figure 20: Principle of Bragg's Law

They pointed out that if the dual of the torsion tensor $T^{\mu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}T_{\nu\alpha\beta}$ is the divergence of a scalar interaction $T^{\mu} = \partial^{\mu}\phi T_{\mu}$, then it can be coupled to electromagnetic interactions in a gauge-invariant manner by $A_{\mu}\tilde{F}^{\mu\nu} = \phi F_{\mu\nu}\tilde{F}^{\mu\nu}$.

Our conclusion on the need to take into account the geometric torsion in the very structure of space-time in order to reconstitute the equivalent of a crystal in 3 dimensions, whereas general relativity, via classical polarizations A^{+} and $A^{\times}$ led to an elastic medium made up of sheets independent of each other, is therefore reinforced by this geometric torsion present in the structure of space-time in the modes of Polarization B of the emitted photons passing through this structure at the time of the Big Bang.

The analysis of the power spectrum in temperature can be seen as the equivalent of an X-ray diffractogram of a crystal. Indeed, the observation of a crystal with X-rays consists of bombarding it with X-rays which, depending on the paths taken by the rays, will either hit the atoms of the crystal's structure or pass through it without hindrance, drawing on the projection screen the very geometric structure of the crystal (Figure 20).

It is then possible to draw X-ray diffractograms as a function of the inclination angle 2θ of the rays and the intensity I of the spots on the screen. The number of peaks, their regular or irregular spacing, provides information on the leaf structure of the middle or not. The width of the peaks provides information on the smallness of the structures (Figure 21).

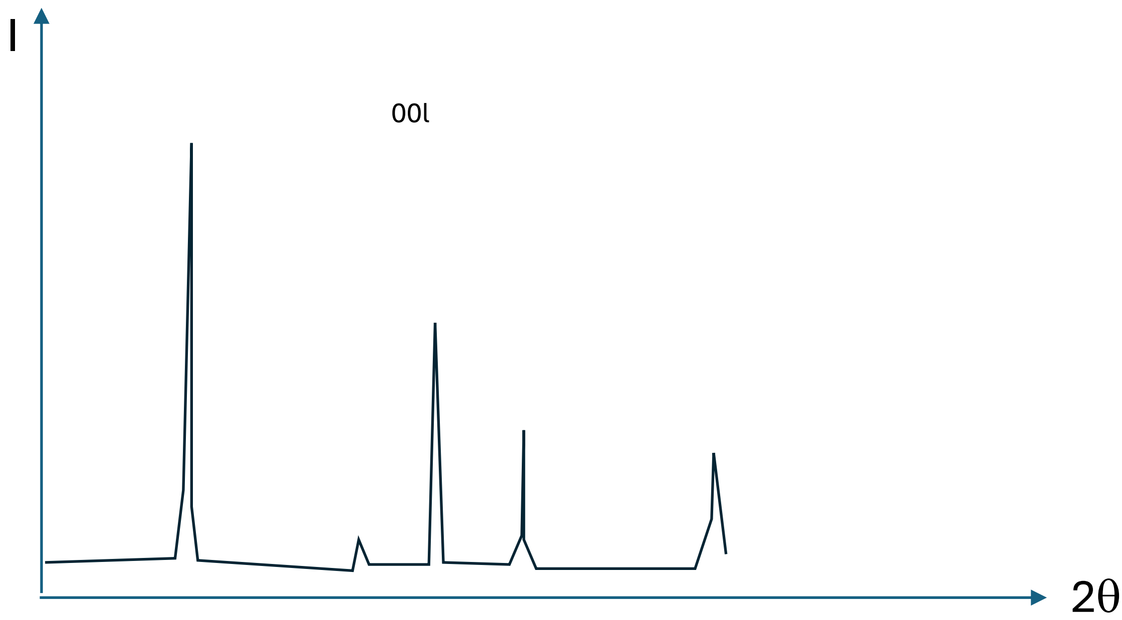


Figure 21: Example of a lamellar clay X-ray diffractogram **[67]**

In the case of the cosmic microwave background, the problem is in a way reversed. The light rays are created in the cosmological equivalent crystal **[49] [50] [14]** by the energy of the big bang (in X-rays they are produced outside the crystal) and leave it being influenced by the very structure of the universe 380000 years after the big bang (in this case the space-time filled with plasma distributed according to its structure and internal dynamics, the distribution of which depends on the structure of the space-time itself, the density of dark matter and baryonic matter etc).

Depending on these structures through which the photons pass, this creates temperature fluctuations (the counterpart of intensity I fluctuations in X-rays) which gives us an image of the structure of plasmas and their polarizations (how they rotate).

Depending on the angle of observation on the x-axis of the power spectra (T or energy, θ), we see different structures via the opening distribution of the different peaks as in the case of Bragg's law and the associated X diffractograms.

From these peaks, as a function of the angle of observation of the light rays, we deduce the internal structure of space-time at that time.

We can thus see the analogy between the two graphs as shown in tables 4 in temperature and 5 in polarizations below. The similarities of the analogy are clearly visible.

This approach is developed in **[112]** where a cosmological Bragg law is proposed.

<table>
<tr><th>The case of lamellar HDL clays under X-ray diffractogram</th><th>Case of the power spectrum of the cosmic background temperature</th></tr>
<tr><td>Harmonics associated with the direction of stacking of atomic planes **[67]**.

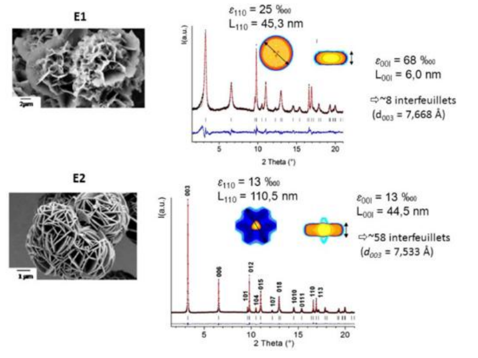
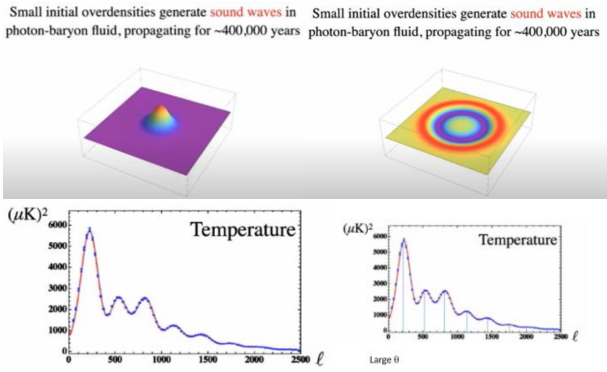
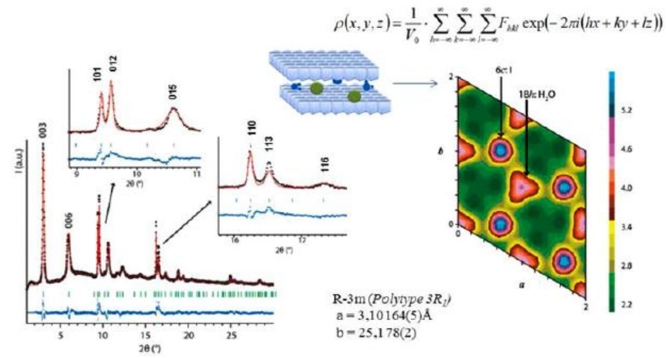
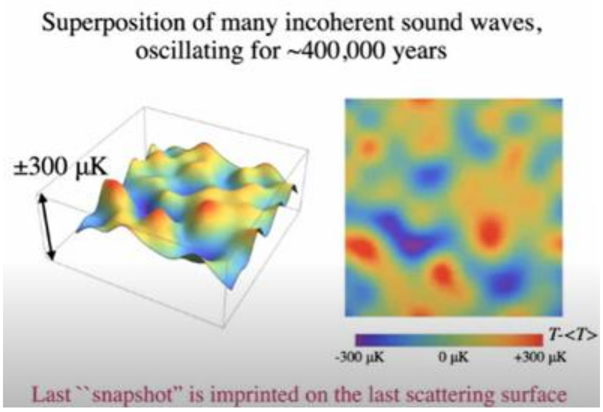



Figure 2. Étude de la microstructure des composés $[Ni_{1,5}Al_{0,5}(OH)_6][1/4\,CO_3^{2-}, 2H_2O]$ à partir de données de diffraction X sur poudre ($\lambda = 0{,}4368$ Å, Cristal -synchrotron SOLEIL) ; ajustement du profil global avec la fonction Thompson Cox-Hastings pseudo-Voigt modifiée (TCH-Z).</td><td>Harmonics associated with density variations in the original plasma **[68]**.
</td></tr>
<tr><td>Light Spot Intensity
</td><td>Intensity of the thermal variation visualized with a light point

Mechanism in the original plasma (the variation in velocity in the plasma generates polarizations and temperature variations

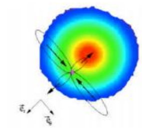



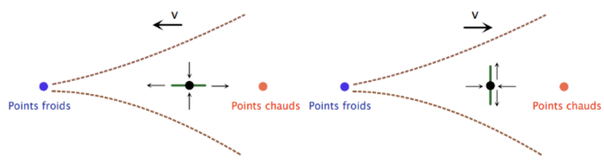
FIGURE 2.5.4: Dans une perturbation scalaire, le flux de photons observé par un électron présente une anisotropie quadrupolaire, ce qui génère de la polarisation linéaire des photons diffusés.
</td></tr>
</table>

Table 4: Analogy between measurements of thermal vibrations of atoms in the case of X-ray diffractogram and temperature variations at the time of photon release in the source cosmic diffuse field **[67], [68], [69]** Credit:EDP sciences : "Étude du mécanisme d'échange et de la structure des matériaux hydroxydes doubles lamellaires (HDL) par diffraction et diffusion des rayons X – 2013 - UVX2012 - 11e Colloque sur les Sources Cohérentes et Incohérentes UV, VUV et X ; Applications et Développements Récents - https://doi.org/10.1051/uvx/201301016 - C. Taviot-Guého and all. »

The analysis of the power spectrum of the cosmic microwave background (Table 4) from this angle reminds us of the X-ray diffractograms of lamellar clays given the regularity of the peaks. The peaks being narrow, this assumes a fine granular structure, as in the case of clays, all things being equal, of course, i.e. the size of the plasma clusters at the time of the big bang, which are very small compared to the size of the universe today. Here again we find the structure that emerged during the analysis of gravitational waves with and without geometric torsion.

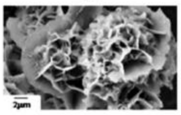
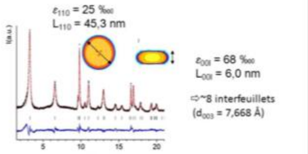
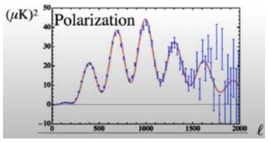
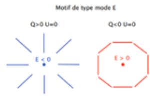
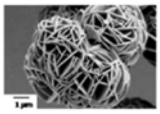
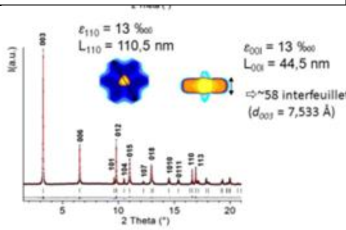
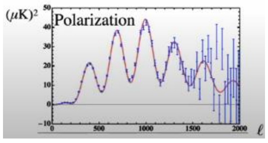
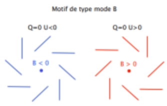
Crystal organization (HDL clay in bands)	X-ray diffractogram of the two clays in HDL sheets of types E1 and E2	Cosmic microwave background polarization power spectrum	Polarizations of the Cosmic Microwave Background
E1 	 2θ	 L	Spiral 
E2 	 2θ	 L	Tourbillon (twist) 

Table 5: Comparison of the analogy between the structures of HDL lamellar clays of type E1 and E2, the associated x-diffractograms, the polarization power spectrum of the cosmic microwave background, the geometries of the 2 associated polarizations E and B of the power spectrum associated with the source **[67]**, **[68]** and **[69]**

The polarization power spectra (Table 5) therefore again show similarities with the X-ray diffractograms of lamellar clays, as if the structure of space-time had this kind of structure 380000 years after the big bang. Note the screw-shaped defects specific to the effects of torsion that seem to be present in the molten plasmas at the time of the big bang.

At the end of this chapter, it seems that the geometric twist necessary to partially correct the anisotropy of the structure of space-time and its crystallography mirror is indeed found if we project ourselves into the past 380,000 years after the big bang in the very structure of the plasma, at least this is the message of the photons that passed through this structure and that have reached us today in the form of a background cosmological diffuse. The presence of this geometric torsion in the B-mode of these photons **[59]** is a striking link with all our previously established reflection, whether in the fluid-like nature of this plasma or rigid as the structure of space-time appears to us today. In the light of these results, it seems essential to us to modify general relativity by including geometric torsion, as was done, for example, in the Einstein-Cartan approach.

Finally, it is interesting to compare the properties of these lamellae clays with the microscopic properties and their macroscopic implications that are emerging for our space-time, at least from the point of view of what gravitational waves tell us, with and without geometric torsions considered:

- These clays are endowed with great plasticity and an ability to self-seal **[70]** in the event of tearing. When we look at what happens now of the coalescence of two black holes, when they rotate relative to each other, space closes immediately behind them, the tear does not stay in place in space-time exactly as in the case of these clays. This is shown by general relativity models of black hole coalescence modeled by finite elements (see official presentation of GW150914 on February 11, 2016)
- These clays, because of their lamellae nature, are subject to liquefaction phenomena under intense dynamic loads **[71]**. It is interesting to note that space-time, under the effect of the rotation of black holes at speeds approaching the speed of light during coalescence, liquefies in a way, thus allowing the formation of waves in it, which are in a way gravitational waves.
- These clays are made up of very fine grains that are sensitive to water and drought and are therefore in perpetual movement, swelling and compaction **[71]**. They are prone to creep (perhaps is the same for space-time? **[87]**). They have a very anisotropic structure due to their lamellae structure. When we look at their Poisson ratio, as shown in Table 6 below, we find variations ranging from 0.3 to 1 depending on the directions considered, and the same is true for their Young's moduli (Table 7). We have already shown in the case of the rigid space-time model that we have a transverse anisotropy with Poisson coefficients of 1 in the transverse planes and 0 or very small (if we consider the torsions in the direction of wave propagation. It remains to be seen whether these significant variations in Young's modulus are found depending on the types of stresses brought to space-time. The stresses to be considered are those associated with the different components of the perturbation tensor of the metric since, as we have shown, this tensor mirrors the deformation tensor of the elastic medium associated with space-time in our elastic and plastic analogy (case of torsion in connection with the theory of defects in crystallography) according to the mechanics of continuous media.

References	E_v (MPa)	E_h (MPa)	ν_{vh}	ν_{hh}	ν_{hv}	G_{hv} (MPa)
Experimental results	400	500	0.4	0.8	0.6- 1.0	-
Proposal for the anisotropic model	280	500	0.3	0.3	0.54	130
François et al (2012)	200	400	0.125	0.125	-	178
Bernier et al (2007)	300	300	0.125	0.125	-	-
Yu et al (2013)	700	1400	0.125	0.125	-	-

Table 6: Elastic parameters determined by local measurements (deformations and stresses) and reference values (used in previous studies with isotropic/anisotropic models) for Boom Clay **[70]**

Mechanical characteristics	E_v (MPa)	E_h (MPa)	ν_{vh}	ν_{hh} (MPa)	G_{hv} (MPa)
Clay Model Type ACC-2A	240	320	0.0625	0.15	100
	300	400	0.125	0.3	178
	360	480	0.25	0.45	250

Table 7: Mechanical characteristics of the ACC-2A clay (source parametric study Table 6.2 **[70]**

The different values are obviously associated with Hooke's law in a transverse anisotropic medium given below. Figure 22 shows the structure of space-time as seen by gravitational waves **[14]**.

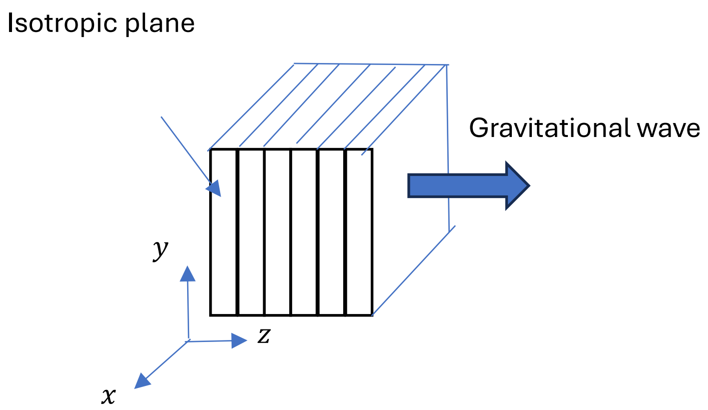


Figure 22: Convention and visualization of space slicks in the case of a gravitational wave

In the case of an anisotropic medium, Hooke's law is then written:

$$\begin{Bmatrix} \varepsilon_h = \varepsilon_{xx} \\ \varepsilon_h = \varepsilon_{yy} \\ \varepsilon_v = \varepsilon_{zz} \\ \varepsilon_{hv} = \varepsilon_{yz} \\ \varepsilon_{hv} = \varepsilon_{xz} \\ \varepsilon_{hh} = \varepsilon_{xy} \end{Bmatrix} = \begin{bmatrix} \begin{matrix} \frac{1}{E_h} & -\frac{\nu_{hh}}{E_h} & -\frac{\nu_{vh}}{E_v} \\ -\frac{\nu_{hh}}{E_h} & \frac{1}{E_h} & -\frac{\nu_{vh}}{E_v} \\ -\frac{\nu_{hv}}{E_h} & -\frac{\nu_{hv}}{E_h} & \frac{1}{E_v} \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} \frac{1}{2G_{hv}} & 0 & 0 \\ 0 & \frac{1}{2G_{hv}} & 0 \\ 0 & 0 & \frac{1+\nu_{hh}}{E_h} \end{matrix} \end{bmatrix} \begin{Bmatrix} \sigma_h = \sigma_{xx} \\ \sigma_h = \sigma_{yy} \\ \sigma_v = \sigma_{zz} \\ \sigma_{hv} = \sigma_{yz} \\ \sigma_{hv} = \sigma_{xz} \\ \sigma_{hh} = \sigma_{xy} \end{Bmatrix}$$

So, at this stage, we need to find a way to study the Young's moduli of space-time in the plane and perpendicular to the plane of stress in our elastic medium analogy to see if we find similarities with the behavior of these clays and therefore with their transverse anisotropic crystal structure and therefore by mirror effect with the geometric torsion included in the modified Einstein-Cartan general relativity. This is what we will see in the next chapter.

9. Development of new complementary rigid elastic models considering the anisotropy in the different deformation planes:

Reproduction of the various components of the space-time strain tensor by decoupling those in the plane (quantum thickness beam lattice for the h_{ij} components) and those perpendicular to the plane (membranes and elastic shells for h_{00} and h_{0i} and h_{i0}) via beam networks or quantum membranes. Use of Young's moduli in the plane and perpendicular to the plane as a variable for fitting results with space-time deformations measured for more than 100 years

Summary

In this chapter, space is modeled with models of the resistances of materials transposed to the cosmological level by considering quantum beams in beam lattices (finite element models of bars) with a quantum thickness for in-plane loads and in membranes and shells (case solutions of structures in continuum mechanics) for loads perpendicular to the plane, this of course in order to remain consistent with the anisotropy studied in the previous chapters. The Young's modulus is used as an adjustment variable by calibrating the deformations obtained in the two types of models with the values measured by 120 years of fine measurements in general relativity (gravitational waves, Newtonian gravitation, deformation by entrainment of the reference frame by Lense-Thirring effect). There are two families of Young's modulus values in the plane (10^{31} Pa, 10^{44} Pa) and out of the plane 10^{20} Pa.

In publications **[12]** to **[14]** and **[8]** we have highlighted the parallelism between Einstein's equations and that of beam mechanics in the form "Curvature² = K x Energy density U or E=RC² with R=1/K = Rigidity of the medium). Tables 8 and 9 highlight this analogy. We will therefore exploit it on examples associated with the components of the perturbation tensor of the metric associated with each plane of deformation of space and spacetime.

Solicitations	**Energy curvature formula**	**Equations to dimensions**
Bending Moment $M_{(x)}$	$\frac{1}{R^2}=\frac{2}{EI}\times\frac{U}{L}=\frac{2}{YI}\times\frac{U}{L}$	$\frac{1}{m^2}=\frac{s^2}{kgm^3}\times\frac{U}{m}$
Torsional Moment $T_{(x)}$	$\frac{1}{R_T^2}=\frac{2}{GI_t}\times\frac{U}{L}=\frac{2}{\mu I_t}\times\frac{U}{L}$	$\frac{1}{m^2}=\frac{s^2}{kgm^3}\times\frac{U}{m}$
Normal Exercise $N_{(x)}$	$\left(\frac{du}{dx}\right)^2=\frac{2}{ES}\times\frac{U}{L}=\frac{2}{YS}\times\frac{U}{L}$	$1=\frac{s^2}{kgm}\times\frac{U}{m}$
Shear Force $V_{(x)}$	$\left(\frac{dy}{dx}\right)^2=\frac{2}{GS_r}\times\frac{U}{L}=\frac{2}{\mu S_r}\times\frac{U}{L}$	$1=\frac{s^2}{kgm}\times\frac{U}{m}$
General relativity	$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}=-\frac{8\pi G}{c^4}T_{\mu\nu}$	$\frac{1}{m^2}=\frac{s^2}{kgm}\times\frac{U}{m^3}$

Table 8: Equations to dimensions for the different formulation curvature of space^2 = K x the energy density **[12] [14] [8]**

Note: in general relativity the curvature is already squared while in mechanics 1/R is squared

Curvature	= K	Energy Density
$\left(\frac{du}{dx}\right)^2$	$=\frac{2}{ES}$	$\frac{U}{L}$
$\left(\frac{dy}{dx}\right)^2$	$=\frac{2}{GS_r}$	$\frac{U}{L}$
$\frac{1}{R^2}$	$=\frac{2}{EI}$	$\frac{U}{L}$
$\frac{1}{z^2}\left[(\varepsilon_{xx})^2+(\varepsilon_{yy})^2+2(1-\nu)\frac{1}{4}\{\varepsilon_{xy}\}^2+2\nu\{\varepsilon_{xx}\varepsilon_{yy}\}\right]$	$=\frac{24(1-\nu^2)}{Eh^2}$	$\frac{dU}{dxdyh}$
$G_{\mu\nu}+\Lambda g_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}$	$=2.0766\ 10^{-43}$	$T_{\mu\nu}$

Table 9: Demonstration of the parallelism of the mechanical equations with the equation of general relativity

Regarding the distortions in the plane

We were interested in the coalescence events of black holes (GW150914) **[7]** and neutron stars (GW170817) **[72]** (component h_{ij} of the perturbation tensor of the spatial metric) as well as the entrainment effect of the Lense-Thirring frame of reference (component h_{0i} and h_{i0} of the perturbation tensor of the spatio-temporal metric).

Modelling of space deformations linked to the coalescence of black holes or neutron stars

The first case considered is that relating to the coalescence of two massive stars such as two black holes or two neutron stars as shown in Figure 23 below**.**

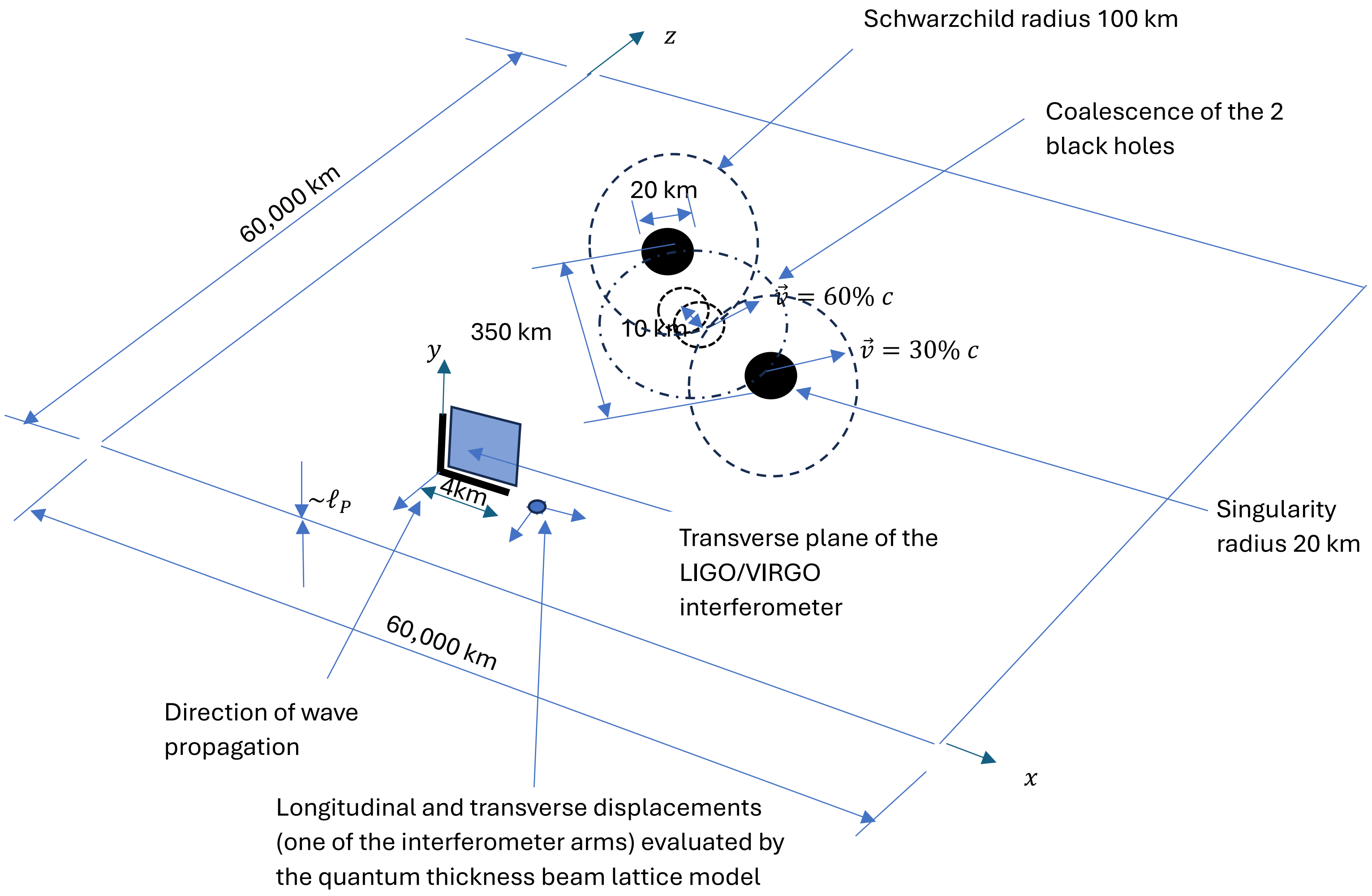


Figure 23: Presentation of the space sheet torsionally charged by the coalescence of two black holes at its center

A model of a planar lattice with bars working in tension/compression (see line in 1/ES in Tables 7 and 8 above) has been made. The calculation of the load of the lattice is given in Figure 24 and detailed below. The geometric and mechanical characteristics of the lattice as well as its full load and support reactions are shown in Figures 25 and 26. In order to find the deformations of space-time measured far from the point of application of the loads, it was necessary to consider Planck-thick bars and a Young's modulus which served as the adjustment value of the different models of Y = E= $10^{44} Pa$ (all calculations done after iterations). In order not to overload the model, only one plane was modeled since we know that the deformations are equal and opposite at an angle of 90°.

The modelling of the load consisted of replacing the dynamic behaviour of the two rotating stars into two equivalent static forces by freezing time (deformation instantaneous) and replacing each star with a force corresponding to its mass multiplied by its acceleration separated by a lever arm of length L_1 (Figure 26).

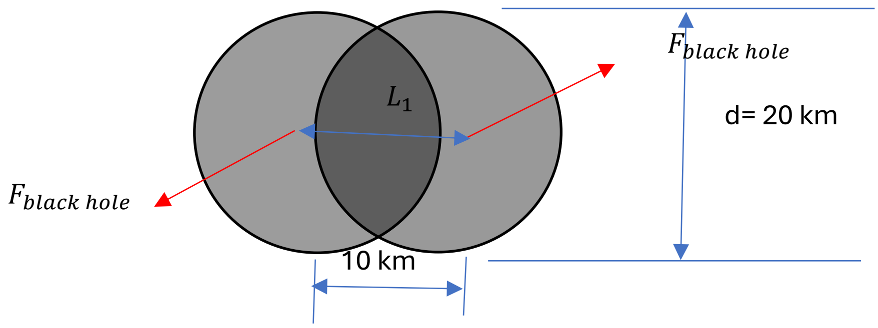


Figure 24: Coalescence of the two black holes replaced by two equal and opposite forces

Thus, we can calculate the acceleration a subjected to each black hole during coalescence, i.e. their velocity variation in fraction of the speed of light c during a coalescence time of about 0.2s according to **[7]**:

$$a = \frac{v_{coalescence} - v_{350km}}{\Delta t} = \frac{(0.6 - 0.3)c}{0.2} = 1.5 \times 299792458 m/s^2$$

According to **[7]**, the mass of the two black holes in the GW150914 event is 36 and 29 solar masses. The solar mass is $2{\times}10^{30}$ kg, so we can consider simplifying a mass of each black hole of m = $6.4{\times}10^{31}$ kg. And so, in the first simplification, the equivalent force applied to each black hole:

$$F_{Black\ hole} = m \times a = 6.4 \times 10^{31} \times 1.5 \times 299792458 = 2.88 \times 10^{40} N$$

The current radius of a stellar black hole (mass between 2 and 150 solar masses) is 30 km. So, for 36 solar masses, we take 20 km. Thus, at mid-coalescence, the distance between the centroid of the two black holes can be taken at 10 km as shown in Figure 24. The torsional torque is related to the coalescence of the two black holes with respect to the center of mass (see figure 24):

$$T = 2F_{Black\ hole} \times \frac{L_1}{2} = 2 \times 2.88 \times 10^{40} \times \frac{(\frac{10}{2}) \times 1000}{1000000} = 2.88 \times 10^{38} MN.m$$

We can now recalculate the torque portion for the center sheet of the thickness of the Planck length. To do this, we need to calculate the number n of thin Planck sheets over the diameter of the black hole of 20km:

$$n = \frac{d_{Black\ hole}}{l_p} = \frac{20000}{10^{-35}} = 2 \times 10^{39}\ concerned\ sheets$$

Thus, the torque applied per Planck sheet is as follows:

$$T_{Planck\ sheet} = \frac{T}{n} = \frac{2.88 \times 10^{38}}{2 \times 10^{39}} = 1.44 \times 10^{-1} \frac{MN.m}{Planck\ sheet}$$

We calculate the load $F_{Planck\ sheet}$ of the elastic model (2 elements of r = 6000 km see figure 25) which gives the same torque:

$$F_{Planck\ sheet} = \frac{T_{Planck\ sheet}}{4r} = \frac{1.44 \times 10^{-1}}{4 \times 6000000} = 6.0 \times 10^{-9} MN$$

It is therefore a model where applied forces seek to recreate a torsional moment applied to a sheet of space by the laws of material strength and to extract the associated deformations. These deformations are then compared with those predicted and measured in general relativity in low field.

To respect the symmetry of the problem, the torsional moment generated in the plane of the lattice by the two rotating stars has been replaced by 4 forces distant from the center of rotation by a distance corresponding to a few moments before the merger of the two black holes, since it is at this moment that gravitational waves are formed. The astrophysical data used come from publications relating to these two gravitational waves GW150914 **[7]** and GW170817 **[72]**. The lattice was taken of large size (mesh of 6000 kmx6000 km on a surface of 60000 x60000 km) to let the local deformations propagate over the entire lattice and avoid the local effects of modelling that could parasitize the results. The CTICM finite element software STRESS was used.

Figure 27 gives the measured displacements of the different nodes of the lattice, the ΔLs associated by 4 fictitious reference forces of 10000 MN are extracted to obtain readable orders of magnitude and to check if the symmetries of the model are indeed found in the results. We then recalibrated the results in displacements obtained according to the exact calculation of charges applied by the black holes defined above.

For a force F of 10000 MN, the displacement of each element is constant of 1.66 m (see Figure 27) and the deformation h_F is as follows:

$$h_{F=10000MN} = \frac{\Delta L}{r} = \frac{1.66}{6000000} = 2.766 \times 10^{-7}$$

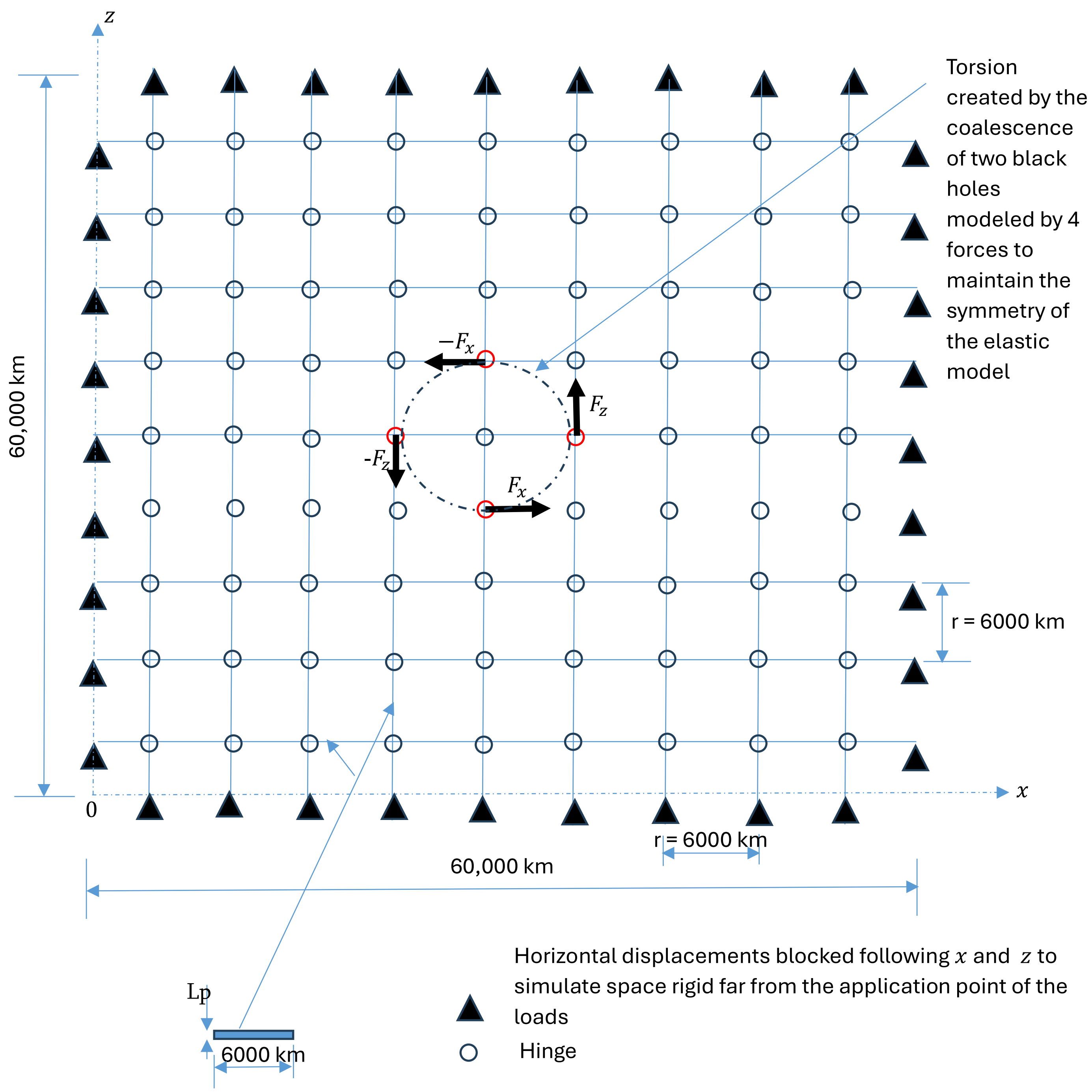
Thus, for the actual deformation in the Planck sheet is longitudinally, for exampleh_F node 44:

$$h_{F_{Planck\,sheet}} = h_{F(for\ F=10000MN)} \times \frac{F_{Planck\ sheet}}{F} = 2.766 \times 10^{-7} \times \frac{6.0 \times 10^{-9}}{10000} = \frac{\Delta L}{L}$$
$$= 1.66 \times 10^{-19}$$

For the transverse nodes 47 49 69 71:

$$h_{F_{Planck\,sheet\,node\,47}} = h_{F(for\ F=10000MN)} \times \frac{F_{Planck\ sheet}}{F} = \frac{6.66}{6000000} \times \frac{6.0 \times 10^{-9}}{10000} = \frac{\Delta L}{L}$$
$$= 6.66 \times 10^{-19}$$

As a reminder, LIGO measured for the event GW150914 a deformation h of $\mathbf{1.0 \times 10^{-21}}$. We are therefore not so far from the order of magnitude of the deformation measured by interferometers with our simplified lattice model. The same model tested on the coalescence of neutron stars GW170817 **[72]** reproduces the 10^{-20} of measured deformation.



$$F_x = -10000MN$$

$$F_z = -10000MN$$

$$A_x = A_z = \ell_p \times r = 10^{-35} \times 6000000$$
$$= 6.0 \times 10^{-29} m^2$$

Y = $3.0 \times 10^{38} MPa$ $(3{,}0 \times 10^{44} Pa)$

Figure 25: Planck plane space model of thickness modeled by a lattice of bars working in compression/tension

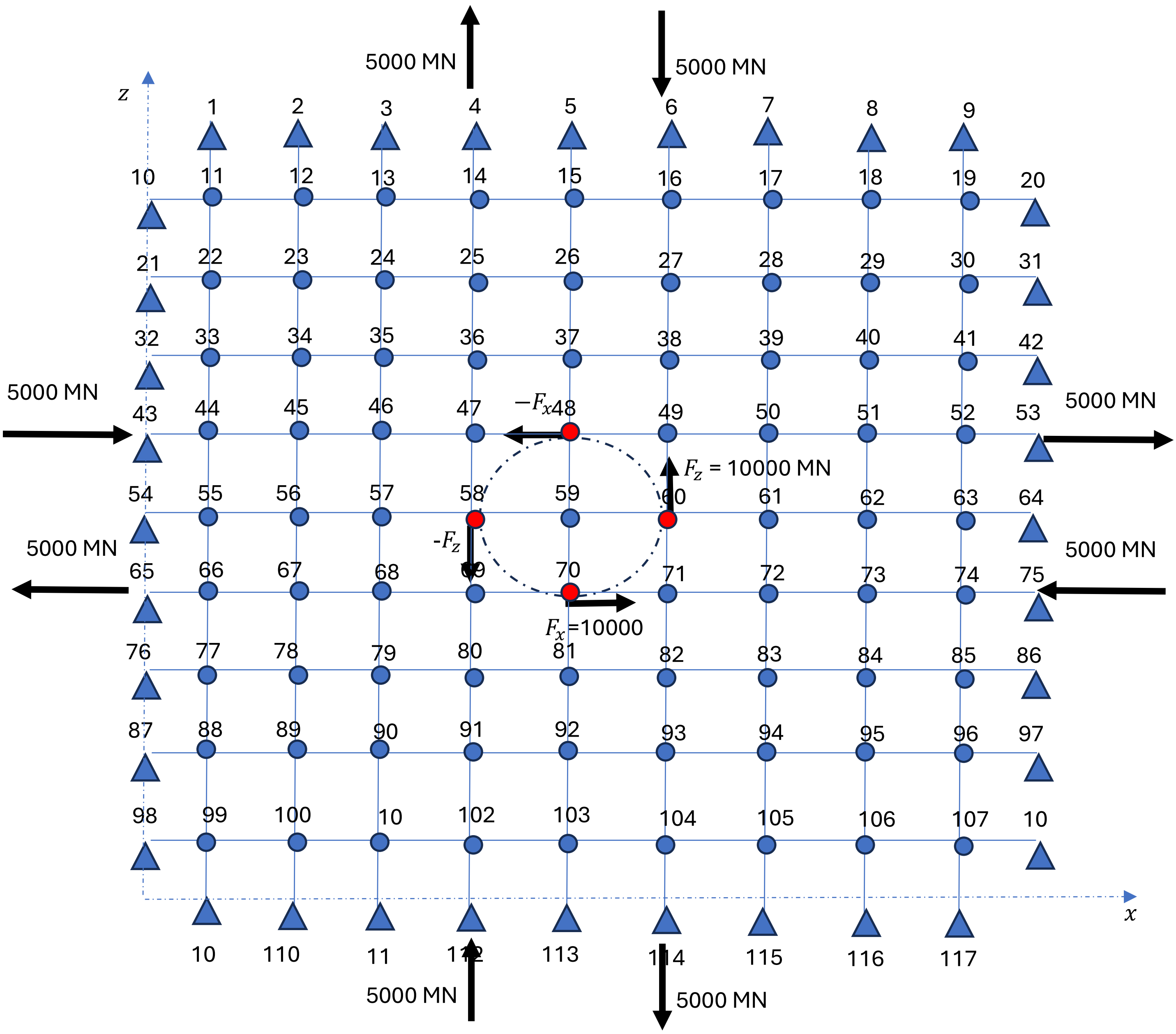


Figure 26: Model loading and support reaction

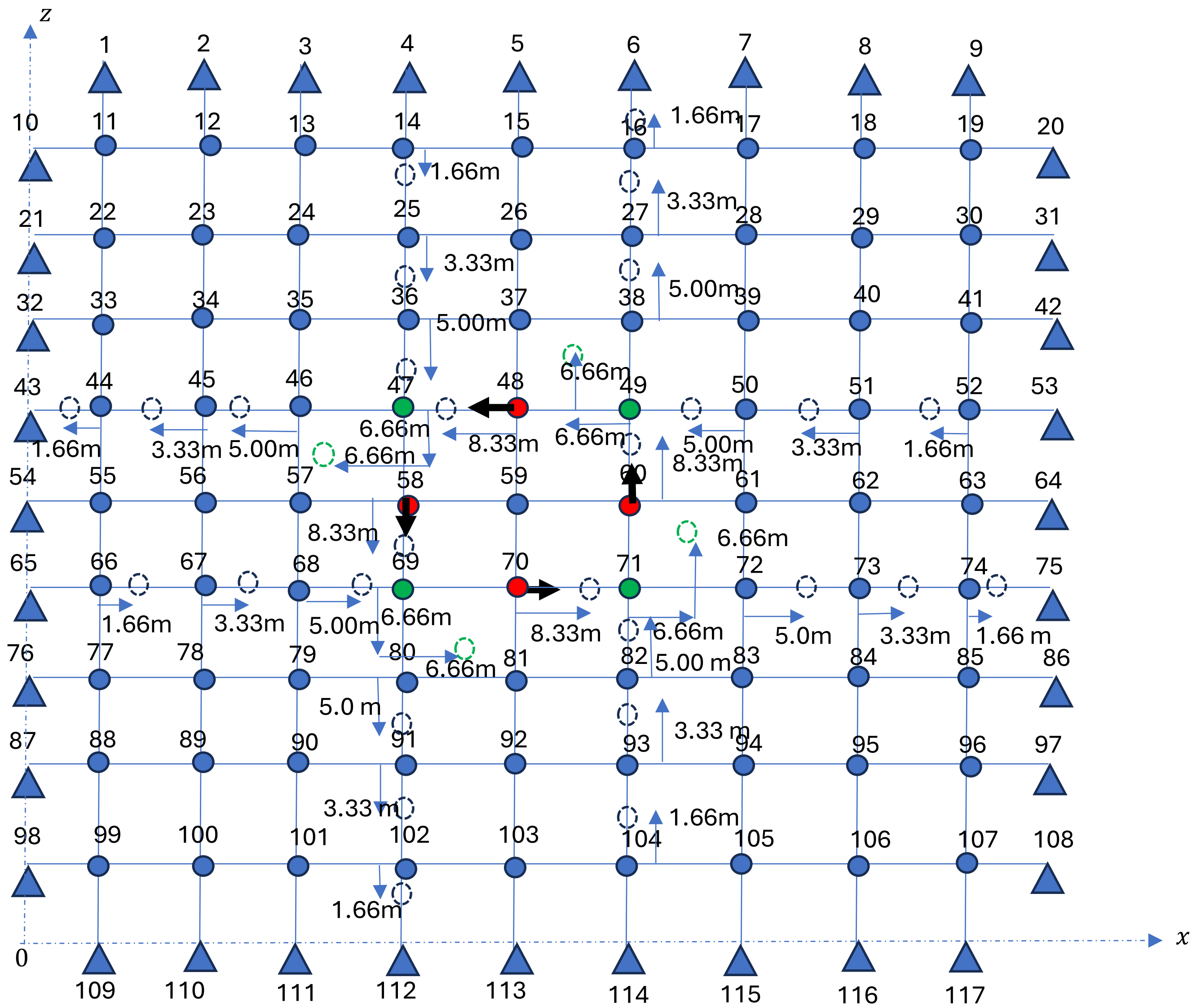


Figure 27: Displacements of the different nodes of the lattice

Modelling of space deformation related to the Lense-Thirring effect around the Earth (entrainment effect part of the reference frame)

The case studied is defined in Figure 28 below. It corresponds to the torsional drive of the space-time sheet by the rotation of the Earth. Effect called Lense-Thirring frame of reference training **[6]**. The measurements are from the Gravity PROBE B experiment **[9]**.

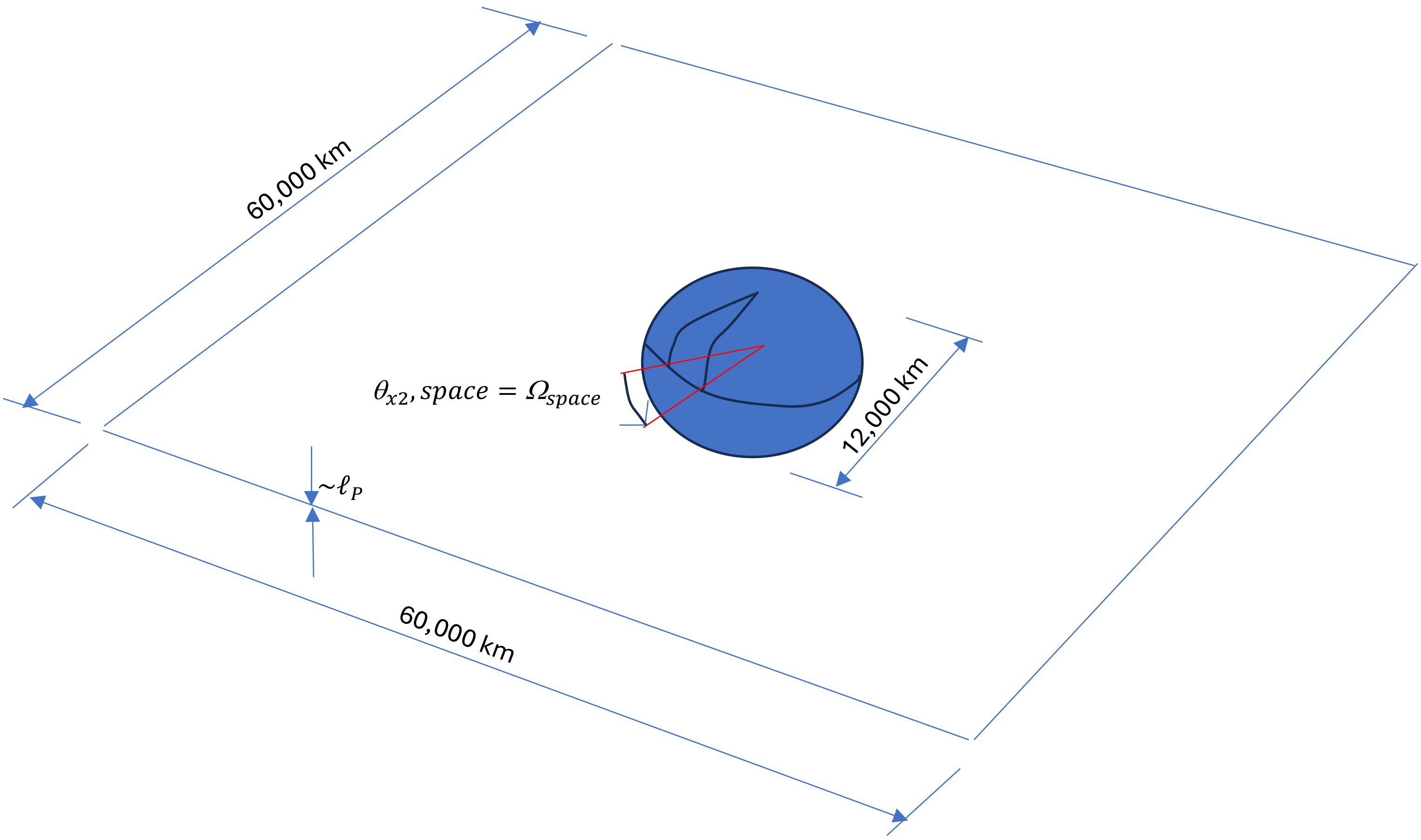


Figure 28: Study of the Lense-Thirring effect (frame of reference entrainment effect) applied to space by the rotation of the Earth

The same lattice model as previously described was used. This time, based on the angular deviation measurements made by the PROBE B satellite, we estimated the part of rotation linked to spatial effects and the part of rotation linked to temporal effects from the publication **[73]** to **[75]**. They are around 50% each. From this rotation imposed on space by the rotating Earth, we have sought the torsional torque which reproduces this rotation by the strength of the materials and have transformed it into equivalent forces which we have applied to the center of the lattice. The torsional torque T applied by the Earth to space is equivalent to **[76]** fig. 252:

$$\mathrm{T} = 2\frac{\pi G d^4 \theta_{x2,\mathrm{space}}}{32\mathrm{L}}$$

Numerical application:

$G = \frac{E}{2(1+\nu)} = \frac{3\times10^{38}}{2(1+1)} = 7.5 \times 10^{37} MPa$; $\theta_{x2,space} = \Omega_{space} = 4 \times 10^{-15} rad/s$

L = 6371000 m for the radius of the Earth ; D = 1274200 m for the diameter of the Earth

$$\mathrm{T} = 2\frac{\pi \times 7.5 \times 10^{37}(12742000)^4 \times 4 \times 10^{-15}}{32 \times 6371000} = 2.44 \times 10^{44} MN.m/s$$

This gives for each of the four applied forces:

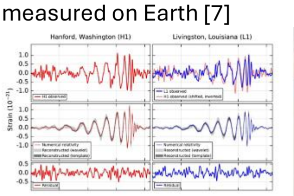
$$F_{x,real} = \frac{T}{4r} = \frac{2.44 \times 10^{44}}{4 \times 6000000} = 1.02 \times 10^{37} MN/s$$

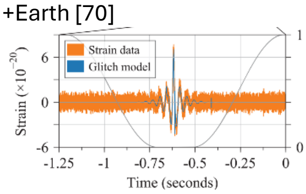
The precision of our model reproduces quite well the spatial angle part of the deformation of space by radiating since we obtain for the spatial part ***1.79×10⁻¹⁵ rad/s*** , and the gravity experiment PROBE B a given for the spatial part ***4×10⁻¹⁵ rad/s*** (estimate from ***[75])*** and for the full space-time Ω = ***6.04 ×10⁻¹⁵ rad/s (space-time).***

The result is that the model works if again the Young's modulus is of the order of $3 \times 10^{38} MPa$ in association with beams of quantum thickness. So, we find an order of magnitude compatible with that obtained for the first model of black hole coalescence.

We also modeled a rotating cylinder of space to re-verify the orders of magnitude obtained above.

Table 10 below summarizes the results obtained (Young's moduli) for different scenarios of deformations resulting from loading in the plane of space.

<table>
<tr><th>Case study of general relativity GW/Lense-thirring</th><th>Type of parameter measured or calculated according to general relativity</th><th>Type of parameter measured or calculated according to general relativity</th><th>Measured results GW => LIGO/VIRGO Lense-Thirring => G PROBE B</th><th>Measured results GW => LIGO/VIRGO Lens-Thirring => G PROBE B</th><th>Young's modulus used for computation (Pa) (spatial aspect)</th></tr>
<tr><td>GW150914 (Coalescence of 2 black holes) (Weak gravitational field)</td><td>Elongations and shortening of the transverse strain h measured on Earth [7]

</td><td>$\pm 10^{-21}$</td><td>$\pm 1 \times 10^{-21}$</td><td>Transversal h
$\pm 1.33 \times 10^{-19}$(1)
Longitudinal h
$\pm 3.32 \times 10^{-20}$ (1)</td><td>3×10^{44}

2×10^{31}</td></tr>
<tr><td>Elongation and shortening of the transverse strain h measured on +Earth</td><td>Elongations and shortening of the transverse strain h measured on +Earth [70]

</td><td>$\pm 10^{-20}$</td><td>$\pm 8 \times 10^{-20}$</td><td>Transversal h
$\pm 3.799 \times 10^{-20}$ (1)
Longitudinal h
$\pm 9.47 \times 10^{-21}$ (1)
$\pm 1.508 \times 10^{-23}$(2)</td><td>$3 \times 10^{44}$

2×10^{31}</td></tr>
<tr><td>Earth-Created Frame of Reference Shift Over Space-Time (Weak gravitational field)</td><td>Horizontal Distortion Angle Ω measured on Earth at r = 6700 km

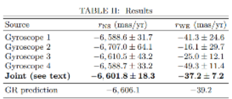
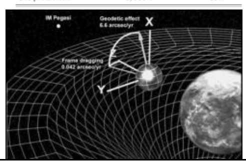



TABLE II: Results
<table><tr><th>Source</th><th>r_{NS} (mas/yr)</th><th>r_{WE} (mas/yr)</th></tr><tr><td>Gyroscope 1</td><td>$-6,588.6 \pm 31.7$</td><td>-41.3 ± 24.6</td></tr><tr><td>Gyroscope 2</td><td>$-6,707.0 \pm 64.1$</td><td>-16.1 ± 29.7</td></tr><tr><td>Gyroscope 3</td><td>$-6,610.5 \pm 43.2$</td><td>-25.0 ± 12.1</td></tr><tr><td>Gyroscope 4</td><td>$-6,588.7 \pm 33.2$</td><td>-49.3 ± 11.4</td></tr><tr><td>Joint (see text)</td><td>$\mathbf{-6,601.8 \pm 18.3}$</td><td>$\mathbf{-37.2 \pm 7.2}$</td></tr><tr><td>GR prediction</td><td>$-6,606.1$</td><td>-39.2</td></tr></table>

</td><td>39.2 milliarc-seconds/year</td><td>37.2 milliarc-seconds/year (space-time)
25.8 milliarc-seconds/year (space estimate only)</td><td>-

Ω = 11.55 milliarc seconds/year (1)

Model (3)</td><td>3×10^{44}

4.73×10^{38}</td></tr>
<tr><td>Frame of reference driven by a neutron star 12 km in diameter, 1.0 solar masses, T = 8s (strong gravitational field)</td><td>Theoretical horizontal distortion angle Ω calculated near neutron stars at r=6 km
[77] NASA's Black Hole mathematics</td><td>3.65 x 10-8 °/s (theoretical)
4143830.4 milliarc second/year (100000 times Ω Earth PROBE B)</td><td>Not applicable</td><td>Theoretical Model (3)</td><td>7.68×10^{44}</td></tr>
<tr><td colspan="6">1. With finite element model 1: 60,000 km x 60,000 km, torsional Planck sheet in its center)
2. With finite element model 2: 3,000,000 km x 3,000,000 km, torsional Planck sheet at its center)
3. Cylinder tangent to neutron stars in torsion in elastic midspace at an imposed angle Ω</td></tr>
</table>

Table 10: Summary of the results obtained by the elastic model of the thin Planck sheet loaded in its plane compared to the measurements made in the domain of classical unmodified general relativity

Concerning deformations perpendicular to the plane

It can be shown that in a weak gravitational field, linearized general relativity is reduced for the component 00 or tt to a Poisson's equation of the following form:

$$\Delta h_{00} = \frac{2}{c^2}\Delta\phi = \frac{8\pi G}{c^4}\rho c^2$$

With the gravitational potential: $\phi = \frac{GM}{R}$, indeed, by multiplying the two sides of the equation by $\frac{c^2}{2}$ we get: $\frac{c^2}{2}\Delta h_{00} = \Delta\phi = 4\pi G\rho$

In **[79]** to **[81]** the author also models space-time as a membrane. Some authors have recently shown, with experiments with supporting measurements, that the elastic membrane model for simulating gravitation is a model that works quite well **[30]**.

In publication **[110]**, the authors demonstrate how a wheeled robot moving on a deformable membrane can accurately replicate the dynamics of curved spacetime. By adjusting its velocity in response to the local curvature of the surface, this active system enables precise mapping of radial and orbital trajectories, analogous to those described by general relativity. The study reveals that such active particles do not necessarily follow geodesics in physical space, but rather in a programmable fiducial spacetime, where parameters such as membrane elasticity and instantaneous velocity shape the metric. This framework provides a simple and accessible robophysical model for simulating relativistic effects—such as those near a black hole—and opens new perspectives for robotic exploration in complex terrains and for understanding the dynamics of active matter.

We can then compare this equation of general relativity in a weak field with that of an elastic membrane:

$$\Delta w = \frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2} = \frac{g\mu}{T} = \frac{1}{R}$$

With:

$w(x,y)$ The vertical displacement of the membrane in m,

T = N/L The force per meter along the membrane in Newton/m,

μ The mass per square meter of the membrane in kg/m²,

$g = \frac{GM}{r^2}$Acceleration g: in m/s².

For a membrane of length L replacing T and μ with the following expression:

$$\sigma = \varepsilon E = \frac{N/L}{S/L} = \frac{T}{S/L} = \varepsilon E = \frac{\Delta L}{L} \quad ; \quad \mu = \frac{M}{L^2}$$

Transferring the different formulas above into the equation of membrane N we obtain:

$$\Delta w = \frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2} = \frac{g\frac{M}{L^2}}{ES\frac{\Delta L}{L^2}} = \frac{1}{R}$$

$$\frac{1}{L}\Delta w = \frac{1}{L}\left[\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right] = \frac{1}{ES} \times \frac{(gM) \times L}{\Delta L \times L^2}$$

Curvature ^{2}= $(1/R)^2$ = flexibility (1/ES=1/N) x energy density N.m/m^3

In analogy with:

$$\Delta h_{00} = \frac{2}{c^2}\Delta\phi = \frac{8\pi G}{c^4}\rho c^2$$

Curvature =Δ = flexibility (1/ES=1/N=>$\frac{8\pi G}{c^4}$) x energy density N.m/m^3

Note the parallelism with the bending of a bending beam:

$$\begin{cases} \dfrac{d^2 y(x)}{dx^2} = \dfrac{M}{EI} = \dfrac{1}{R} \\ U = \dfrac{1}{2}\displaystyle\int_0^L \dfrac{M^2}{EI} dx \end{cases} \rightarrow \frac{1}{R^2} = \frac{1}{EI}\frac{U}{L}$$

Thus, phenomena of gravitation on Earth or of the deformation or curvature of the web of space-time due to the presence of the Earth or the Sun can be modeled by a membrane charged perpendicular to its plane.

Application of the membrane approach to Earth

We can show from **[78]** that the perturbation of the metric is equivalent to a variation in the radius of curvature of the space-time web of:

$$h_{00} \approx 2\frac{GM}{rc^2} = \frac{2\phi}{c^2} = 1.392215637 \times 10^{-9}$$

$$\Delta R = \frac{1}{6}R_S = \frac{h_{00}Rc^2}{6c^2} = \frac{h_{00}R}{6} = 0.001478m$$

$$\frac{\Delta R}{R} = \varepsilon = \frac{h_{00}}{6} = 2.320359395 \times 10^{-10}$$

$$u_r = \Delta L = 2\pi R\frac{h_{00}}{6} = \pi\frac{h_{00}}{3}R = 0.009288m$$

We can therefore consider that this variation in radius corresponds to the variation in deflection of an equivalent membrane as shown in Figure 29 below.

The number of leaves is evaluated from the volume of the Earth as part of the equivalent surface cylinder πr^2that surrounds the Earth at a height of $\frac{4r}{3}$: $V = \frac{4r}{3} \times \pi r^2$

Thus, the number of leaves is:

$$n = \frac{\frac{4}{3}r}{\ell_p} = \frac{\frac{4}{3} \times 6371000}{1.626 \times 10^{-35}} = 5.22 \times 10^{41} concerned\ sheets\ of\ space$$

The mass of the Earth charging each leaf:

$$m = \frac{Mass\text{ of the Earth}}{n} = \frac{5.972 \times 10^{24}}{5.22E41 \times 10^{42}} = 1.143 \times 10^{-17} kg/sheet$$

So, due to gravitation, have a load on the area of the cylinder equivalent of area:πr^2

$$p/Sheet = \frac{mg}{\pi r^2} = \frac{9.81 \times 1.143 \times 10^{-17}}{\pi \times 6371000^2} = 8.79 \times 10^{-31} N/m^2/sheet$$

We assume that a span of the diameter of the Earth d charged by a pressure p which gives a linear charge q: $\frac{q}{Sheet} = p \times d = 8.79 \times 10^{-31} \times 12742000 = 1.12 \times 10^{-23} \frac{N}{m}/sheet$

The support response for an equivalent range of 12742 km is as follows:

$$R = \frac{qd}{2} = \frac{1.12 \times 10^{-23} \times 12742000}{2} = 7.13 \times 10^{-17} N/sheet$$

The horizontal force (membrane effect with a vertical deflection equal to the variation R seen above): $H = \frac{qd^2}{8f} = \frac{1.12 \times 10^{-23} \times 12742000^2}{8 \times 0.00147792} = 1.538 \times 10^{-7} N/sheet$

The resulting force T that puts the membrane in tension is:

$$T = \sqrt{R^2 + H^2} = 1.5389 \times 10^{-7} N/sheet$$

The stress in the Planck thickness sheet and width of the Earth's diameter is:

$$\sigma = \frac{T}{d \times \ell_p} = \frac{1.5389 \times 10^{-7}}{12742000 \times 1.62 \times 10^{-35}} = 7.455 \times 10^{20} Pa/sheet$$

The Young's modulus of the leaf is like this if we consider the deformation:

$$\varepsilon = \frac{h_{00}}{6} = \frac{1.39222 \times 10^{-9}}{6} = 2.320 \times 10^{-10}$$

We obtain for Y associated with the time component: $Y = \frac{\sigma}{\varepsilon} = \frac{7.455 \times 10^{20}}{2.320 \times 10^{-10}} = 3.21 \times 10^{30} Pa$

For the record, R Weiss in his Nobel Prize reading **[5]**, proposes Y = $10^{20} \times Y_{steel}. = 2.1 \times 10^{31} Pa$ se which is very close to our result.

Application of the membrane approach for the sun

The same approach is being carried out; $h_{00} \approx 2\frac{GM}{rc^2} = \frac{2\phi}{c^2}$

Numerical application for the Sun **[78]**, the value of the deformation is:

$$h_{00} = 4.244 \times 10^{-6}$$

$$\Delta R = \frac{1}{6}R_S = \frac{h_{00}Rc^2}{6c^2} = \frac{h_{00}R}{6} = \frac{4.244 \times 10^{-6} \times 695990000}{6} = 492m$$

$$\frac{\Delta R}{R} = \varepsilon = \frac{h_{00}}{6} = \frac{4.244 \times 10^{-6}}{6} = 7.0733 \times 10^{-7}$$

$$u_r = \Delta L = 2\pi R\frac{h_{00}}{6} = \pi\frac{h_{00}}{3}R = 3093.19m$$

We then obtain a Young's modulus of $Y = 2.658 \times 10^{26} Pa$

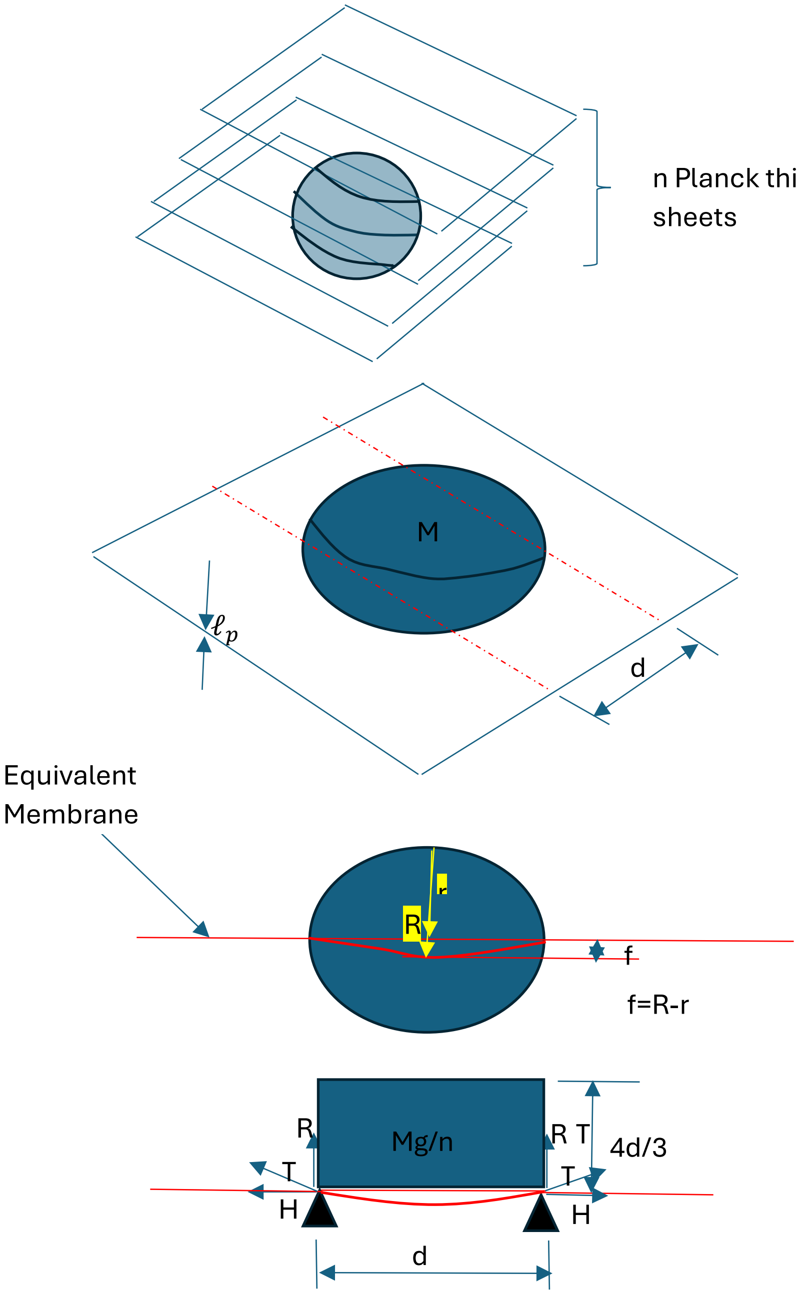


Figure 29: Visualization of the membrane model of a thin sheet of Planck bent by the Earth

Modeling of low-field gravitation by spherical shells

It is well known that gravitation is a phenomenon that acts in 4 dimensions and in weak fields it can be approximated by a Poisson's equation which gives Newton's equations in 3 dimensions. We therefore wanted to test this hypothesis using models of elastic shells see Figure 30. Figure 32 gives an overview of the analogy of the deformed and curved elastic medium in the case of a hull **[82]**. The data on the sphere with internal pressure are given in Figure 31.

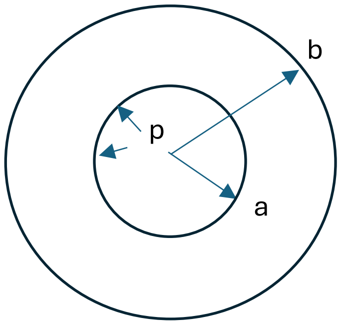


Figure 30: Notation of a sphere charged by an internal pressure

In elasticity, we have the differential equation **[82]**:

$$\frac{d^2u_r}{dr^2} + \frac{2}{r}\frac{du_r}{dr} = \frac{2}{r^2}u_r$$

Thus, the beginning of the equation is presented in the following form:

$$\Delta\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{2}{r} \times \frac{\partial\phi}{\partial r} + \frac{1}{r^2} \times \frac{\partial^2\phi}{\partial\theta^2} + \frac{1}{r^2 tan\theta} \times \frac{\partial\phi}{\partial\theta} + \frac{1}{r^2 sin^2\theta} \times \frac{\partial^2\phi}{\partial\varphi^2}$$

$$\Delta_{u_r} = \frac{2}{r^2}u_r$$

So, this is an expansion of the Poisson's equation that is modified by a distribution f that is not constant. The solution can be found on the form:

$$u_r = C_1 r + \frac{C_2}{r^2}$$

With for the two constants:

$$C_2 = \frac{1+\nu}{2(1+2\nu)} b^3 C_1 \qquad C_1 = \frac{(1-2\nu)}{E}\frac{a^3}{b^3-a^3} p$$

We know the displacement for space-time calculated from u_r **[78]**, so we can extract the Young's modulus E = Y:

$$E = Y = \frac{a^3 p}{u_r(b^3 - a^3)}\left[(1-2\nu)r + (1+\nu)\frac{b^3}{2r^2}\right]$$

Application in the case of the Earth

If with the study of the Earth's gravity effect, we see that in the sphere of influence (area where we can estimate that the gravitational effects are considered weak) is about $2x10^7$ m (Table 11).

R (m)	G (m^3/kgs^2)	Mass of the Earth (kg)	g (m/s^2)
6371000	6.6743E-11	5.972E+24	9,819973426
7000000	6.6743E-11	5.972E+24	8,134473388
8000000	6.6743E-11	5.972E+24	6,227956188
9000000	6.6743E-11	5.972E+24	4,920854272
10000000	6.6743E-11	5.972E+24	3,98589196
12000000	6.6743E-11	5.972E+24	2,767980528
14000000	6.6743E-11	5.972E+24	2,033618347
16000000	6.6743E-11	5.972E+24	1,556989047
18000000	6.6743E-11	5.972E+24	1,230213568
20000000	6.6743E-11	5.972E+24	0,99647299
25000000	6.6743E-11	5.972E+24	0,637742714

Table 11: Zone of influence of the Earth's gravity

Thus, we will consider:

b=20000 km, a=6371 km, $\nu = 1$ **[11]** to **[14]**

$u_r = 0.00928m$ (see the details of the calculations via **[78]**)

$P = \frac{gM}{4\pi r^2} = \frac{9.81\times5.972\times10^{24}}{4\pi\times6371000^2} = 1.14859 \times 10^{11}$ Pa

The pressure p is also given in Figure 31.

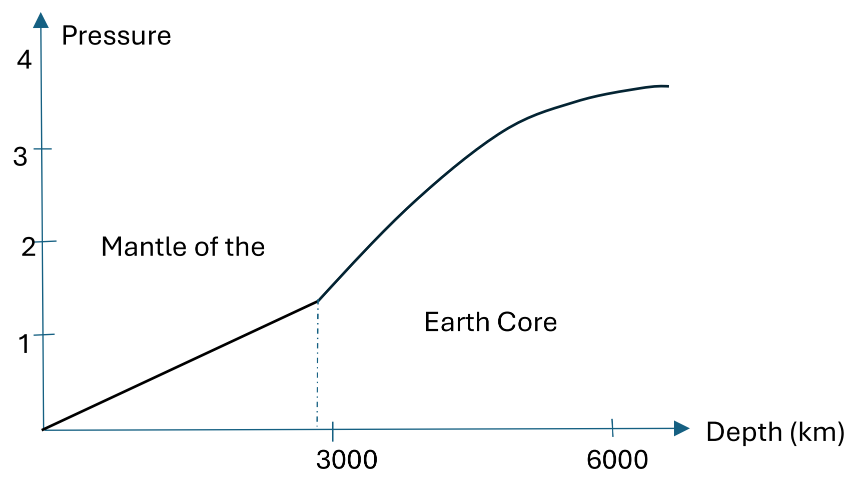


Figure 31: Pressure applied by Earth's gravity **[83]**

$$E = Y = \frac{a^3 P}{u_r(b^3 - a^3)}\left[(1 - 2\nu)r + (1 + \nu)\frac{b^3}{2r^2}\right]$$

The numerical application gives for the Young's modulus of space:

$$E = Y = \frac{6371000^3 \times 3.6 \times 10^{11}}{0.00928 \times (20000000^3 - 6371000^3)}\left[-6371000 + \frac{20000000^3}{6371000^2}\right]$$
$$= 2.4715 \times 10^{20} Pa$$

This value is lower than that obtained with deformations in the plane.

Application in the case of the sun

In the case of the sun, a similar approach gives:

$$E = Y = \frac{a^3 P}{u_r(b^3 - a^3)}\left[(1 - 2\nu)r + (1 + \nu)\frac{b^3}{2r^2}\right]$$

$$E = Y = \frac{695990000^3 \times 6.0 \times 10^{14}}{3093.19 \times (10000000000^3 - 695990000^3)}\left[-695990000 + \frac{10000000000^3}{695990000^2}\right]$$
$$= 1.35 \times 10^{20} Pa$$

The order of magnitude is therefore close to that obtained in the case of the Earth.

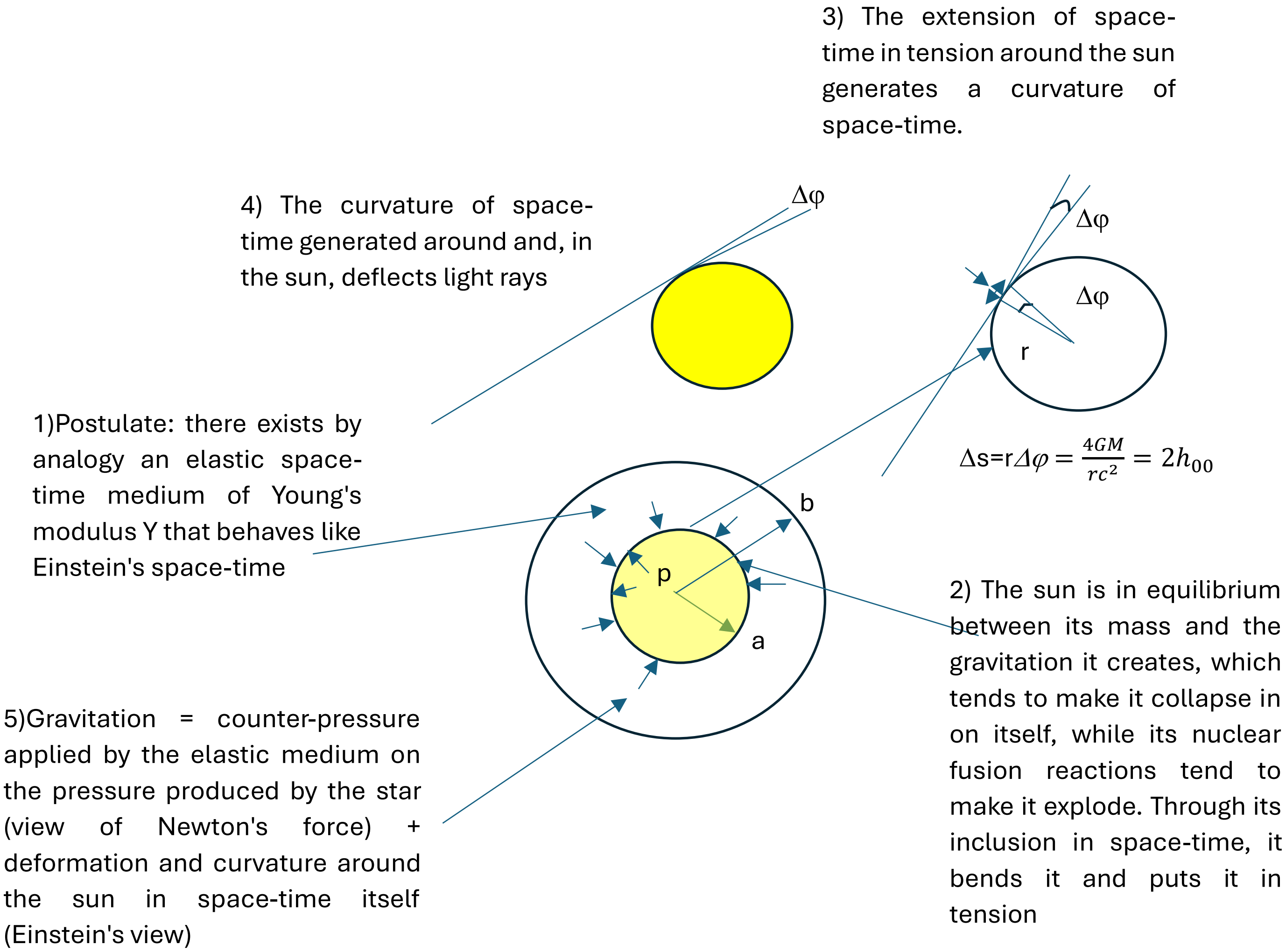


Figure 32: Visualization of the analogy and effects of the curvature of space in the case of a shell

Table 12 below summarizes the results obtained (Young's modulus) for different scenarios of deformations resulting from loading perpendicular to the plane of space

The case of general relativity	**Type of parameter measured or calculated according to general relativity**	**Theoretical results of general relativity**	**Measured results**	**Mechanical model of the Planck sheet associated with Planck-thick membranes or elastic shells**	**Young's modulus used for computation (Pa) (time aspect)**
Calculation of the curvature of space-time for the Earth (weak gravitational field)	Increase in Earth radius due to curvature **[78]** $\Delta R = 1.477mm$	$\Delta R = \frac{1}{6}R_S = \frac{h_{00}R}{6} = \frac{GM}{3c^2}$	Not applicable	Membrane charged perpendicular to its plane	$\mathbf{3.21 \times 10^{30}}$
Calculation of the curvature of space-time for the Sun (weak gravitational field)	Increased radius of the sun due to curvature, **[78]** $\Delta R = 492\ m$ $\Delta\varphi_{beam\ light\ measured} = 2.784 \times 10^{-9} rad$ $\Delta\varphi_{Schwarchild} = 2.32 \times 10^{-10} rad$	$\Delta R = \frac{1}{6}R_S = \frac{h_{00}R}{6} = \frac{GM}{3c^2}$ $\Delta\varphi_{exact\ beam\ light} = \frac{4GM}{rc^2} = 2h_{00}$ $\Delta\varphi_{approach\ scharwchild} = \frac{h_{00}}{6}$	Solar beam deflection	Membrane charged perpendicular to its plane	$\mathbf{2.658 \times 10^{26}}$
Calculation of the curvature of space-time in the Earth's interior (weak gravitational field)	Calculation of the curvature of space-time for the Earth, **[78]** $\varepsilon = 2.320 \times 10^{-10}$ $u_r = 0.00928m$	$\varepsilon = \frac{h_{00}}{6}$ $u_r = \Delta L = 2\pi R\frac{h_{00}}{6} = \pi\frac{h_{00}}{3}R$	Not relevant	Sphere with internal pressure	$\mathbf{2.471 \times 10^{20}}$
Calculating the curvature of space-time for the Sun	Calculation of the curvature of space-time for the , **[78]** $\varepsilon = 7.073 \times 10^{-7}$ $u_r = 3093.19m$	$\varepsilon = \frac{h_{00}}{6}$ $u_r = \Delta L = 2\pi R\frac{h_{00}}{6} = \pi\frac{h_{00}}{3}R$	Not applicable	Sphere with internal pressure	$\mathbf{1.35 \times 10^{20}}$
Geodesic effect created by the Earth on space-time (Weak gravitational field)	Geodetic angle measured on Earth at r = 6700 by gravity PROBE B, **[9]**	$\Omega = \frac{3GM}{2c^2R^3}(R \times v) + \frac{GI}{c^2R^3}\left[\frac{3R}{R^2}(\omega \cdot R) - \ldots\right.$ calculated with an equivalent membrane of deflection f and span of gravity Influence of the Earth: $f = \Delta R = \frac{1}{6}R_S = \frac{h_{00}Rc^2}{6c^2} = \frac{h_{00}R}{6} = 0.001477m$	6600 milliarc seconds/ year (space estimate)	6600 milliarcsecond/year Membrane of the model loaded perpendicular to its plane Area of influence141000 km (0.02g)	$\mathbf{3.96 \times 10^{32}}$

Table 12: Summary of results for deformations perpendicular to the plane

10. New interconnections between spatial and temporal aspects related to anisotropy:

The need to take these four dimensions into account simultaneously to predict all the distortions of space-time – didactic and predictive implications.

Summary

In this chapter we start with the results obtained for the two families of Young's moduli in the plane and perpendicular to the plane in order to search for a connection/correlation law between these two families of values. We obtain this law by transforming the expression of the ds^2 interval of special relativity according to an energetic mechanical law by connecting the speed of light to the Young's modulus and to the density of the vacuum for the 4 space-time dimensions in quasi-flat geometry. The law we obtain in energy works well if we combine the results obtained in the plane with those obtained by spatial models perpendicular to the plane. We also show, to complete with our initial hypothesis on the addition of an energy-momentum tensor related to vacuum deformations, that it is possible to write such an elastic tensor whose trace precisely gives the law connecting the Young's moduli in the plane and perpendicular to the plane established from the models of quantum beam and shell lattices.

In Chapter 8 we have assumed by analogy with the anisotropic behavior of space-time, the mirror of which is that of lamellar clays, that the Young's modulus of the structure of space-time must vary according to the plane considered. The various examples conducted in weak gravitational fields in Chapter 9 highlight this phenomenon and thus confirm the transverse anisotropy.

Table 13 below shows a summary of these different values obtained.

General Relativity event	Gravitation	Case studied	Strain	Type	Strain values	Unit	Mechanical model	Type of loading	Y (Pa)	Direction
GW150914	Weak	Black hole coalescence 1	hij (x,y)	ε	1,00E-21	-	Truss in torsion	in plane	1,00E+44	x or y
GW150914	Weak	Black hole coalescence 2	hij (x,y)	ε	1,00E-21	-	Truss in torsion	in plane	1,00E+44	x or y
GW170817	Weak	Neutron star coalescence	hij (x,y)	ε	1,00E-20	-	Truss in torsion	in plane	1,00E+44	x or y
NASA example	Strong	Frame dragging Neutron star	h0i;hj0	θ	6,37E-10	rad/s	Cylinder in torsion	in plane	7,70E+44	t, x or y or z
Gravity prob B	Weak	Frame dragging Earth	h0i;hj0	θ	4,00E-15	rad/s	Cylinder in torsion	in plane	4,73E+38	t, x or y or z
Gravity prob B	Weak	Frame dragging Earth	h0i;hj0	θ	4,00E-15	rad/s	Truss in torsion	in plane	3,00E+44	t, x or y or z
Gravity prob B	Weak	Geodetic Earth	h0i,hj0	β	1,00E-12	rad/s	Rectangular membrane uniformly loaded (max)	Perpendicular at the plane	3,95E+32	t, or y or z
Gravity prob B	Weak	Geodetic Earth	h0i,hj0	β	1,00E-12	rad/s	Rectangular membrane uniformly loaded (repartition load on all the membrane)	Perpendicular at the plane	2,80E+31	t, or y or z
Newton/GR	Weak	Earth Gravitation	h00	ε	2,32E-10	-	Rectangular membrane uniformly loaded (max)	Perpendicular at the plane	3,21E+30	tt
Eddington eclipse	Weak	Sun Gravitation	h00	ε	7,07E-07	-	Rectangular membrane uniformly loaded (max)	Perpendicular at the plane	2,66E+26	tt
Newton/GR	Weak	Earth Gravitation	h00	ε	2,32E-10	-	Sphere with internal pression	Perpendicular at the plane	2,47E+20	tt
Eddington eclipse	Weak	Sun Gravitation	h00	ε	7,07E-07	-	Sphere with internal pression	Perpendicular at the plane	1,35E+20	tt
Newton/GR	Weak	Earth Gravitation	h00	ε	2,32E-10	-	Circular membrane (R=R Earth)	Perpendicular at the plane	8,03E+39	tt
Eddington eclipse	Weak	Sun Gravitation	h00	ε	7,07E-07	-	Circular membrane (R=R Sun)	Perpendicular at the plane	2,19E+38	tt
Newton/GR	Weak	Earth Gravitation	h00	ε	2,32E-10	-	Circular membrane (R=R Earth impact)	Perpendicular at the plane	3,15E+40	tt
Eddington eclipse	Weak	Sun Gravitation	h00	ε	7,07E-07	-	Circular membrane (R=R Sun impact)	Perpendicular at the plane	3,14E+39	tt

Table 13: Overview of the different values of the Young's modulus obtained with the different models in the plane and perpendicular to the plane

In view of these results, it seemed logical to try to find out why such differences appear for Young's modulus and whether it was possible to find a link between these different values.

We will therefore look for an expression that allows us to connect the Young moduli in the plane and perpendicular to the plane.

In the case of low gravity fields, the metric is written as a perturbation of the Minkowski flat metric. Which gives for time:

$$g_{00} = \eta_{00} + h_{00}$$

With $\eta_{00} = -1$ and we obtained for the case of the Earth $10^{-10} \leq h_{00} \leq 10^{-9}$and $h_{00} \approx 10^{-6}$ for the Sun that we found via a membrane or sphere model leading us to a Young's modulus related to the deformations of the time between $10^{20}$and 10^{40}Pa.

And for space:

$$g_{ij} = \eta_{ij} + h_{ij}$$

With $\eta_{ij} = (1,1,1)$ et $h_{ij} \rightarrow h \approx 10^{-21}$and for gravitational waves that we found via a lattice of bars working in tensile compression, leading us to a Young's modulus bound to the 10^{44}Pa space.

Finally, by the expression $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$ T Tenev and M Horstemeyer **[11] [17]** postulated that there are deformations related to time as for space.

$$\varepsilon_{00} = \varepsilon^{3D} = \varepsilon_i^i$$

Since we are in the context of very, very small deformations, space-time is in fact almost flat at 10^{-21} ready in the case of gravitational waves (by placing itself far from their sources). We have shown that a flat lattice model can reproduce these space deformations. We will therefore start with the interval of special relativity, i.e. the Minkowski metric, and see if it is not possible to reformulate it in the form of an expression involving mechanical parameters of our elastic analogy of space-time, namely the Young's moduli.

The interval of special relativity is therefore written:

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

We have seen that there is a passage relation, a second principle of equivalence between the energy density ρc^2 of an elastic medium potentially constituting this elastic space-time and its Young's moduli in the elastic analogy in the case of compression waves or shear waves respectively.

$$Y = \rho c^2$$

$$\mu = \frac{Y}{2(1+\nu)} = \rho c^2$$

Having modeled space as a lattice working in tensile compression, we retain the first expression. Replacing c^2 in the interval with its above expression in the context of the analogy of space as an elastic medium, we obtain with Y_t the Young's modulus of the time part of space-time and ρ its density which can be related to the energy of the vacuum via$\frac{E}{V} = m/Vc^2 = \rho c^2$:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right)dt^2 + dx^2 + dy^2 + dz^2$$

In addition, we can express dx, dy, dz in terms of the displacements u_x, u_y, u_z in these different directions:

$$\varepsilon_{xx} = \frac{u(x+dx) - u(x)}{dx} = \frac{du}{dx}$$

The interval then becomes replacing dx, dy, dz by their expression function of the strain:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right)dt^2 + \left(\frac{du_x}{\varepsilon_{xx}}\right)^2 + \left(\frac{du_y}{\varepsilon_{yy}}\right)^2 + \left(\frac{du_z}{\varepsilon_{zz}}\right)^2$$

Considering Hooke's law:

$$\sigma_{xx} = \varepsilon_{xx} Y_x$$

By replacing the strains with their expressions as a function of the stresses, the interval becomes:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right)dt^2 + \left(\frac{du_x}{\sigma_{xx}}Y_x\right)^2 + \left(\frac{du_y}{\sigma_{yy}}Y_y\right)^2 + \left(\frac{du_z}{\sigma_{zz}}Y_z\right)^2$$

We have shown that the stress tensor and thus the normal stresses can be expressed as a function of velocities v_i and v_j as follows **[12]**:

$$\sigma_{ij} = \rho v_i v_j$$

By substituting the normal stresses for their density and velocity expressions in the interval ρ , we obtain:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right)dt^2 + \left(\frac{du_x}{\rho v_x^2}Y_x\right)^2 + \left(\frac{du_y}{\rho v_y^2}Y_y\right)^2 + \left(\frac{du_z}{\rho v_z^2}Y_z\right)^2$$

Substituting one of the velocities in each term $\frac{d_{xi}}{dt}$ for the interval, we obtain for the interval:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right)dt^2 + \left(\frac{du_x}{\rho dx v_x}Y_x dt\right)^2 + \left(\frac{du_y}{\rho dy v_y}Y_y dt\right)^2 + \left(\frac{du_z}{\rho dz v_z}Y_z dt\right)^2$$

To have an expression similar to the one we have for the time component, we factor the ratio $\frac{Y_i}{\rho}dt^2$ and replace $\frac{du_i}{dx}$ by ε_{ii} in the interval:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right)dt^2 + \left(\frac{\varepsilon_{xx}^2}{\rho v_x^2}Y_x\right)\left(\frac{Y_x}{\rho}\right)dt^2 + \left(\frac{\varepsilon_{yy}^2}{\rho v_y^2}Y_y\right)\left(\frac{Y_y}{\rho}\right)dt^2 + \left(\frac{\varepsilon_{zz}^2}{\rho v_z^2}Y_z\right)\left(\frac{Y_z}{\rho}\right)dt^2$$

Using Hooke's law again:

$$\sigma_{xx} = \varepsilon_{xx} Y_x$$

We obtain so:

$$ds^2 = -\left(\frac{Y_t}{\rho}\right)dt^2 + \left(\frac{\sigma_{xx}}{\rho v_x^2}\varepsilon_{xx}\right)\left(\frac{Y_x}{\rho}\right)dt^2 + \left(\frac{\sigma_{yy}}{\rho v_y^2}\varepsilon_{yy}\right)\left(\frac{Y_y}{\rho}\right)dt^2 + \left(\frac{\sigma_{zz}}{\rho v_z^2}\varepsilon_{zz}\right)\left(\frac{Y_z}{\rho}\right)dt^2$$

Again, using the relationship between Young's modulus and density and velocities.

$$ds^2 = -\left(\frac{Y_t}{\rho}\right)dt^2 + \left(\frac{\sigma_{xx}}{Y_x}\varepsilon_{xx}\right)\left(\frac{Y_x}{\rho}\right)dt^2 + \left(\frac{\sigma_{yy}}{Y_y}\varepsilon_{yy}\right)\left(\frac{Y_y}{\rho}\right)dt^2 + \left(\frac{\sigma_{zz}}{Y_z}\varepsilon_{zz}\right)\left(\frac{Y_z}{\rho}\right)dt^2$$

Using Hooke's law again $\varepsilon_{ii} = \frac{\sigma_{ii}}{Y_i}$:

$$\boldsymbol{ds^2 = -\left(\frac{Y_t}{\rho}\right)dt^2 + \varepsilon_{xx}^2\left(\frac{Y_x}{\rho}\right)dt^2 + \varepsilon_{yy}^2\left(\frac{Y_y}{\rho}\right)dt^2 + \varepsilon_{zz}^2\left(\frac{Y_z}{\rho}\right)dt^2}$$

And that the energy density of the deformation of spacetime in a weak field is written:

$$\boldsymbol{U = \frac{1}{2}\rho(c)^2 = -\frac{1}{2}\varepsilon_{tt}^2 Y_t + \frac{1}{2}\varepsilon_{xx}^2 Y_x + \frac{1}{2}\varepsilon_{yy}^2 Y_y + \frac{1}{2}\varepsilon_{zz}^2 Y_z}$$

Thus, the above equation in Young's moduli of space and time is just a kind of energy density equation associated with a deformable elastic medium consisting of a virtual spatio-temporal network of quantum-thick beams assembled into lattices, membranes or shells.

Thus, if we consider a particular space-time like light (gravitational waves travel at the speed of light and the deformations that materialize in space are very small (sun, Earth). The Pythagorean length is equal to the time traveled by light in this almost flat space $c^2dt^2=dx^2+dy^2+dz^2$

We have seen above that in weak field taking into account the verry small value of h, we can by simplification in the mechanic model consider here $g_{\mu\nu} \cong \eta_{\mu\nu}$.

So, Pythagoras can be applied for the mechanical model, and we have:

$$d_{space}^2 = dx^2 + dy^2 + dz^2$$

In addition, we know with the coalescence of the two neutron stars GW170817 that emitted both light and gravitational waves **[72]** move at the speed of light. Indeed, in **[72]** the gravitational wave and the electromagnetic wave propagate in space at the speed c. They arrive on Earth at the same time.

So, we have:

$$c^2dt^2 = d_{space}^2 = dx^2 + dy^2 + dz^2$$

If we consider that we are in a vacuum, in a plane receiving distortions in one direction x, we have the so-called light-like interval in general relativity **[84] [85]** which is written:

$$ds^2 = c^2dt^2 - d_{space}^2 = 0$$

So, we have in the sense x (it would be the same in the sense y):

$$0 = -c^2dt^2 + dx^2$$

Thus: $$c^2dt^2 = dx^2$$

We can show with $Y = \rho c^2$ and thus replace c with the parameters of mechanics and density of the vacuum. Considering Hooke's law $\sigma = \varepsilon Y$, we can show that:

$$0 = -\varepsilon_{tt}^2 Y_t + \varepsilon_{xx}^2 Y_x$$

In **[79]**, the author adopts the same approach to study a space-time membrane. He confirms that this approach is possible and realistic in weak field and near-flat metrics.

In this case, there is symmetry between spatial and temporal distortions.

$$\varepsilon_{tt}^2 Y_t = \varepsilon_{xx}^2 Y_x$$

Remark

As a reminder, relativistic results of general relativity often duplicate Newton's non-relativistic effects (e.g., the deflection of light rays near the sun).

Let be the following relation between the time-related Young's modulus and the space-related Young's modulus:

$$Y_x = \frac{\varepsilon_{tt}^2 Y_t}{\varepsilon_{xx}^2}$$

Numerical application

We recall the orders of magnitude of the different terms in each direction of the network:

$Y_{x-space} = Y_{y-space} < 3 \times 10^{44}\ Pa$ Estimated in chapter 9 to best find the deformations of space in the plane measured by LIGO and VIRGO.

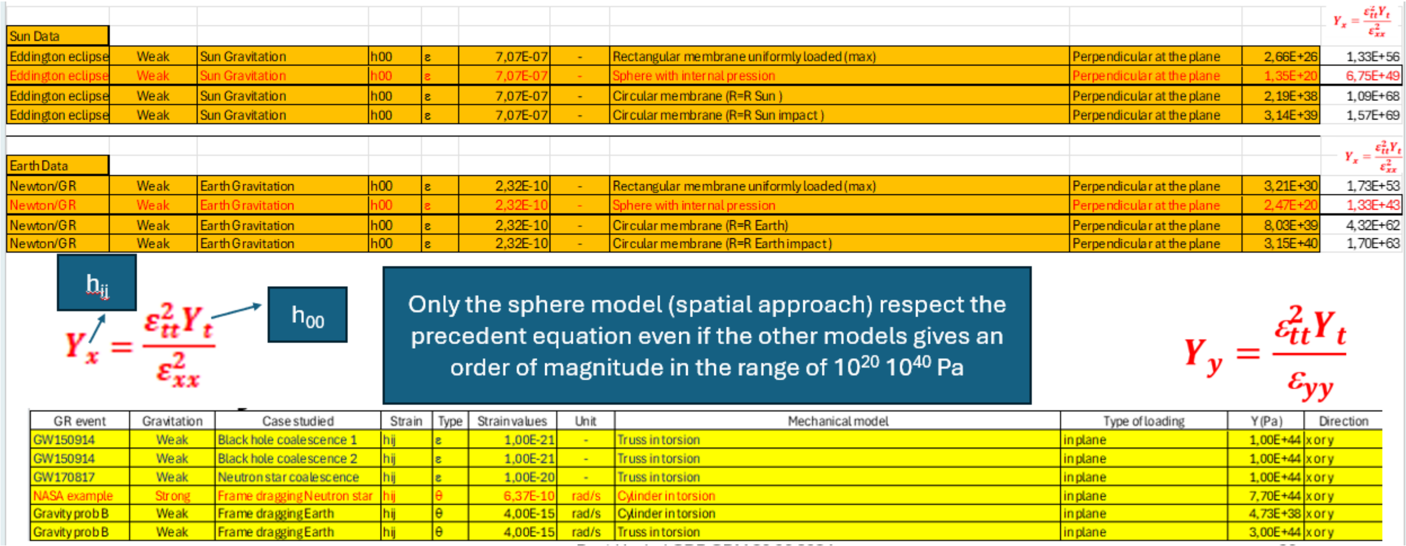
$\varepsilon_{xx} = \varepsilon_{yy} = 1 \times 10^{-21}$ measured by LIGO and VIRGO interferometers. And in the case of membrane or sphere models (the deformations associated with T_{00}):

$1.35 \times 10^{20} Pa < Y_{time} < 3.96 \times 10^{32}\ Pa$ Estimated in chapter 9 to best find the deformations perpendicular to the plane associated with time measured by gravity PROBE B or during the deflections of light rays near the sun.

$7.03 \times 10^{-7} < \varepsilon_{tt} < 2.32 \times 10^{-10}$ Calculated from formulations of general relativity

We therefore use the formulation demonstrated in the previous chapter: $\boldsymbol{Y_x} = \frac{\boldsymbol{\varepsilon_{tt}^2 Y_t}}{\boldsymbol{\varepsilon_{xx}^2}}$

Thus, with $\boldsymbol{\varepsilon_{tt}}$; $\boldsymbol{Y_t}$; $\boldsymbol{\varepsilon_{xx}}$ the following results taken from Tables 10, 12 and 13, we obtain the results given in Figure 33:



Sun Data										$Y_x = \frac{\varepsilon_{tt}^2 Y_t}{\varepsilon_{xx}^2}$
Eddington eclipse	Weak	Sun Gravitation	h00	ε	7,07E-07	-	Rectangular membrane uniformly loaded (max)	Perpendicular at the plane	2,66E+26	1,33E+56
Eddington eclipse	Weak	Sun Gravitation	h00	ε	7,07E-07	-	Sphere with internal pression	Perpendicular at the plane	1,35E+20	6,75E+49
Eddington eclipse	Weak	Sun Gravitation	h00	ε	7,07E-07	-	Circular membrane (R=R Sun)	Perpendicular at the plane	2,19E+38	1,09E+68
Eddington eclipse	Weak	Sun Gravitation	h00	ε	7,07E-07	-	Circular membrane (R=R Sun impact)	Perpendicular at the plane	3,14E+39	1,57E+69

Earth Data										$Y_x = \frac{\varepsilon_{tt}^2 Y_t}{\varepsilon_{xx}^2}$
Newton/GR	Weak	Earth Gravitation	h00	ε	2,32E-10	-	Rectangular membrane uniformly loaded (max)	Perpendicular at the plane	3,21E+30	1,73E+53
Newton/GR	Weak	Earth Gravitation	h00	ε	2,32E-10	-	Sphere with internal pression	Perpendicular at the plane	2,47E+20	1,33E+43
Newton/GR	Weak	Earth Gravitation	h00	ε	2,32E-10	-	Circular membrane (R=R Earth)	Perpendicular at the plane	8,03E+39	4,32E+62
Newton/GR	Weak	Earth Gravitation	h00	ε	2,32E-10	-	Circular membrane (R=R Earth impact)	Perpendicular at the plane	3,15E+40	1,70E+63

GR event	Gravitation	Case studied	Strain	Type	Strain values	Unit	Mechanical model	Type of loading	Y (Pa)	Direction
GW150914	Weak	Black hole coalescence 1	hij	ε	1,00E-21	-	Truss in torsion	in plane	1,00E+44	x or y
GW150914	Weak	Black hole coalescence 2	hij	ε	1,00E-21	-	Truss in torsion	in plane	1,00E+44	x or y
GW170817	Weak	Neutron star coalescence	hij	ε	1,00E-20	-	Truss in torsion	in plane	1,00E+44	x or y
NASA example	Strong	Frame dragging Neutron star	hij	θ	6,37E-10	rad/s	Cylinder in torsion	in plane	7,70E+44	x or y
Gravity prob B	Weak	Frame dragging Earth	hij	θ	4,00E-15	rad/s	Cylinder in torsion	in plane	4,73E+38	x or y
Gravity prob B	Weak	Frame dragging Earth	hij	θ	4,00E-15	rad/s	Truss in torsion	in plane	3,00E+44	x or y

Figure 33: Test of the formula linking Young's modules and deformations from temporal approaches with Young's moduli and spatial deformations

We have therefore found with $Y_x = \frac{\varepsilon_{tt}^2 Y_t}{\varepsilon_{xx}^2}$ a mechanical expression that allows us to relate the different Young's moduli characteristic of the transverse anisotropy of the elastic spatio-temporal medium if we consider the spatial approach of gravitation. At this stage we can use the previous results to try to precise the nature of $t_{\mu\nu,el}$. We therefore propose the following structure for this four-dimensional elastic vacuum deformation tensor $t_{\mu\nu}$:

$$t_{\mu\nu} = Y\,\varepsilon_{\mu\alpha}\varepsilon^{\alpha}{}_{\nu}$$

With this tensor form, t_{tt} corresponds to the energy density stored in static curvature (Newtonian potential), while t_{xx}, t_{yy} capture transverse strain from gravitational wave polarizations. In a Cartesian basis (t, x, y, z), the expanded version of this tensor $t_{\mu\nu}$ gives:

$$Y\begin{pmatrix} \varepsilon_{tt}^2 + \varepsilon_{tx}^2 + \varepsilon_{ty}^2 + \varepsilon_{tz}^2 & \varepsilon_{tt}\varepsilon_{tx} + \varepsilon_{tx}\varepsilon_{xx} + \varepsilon_{ty}\varepsilon_{yx} + \varepsilon_{tz}\varepsilon_{zx} & \cdots & \cdots \\ . & \varepsilon_{xt}^2 + \varepsilon_{xx}^2 + \varepsilon_{xy}^2 + \varepsilon_{xz}^2 & \cdots & \cdots \\ . & . & \varepsilon_{yt}^2 + \varepsilon_{yx}^2 + \varepsilon_{yy}^2 + \varepsilon_{yz}^2 & \cdots \\ . & . & . & \varepsilon_{zt}^2 + \varepsilon_{zx}^2 + \varepsilon_{zy}^2 + \varepsilon_{zz}^2 \end{pmatrix}$$

This first proposed structure for this tensor $t_{\mu\nu}$ is a pure quadratic product of the deformations. It contains all the directional information: this tensor is non-isotropic. It does not separate shear and expansion energy. Everything is included in it. It corresponds well to a gross directional energy density. It may have a non-zero trace. This symmetric tensor encodes the distribution of energy related to geometric deformations of space-time, according to an analogy with a continuous elastic medium. It has ten independent components, including diagonal terms e.g. (tt, xx, yy, zz): and non-diagonal (shear).

However, according to our mechanical models — lattices in the transverse planes seen at chapter 9a (xx, yy component) and membranes representing the gravitational potential seen at chapter 9b (component tt) — the following behaviors are observed:

- The main deformations appear in the transverse directions (typically xx and yy), corresponding in the models to the elongation or contraction of the horizontal members of the lattice:

$$\varepsilon_{xx}, \varepsilon_{yy} \text{ are dominant}$$

- The longitudinal deformation in the direction of propagation of the waves (e.g. the axis z) is very small (zero according to traditional general relativity, non-zero but small in modified general relativity with Einstein Cartan-type torsion **[14]** :

$$\varepsilon_{zz} \approx 0$$

- The temporal component ε_{tt}, on the contrary, is significant in static configurations, and models classical Newtonian gravitation in the form of a vertical subsidence of an elastic membrane.

- The off-diagonal components (e.g.: $\varepsilon_{tx}, \varepsilon_{ty}, \varepsilon_{tz}$) are negligible in our simulations if we compare the difference between the gravitational wave signal predicted by general relativity and the observed signal **[7]**, **[14]**. These terms would correspond to shear or torsional effects, which appear only in geometric extensions of relativity (geometric torsion of the Einstein-Cartan type, non-metricality).

In this configuration, the tensor $t_{\mu\nu}$ becomes almost diagonal, and its trace is reduced to:

$$t^{\mu}{}_{\mu} \approx -Y_t \varepsilon_{tt}^2 + Y_x \varepsilon_{xx}^2 + Y_y \varepsilon_{yy}^2 + Y_z \varepsilon_{zz}^2$$

Given that $\varepsilon_{zz} \approx 0$, we find the scalar expression of energy already introduced by analysis of the coherence between the Young's moduli obtained in the plane and perpendicular to the plane in chapter 9a and 9b calibrated to obtain the strain measured in general relativity in weak field and developed at the beginning of this chapter.

$$t^{\mu}{}_{\mu} \approx -Y_t \varepsilon_{tt}^2 + Y_x \varepsilon_{xx}^2 + Y_y \varepsilon_{yy}^2$$

This correspondence justifies the use of equations developed in first part of this chapter as an effective scalar energy density, supported by both mechanical and numerical models, and consistent with the covariant structure of the elastic energy tensor $t_{\mu\nu}$.

This result is then consistent with the tensor approach of the elastic energy of the gravitational vacuum is written:

$$U = \frac{1}{2} Y \varepsilon_{\mu\nu} \varepsilon^{\mu\nu}$$

The tensor $t_{\mu\nu} \sim \varepsilon_{\mu\alpha} \varepsilon^{\alpha}{}_{\nu}$ trace gives $\varepsilon_{\mu\nu} \varepsilon^{\mu\nu}$ a good measure of the overall energy content (all modulus of deformation combined).

$$2U = t^{\mu}{}_{\mu}$$

We therefore have, in relation to our calculations made in chapter 9a and chapter 9b, a completely coherent link between the deformation energy of the vacuum and the $t_{\mu\nu}$ tensor trace, provided that the principal components are diagonal and the non-diagonal components are zero or very small **[7]**, **[14]**.

11. Limitations of the experimental devices in place and the need for new techniques for measuring gravitational waves to validate the predictions of continuum mechanics models

Detection of angular distortions, complementary polarizations – interest of space-based interferometers (LISA) and pulsar arrays.

Summary

In this chapter, we detail the two main observables from our model, namely the lateral movements of the interferometer arms and the complementary polarizations, especially in the direction of gravitational wave propagations if we consider the geometric torsion. We also explain which test device would allow to detect these two observables (future LISA interferometers, multiple pulsar interferometers).

For our approach to anisotropic space modeling to be confirmed, we need to identify model predictions that could be tested repeatedly and with certainty.

Our approach leads to two observables:

- Traditional $A^{+}$and $A^{\times}$ polarizations can be seen by mirror effects as the two expressions of a pure torsional strain tensor **[11]** to **[14]**. This implies elongations and shortenings of 10^{-20} to 10^{-21} **[7]** **[72]** which are already measured by the LIGO and VIRGO interferometers, but also according to a 45° facet of shears and angular distortions, i.e. lateral movements of the interferometer arms of the same order of magnitude as elongations and shortenings. Having discussed this subject with R Weiss, current interferometers were not designed for this. On the other hand, LISA-type interferometers with 3 arms or the connections of the returns of several pulsars should allow us to see these deformations predicted by our transverse isotopic model.

- The need to reconstruct a matrix of the elastic medium at least in 3 dimensions of space, implies by analogy with crystallography, the theory of defects **[33]** **[35]** to have deformations associated with a certain cohesion of the separate transverse space sheets in classical linearized general relativity in the case of the propagation of gravitational waves in a vacuum. The mirror between this theory of defects and the geometric torsion associated with Einstein Cartan's modified general relativity, (but also for other theories of general relativity with torsion) implies if this torsion is really to be taken into account, complementary polarizations in the direction of propagation of gravitational waves **[38]** and a plastic interpretation of complementary deformations as in crystallography the theory of defects **[39]**. Here again, if these new polarizations exist, the new generation of interferometer should make it possible to see them.

These are the two observables predicted by our anisotropic elastic space model.

12. Consequences and limitations of anisotropic elastic models on the major questions in physics: pedagogical and predictive aspects (dark matter, dark energy, quantum gravity, etc.)

Summary

In this important chapter we summarize what the analogy of the elastic medium modeling space-time brings from a predictive point of view (lateral movements of the interferometer arms, complementary polarizations). We also specify what the analogy brings from a didactic point of view (possible origin of the limit value of light, possible coupling of the gravitational constant with mechanical parameters of the vacuum, mechanization of the coupling constant κ, why there are potentially only two polarizations in unmodified general relativity with torsion, why it is essential to consider 4 dimensions even in mechanics, how to model the vacuum as a coherent Timoshenko-type engineering structure given the very small deformations and infinitely large Young's modulus gathered in a four-dimensional Hooke's law, how the extension of the cosmological crystal model could provide clues to solve what dark energy and dark matter would be in the same mechanical paradigm). In this chapter we also list the limits of the analogy (anisotropy of the elastic medium contrary to all the isotropy hypotheses made in physics about the cosmos, the need to add a tensor related to the deformations of the vacuum to preserve Hooke's law in a vacuum, values of mechanical parameters outside the usual values of terrestrial materials, potential foliated constitutions of space). This chapter also lists some avenues to go beyond this thesis.

Our approach to general relativity by the analogy of the anisotropic elastic medium leads therefore if it is correct:

a- From a predictive point of view:

Has an anisotropic elastic medium model that predicts two physical phenomena:

- Lateral movements of the interferometer arms **[12]** to **[14]**
- Complementary polarizations in the direction of propagation of gravitational waves in the case of modified general relativity with torsion **[14]**, **[38]**, **[47]**, **[48]** or according to a second-order approach to general relativity (gravitomagnetism) **[51]**, **[52]** see in Hydro acoustics **[56]**.

b- From a didactic point of view:

Has an anisotropic elastic medium model **[14]** that offers the following didactic explanations:

- The speed of light is correlated with Young's modulus and the density of the elastic medium, $c = \sqrt{\frac{Y}{\rho}}$ in the plane or $c = \sqrt{\frac{\mu}{\varepsilon}}$ perpendicularly at the plane **[12] [13]**. The speed of light becomes a physical feature of space-time itself. The analogy is as follows. Or a bullet fired into a pile of sand, depending on the density and size of the grains it will go quickly in its progression in the pile of sand until a certain impassable speed limit. The very texture of the sand, which is much smaller than the ball, will slow it down and finally stop it. The photon is the ball, the pile of fine-grained sand making a screen an impassable wall at the speed of light is space-time. So, the speed of light, so our model, is not a matter of vibrating a medium, but a particle that propagates in an elastic deformable medium that influences, depending on its elasticity and density, its speed of propagation **[12] [13]**. Thus, space-time would approach a fluid at low velocity and a solid at speeds close to the speed of light.

- The gravitational constant becomes correlated with the natural frequency of the elastic medium and the density of the medium $G = \frac{\pi f^2}{\rho}$. Einstein took the constant G to force these equations to give Newton back to a weak field. But we know that Newton's forces are only an illusion, gravity is the deformation of space-time geometrically modeled by general relativity. In our elastic model, these deformations depend on the characteristics of the medium and therefore on its Young's modulus and its density ρ, which can be related to the deformation energy of the vacuum by $Y=\rho c^2$. Moreover, the equation with the dimensions of G ($m^3/(kgs^2)$) should raise questions about the possible assembly of several physical aspects within it **[12]** to **[14]**.

- Einstein's constant κ becomes correlated with Young's modulus E=Y or G=μ for the shear associated with the Poisson's ratio of the elastic medium as well as the proper pulsation ω=2πf of the medium, $\kappa = 2\rho\left(\frac{\omega}{E}\right)^2$ or $\kappa = 2\rho\left(\frac{\omega}{\mu}\right)^2$. If space-time is an elastic medium, then it must have flexibility. This is what is reflected κ in the analogy approach with an elastic medium **[12] [13]**.

- The two polarizations associated with gravitational waves become correlated with the two expressions of the pure torsional strain tensor as a function of the facet considered.

This is the direct consequence of the mechanics of continuous media **[11] [14]**. This is logical since the coalescence of two black holes, by their rotation with respect to each other, generates a torsion of the medium.

- The fourth dimension (c dt) is understood as deformations of space that are "as if arriving" at a point of measurement as a function of the propagation time of the signal within the web of space, given its texture. Space-time is indeed a dynamic object that is always in motion, making it impossible to measure in statics **[11]** to **[14]**.
- It is possible to model space with structural engineering techniques (see chapter 9) via quantum beams in equivalent static (the dynamics of space-time can be interpreted as a succession of instantaneous spatial slices, each statically strained like a photograph, evolving dynamically over discrete time intervalsand) not quantum strings, these beams have a thickness, Planck. They also have complementary rigidities compared to strings **[12]** to **[14]** that is more compatible with the extremely high space-time rigidity (1/κ).
- Young's modules of spacetime are different for the temporal and spatial components. By the formula $\frac{Y}{c^2} = \rho$ this implies a strong anisotropy between the behavior of space-time under the time solicitations 00 and the spatial components ij and therefore different vacuum densities according to the two aspects.
- Moreover, we have shown as an aside from the thesis that dark energy can be understood as a thermal curvature of an elastic plate taken as an analogy as space-time **[86].**
- Moreover, we have also shown, as an aside from the thesis, that dark matter can be understood as a creep of the texture of the elastic medium taken as an analogy as space-time **[89].**
- All numeric values are consistent. Small deformation 10^{-21} of elongation and shortening of space in the case of gravitational waves GW150914 and GW170817 for example **[7] [72]** or angles via gravity PROBE B **[9]**, small flexibility of space-time; κ=2.0766x10^{-43} N^{-1} **[12] [13]**, large value of the Young's modulus of the order of 10^{31} / 10^{38} Pa **[5] [86]**.

c- The limits of this analogy

The limits of the analogy lie in its foundation from the outset on the existence of a stress energy-tensor $t_{\mu\nu}$ associated with the structure itself of space-time in a weak field. This amounts to modifying Einstein's equation according a conservative law to the form **[21]** to **[24]**:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \left(T_{\mu\nu} + t_{\mu\nu} \right)$$

This complementary tensor reflects the return of a certain ether that Einstein banished in 1915 **[1]** **[2]** since light does not need a medium that it would make resonate in order to propagate thanks to Maxwell's equations in electromagnetism on the one hand or according to the experiment of Michelson and Morley, but which he also partly rehabilitated in 1920 during his Leiden conference **[87]** or he considered the existence of another kind of Ether $g_{\mu\nu}$, (a new good name to avoid any misunderstanding with the old ether luminifer should be "Elasther" sum of elastic and old Ether) translating the distortions of the metric and therefore the curvature of space-time as a function of the energy masses that are within it, space-time which has definitively ceased to be rigid and undeformable as Newton supposed.

"Recapitulating, we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an ether. According to the general theory of relativity, space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this ether may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it.

Albert Einstein, Leiden, 1920" **[88]**

The main point of divergence of the analogy with the physics of general relativity is that our model, in the light of the deformations measured by the passage of gravitational waves, leads inexorably to an anisotropic medium that seems to consist of lamellae or sheets of infinitely small space the size of the Planck length, which is completely contradictory to the hypothesis of an isotropic and homogeneous medium generally taken for the universe.

d- Consequence of our analogy on general relativity

The presence of geometric torsion in the B polarizations of the cosmic microwave background 380,000 years after the big bang in primordial space-time **[59]**, as well as the mechanical need to have complementary deformation components to connect the sheets of deformed transverse space independently of each other during the passage of gravitational waves **[14]**, **[39]**, as well as the associated complementary polarizations in the direction of propagation of these waves **[38]**, implies in our opinion the imperative need to definitively introduce geometric torsion into the equations of general relativity, as has been the case in the approach of Einstein Cartan or other theories with torsion **[33]**.

On the other hand, in the strong field, our thesis says nothing and everything remains to be demonstrated.

e- Beyond this thesis

Our entire approach to unifying torsionally modified general relativity with continuum mechanics and quantum mechanics via Planck thickness involves:

- to develop in theoretical physics a new theory of quantum beams in static and dynamic **[111]** which would be the sum of the knowledge of string theory rigidified by the sum of knowledge of the mechanics of continuous media (static, dynamic, instability, plasticity),

- extend these common principles based on $E=RC^2$ (with E the elastic strain energy, R the rigidity of the elastic medium and C the curvature in the senses of the strength of material 1/R) linking mechanics and general relativity **[8]** to all sectors where Newton's gravitational principles are used (economic gravity in commercial financial exchanges, biophysics for geometric structures expressible in the form of metrics, AI GSA (Gravitational Search Algorithm) algorithm.

13 Conclusion – thesis results

This thesis explores the analogy of space-time deformation in a field of weak gravity as defined in the Einstein field equation with and without geometric torsion in the Einstein-Cartan sense with an equivalent elastic medium. The scientific approach aims to highlight both the points for which the analogy works on the basis of a solid and complete state of the art (tensor analogy of pure torsional deformations and the perturbation of the metric during the passage of the gravitational wave over test masses distributed on a circle, stress tensor/stress energy tensor, etc.) than those for which they diverge (anisotropy related to the Poisson ratio equal to 1 contrary to the homogeneous and isotropic hypothesis).

This thesis also highlights the fact that gravitational wave polarizations for a relativistic physicist can be read in terms of a strain tensor for mechanics. It appears that in the case where the geometric torsion in the sense of Einstein Cartan is taken into account, the complementary polarizations that appear in the direction of propagation of the gravitational wave can be interpreted in terms of shear deformations (distortion) in 3 dimensions that raise the notion of spatial structure made up of planes independent of each other to find a complete anisotropic medium. Thus, the analogy goes in the same direction as for the relativistic calculations of complementary polarizations of gravitational waves (Einstein-Cartan approach) with the need to have an anisotropic 3D medium presenting both elongation and shortening deformations perpendicular to the direction of propagation of gravitational waves but also angular distortions, particularly in the direction of wave propagation. These observations also appear when we consider a fluid elastic medium solicited by an acoustic binary (analogy of the acoustic theory in fluid elastic medium) or when we consider Einstein's linearized general relativity in the second order.

We also show that, since the B polarization of the cosmic microwave background contains the Einstein Cartan twist, the temperature power spectrum of this background has a structure and is reminiscent of the X-ray diffractograms of lamellar clays. In addition, the properties of the latter are found in the behavior of space (liquefaction under dynamic actions/gravitational waves, self-clogging property/black holes do not leave tears behind when they move). The analogy of space consisting of thin sheets of Planck thickness modeled by an elastic lattice also makes it possible to reproduce the deformations of space measured during the passage of the gravitational wave GW150914 on Earth and the angular deformations of space by the Lense-Thirring effect (frame drag effect).

This thesis confirms that LISA type interferometers, or a better interaction between terrestrial interferometers, are essential to measure the deformations in the 3 directions during the passage of gravitational waves, especially since those in the direction of propagation appear extremely small compared to those in the plane.

It is important to emphasize that the proposed mechanical analogy does not challenge the cosmological principle of a homogeneous and isotropic Universe on large scales. Instead, it highlights that the local response of spacetime to specific sources—such as gravitational waves—can exhibit an effective anisotropy. This anisotropy is not an intrinsic property of the spacetime vacuum but emerges from the way it is excited by particular matter-energy configurations. Thus, the privileged direction induced by a gravitational wave results from the source's dynamics, not from a fundamental symmetry breaking in spacetime's texture. Our transversely isotropic model and the introduction of geometric torsion aim precisely to capture this directional response without abandoning the standard cosmological framework.

The addition of a supplementary tensor $t_{\mu\nu}$ to the right-hand side of Einstein's equations requires careful consideration regarding general covariance and conservation laws. To prevent any violation of covariant invariance, we assume that $t_{\mu\nu}$ derives from an effective Lagrangian describing the intrinsic elasticity of the vacuum. Furthermore, the conservation condition $\nabla^{\mu}(T_{\mu\nu} + t_{\mu\nu}) = 0$ must be respected, implying that $t_{\mu\nu}$ is not independent but coupled to the matter energy-momentum tensor. In our approach, $t_{\mu\nu}$ interacts with the spacetime geometry in a covariant manner, and its interpretation as a vacuum deformation energy tensor allows it to respect the geometric constraints imposed by the Einstein tensor. This extension, while bold, is grounded in a logic that preserves the fundamental principles of general relativity.

The proposed model is based on an analogy between space-time and a rigid, anisotropic, structured elastic medium, capable of deforming under the effect of mass-energies. This approach might, at first glance, seem to be in tension with the principle of general covariance of general relativity, which requires that physical laws be invariant under any change of coordinates (general diffeomorphism), and that space-time does not rest on any fixed background structure or preferred direction.

However, this apparent tension is lifted when we consider that the mechanical properties attributed to the "space-time medium" (Young's moduli, Poisson coefficients, anisotropy, etc.) do not constitute a rigid and fixed background structure, but rather a dynamic, covariant and geometrical structure, in the same way as the metric itself. The medium is not an absolute reference frame but a tensor field of elasticity that varies locally and dynamically, and whose properties are described in covariant language (via deformation, torsion, or curvature tensors).

This thesis also shows that in order to work, the analogy of considering Einstein's gravitational field equation in a weak field as a Hooke's law of an equivalent deformable elastic medium (elasther) must be completed both by an additional tensor of elastic deformation of the vacuum that remains active when spacetime is discharged to propagate the deformations of gravitational waves on the one hand and to complete general relativity with torsion geometric to ensure a three/four-dimensional behavior, thus ensuring cohesion between the sheets that seem to constitute the elastic medium modeling the functioning of space-time.

This view is consistent with other generalizations of general relativity, such as the Einstein–Cartan theory, where spacetime has a dynamic twist in addition to curvature, or bimetric or emergent geometry approaches, where additional degrees of freedom are assigned to the structure of the vacuum.

Thus, as long as the fundamental equations derived from the model (equations of deformation, motion, conservation) are expressed in covariant tensor form, the principle of general covariance remains respected. The proposed model does not introduce a break in covariance, but an extension of the physical content of the geometry of space-time, by integrating a mechanical response (elastic, plastic, thermal) interpreted as geometric fields coupled to the metric.

All our results tend to show that it would be interesting to develop a new theory of quantum beams resulting from a unification between quantum string theory and mechanical beams.

Finally, the principles developed in this thesis could be extended wherever the principles of Newtonian gravitation have been exported, such as economic gravity models, bio-physics models, gravity-based algorithms such as GSA.

14 Main publications

David Izabel, Yves Remond, Matteo Lucas Ruggiero (2025) Some geometrical aspects of gravitational waves using continuum mechanics analogy: State of the art and potential consequences MEMOCS Vol. 13, (2025), No. 2, 201–236
DOI: 10.2140/memocs.2025.13.103

David Izabel, Yves Remond, Matteo Lucas Ruggiero (2025) Analogy of space-time as an elastic medium - Study of the perturbation tensor of the metric $\boldsymbol{h}_{\mu\nu}$ through the prism of the analogy of the theory of elasticity- Analysis and potential consequences - International Journal of Theoretical Physics Vol. 64, (2025), No. 186,
DOI: 10.1007/s10773-025-06050-1

David Izabel, (2023) Analogy of spacetime as an elastic medium—Can we establish a thermal expansion coefficient of space from the cosmological constant Λ? International Journal of modern physic Vol.32, No.13,2350091 D
DOI: 10.1142/S0218271823500918

David Izabel, (2020) Mechanical conversion of the gravitational Einstein's constant κ, PRAMANA 94:119, DOI: 10.1007/s12043-020-01954-5

References

[1] A. Einstein, (1915) Die Feldgleichungen der Gravitation. Sitzungsberichte der Königlich Preußischen *Akademie der Wissenschaften (Berlin), Seite, 844-847*

[2] A. Einstein, (1916). Näherungsweise Integration der Feldgleichungen der Gravitation. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin*, physikalisch-mathematische Klasse, 688–696.

[3] A.D. Sakharov, (1968) Vacuum Quantum Fluctuations in Curved Space and the Theory of Gravitation. Soviet Physics, Doklady, 12, 1040-1041.

[4] T. Damour, (2016) If Einstein was told to me. ed Champ science, Conference, Paris

[5] R. Weiss, LIGO and the Discovery of Gravitational Waves, Nobel Lecture, December 8, 2017 by Rainer Weiss Massachusetts Institute of Technology (MIT), Cambridge, MA, USA

[6] J. Lense, H. Thirring, (1918) Über den Einflub der Eigenrotation der Zentrlkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen gravitatios theorie. Physik Zeitschr XIX, 156

[7] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso et al. (LIGO Scientific Collaboration and VIRGO Collaboration), (2016) Observation of Gravitational Waves from a Binary Black Hole Merger. Physical review letter 116, 061102

[8] D. Izabel, Y. Remond, M. L. Ruggiero (2025) Analogy of space-time as an elastic medium - Study of the perturbation tensor of the metric $\boldsymbol{h}_{\mu\nu}$ through the prism of the analogy of the theory of elasticity- Analysis and potential consequences. International Journal of Theoretical Physics Vol. 64, No. 186

[9] C. W. F. Everitt, D. B. DeBra, B. W. Parkinson, J. P. Turneaure, J. W. Conklin, M. I. Heifetz, G. M. Keiser, A. S. Silbergleit, T. Holmes, J. Kolodziejczak, M. Al-Meshari, J. C. Mester, B. Muhlfelder, V. Solomonik, K. Stahl, P. Worden, W. Bencze, S. Buchman, B. Clarke, A. Al-Jadaan, H. Al-Jibreen, J. Li, J. A. Lipa, J. M. Lockhart, B. Al-Suwaidan, M. Taber, S. Wang, (2011) Gravity PROBE B: Final Results of a Space Experiment to test General Relativity. Physical Review Letter, 106, 221101

[10] I. Newton, (1687) Philosophiae naturalis principia mathematica

[11] T. G. Tenev, M. F. Horstemeyer, (2018) Mechanics of spacetime — A Solid Mechanics perspective on the theory of General Relativity. International Journal of Modern Physics D, 27, 1850083, 2018

[12] D. Izabel, (2020) Mechanical conversion of the gravitational Einstein's constant κ. PRAMANA 94:119

[13] D. Izabel, (2021) What is space-time made of? EDP sciences

[14] D. Izabel, Y. Remond, M. L. Ruggiero, (2025) Some geometrical aspects of gravitational waves using continuum mechanics analogy: State of the art and potential consequences, Mathematics and Mechanics of Complex Systems, Vol. 13, No. 2, 201–236

[15] F. W. Dyson, A. S. Eddington and C. Davidson(1920) A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations Made at the Total Eclipse of May 29, 1919 Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character Vol. 220 (1920), 291-333, Published By: Royal Society

[16] S. Timoshenko, S. Woinowsky-Krieger (1951) Theory of plates and shells. McGRAW -Hill book company Newyork

[17] T.G Tenev, PhD Thesis (2018) An elastic constitutive model of spacetime and its applications, Mississippi State University, USA

[18] C.W Misner, K.S Thorne, J.A Wheeler, (1973) *Gravitation*. San Francisco: W. H. Freeman and Company. ISBN 0-7167-0344-0.

[19] T.A. Moore (2015) General Relativity Translated by Richard Taillet, Ed. De Boeck

[20] S. Antoci and L. Mihich (1999) Four-dimensional Hooke's law can encompass linear elasticity and inertia, Il Nuovo Cimento B, Issue 8, p873-880

[21] L. Levrino, A. Tartaglia, (2012 and 2014) From the elasticity theory to cosmology and vice versa » and sience. China physics mechanics and astronomy, vol 57, p597-603

[22] A. Tartaglia and N. Radicella, (2010) Space-time as a deformable continuum. J. Phys.: Conf. Ser. 222 012028

[23] A.Tartaglia and N.Radicella, (2009) From Elastic Continua to Space-time. arXiv:0911.3362v1

[24] M. R. Beau, (2015 and 2018) Constraint and Deformation Field Theory in General Relativity and Cosmological Expansion. Foundations of Physics manuscript arXiv:1209.0611v2 p4 and Annales de la Fondation Louis de Broglie, Volume 40

[25] S.M. Caroll, (2004) An introduction to general relativity space time and geometry. Cambridge University Press (2019)

[26] S. A. Balbus, (2016) Simplified derivation of the gravitational wave stress tensor from the linearized Einstein field equations. Physical Sciences 113 (42) 11662-11666

[27] R. Rubenzahl, (2017) *Gravitational Wave Radiation by Binary Black Holes*. Final paper for PHY 413: Gravitation, University of Rochester

[28] K.T. McDonald, (2018) What is the stiffness of the space-time J. Henry. Lab. Princ. Univ. NJ08544, 1 (2018)

[29] A.C. Melissinos, (2018) Upper limit on the Stiffness of space-time arXiv:1806.01133

[30] S. Catheline, a V Delattre, G. Laloy-Borgna, F. Faure, and M. Fink, (2022) Gravitational lens effect revisited through membrane waves. American Journal of Physic *Am. J. Phys.* 90:47–50

[31] Kokarev, SS (1999) Space-time as a strongly bent plate *Il Nuovo Cimento B*, **114**(8), pp. 903–922. DOI: 10.1007/BF03035699

[32] H. Kleinert, (1989) Gauge Fields in Condensed Matter, II. Stresses and Defects, World Scientific, Singapore

[33] M. L. Ruggiero, A. Tartaglia, (2003) Einstein-Cartan theory as a theory of defects in spacetime, American Journal of Physics 71, 1303-1313

[34] H. Kleinert, (2005) Emerging gravity from defects in world crystal. Brazilian Journal of Physics, 35 (2a)

[35] S. Capozziello, G. Lambiase, C. Stornaiolo, (2001) Geometric classification of the torsion tensor in space-time. Annalen der Physik, 10, 713-727

[36] S. Shrikanth, K. M. Knowles, S. Neelakantan, R. Prasad, (2020) Planes of isotropic Poisson's ratio in anisotropic crystalline solids, International Journal of Solids and Structures, 191-192, 628-645

[37] S. M. Carroll, (2002) De la curvature des espaces (manifolds Riemanniennes) Jacques Fric March 2002, translation from Lecture notes on general Relativity" by Sean M Carroll : http://pancake.uchicago.edu~carroll/notes/ and https://astromontgeron.fr/MIT-RG3F.pdf

[38] E. Elizalde, F. Izaurieta, C. Riveros, G. Salgado, O. Valdivia, (2022) Gravitational Waves in ECSK theory: Robustness of mergers as standard sirens and nonvanishing torsion. arXiv:2204.00090

[39] F. L. Carneiro, S. C. Ulhoa, J. W. Maluf, J. F. da Rocha-Neto, (2021) Non-linear plane gravitational waves as space-time defects. The European Physical Journal C, 81, 67

[40] J. W. Maluf, J. F. da Rocha-NetoS. C. Ulhoa, F. L. Carneiro, (2018) Variations of the Energy of Free Particles in the p-p Wave Space-times. Universe, 4, 7, 74

[41] C.F. Gauss, (1828) Disquisitionnes générales superficies curvas. Ed. Göttingen, Dieterich, 1828

[42] D. Buskulic, (2013) Gravitational Waves, Theoretical and Experimental Aspects, Gravitation Theory and Experiments, Proceedings, 3rd School of Theoretical Physics, Jigel, Algeria, 29 September – 3 October 2009, Ed. Herman, 231-289

[43] P. M. Zhang, C. Duval, W. Gibbons, P. A. Hovarthy, (2017) The memory effects for plane gravitational waves. Physics Letters B, 772, 743-746

[44] P. M. Zhang, C. Duval, G. W. Gibbons, P. A. Hovarthy, (2017) Soft gravitons and the memory effect for plane gravitational waves. Physical Review D, 96, 064013

[45] C. Duval, G. W. Gibbons, P. A. Hovarthy, P. M. Zhang, Carroll (2017) symmetry of plane gravitational waves. Classical and Quantum Gravity, 34, 175003

[46] P. M. Zhang, M. Cariglia, C. Duval, M. Elbistan, G. W. Gibbons, P. A. Hovarthy, (2018) Ion traps and the memory effect for periodic gravitational wave. Physical Review D, 98, 044037, 2018, and Erratum 089901

[47] L. A. Philippoz, PhD, (2018) On the Polarization of Gravitational Waves, Universität Zürich

[48] S. Mathur, Gravitational Wave Polarizations (2020) A test of General Relativity using Binary Black hole mergers. Thesis, California Institute of Technology

[49] M. O. TAHIM, R. R. LANDIM C. A. S. ALMEIDA (2009) space time as a deformable solid. Modern Physics Letters A Vol. 24, No. 15, pp. 1209-1217 Research Papers

[50] F. S. N. Lobo, G. J. Olmo, and D. Rubiera-Garcia (2015) Crystal clear lessons on the microstructure of space-time and modified gravity. Phys. Rev. D 91, 124001

[51] D. Baskaran, L. P. Grishchuk, (2004) Components of the gravitational force in the field of a gravitational wave, Classical and Quantum Gravity, 21, 17, 4041-4062

[52] M. L. Ruggiero, (2022) Gravitomagnetic induction in the field of a gravitational Wave, General Relativity and Gravitation, 54, 9, 97

[53] W. Zhou et al, (2025) 3D polycatenated architected materials. Science Vol 387, Issue 6731pp. 269-277, DOI: 10.1126/science.adr9713

[54] C. Barcelo, (2011) Analogue Gravity. Living Reviews in Relativity volume 14, Article number: 3

[55] S. Datta (2018) Acoustic analog of gravitational wave. Phys. Rev. D 98, 064049

[56] D. Masovic, (2022) 'Acoustic analogies with general relativity quantum felds, and thermodynamSics' Technische Universität Berlin Fakultat V - Verkehrs- und Maschinensysteme Institut für Strömungsmechanik und Technische Akustik Fachgebiet Technische Akustik Sekr. TA 7 - Einsteinufer 25 - 10587 Berlin

[57] Bigot. Sazy, (2013) Thesis M A Measurement of the polarization anisotropies of the cosmic microwave background with the QUBIC bolometric interferometer

[58] AstroSaône The Microwave Background Cosmologiquehttps://www.astrosaone.fr/spip/IMG/pdf/CMB_resume.pdf 535

[59] Das Moumita, S. Mohanty and A.R. Prasanna, (2013) Constraints on background torsion from birefringence of CMB polarization. Int. J. Mod. Phys. *D* 22 (2013) 1350011

[60] Collectif (2018) Planck 2018 results VI. Cosmological parameters Astronomy and astrophysic review

[61] Planck Collaboration (2014) Planck 2013 results. I. Overview of products and scientific results» Astronomy & Astrophysic Volume 571

[62] Planck Collaboration (2020) Planck 2018 results. V. CMB power spectra and likelihoods» Astronomy & Astrophysic Volume 641

[63] L. Nash, (2024) Curved Space-Time at the Planck Scale. Journal of High Energy Physics, Gravitation and Cosmology, 2024, 10, 167-179

[64] M. Novello, (1976) *Scalar and massless vector fields in Cartan space*. *Physics Letters A*, **59**(2), pp. 105–106.

[65] V. De Sabbata and M. Gasperini, (1981) *Propagating torsion and electromagnetic gauge invariance*. *Physics Letters A*, **83**(3), pp. 115–117

[66] M. J. Duncan, N. Kaloper and K. A. Olive, (1992) *Axion hair and dynamical torsion from anomalies*. *Nuclear Physics B*, **387**(1), pp. 215–235.

[67] C. Taviot-Guého, C.F. Leroux Goujon, F.P. Malfrey, and R. Mahiou, (2013) Study of the exchange mechanism and structure of double lamellar hydroxide (HDL) materials by X-ray diffraction and scattering. EDP sciences

[68] Y. Ali-Haïmoud, (New York University, USA (2021) conf Physics of the Cosmic Microwave Background https://www.youtube.com/watch?v=meWV33xKXJ8

[69] M.A. Bigot-Sazy, (2013) *Measurement of Cosmic Microwave Background Polarization Anisotropies with the QUBIC Bolometric Interferometer*. Doctoral thesis, Université Paris Diderot – Paris 7

[70] L. Quyen DAO, (2015) *Étude du comportement anisotrope de l'argile de Boom*. Thèse de doctorat, spécialité géotechnique, sous la direction de Yu Jun Cui, soutenue à l'Université Paris-Est

[71] G. Veylon, (2017) *Modélisation numérique du mécanisme de liquéfaction des sols : application aux ouvrages hydrauliques*. Thèse de doctorat, Université Grenoble Alpes.

[72] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikariet al. (LIGO Scientific Collaboration and VIRGO Collaboration) (2017) GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral Phys. Rev. Lett. **119**, 161101

[73] Lucia, U.; Grisolia, G. (2020). Time & clocks: A thermodynamic approach. Results in Physics,**2020**, *16*, 102977

[74] Atanu Chatterjee, Germano Iannacchione, (2020) Time and thermodynamics extended discussion on “Time & clocks: A thermodynamic approach”. Results in Physics,Volume 17,103165

[75] I. Ciufolini, (2013) *Time travel, Clock Puzzles and Their Experimental Tests*. *European Journal of Physics Web of Conferences*, **58**, 01005.

[76] J.Muller, (1977) General Mechanics Technical Form, 16th edition

[77] S. Odenwald, (2019) *Space Math @ NASA: Black Hole Math Problems*. NASA Goddard Space Flight Center.

[78] R. Johnston, (2008) Calculation on space-time curvature within the Earth and Sun

[79] H.A. Perko, (2019) G*ravitation in the Surface Tension Model of Spacetime*. *Journal of Physics: Conference Series*, **1239**

[80] H.A. Perko, (2003) *Introducing surface tension to spacetime*. *Journal of Physics: Conference Series*, **845**, 012003

[81] H.A Perko, (2021) *Dark Matter and Dark Energy: Cosmology of Spacetime with Surface Tension*. *Journal of Physics: Conference Series*, **1956**

[82] Collectif Lecture SPHERICAL ENVELOPE SUBJECT TO INTERNAL PRESSURE ENSmMP http://mms2.ensmp.fr/mms_paris/plasticite3D/exercices/f_Sphere.pdf

[83] L. Volgyesi, M. Moser, (1982) The Inner Structure of the Earth. Periodica Polytechnica Chemical Engineering **26**(3)

[84] W. Roman Wiszniewski, (2006) *Time, Quasi-Temporal Change and Imaginary Numbers*. Doctoral thesis, University of New South Wales, 638 pages

[85] E. Witten, (2022) A Note On Complex Spacetime Metrics. arXiv:2111.06514v2 [hep-th] 11 Feb 2022

[86] D. Izabel (2023) Analogy of spacetime as an elastic medium—Can we establish a thermal expansion coefficient of space from the cosmological constant Λ? International Journal of modern physic D Vol. 32, No. 13, 2350091 (2023)Research Paper

[87] A. Einstein, Leiden Conference (1920), The Ether and the Theory of Relativity, Geometry and Experience", translated from German into French by M. Solovine, vol 22x14 de 29pp, Paris, Gauthier-Villars, 1953

[88] A. Einstein (1918) Über Gravitationswellen *Sitzungsber.Preuss.Akad.Wiss.Berlin (Math.Phys.)* 1918 (1918) 154-167

[89] D. Izabel (2025) Analogy of spacetime as an elastic medium — Estimation of a creep coefficient of space from space data via the MOND theory and the gravitational lensing effect — the ball cluster — and via time data from the GPS effect — comparison, discussion and implication of the results for dark matter and Einstein’s field equation. International Journal of modern physic D Vol. 34, No. 02, 2450070 (2025)Research Paper

[90] M. Chapon, L. Darondeau, R. Desmorat, C. Ecker, and B. Kolev, (2024) General covariant relativistic gradient hyperelasticity, available at https://hal.science/hal-04792877. Ffhal-04792877f.

[91] B. Kolev, (2016) Éléments de géométrie différentielle à l'usage des mécaniciens", Cours, 5ème école d'été de mécanique théorique, CNRS, Quiberon, available at https://hal.science/hal-03330418.

[92] B. Kolev, (2023) Covariance générale et objectivité, Conférence du GDR Géométrie différentielle et mécanique, CNRS, La Rochelle

[93] J.-F. Bennoun, (1965) Étude des milieux continus élastiques et thermodynamiques en relativité générale. Ann. Inst. Henri Poincaré III:1 41–110.

[94] J. D. Brown, (2021) Elasticity theory in general relativity, Classical and Quantum Gravity 38:8, art.id.085017.

[95] B. Carter and H. Quintana, (1972) Foundations of general relativistic high-pressure elasticity theory. Proc. R. Soc. Lond. A 331:1584, 57–83.

[96] A. Einstein, (1956) The meaning of relativity. Princeton University Press, Princeton, NJ, 1988. Reprint of the 1956 edition.

[97] R. A. Grot and A. Eringen, (1966) Relativistic continuum mechanics. part I: Mechanics and thermodynamics", Int. J. Eng. Sci. 4:6 611–638.

[98] J. Kijowski and G. Magli, (1992) Relativistic elastomechanics as a Lagrangian field theory, J. Geom. Phys. 9:3, 207–223.

[99] B. Kolev and R. Desmorat, Souriau's general covariant formulation of relativistic hyperelasticity revisited. J. Mech. Phys. Solids 181 (2023), art.id.105463.

[100] B. Kolev and R. Desmorat, (2024) Objective rates as covariant derivatives on the manifold of Riemannian metrics, Arch. Ration. Mech. Anal. 248:4, 66.

[101] R. Al Nahas, M. Wang, B. Panicaud, E. Rouhaud, A. Charles, and R. Kerner, (2022) Covariant spacetime formalism for applications to thermo-hyperelasticity. Acta Mech. 233:6 (2022), 2309–2334.

[102] J. D. Norton, (1993) General covariance and the foundations of general relativity: Eight decades of dispute. Rep. Prog. Phys. 56:7, 791–858.

[103] J.-M. Souriau, (1958) La relativité variationnelle. Publ. Sci. Univ. Alger. Sér. A 5, 103–170.

[104] J.-M. Souriau, (1964) Géométrie et relativité. Enseignement des Sciences, VI, Hermann, Paris

[105] J. L. Synge, (1959) A theory of elasticity in general relativity. Math. Z. 72:1 82–87.

[106] M. R. Beau, (2015) Théorie des champs des contraintes et des déformations en relativité générale et expansion cosmologique. Ann. Fond. Louis de Broglie 40, arXiv 1209.0611v2

[107] M. O. Tahim, R. R. Landim, and C. A. S. Almeida, (2009) Spacetime as a deformable solid, Modern Phys. Lett. A 24:15, 1209–1217.

[108] A. Tartaglia and N. Radicella, (2010) From elastic continua to space-time, AIP Conference Proceedings 1241, 1156–1163.

[109] F.L Carneiro, B.C.C. Carneiro, D.L. Azevedo, Ulhoa, S.C. (2025): On nanocones as gravitational analog systems. Ann. Phys. (Leipzig) Annalen der Physic

[110] S. Li, H.N. Gynai, S.W. Tarr, and al. (2023) A robophysical model of spacetime dynamics. Sci Rep 13, 21589, 2023.

[111] D. Izabel (2025) Mechanical analogy between the second-order Schrödinger equation without potential for the case of a particle in an ideal infinite well with the fourth-order Schrödinger equation in connection with the potential manifestation of negative mass in Bose–Einstein condensates and exciton–polaritons in cavity. European physical journal H Volume 50, article number 19, (2025)

[112] D. Izabel (2025) A Cosmological Bragg Law: Interpreting the CMB as a Diffractogram of Foliated Spacetime, European Physical journal plus (sous presse)

Abstract

This work is placed in the framework of the analogy between the theory of elasticity of the mechanics of continuous media and the propagation of gravitational waves, following the deformations measured by the VIRGO and LISA interferometers.

Some authors have tried to present general relativity in the formalism of continuum mechanics, while others have generalized this continuum mechanics in 4D by introducing a "mechanistic" metric that includes the effect of time. The last work that, to our knowledge, provides an up-to-date assessment of this subject was given by **[90, 91, 92]**, based on the work of **[93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105]**. Summarizing the theoretical context in which this work is situated, we will assume that both general relativity and continuum mechanics are based on the principle of general covariance, which, as summarized well by B. Kolev in **[92]**, requires the introduction of three components: *(i)* Lagrangian-type functionals, depending on the metric *g* defined on the manifold *M* of the universe in 4 dimensions and dependent on various fields; *(ii)* tensors, such as the energy-momentum tensor dependent on these fields; *(iii)* field equations.

Our approach is therefore part of the general covariance of field equations by the group of diffeomorphisms. Or expressed explicitly, a Lagrangian L(g, Ψ,..) is a general covariant if it satisfies L(φ∗g, φ∗Ψ,..) = L(g, Ψ,..) for any φ diffeomorphism. It should be noted, again with B. Kolev, that the functional H(g) =∫ RgVolg, where Rg represents the scalar curvature, is a general covariant. Its L2 gradient, the Einstein tensor *G(g)* is also a general covariant. It therefore satisfies ∀φ, Gφ∗g = φ∗G(g), and divgG(g) = 0. An energy-momentum tensor T(g, Ψ) satisfying Einstein's equation G(g) = T(g, Ψ) therefore satisfies the mechanical equation: *div T = 0*. The rest of this work therefore focuses on the energy-momentum tensors (and their variations) satisfying these two properties. The reader is referred to the work of J. M. Souriau **[103, 104]** and his successors already cited, for the definitions of the perfect field of matter (as a section of a vector beam) and of conformations, allowing the proposal of relativistic constituents.

In addition to these assumptions, we recall the discussions on the validity of relativistic equivalence, in particular for the stress field used in continuum mechanics. Our approach adds additional elements to this debate. However, when deformations occur in a vacuum away from the space-time charge, as is the case with gravitational waves arriving on Earth, the energy-momentum tensor is zero: Tμν = 0. To maintain a consistent Hooke's law, it is then necessary to consider an elastic strain energy tensor of the vacuum itself: $T_{e,\mu\nu}$, related to the deformations correlated with hμν (small perturbation of the Minkowski tensor). It is a way of calculating the energy of gravitational waves or of studying space-time itself as an equivalent elastic medium **[106, 107, 108].**

Following these preliminary remarks, we can define this "elastic gravitational analogy" based on the following three principles of equivalence. *(i)* The hij perturbations of space in the presence of gravitational waves are related to the covariant Green–Lagrange tensor:$D = \frac{1}{2}(\varphi^{*\varphi}g - g)$, which will be assimilated in the following, under infinitesimal deformations, to the geometric linearization of the deformation tensor D = ε = εij. *(ii)* Einstein's equation relates, within the medium, the strain tensor (with respect to the metric perturbation in a weak field) to the equivalent stress field, like a Hooke's law, with the help of an equivalent conformity matrix. *(iii)* The energy density of space-time itself ρc^2 is correlated, through quantum field theory or the Casimir effect, with a non-zero vacuum energy density. This energy density of the vacuum is reflected in this analogy by the Young's modulus Y of the equivalent elastic medium, given by the equation $Y = \rho c^2$.

The results of different mechanical models are compared to experimental observations within the framework of the Einstein-Cartan theory. Despite some limitations, gravitational wave polarizations can be considered as expressions of a strain tensor of an equivalent elastic medium. Moreover, an anisotropy of the properties of space seems inevitable at the point of gravitational wave measurement if we rely on the current first-order general relativity, which predicts that gravitational waves generate deformations only in the transverse planes. Classical polarizations can be associated with a state of pure torsion in the equivalent elastic medium. This approach involves a transverse isotropic medium composed of independent sheets that deform perpendicular to the direction of propagation of these waves. By considering this geometric torsion, we can examine complementary polarizations in the direction of wave propagation. This makes it possible to link these sheets and reconstruct a 3D environment.

Thus, as long as the fundamental equations from which the model is derived (conservation equations) can be expressed in covariant tensor form, the principle of general covariance remains respected. The proposed model does not introduce a break in covariance, but an extension of the physical content of the geometry of space-time, by integrating a mechanical response (elastic, plastic, thermal) interpreted as geometric fields coupled to the metric.

Annex A – Publications –

International Journal of Theoretical Physics (2025) 64:186
https://doi.org/10.1007/s10773-025-06050-1

REVIEW

Elastic Medium Analogy of Spacetime: $h_{\mu\nu}$ Metric Perturbation Tensor Analysis and Theoretical Implications

David Izabel[1,3] · **Yves Remond**[1] · **Matteo Luca Ruggiero**[2]

Received: 9 April 2025 / Accepted: 10 June 2025

Abstract
A state-of-the-art review of the different deformations of space–time measured over more than a hundred years within the framework of general relativity in the weak-field regime is presented. The general relativity phenomena considered in this low field context include gravitational waves, the Lense-Thiring effect, gravitational lensing, gravitation around the Earth or the Sun. This overview of various deformations highlights the different active components of the perturbation tensor of the metric $h_{\mu\nu}$. The authors demonstrate that each phenomenon corresponds to one or more distinct components of this tensor. They also show that the various components can be interpreted, within the elastic analogy of space–time, as coherent components of an associated strain tensor $\varepsilon_{\mu\nu}$ in terms of elongation, compression or angular distortion of an equivalent elastic medium modeling the behavior of space–time. Through this synthetic ensemble approach and elastic analogy, it becomes evident—for the first time—that some components of the tensor $h_{\mu\nu}$ remain to be identified and measured potentially corresponding to new phenomena or modified versions of general relativity in the weak-field limit.

Keywords General relativity · Gravitational waves · Frame dragging · Geodetic effect · Lense-Thirring effect · Sun gravitation · Earth gravitation · Gravitational lens · Elasticity · Continuum mechanics

1 Introduction

The theory of general relativity [1, 2] is now more than a century old and has been tested and validated in numerous situations through increasingly precise experiments [3–7], It was proven to be highly successful, and conceptually, gravitation is now understood as a manifestation of deformation or curvature of space–time. This distortion of space–time is associated with the establishment of the metric tensor, $g_{\mu\nu}$, which is determined by solving

✉ David Izabel
david.izabel@etu.unistra.fr

1 University of Strasbourg, CNRS, Strasbourg, France

2 University of Turin, Turin, Italy

3 University of L’Aquila, CNRS, L’Aquila, Italy

Published online: 26 June 2025

the nonlinear tensor differential equation of the gravitational field formulated by A. Einstein in 1915 [1].

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1}$$

In this expression, $R_{\mu\nu}$ is the Ricci curvature tensor, R is the tensor contraction of the Ricci tensor, $T_{\mu\nu}$ the energy–momentum tensor that somehow shapes space–time, G is Newton's gravitational constant, and c is the speed of light.

In weak gravitational fields, the metric $g_{\mu\nu}$ can be written as:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{2}$$

where $\eta_{\mu\nu}$ is the Minkowski tensor of flat space–time and $h_{\mu\nu}$ a very small perturbation ($\left|h_{\mu\nu}\right| \ll \left|\eta_{\mu\nu}\right|$).

The purpose of this article is to conduct an in-depth study of the various components of $h_{\mu\nu}$, in the light of the numerous tests of general relativity that have been carried out over the past 100 years.

Some components of $h_{\mu\nu}$ are activated and their intensities are established according to the physical phenomena soliciting space–time. Thus, depending on the type of space–time stress, the activated components, i.e. those necessary for representating the phenomenon under study and the associated deformations concerned, are framed together in Fig. 1. The phenomena of general relativity in low fields targeted are gravitation around the Earth or

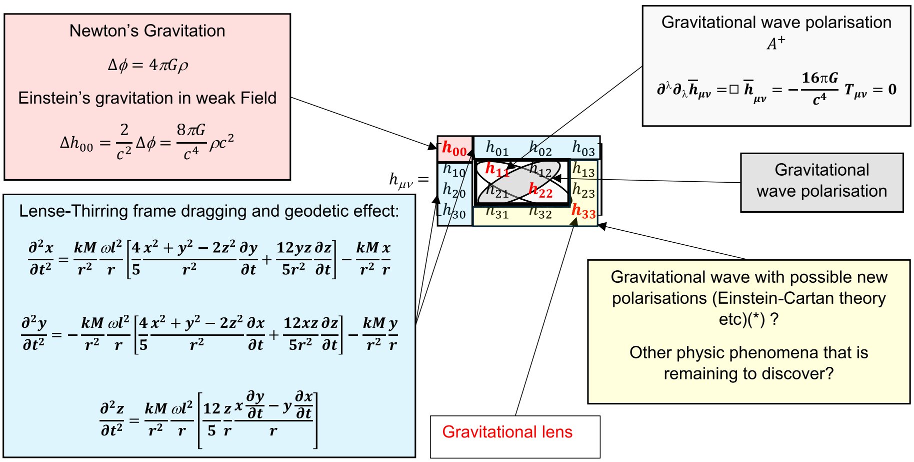


Fig. 1 Physical phenomena associated with each component of the perturbation tensor of the metric $h_{\mu\nu}$—Note (*) It is worth noting that while our analogy is developed within the classical linearized framework of general relativity and the Einstein–Cartan theory, similar correspondences have been observed in the context of exact solutions to Einstein's equations, such as pp-wave spacetimes. In particular, these solutions allow for non-trivial polarization states beyond those derived from the linearized metric perturbation hμν, as the function H(u,x,y) in the pp-wave metric satisfies a two-dimensional Laplace equation. These exact (and non-linearized) solutions to Einstein's equations, such as pp waves (plane-fronted waves with parallel rays), allow for other polarization states. These states are not derived from a linearized version, but from an exact solution, such as that given by the metric: $ds^2 = H(u,x,y)du^2 + dx^2 + dy^2 - 2dudv$ with $\left(\partial_x^2 + \partial_y^2\right)$ H = 0. This opens the possibility of extending the mechanical analogy, especially the link with plastic deformations in crystals, to stronger gravitational regimes. We acknowledge this broader perspective and refer the interested reader to [8, 9] for an example of such a connection in the context of exact solutions

the Sun, gravitational waves, Lense-Thirring effects (frame dragging, geodetic effect). These effects involve different components of $h_{\mu\nu}$ which will now be examined and analyzed through a synthetic overview. Each of these components are particular deformations of space–time (elongations, compression, angular variations) that have been precisely measured via tests carried out in recent years. The objective of this paper is therefore to provide a comprehensive and unified view of the various components of the tensor $h_{\mu\nu}$ and to interpret them within the framework of the elastic analogy, using a four-dimensional elastic strain tensor $\varepsilon_{\mu\nu}$. The aim is to assess wether these two tensors indeed describe analogous mechanical phenomena – sharing the same number of components—and to explore wether certain components remain to be investigated, potentially indicated new physical phenomena or new polarizations associated with additional degrees of freedom in the context of elastic medium analogy.

2 Methods

The methodology followed by the authors is as follows.

- For each phenomenon of general relativity in weak-field regime, identify the active components of the perturbation tensor $h_{\mu\nu}$ of the metric that are involved,
- To construct a unified perspective of the different components of the tensor as a whole, thereby identifying the active components involved in observed phenomena and the inactive ones that may correspond to unexplored or theoretical effects,
- To explicitly construct the correspondences between the specific components of space–time deformations, as positioned within the perturbation tensor of the metric $h_{\mu\nu}$, and the corresponding deformations in the elastic strain tensor $\varepsilon_{\mu\nu}$, within the context of the analogy that models space–time as an elastic medium,
- To use the analogy between the deformation tensor of the equivalent elastic medium and the perturbation tensor of the metric to identify potential new physical phenomena or space–time deformations that have yet to be observed or measured,
- To conclude by exploring the potential implications for the low-energy regime of general relativity, particularly regarding the theoretical extensions or modifications required to incorporate these additional, previously unaccounted-for components of the metric perturbation tensor.

3 Presentation and Structuring of the Different Components of the Perturbation Tensor of the Metric According to the Physical Phenomenon of General Relativity in the Weak Field Studied

3.1 Concerning the Activation of the Component h_{00}– Newton's Classical Gravitation

This framework was developed by Einstein in 1915 [1]. In order for these equations to restore Newtonian gravity in a weak field, Einstein calibrated his passage constant $\kappa = \frac{8\pi G}{c^4}$ based on the components tt (or 00) of the tensors constituting his gravitational field equation [1].

So, the componennt 00 of the metric g_{00} takes the following form. In this expression ϕ represents Newton's gravitational potential.

$$g_{00} = \eta_{00} + h_{00} = 1 + \frac{2\phi}{c^2} \tag{3}$$

Taking the Laplacian noted Δ from the above expression, we obtain:

$$\Delta h_{00} = \frac{2}{c^2}\Delta\phi \tag{4}$$

Moreover, we know that in a weak field, the gravitational field satisfies the Poisson's equation:

$$\Delta\phi = 4\pi G\rho \tag{5}$$

Thus we find:

$$\Delta h_{00} = \frac{2}{c^2}\Delta\phi = \frac{8\pi G}{c^4}\rho c^2 = \frac{8\pi G}{c^4}T_{00} = \kappa T_{00} \tag{6}$$

This equation is the transposition of Einstein's equation into a weak gravitational field. It gives Newton's gravitation, i.e. the Poisson's equation.

Space–time becomes deformable, it becomes a physical object subject to deformation when it is stressed, just like any material elastic medium that can be studied with the theory of elasticity. It is therefore within the regime of weak gravitational fields, and through the lens of the elastic analogy, that general relativity will be analyzed and interpreted in this paper.

As shown in our publication [10], the analysis of deformations associated with gravitational waves, the Lense-Thirring effect, and the deflection of starlight behind the Sun during a solar eclipse, all falls within the elastic regime, as these deformations are reversible. Spacetime returns to its initial configuration once the deformation energy has dissipated. However, this analysis reveals a foliated medium lacking coherence between layers, especially in the case of gravitational waves (in planes transverse to the direction of wave propagation). In this respect, our model partially aligns in part with those proposed by ADM [11] and by Tenev and Hortemeyer [12]. The issue is that in continuum mechanics, the medium is coherent in all directions. Although anisotropy is permitted, the layers seem to be nonetheless interconnected to some degree. To address this discrepancy and reconstruct a truly three-dimensional—or even four-dimensional -, continuum resembling a crystalline solid, it becomes necessary to introduce geometric torsion into general relativity, for example, through the Einstein–Cartan theory). In solid-state physics, the corresponding concept is the theory of defects [13]. In this context, such defects correspond to local plasticity, specifically, interlayer sliding. The medium thus becomes elasto-plastic: elastic in the transverse planes, and plastic between the planes, along the direction of gravitational wave propagation. In this sense, space–time behaves as an elastic medium within the framework of classical general relativity, but exhibits plastic behavior in the context of modified general relativity. This is what is discussed in publication [10].

To visualize this physical object embedded in transparent space–time—given that the intergalactic vacuum fills the space surrounding the stars—an effective analogy is to consider a two-dimensional membrane represented as a grid-like fabric, on which one can observe how the Sun and the Earth deform it. This membrane or plate approach [14–18] certainly has its limitations, but it allows us to clearly visualize the effects of gravitation, such as the apparent shift of stars behind the sun. This was shown in the publication [19]. The balls thrown onto this surface following the curvature of the canvas will inexorably stick to a heavy ball placed in the center simulating the appearance of a massive object like the sun, giving the impression of a Newtonian force which attracts them towards each other.

In publication [20], the authors demonstrate how a wheeled robot moving on a deformable membrane can accurately replicate the dynamics of curved spacetime. By adjusting its velocity in response to the local curvature of the surface, this active system enablesprecise mapping of radial and orbital trajectories, analogous to those described by general relativity. The study reveals that such active particles do not necessarily follow geodesics in physical space, but rather in a programmable fiducial spacetime, where parameters such as membrane elasticity and instantaneous velocity shape the metric. This framework provides a simple and accessible robophysical model for simulating relativistic effects—such as those near a black hole—and opens new perspectives for robotic exploration in complex terrains and for understanding the dynamics of active matter.

But since gravitation is a three-dimensional phenomenon in Newtonian physics and a four-dimensional phenomenon for Einstein, this analogy necessarily has limit: namely, according to Einstein both space and time are deformed [21]. Moreover, Einstein adjusted his field equation to the temporal component h_{00} of the metric perturbation with Newton's gravitational potential, a correspondence that yields highly accurate results—especially within the context of solar system dynamics [22].

3.2 Concerning the Activation of the Components h_{00}, h_{xx}, h_{yy}, h_{zz}, Associated with Newton's Classical Gravitation Applied to Gravitational Lensing – Case on Single Ray of Light

If a mass–energy slightly deforms space [23, 24], a gravitational field arises, and the space–time interval in this weakly perturbed regime is then expressed as follows [25]:

$$ds^2 = -\left(1 + \frac{2\phi}{c^2}\right)c^2dt^2 + \left(1 - \frac{2\phi}{c^2}\right)\left(dx^2 + dy^2 + dz^2\right) \tag{7}$$

The metric of this space–time is then written from the Minkowski tensor and perturbation tensor of the metric:

This metric is used in the case of gravitational lensing [25]. The interval is of the light type $ds^2 = 0$

In this case according to [25], we can obtain the propagation coordinate time:

$$t = \frac{1}{c}\int_S^0 \left(1 - \frac{2\phi}{c^2}\right)dl = \frac{D}{c} - \frac{2}{c^3}\int_S^0 \phi dl \tag{8}$$

So, gravitational lensing depends both on the Newtonian potential h_{00} and on the space curvature h_{ii}.

Thus $h_{\mu\nu}$ can be divided into 3 parts: the temporal components h_{00}, the spatiotemporal components h_{0i} and h_{j0}, and the purely spatial components h_{ij}. Each of these components corresponds to a specific phenomenon in general relativity within the weak-field regime. We will therefore examine these components in detail in the following section.

3.3 Concerning the Components $h_{ij}\left(\mathrm{i,j} \rightarrow x, y\right)$– Gravitational Waves

This was developed by A. Einstein in 1916 [2] and then in 1918 [26]. The components h_{ij} of the metric perturbation tensor are associated with gravitational waves.

In weak-field approximation, Einstein's field equation becomes [26]:

$$\partial^{\lambda}\partial_{\lambda}\overline{h}_{\mu\nu} = \Box\overline{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu} \tag{9}$$

In the specific case of gravitational waves, when considered in vacuum where: $T_{\mu\nu} = 0$, the field equation reduces to:

$$\partial^{\lambda}\partial_{\lambda}\overline{h}_{\mu\nu} = \Box\overline{h}_{\mu\nu} = 0 \tag{10}$$

h is the trace of $h_{\mu\nu}$, $\Box$: the D'Alembert's operator. The gauge condition taken is $\partial^{\lambda}\overline{h}_{\mu\lambda} = 0$.

$\overline{h}_{\mu\nu}$ represents the metric perturbation defined through the following variable transformation:

$$\overline{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{h} \tag{11}$$

The solution of this differential equation is:

$$\overline{h}_{\mu\nu} = -\frac{4G}{c^4}\iiint_{Source}\frac{T_{\mu\nu}\left(x^0 - x_s^0 - \|\vec{x} - \vec{x}_s\|\right)}{\|\vec{x} - \vec{x}_s\|}d^3\vec{x}_s \tag{12}$$

In the case of gravitational waves in vacuum, the solution of the above equation can be writren as:

$$\overline{h}_{\mu\nu} = A_{\mu\nu}cos\left(k_{\sigma}k^{\sigma}\right) \tag{13}$$

With:

$$A_{\mu\nu} = A_{+}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + A_{\times}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{14}$$

The non-zero terms are therefore at the coordinates xx, yy, xy, yx. Accordingly, we focus on the perturbation of the spatial metric h_{ij}, rather than the spatiotemporal components. Under the imposed gauge conditions, only the spatial part—associated with the deformations caused by gravitational waves—remains. These components are directly related to the sources of gravitational waves [26]; indeed, we have:

$$\overline{h}_{ij(t)} = h_{ij(t)} = \frac{2G}{Rc^4}\frac{d^2}{dt^2}I_{ij}\left(t - \frac{R}{c}\right) = \frac{2G}{Rc^4}\ddot{Q}_{ij}\left(t - \frac{R}{c}\right) \tag{15}$$

Here, R denotes the distance between the observer and the source, I_{ij} is the mass quadrupole moment, $Q_{ij(t)}$ the quadrupole moment. The deformations induces by gravitational waves are thus purely spatial and manifest as elongations and contractions in the (x, y) planes, which are perpendicular to the direction of wave propagation z as considered in this article. For simplicity, we will focus on deformations occurring in the plane throughout this article. Gravitational waves were first directly detected in 2015 through the GW150914 event [6]. Gravitational and electromagnetic waves were observed simultaneously for the first time during the GW170817 event [7]. This dual observation confirmed that gravitational waves propagate at the speed of light.

3.4 Concerning the Off-Diagonal Components $h_{0i}h_{j0}$: Frame-Dragging Effect and Geodesic Precession

This effect was first described by J. Lense and H. Thirring in 1918 [3]. It concerns the dragging of the frame of reference caused by a massive rotating object, such as the Earth or the Sun. In paper [3], formulae 4 (regarding $T_{\mu\nu}$) and 10 (regarding $g_{\mu\nu}$) show that the off-diagonal component h_{0i} and h_{j0} characterize this effect. More recent studies [27] revisit these phenomena using updated notation. The two Lense-Thirring effects—frame dragging and geodetic precession—were measured in the Gravity Probe B experiment [5].

4 Summary and Overview of the Different Components of the Perturbation Tensor of the Metric

As discussed in the previous Chapter 3, in a weak gravitational field, general relativity predicts deformations—such as elongations, shortenings, and angular distortions—both in the plane and perpendicular to it. Figure 1 above illustrates each phenomenon associated with the corresponding components of the metric perturbation tensor $h_{\mu\nu}$ where ($0 = t, 1 = x;2 = y;3 = z$) studied in this article. By superimposing these different components of $h_{\mu\nu}$ to provide, for the first time, a global and synthetic overview, we observe that some components remain inactive—specifically h_{0z}, h_{iz}, h_{zj}—with respect to the classical phenomena measured and described in chapter 3. Figure 1 above unified so structural interpretation of the components of the metric perturbation tensor $h_{\mu\nu}$, correlating known relativistic phenomena with specific elastic deformations. This diagram outlines both experimentally validated effects (gravity, gravitational waves, frame dragging) and yet-unmeasured components, suggesting new degrees of freedom possibly accessible through extended theories (Einstein–Cartan, teleparallel gravity). This visual framework proposes a structural reading of spacetime curvature and offers a roadmap for future gravitational experiments.

5 Deep Co-Correspondence Between the Deformations of Spacetime from the Perturbation Tensor of the Metric and a Deformation Tensor of an Equivalent Elastic Medium

5.1 Founding Principles and Hooke's Law Associated with the Elastic Analogy of Space–Time in Weak Field

The fundamental principles of equivalence are as follows. They are derived from well-established publications and are reinterpreted here in the context of the elastic analogy of the space–time medium.

5.1.1 Equivalence principle n°1: Correspondence Between the Metric Perturbation Tensor and the Strain Tensor Within the Elastic Medium Analogy

This equivalence principle described in [12, 28] and [29] is as follows:

$$h_{\mu\nu} = 2\varepsilon_{\mu\nu} \tag{16}$$

With:

$h_{\mu\nu}$ the metric perturbation tensor such that defines in the metric of the general relativity from the flat Minkowski metric.

$\varepsilon_{\mu\nu}$ the strain tensor in four dimensions.

Both are symmetric tensors.

In this analogy, the strain elastic tensor $\varepsilon_{\mu\nu}$ corresponds to the space–time metric pertubation tensor $h_{\mu\nu}$ related by a factor of 2.

5.1.2 Equivalence Principle n°2: Equivalence Between the Stress Energy Tensor and the Stress Tensor in Elasticity

This equivalence principle described in [28] is as follows:

In four dimensions, we have $T_{\mu\nu} = \rho_{matter} u_u u_v$. In three dimensions, the stress tensor is given by $\sigma_{ij} = \rho v_i v_j$.

In these expressions, $T_{\mu\nu}$ is the tensor energy–momentum or stress energy tensor, u_u and u_v the four-velocity of the matter distribution, σ_{ij} the stress tensor, v_i, and v_j the components of the velocity vector in space.

In our analogy, the stress tensor σ_{ij}, when extended in four dimensions $\sigma_{\mu\nu}$, corresponds to the stress energy tensor $T_{\mu\nu}$ in general relativity that is equivalent at the loading of space time.

5.1.3 Equivalence principle n°3: Einstein's Constant Characterizes the Flexibility of Space–Time

This equivalence principle, described in [28] and [29], is as follows:

–For an elastic and isotropic Timoshenko bar [30] of length L, with a cross-sectional area S, and Young's modulus Y = E, subjected to a normal force N, the bar undergoes a longitudinal displacements u_x along the direction x and therefore strains,ε_{xx} ,ε_{yy}, and ε_{zz}. If U represents the elastic energy stored in the bar, then we have:

$$\varepsilon_{xx}^2 = \left(\frac{du_x}{dx}\right)^2 = \frac{2}{ES}\frac{U}{L} = \frac{2}{YS}\frac{U}{L};\varepsilon_{yy} = \varepsilon_{zz} = -\nu\varepsilon_{xx} \tag{17}$$

$$U = \frac{1}{2}\int_0^L \frac{N^2}{ES}dx \tag{18}$$

–For an elastic and isotropic Timoshenko bar [30] of length L, with a reduced cross-sectional area S_r due to the shear loading, a shear modulus G = μ, subjected to a shear force V, the bar undergoes rotation of its cross-section. This results in shear strains ε_{xy}, and ε_{yx} associated with angular distortion γ_{xy}, and γ_{yx}:

$$\gamma_{xy}^2 = \gamma_{yx}^2 = (\frac{du_x}{dy})^2 = \left(\frac{du_y}{dx}\right)^2 = \frac{2}{GS_r}\frac{U}{L} = \frac{2}{\mu S_r}\frac{U}{L} \tag{19}$$

$$U = \frac{1}{2}\int_0^L \frac{V^2}{GS_r}dx \tag{20}$$

The classical terms $2/ES$ and $2/GS_r$ are the flexibilities of the bar. They share the same unit (N^{-1}) as the flexibility constant $\kappa = \frac{8\pi G}{c^4}$ (with G the gravitational constant), in four-dimensional space–time, as appears in Einstein's field equation.

In our analogy, the medium's flexibility—proportional to 1/ES – serves as the counterpart to the space–time flexibility κ in general relativity.

Let us generalize Hooke's law for spatial components alone, following reference [12]:

$$U_\xi = \frac{1}{2}\sigma^{ij}\varepsilon_{ij} = \frac{1}{2}C^{ijkl}\varepsilon_{ij}\varepsilon_{kl} \tag{21}$$

With C^{ijkl} the fourth rank elasticity tensor:

$$C^{ijkl} = \frac{Y}{1+\nu}\left(\frac{\nu}{1-2\nu}g^{ij}g^{kl} + g^{ik}g^{jl}\right) \tag{22}$$

The estimated value of the space–time's Young's modulus varies significantly among authors. Tenev and Horstemeyer [31], using quantum field theory, report Y = 10^{113} Pa. M. Beau [32], finds K = 1.6×10^{109} Pa, a magnitude also supported by Izabel in [28]. In contrast, R. Weiss, during his Nobel Prize lecture, derives a value of Y = $10^{20}Y_{steel}$, or approximately 2.1×10^{31} Pa, based on the energy of gravitational waves. Similarly, K. McDonald [33] arrives at a comparable estimate. For the authors of this paper, the Young's modulus is on the order of 10^{44} Pa in the transverse planes of gravitational waves. We have to note that the deformations are extremely small—on the order of 10^{-21}. If one takes the constant 1/κ and assumes it applies per unit area, one obtains a rigidity approximately of 4.8×10^{42} Pa. In publication [34] the authors demonstrate that graphene exhibits mechanical behavior with noteworthy parallels to spacetime, estimating graphene's Young's modulus at 10^{12} Pa. They further propose that certain graphene structures may provide insight into the microstructure of spacetime itself, within the Teleparallel Equivalent of General Relativity (TEGR) framework originally suggested by Einstein in his attempts to unify electromagnetism with gravitation. The authors demonstrate that the coupling constant κ converges toward its value in general relativity, thereby illuminating both the possible microstructure of spacetime and the origin of its intrinsic stiffness. Their approach explicitly incorporates geometric torsion—which, as we showed in [10], is essential for maintaining a coherent stacking of spacetime 'sheets. Their final results are as follows:

> "If the preceding hypothesis is accurate, as substantiated by these findings, and we conceptualize spacetime as a cellular structure comprised of minuscule particles, we can deduce that the force between these particles corresponds to the coupling constant. Expressed in SI units, this equation results in $c\kappa = c^4/16\pi G = 2.41596 \times 10^{42}$ N. The immense magnitude of this force indicates that spacetime possesses an extremely

high resistance to deformation (about 10^{30} times greater than that of a one-square-meter graphene sheet). Consequently, gravity, which arises from the deformation of spacetime, is a remarkably feeble phenomenon. If the force of deformation is insufficient, meaning that smaller amounts of energy cannot cause considerable deformation of spacetime. This aligns with our understanding of gravity."

5.1.4 Equivalence Principle n°4: In the Weak-Field Limit, Einstein's Field Equations are formally Equivalent to a Generalized Hooke's Law

According to [12, 28, 29], and [35] in the weak-field approximation, Einstein's field equations take the form shown in Eq. 9.

In the transverse gauge, the trace of the perturbation tensor $h_{\mu\nu}$ vanishes, so h = $\overline{h}$= 0 and therefore $\overline{h}_{\mu\nu} = h_{\mu\nu}$= 2 $\varepsilon_{\mu\nu}$, according to principle 1.

Moreover, $T_{\mu\nu}$ is related to the stress tensor in accordance with principle 2.

Since $\frac{16\pi G}{c^4} = 2\kappa$, and by Principle 3, this term is related to the flexibility of the equivalent elastic medium via 1/E = 1/Y.

The entire framework becomes conceptually equivalent to Hooke's law for an elastic medium modeling spacetime [36]:

$$\varepsilon = \frac{1}{E}\sigma \tag{23}$$

$$\gamma = \frac{1}{G}\tau \tag{24}$$

5.1.5 Equivalence Principle n°5: The Young's Modulus of the Elastic Medium can be Related to its Energy Density

In the case of elastic waves, it is shown in [12, 28] and [29] that:

–In the case of longitudinal waves:

$$Y = \rho_{vacuum}c^2 \tag{25a}$$

–In the case of shear waves:

$$\mu = \rho_{vacuum}c^2_{shear} \tag{25b}$$

Here, ρ denotes the density of the medium (the vacuum, for the purposes of this article). Thus, in our analogy, the Young's modulus of spacetime serves as a bridge to the elastic strain energy density of the vacuum itself. So vacuum as an intrinsic energy as demonstrated by the Casimir force [37–41] that becomes elastic in our model

5.1.6 Principle n°6: The Polarization Modes of Gravitational Waves can be Interpreted as Components of a Strain Tensor in the Vacuum, Modeled as an Elastic Medium

This principle described in [28] is based on principle 1 and is as follows:

$$h_{\mu\nu} = A_{+}\cos\left(\frac{\omega}{c}(ct-z)\right)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \varepsilon_{xy(A_{+})} = \frac{1}{2}A_{+}\cos\left(\frac{\omega}{c}(ct-z)\right)\begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$h_{\mu\nu} = A_{\times}\cos\left(\frac{\omega}{c}(ct-z)\right)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \varepsilon_{xy(A_{\times})} = \frac{1}{2}A_{\times}\cos\left(\frac{\omega}{c}(ct-z)\right)\begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{26}$$

From the perspective of deformation—and hence from a mechanical viewpoint—these two expressions describe a state of pure torsion in the elastic medium, manifested as two shear modes oriented at 45° to one another. This formulation is fully consistent with the relativistic origin of gravitational-wave polarizations, namely the orbital motion of a binary black-hole system, which induces a torsional distortion in spacetime [10, 28, 29].

5.1.7 Postulate n°1 The Necessity of Accounting for the Intrinsic Deformation Energy of Vacuum Spacetime Itself

Classical general relativity describes how spacetime responds when "charged" by mass or energy. However, in a true vacuum the classical energy–momentum tensor $T_{\mu\nu}$ vanishes. To retain a Hooke's-law–type relation between stress and strain via a proportionality constant, one may introduce an additional tensor $t_{\mu\nu,el}$ that represents the vacuum's elastic strain energy—an approach developed in [42] and [43]. In this extended framework, Einstein's field equations take the form:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} + t_{\mu\nu}\right) \tag{27}$$

And in the vacuum:

$$\begin{cases} T_{\mu\nu} = 0 \\ t_{\mu\nu} \neq 0 \end{cases} \tag{28}$$

where $t_{\mu\nu} \sim Y\varepsilon_{\mu\alpha}\varepsilon^{\alpha}_{\nu}$, with Y the effective modulus of the vacuum, and $\varepsilon_{\mu\nu}$, the strain tensor. Assuming the off-diagonal components are negligible compared to the principal strains ε_{tt}, ε_{xx}, ε_{yy}, and ε_{zz}, the trace $t_{\mu\nu}$ may be interpreted as the elastic strain energy:

$$U = \frac{1}{2}\rho(c)^2 = -\frac{1}{2}\varepsilon_{tt}^2 Y_t + \frac{1}{2}\varepsilon_{xx}^2 Y_x + \frac{1}{2}\varepsilon_{yy}^2 Y_y + \frac{1}{2}\varepsilon_{zz}^2 Y_z \tag{29}$$

The tensor $t_{\mu\nu}$ can be interpreted as an internal stress perturbation of the vacuum. It may also be viewed as the Hookean response of an elastic medium to geometric deformations—an idea dating back over a century and echoing Sakharov's induced-gravity concept

[44]. This additional vacuum elastic-energy tensor is thus intimately linked to gravitational energy in empty space, a subject that remains unresolved in modern physics.

Indeed, in classical general relativity the gravitational field lacks a local energy–momentum tensor because of the equivalence principle. Various approaches have been proposed to define gravitational energy. The earliest are the energy–momentum pseudotensors t^{ik} of the gravitational field (Einstein, Landau–Lifshitz) [45], where in chapter 11, one write:

$$(-g)\left(T^{ik} + t^{ik}\right) = \frac{\partial h^{ikl}}{\partial x^l}$$

with $t^{ik} = t^{ki}$ the symmetric pseudo-tensor. Later, the ADM energy formalism [46, 47] was developed by decomposing the spacetime metric into three spatial dimensions and one temporal dimension, thereby facilitating the analysis of gravitational dynamics and energy. One writes: $ds^2 = -N^2dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)(dx^j + N^j dt)$ where N is the lapse function and γ_{ij} is the 3D spatial metric. In this approach, gravitational energy takes the form: $E_{ADM} = \frac{1}{16\pi G} lim_{r\to\infty} \int_{S_r} \left(\partial_j\gamma_{ij} - \partial_i\gamma_{jj}\right) n^i dS$.

This direction is consistent with an anisotropic space medium composed of transversely strained sheets under longitudinal gravitational plane waves, as studied by the authors in [10].

We also have the Komar energy (or equivalently, mass integrals) [48]: $m = \int_V \left(2T_{ab} - Tg_{ab}\right) u^a \xi^b dV$ where T_{ab} is the stress-energy tensor, T its trace, u^a a velocity vector, and ξ^b a Killing vector field. Finally, quasi-local energy definitions (Hawking, Penrose, Brown-York) [49, 50], and [51] attempt to define gravitational energy within a finite spacetime region bounded by a closed surface (typically a 2-sphere). These approaches aim to circumvent the fact that gravitational energy cannot be localized (due to the absence of a well-defined energy tensor for gravity) by defining it consistently over a finite domain.

These definitions are either coordinate-dependent (as with pseudo-tensors) or limited to asymptotic or stationary regimes (ADM, Komar) [46–48], and thus do not offer a general covariant local formulation [10, 52].

Therefore, in the context of weak fields and the elastic analogy, we propose a symmetric tensor $t_{\mu\nu}$ to model the intrinsic elastic energy of spacetime deformations—even in vacuum, i.e., in the absence of matter. This tensor plays a role similar to the effective stress-energy content used in Balbus averaging of gravitational waves [53]. Thus, in our elastic gravity framework, we encounter the same foundational challenge as in classical GR regarding the definition of gravitational energy. However, since $Y = \rho c^2$—the vacuum energy density related to mass density ρ and the Young's modulus of the medium—our elastic approach is conceptually closest to the pseudo-tensor formulation.

In the case of the pseudotensor approach—such as when computing the energy of gravitational waves in vacuum—the Einstein field equations take the form:

$$G^{(1)}_{\mu\nu} = -\frac{8\pi G}{c^4}\left(T_{\mu\nu} + t_{\mu\nu}\right) \tag{30}$$

Here, $G^{(1)}_{\mu\nu}$ contains only the term $G_{\mu\nu}$ linear in $h_{\mu\nu}$ (see Eq. 4 of [54]) and the pseudo tensor $t_{\mu\nu}$ is defined by:

$$t_{\mu\nu} = T^{GW}_{\mu\nu} = \frac{c^4}{8\pi G}\left[G^{(2)}_{\mu\nu} + \dots\right] \tag{31}$$

where $G^{(2)}_{\mu\nu}$ comprises the quadratic terms of $h_{\mu\nu}$ (see Eq. 5 of [54]):

5.2 Quantum Beams and the Mechanics of Spacetime: A Unified Structural Framework for the Universe

Basing on the principles definied above, and to illustrate our elastic analogy, imagine that, in the weak-field limit, spacetime is a network of Timoshenko beams [30], each with a thickness on the order of the Planck length [12, 31]—effectively quantum-scale beams much stiffer than simple strings. These beams bear mass (or an energy density $\frac{E}{V} = \frac{Mc^2}{V} = \rho c^2$) at specific nodes that undergo continuous dynamic motion. In 1915–1916, Einstein effectively proposed a mechanical model of this structure, in which the displacements of these nodes are determined by the spacetime metric $g_{\mu\nu}$, and hence by geometry. The interval ds^2 then corresponds to the squared length of an infinitesimal beam element. The constant κ could be interpreted as a measure of flexibility of these bars. This formulation enables the calculation of node displacements through a law analogous to: $E = RC^2$ (Energy density = Rigidity × Curvature^2 for a bending beam, where $\frac{U}{L} = \frac{1}{2}EI\frac{1}{R^2}$).

Notice: The Einstein field equation in general relativity is often written in the form: curvature = κ × energy density, where the unit of curvature is 1/m^2. In this context, curvature refers to Gaussian curvature, originally introduced by Gauss [55] and later generalized by Riemann. It characterizes the intrinsic curvature of a surface as perceived from within and is derived from second derivatives of the metric tensor, hence its unit of 1/m^2. In the mechanical analogy using beams or Timoshenko plates, curvature typically refers to the inverse of the radius of curvature, 1/R. From beam theory, the bending equation $\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{1}{R}$ relates curvature to the bending moment M, Young's modulus E, and the moment of inertia I. When this is combined with the expression for the elastic strain energy $U = \frac{1}{2}\int_0^L \frac{M^2}{EI}dx$, we obtain a result in one or two dimensions (beam or plate) of the form: $\frac{1}{R^2} = \frac{2}{EI}\frac{U}{L}$. This yields a term on the left-hand side with units of 1/m^2, aligning precisely with the units of curvature in general relativity.

Thus, although curvature is initially expressed differently in the two contexts—1/R in mechanics and via second derivatives of the metric in general relativity—both ultimately converge to a quantity with the same physical units when squared. This formal resemblance is striking and reinforces the validity of the analogy. However, it's important to be aware that the term"curvature"carries different conceptual meanings in the two frameworks, which can sometimes lead to confusion. Thus, the curvature of space–time (expressed via $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ in GR) arises from the load applied (expressed via $T_{\mu\nu}$), just as in beam theory. For example, in general relativity, the scalar curvature of a sphere is exactly $\frac{2}{R^2}$, which directly parallels the mechanical result when squared curvature is involved. From this perspective, general relativity can be interpreted as a displacement method: it allows the calculation of both instantaneous local displacements of nodes in 3D space and delayed displacements—i.e., strains $\varepsilon_{\mu\nu}$ or metric perturbations $h_{\mu\nu}$ —that propagate over time at the speed of light c, governed by the mechanical properties (e.g., stiffness, density of the medium) of the structural elements of spacetime that constitute a sort of cristal [34, 56, 57]. That is why GR is well exprimed in 4 dimensions to take into account of these two types of deformations instantaneous and differed. In this elastic analogy, $t_{\mu\nu}$ represents the internal stresses associated with deformations and displacements. It includes contributions from the self-weight or intrinsicstress of the structure itself—identified here with the quantum vacuum, characterized by a vacuum energy density ρ_{vacuum}. including those due to the self-weight of the structure—namely, the quantum vacuum with density ρ_{vacuum}.

This structural model would also be sensitive to thermal gradients between the cosmic web—a relatively warm, matter-rich environment—and the much colder quantum vacuum.

Such gradients could drive the expansion of space and result in a residual thermal curvature present throughout the universe. This effect could manifest as a form of mysterious energy that evolves over time, consistent with recent observations made by the Dark Energy Spectroscopic Instrument (DESI) in 2025 [58]. The resulting temperature differential between the warm cosmic web and the cold vacuum introduces a form of thermally driven deformation in the spacetime structure.

Moreover, under long-term stress, this spacetime"material"would exhibit creep behavior—a slow, continuous deformation under constant loading. This phenomenon may offer a physical analogy for the presence of dark matter, whose gravitational effects imply more deformation (curvature) than can be explained by visible matter alone [59]. In this framework, dark matter emerges not as an unknown particle, but as a manifestation of structural response—a deeper deformation within the elastic medium of spacetime.

This structure may also exhibit defects and localized plastic behavior, much like a crystal, resulting in anisotropic responses under specific loading conditions. This behavior is analogous to the presence of geometric torsion and defect structures in the Einstein–Cartan theory of gravity [10, 13] with all associated polarizations longitudinally (remaining to measure) and transversally (already measured).

Furthermore, the eigenfrequencies and eigenmodes of these quantized quantum beams could be directly related to the energy levels and wavefunctions of associated quantum wells [60]. In this context, wave propagation through the structure—triggered by violent, localized disturbances—mirrors the dynamic response of beam-column systems under seismic loading. This is the mechanical analogue of gravitational waves, which are generated by cataclysmic astrophysical events such as black hole mergers [10, 12, 28].

If this structure becomes locally overloaded at a given node, plastic hinges may form at that location. In such cases, the connected beams behave like a mechanism, allowing relative rotations. As loading continues, the internal fibers may exceed their elastic limit, and the characteristic triangular stress distribution transitions to a bi-rectangular profile, indicating the onset of plasticity. The hinge localizes into a single point, where a singularity emerges in the structure. This phenomenon serves as a mechanical analogue of black holes, as described in [29].

Moreover, when the beams are subjected to high compressive forces, they may undergo buckling. There exists a compelling analogy between buckling phenomena in mechanics and spontaneous symmetry breaking in quantum field theory, as discussed by J. Iliopoulos in the context of the Higgs boson [61].

So, quantum mechanics emerges so as the dynamic theory of an elastic microstructure of spacetime: energy levels correspond to vibrational modes of quantum-scale beams, wavefunctions to deformation shapes, and spontaneous symmetry breaking to mechanical buckling. Quantization is not imposed axiomatically but arises naturally as the modal behavior of a structured medium.

Finally, in this framework, all fundamental physical phenomena emerge and are unified from the mechanical behavior of a quantum-scale beam network structuring spacetime. General relativity becomes the static deformation of this lattice—its curvature representing gravitational effects. Gravitational waves correspond to vibrational modes propagating through the beam structure. Quantum mechanics arises naturally as the quantization of these vibrational states, with energy levels corresponding to modal eigenfrequencies. Quantum field theory is interpreted as the study of instability and buckling within this network, leading to symmetry breaking as observed in the Higgs mechanism. Dark energy appears as a global thermal curvature—analogous to a thermal expansion of the spacetime framework—while dark matter results from the long-term viscoelastic creep of the medium. Finally, black holes are understood as plastic hinges, rupture zones where the structural continuum collapses. This unified vision reframes the universe not as abstract geometry, but as a structured, dynamic, and responsive mechanical entity. Ultimately, this perspective suggests that the universe is not a mere abstract mathematical construct, but rather an immense feat of engineering, a sort of cosmic Eurocode — a vast, structured edifice whose fabric, dynamics, and evolution obey the deep laws of mechanics. So, in our analogy, The Fabric of the Universe become an Elastic Lattice made of quantum beams: A New Paradigm.

5.3 The Mechanical Constitution of the Equivalent Elastic Medium Behaving like Space–Time

The literature surrounding the definition of this equivalent elastic medium is extensive and multifaceted. Notable contributions include the works of Tenev and Horstemeyer [12], Izabel [28, 29, 58], and many others [10, 35, 44, 62–66]. Generally, two principal estimations of the Young's modulus of spacetime emerge. The first, derived from quantum field theory and the vacuum energy density, places it around 10^{113} Pa [12, 28, 29, 58, 62]. The second, based on the elastic energy associated with spacetime deformations due to gravitational waves, reaches much lower values—on the order of 10^{31} Pa [67].

It is also important to highlight that the mechanistic approach to spacetime—constructed as a network of quantum-scale beams—not only provides a structural framework for interpreting general relativity and quantum mechanics, but also offers a natural convergence point for the fundamental constants of physics. In this perspective, key constants such as the speed of light c, Planck's constant $\hbar$, and Newton's gravitational constant G become interrelated through the mechanical properties of spacetime itself. For instance, in the work of Tenev and Horstemeyer [12, 31] confirmed by Izabel [28, 29] the elastic modulus of the quantum vacuum is expressed as $Y = \frac{6c^7}{2\pi\hbar G^2}$, an equation remarkably similar in structure to the Hawking temperature formula for black hole radiation. This convergence suggests that mechanical properties such as stiffness, energy density, and curvature are not merely analogues but are physically and dimensionally linked to the constants that govern the deepest layers of reality. Thus, the structural model not only unifies physical theories but also binds together the constants that define them.

Taking into account that the Young's modulus Y can be related to an energy density through the expression $Y = \rho c^2$, we encounter a situation analogous to the well-known discrepancy between the quantum vacuum energy predicted by quantum field theory and the energy density associated with the cosmological constant Λ. This is the infamous"vacuum catastrophe,"where the theoretical prediction exceeds the observed value by approximately 10^{120} orders of magnitude [68]. However, within the elastic analogy framework, this discrepancy is somewhat reduced: the gap is on the order of 10^{80}, which—while still enormous—represents a narrowing of the problem when interpreted through the lens of mechanical elasticity.

The literature [12, 28, 29, 58] consistently reports a Poisson's ratio of 1 when analyzing the characteristic deformations induced by gravitational waves. This reflects a behavior in which space deforms transversely without any longitudinal contraction or expansion, a hallmark of incompressibility under transverse strain. Additionally, the"particle size"of the equivalent elastic medium—often interpreted as a fundamental scale of granularity—tends toward the Planck length, as discussed in [12, 28, 29, 58, 62], and [69].

A more detailed mechanical analysis of gravitational waves [10] reveals that this medium exhibits anisotropic behavior. Specifically, deformations occur primarily in planes perpendicular to the wave propagation direction, while there is virtually no deformation along the direction of propagation. This dynamic response suggests that spacetime behaves as if composed of layered sheets—or"leaves"—which lack cohesion in the longitudinal direction. Such behavior is indicative of a laminated or stratified medium with weak coupling between layers, echoing models in elasticity theory where out-of-plane coherence is minimal or absent.

When geometric torsion is added to the Riemann curvature tensor, the framework of general relativity is modified. This extension gives rise to additional gravitational wave polarizations and, following Principle 6, leads to complementary deformations between the spatial sheets that make up spacetime [67, 70, 71]. Such phenomena are also observed in gravito-electromagnetism, when general relativity is expanded to second-order approximations [13, 72], or when spacetime is modeled as a fluid medium, with light behaving analogously to acoustic waves in hydrodynamics [73].

These approaches collectively allow us to reconstruct a coherent elastic medium in three dimensions, capable of transmitting not only curvature-induced deformations but also shear and torsional effects—attributes that classical general relativity (which neglects torsion) does not capture.

Notably, the mathematical formalism of geometric torsion closely mirrors that of defect theory and plastic crystallography [13, 67, 70–72], suggesting that spacetime could be understood as a medium with internal defects and dislocations—much like a crystalline solid under stress.

In this context, Ref. [74] offers a rigorous geometric interpretation of dislocations and disclinations within the framework of Riemann–Cartan geometry. In this formalism, torsion and curvature serve as the geometric analogs of Burgers vectors and Frank vectors, respectively, interpreted as surface densities. This geometric correspondence reinforces the analogy with defect theory in solid-state physics, where dislocations and disclinations represent discontinuities in the lattice structure.

Moreover, these results confirm that asymmetric elasticity theories and chiral field models can arise naturally from such a geometric framework. This supports the idea that torsion should be incorporated into the geometry of spacetime—not merely as a mathematical extension, but as a physically motivated necessity for describing microscopic deformations and interactions beyond the linear regime of classical general relativity.

Based on this state of the art, it is therefore reasonable to consider that spacetime can be modeled as an elastic medium which, in the regime of weak gravitational fields, obeys a form of Hooke's law. However, this medium exhibits extremely low—virtually negligible—flexibility, corresponding to an almost perfectly rigid behavior. This extreme stiffness is consistent with the very high equivalent Young's modulus inferred from vacuum energy densities and the observed dynamics of gravitational waves. Such a model provides a coherent mechanical analogy for understanding spacetime deformations, while also capturing the geometric richness of general relativity.

5.4 Systematic Correspondences Between Measured Weak-Field Space–Time Deformations and those of an Equivalent Strain Tensor

It is well known in elasticity theory that the diagonal components of the strain tensor are related to elongations and shortening of an elementary volume and that the other components are related to angular deformations.

If we superimpose the various components of the tensor $h_{\mu\nu}$ with the tensor $\varepsilon_{\mu\nu}$, (see Fig. 1), we observe the following:

h_{00} associated with the product $c.t$, is the first diagonal term related to the time component of the tensor. It corresponds to an isotropic compression in all directions—representing classical gravitation, which binds together the Earth, planets, stars, and other celestial bodies,

$h_{ii} = -h_{jj}$: These diagonal terms associated with the spatial components of the tensor and the A^+ polarization, correspond to the spatial elongations and contractions measured by interferometers such as Ligo and Virgo during the passage of a gravitational waves,

$h_{ij} = h_{ji}$: These are the off-diagonal transverse spatial terms, associated with the $A^\times$ polarization. They correspond to shear (or torsional) deformations depending on the orientation of the facets (typically at 45°). These angular distortions are still to be measured by interferometers during the passage of a gravitational wave.

h_{0i};h_{j0}: These time–space components correspond to angular distortions generated both within the orbital plane and perpendicular to it by the Lense–Thirring effect—namely, frame dragging and the geodetic effect caused by a rotating massive body.

Note: We consider here various components of the metric perturbation. However, since they are expressed in different reference frames—such as the Earth's center for the Lense–Thirring effect, and the transverse-traceless (TT) gauge for gravitational waves—they cannot be directly compared within a single tensor without first being transformed into a common coordinate system. Nevertheless, the key point is that the active components of a general $h_{\mu\nu}$ tensor, once expressed within a unified and consistent framework encompassing all relevant phenomena, can be meaningfully associated with specific mechanical deformations. This allows for an analogy with a four-dimensional strain tensor of an elastic spacetime medium.

$$\begin{bmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{10} & h_{11} & h_{12} & h_{13} \\ h_{20} & h_{21} & h_{22} & h_{23} \\ h_{30} & h_{31} & h_{32} & h_{33} \end{bmatrix} \rightarrow \begin{bmatrix} GravitationorGLense & Lense-Thirring & Lense-Thirring & Lense-Thirring \\ Lense-Thirring & GWLIGOorGLense & GW & NewGW? \\ Lense-Thirring & GW & GWLIGOorGLense & NewGW? \\ Lense-Thirring & NewGW? & NewGW? & NewGW?orGLense \end{bmatrix} \tag{32a}$$

Physicist's and General Relativity's Viewpoint

$$\begin{bmatrix} h_{tt} & h_{tx} & h_{ty} & h_{tz} \\ h_{xt} & h_{xx} & h_{xy} & h_{xz} \\ h_{yt} & h_{yx} & h_{yy} & h_{yz} \\ h_{zt} & h_{zx} & h_{zy} & h_{zz} \end{bmatrix} \rightarrow 2 \times \begin{bmatrix} \varepsilon_{tt} & \frac{1}{2}\gamma_{tx} & \frac{1}{2}\gamma_{ty} & \frac{1}{2}\gamma_{tz} \\ \frac{1}{2}\gamma_{xt} & \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yt} & \frac{1}{2}\gamma_{yx} & \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zt} & \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_{zz} \end{bmatrix} \tag{32b}$$

Physics and Mechanical Analogy: Elastic Medium Behavior in Weak Gravitational Fields

6 Consequences of Analogy—Identification of Components not yet Associated with Physical Phenomena in General Relativity and Possible New Deformations, Degrees of Freedom of Space, Time and Polarization

As demonstrated in publication [28], the components h_{ij}, by analogy with the presumed distortion tensor associated with torsion of space caused by the rotation of two stars orbiting each other, correspond to angular movements in the planes of the interferometer arms. These angular deformations have yet to be measured and are expected to be detectable with next-generation interferometers such as LISA.

The components h_{0z}, h_{xz}, h_{yz} do not correspond to deformations that have been identified or measured within classical general relativity.

It is also important to remember that the degrees of freedom associated with gravitational waves deformations correspond physically to the polarizations of those waves. Thus, if the $h_{0z}, h_{xz}, h_{yz}, h_{zz}$ are linked to complementary polarizations, this would imply additional degrees of freedom in the mechanical analogy—specifically, components of angular deformation (for $h_{0z}, h_{xz}, h_{yz}, h_{zz}$) as well as elongation or contraction along the direction of wave propagation, as demonstrated in [10].

These complementary polarizations have not yet been measured and therefore remain speculative at this stage. It should be remembered that they do not emerge within the framework of classical general relativity but arise only in modified theories such as Einstein-Cartan gravity or related extensions.

Summarizing the theoretical context in which our work is situated, we will assume that both general relativity and continuum mechanics are based on the principle of general covariance, which, as B. Kolev aptly summarizes in [75], requires the introduction of three components:

- Lagrangian-type functionals $\mathcal{L}$, depending on the metric g defined on the 4-dimensional universe manifold M, and depending on various fields Ψ;
- Tensors, such as the energy–momentum tensor, dependent on these fields;
- Field equations.

Our approach is therefore placed in the framework of the general covariance of field equations by the group of diffeomorphisms. Or expressed explicitly, a Lagrangian $\mathcal{L}(g, \Psi, ..)$ is general covariant if it verifies $\mathcal{L}(\boldsymbol{\varphi}^* g, \boldsymbol{\varphi}^* \Psi) = \mathcal{L}(g, \psi)$ for any diffeomorphism $\boldsymbol{\varphi}$. Note, still with B. Kolev, that the functional$\mathfrak{H}(g) = \int R_g Vol_g$, where R_g represents the scalar curvature, is general covariant. Its gradient L^2, the Einstein tensor $G(g)$ is also general covariant. It consequently verifies:$\forall \boldsymbol{\varphi}, G_{\boldsymbol{\varphi}^* g} = \boldsymbol{\varphi}^* G(g)$, and $div^g G(g) = 0$. An energy–momentum tensor $T(g, \Psi)$ verifying the Einstein equation $G(g) = T(g, \Psi)$ therefore verify the mechanics type equation:$divT = 0$. The rest of this work will therefore concern energy–momentum tensors verifying these two properties. The reader is referred to the work of J. M. Souriau [76, 77] and his successors already cited, for the definitions of the perfect matter field (as a section of a vector bundle) and the conformations, allowing the proposal of relativistic constitutive laws [78].

The problem of defining gravitational energy in general relativity remains open. While matter and non-gravitational fields are described by $T_{\mu\nu}$, the gravitational field itself resists a covariant and local description. The introduction of a vacuum energy tensor $t_{\mu\nu}$ is motivated by this gap, and aims to provide a mechanical representation of spacetime response, within the elastic analogy, beyond the constraints of pseudo-tensors. This analogy aligns with the spirit of quasi-local definitions [52] and extends it to a tensorial framework inspired by elasticity.

7 Possible Experimental Validation: Optimizing Gravitational Wave Detection to Reveal Additional Polarizations and Associated Strain and Metric Perturbations

To complete the description of the components of the metric perturbation tensor—particularly h_{0z},h_{xz}, h_{yz}, and their symmetric counterparts—several avenues can be explored.

- Complementary polarizations in the direction of gravitational wave propagation of that have not yet been experimentally observed [12] to [10, 58].

- Hypothetical pure shear stresses within the fabric of space–time, suggesting new forms of deformation beyond those currently accounted for in classical general relativity.

In any case, as Paul Langevin aptly noted, "tensors are always one step ahead of physicists"—the implication being that the currently unpopulated components of the metric perturbation tensor are unlikely to remain empty forever. It is highly plausible that physical phenomena not yet described by classical or modified general relativity remain to be discovered, and these could naturally fill in the missing components.

Next-generation instruments such as the LISA interferometer, the Einstein Telescope, and pulsar timing arrays are designed with the sensitivity required to detect these potential new gravitational wave polarizations or complementary spacetime distortions. In particular, they may also enable the detection of lateral angular deviations of the interferometer arms—an effect predicted by analogy with torsional deformations [10, 28].

8 Conclusion

We show that a simultaneous analysis of the different components of the metric perturbation tensor, $h_{\mu\nu}$ which corresponds—within the elastic analogy of gravitation – to twice the elastic strain tensor $\varepsilon_{\mu\nu}$, provides a deeper understanding of general relativity through the lens of the continuum mechanics in the weak-field regime.

The components h_{00}, h_{0i}, h_{j0}, $h_{ij}(x, y)$ enable the characterization of physical effect—such as linear elongations, compressions or angular distorsions—that are fully consistent, under the elastic analogy, with the behavior of a strain tensor describing space–time as an elastic medium in the weak-field approximation, provided that all relevant components of $h_{\mu\nu}$. are consistently expressed within the same reference frame. Einstein's field equation can be interpreted as a generalized form of Hooke's law in four equivalent spatial dimensions, since in the interval ds^2, time is scaled by the speed of light, effectively reconstructing a fourth dimension with the squared length term c^2dt^2. The speed of light acquires its value as the square root of the ratio between the effective Young's modulus of the space–time fabric and the density of the equivalent medium. Since space–time is a dynamic entity, instantaneous and absolute deformations are not physically possible. Some deformations at a given point can be thought of as'in transit'—still propagating through the medium [79, 80]. This highlights the necessity of treating space–time as a genuinely four-dimensional mechanical system, rather than a merely three-dimensional one, to accurately describe its behavior [81, 82]. In classical general relativity, gravitational waves exhibit two polarization modes, corresponding to the two expressions of the strain tensor under pure shear—each oriented along facets rotated 45° with respect to the other. This could potentially give rise to complementary lateral displacements of the interferometer arms—undetectable by LIGO or VIRGO, but possibly observable with next-generation detectors such as the Einstein Telescope or LISA. The interval of special relativity can be interpreted as an elastic strain equation that incorporates both aspects: the spatial deformations that have already arrived at a given point and time, and those that are still in transit. In this sense, what we measure are local spatial deformations minus those that have not yet reached the observer—leading to differences in perception and quantification depending on the observer's position. Space–time exhibits mechanical properties fundamentally different from those of conventional terrestrial materials [83]. Its effective Young's modulus varies significantly depending on the direction of deformation: estimates suggest values on the order of 10^{20} Pa for deformations perpendicular to the plane of propagation and 10^{40} Pa within the plane itself [29, 58]. A Poisson's ratio of 1 is observed in-plane. These anisotropic characteristics imply that the'engine'driving time differs from that governing spatial behavior—if one accepts the correlation between Young's modulus and the energy density of the medium.

This analysis of the perturbation tensor of the weak-field metric—based on well-documented experimental results in general relativity (such as gravitational waves GW150914 and GW170817, Gravity Probe B, and Earth- or Sun-based gravitation [84])—also demonstrates that the elastic analogy of gravitation plays a predictive role. It enables the consideration of additional space–time deformations, extremely small in magnitude (on the order of 10^{-21} for gravitational waves), which classical general relativity without geometric torsion appears to overlook. In particular, additional polarization modes seem necessary beyond the two canonical ones, involving components such as $h_{0z}, h_{xz}, h_{yz}, h_{zz}$ and their symmetric counterparts. These suggest a complementary set of strain tensor components, analogous not to a string model but rather to a Timoshenko bar under pure compression [30], incorporating shear and angular distortions.

According to our elastic analogy, the weak-field deformations of space–time, represented by the metric perturbation tensor $h_{\mu\nu}$ can be interpreted as those of a four-dimensional elastic medium governed by a strain tensor $\varepsilon_{\mu\nu}$.This tensor captures longitudinal deformations—such as elongation and compression—through its diagonal terms, and shear or angular distortions through its off-diagonal terms. Some components of the perturbation tensor have already been measured and experimentally verified – such as h_{00} for gravitation, h_{ij} for gravitational waves, and h_{0i}, h_{j0} for frame-dragging effects. Others, however, remain to be discovered, particularly those associated with the direction of gravitational wave propagation – namely $h_{0z}, h_{xz}, h_{yz}, h_{zz}$. Such is the central message of this article, assuming the validity of the proposed elastic analogy. By establishing a direct correspondence between the metric perturbation tensor and the elastic strain tensor—$h_{\mu\nu} = 2\varepsilon_{\mu\nu}$— the elastic analogy in weak gravitational fields provides theoretical support for ongoing investigations into the existence of additional space–time polarizations, as suggested by several advanced extensions of general relativity [85–90].

Acknowledgements Thank you verry much for his advises, verry interesting references that we have added in the paper

Author Contribution D.I Wrote the paper, Y.Remond and M.L Ruggiero have reviewed the manuscript.

Funding No funding.

Data Availability No datasets were generated or analysed during the current study.

Declarations

Competing Interests The authors declare no competing interests.

References

1. Einstein, A.: Die Feldgleichungen der Gravitation, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), Seite, vol. 48, pp. 844–847 (1915)

2. Einstein, A.: Näherungsweise Integration der Feldgleichungen der Gravitation, Sitzung der physikalisch mathematischen Klasse, pp. 688–696 (1916)
3. Lense, J., Thirring, H.: Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie. Physikalische Zeitschrift **19**, 156–163 (1918)
4. Dyson, F.W., Eddington, A.S., Davidson, C.: A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations Made at the Total Eclipse of May 29, 1919. Philos. Trans. R. Soc. **220**, 291–333 (1920)
5. Everitt, C.W.F., DeBra, D.B., Parkinson, B.W., Turneaure, J.P., Conklin, J.W., Heifetz, M.I., Keiser, G.M., Silbergleit, A.S., Holmes, T., Kolodziejczak, J., Al-Meshari, M., Mester, J.C., Muhlfelder, B., Solomonik, V., Stahl, K., Worden, P., Bencze, W., Buchman, S., Clarke, B., Al-Jadaan, A., Al-Jibreen, H., Li, J., Lipa, J.A., Lockhart, J.M., Al-Suwaidan, B., Taber, M., Wang, S.: Gravity Probe B: Final Results of a Space Experiment to test General Relativity. Physical Review Letter **106**, 221101 (2011)
6. Abbott, B.P., et al.: (LIGO scientific collaboration and Virgo collaboration), Observation of Gravitational Waves from a Binary Black Hole Merger GW150914. Physical Review Letter **116**, 061102 (2016)
7. Abbott, B.P., et al.: (LIGO scientific collaboration and Virgo collaboration), GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral. Physical Review letter **119**, 161101 (2017)
8. Podolsky, J., Steinbauer, R., Svarc, R.: Gyratonic pp waves and their impulsive limit. Phys. Rev. D. **90**, 044050 (2014)
9. Carneiro, F.L., Ulhoa, C., Maluf, J.W., da Rocha-Neto, J.F.: Non-linear plane gravitational waves as space-time defects. Eur. Phys. J. C. **81**, 67 (2021)
10. Izabel, D., Remond, Y., Ruggiero, M.L.: Some geometrical aspects of gravitational waves using continuum mechanics analogy: State of the art and potential consequences. Mathematics and Mecanics of Complex Systems **13**, 2 (2025)
11. Arnowitt, R., Deser, S., Misner, C.W.: The Dynamics of General Relativity. Gen. Rel. Grav. **40**(2008), 1997–2027 (1962)
12. Tenev, T.G., Horstemeyer, M.F.: Mechanics of spacetime — A solid mechanics perspective on the theory of general relativity. Int. J. Mod. Phys. D **27**, 1850083 (2018)
13. Ruggiero, M.L., Tartaglia, A.: Einstein-Cartan theory as a theory of defects in spacetime. Am. J. Phys. **71**, 1303–1313 (2003)
14. Kokarev, S.S.: space time as strongly bent plate. Nuovo Cim. B **114**, 903–921 (1999)
15. Perko, H.A.: Introducing surface tension to spacetime;J. Phys. Conf. Ser **845**, 012003 (2017)
16. Perko, H.A.: Gravitation in the surface tension model of space-time. J. Phys. Conf. Ser. **1239**, 012010 (2019)
17. Hencky, H.: Über Den Spannungszustand in Kreisrunden Platten Mit Verschwindender Biegungssteigkeit. Zeitschrift fur Mathematik und Physik **63**(1915), 311–317 (1915)
18. Perko, H.A.: Dark matter and dark energy: Cosmology of spacetime with surface tension. Communication présentée à la 12th Biennial Conference on Classical and Quantum Relativistic Dynamics of Particles and Fields, Prague, Czechia (2021). Disponible en ligne : https://v2020.iard-relativity.org/assets/articles/perko_paper.pdf
19. Catheline, S., Delattre, V., Laloy-Borgna, G., Faure, F., Fink, M.: Gravitational lens effect revisited through membrane waves. Am. J. Phys. **90**(1), 47–50 (2022)
20. Li, S., Gynai, H.N., Tarr, S.W., et al.: A robophysical model of spacetime dynamics. Sci. Rep. **13**, 21589 (2023)
21. Kersting, M.: Free fall in curved spacetime—how to visualise gravity in general relativity. Phys. Educ. **54**(03), 5008 (2019)
22. Touboul, P., et al.: (MICROSCOPE Collaboration) MICROSCOPE mission: final results of the test of the equivalence principle. Phys Rev. Lett. **129**, 121102 (2022)
23. Robertson, S.: Optical Kerr effect in vacuum. Phys. Rev. A **100**, 063831 (2019)
24. Robertson, S., Mailliet, A., Sarazin, X., Couchot, F., Baynard, E., Demailly, J., Urban, M.: Experiment to observe an optically induced change of the vacuum index. Phys. Rev. A. **103**(2), 023524 (2021)
25. Claeskens, J.F.: Aspects statistiques du phénomène de lentille gravitationnelle dans un échantillon de quasars très lumineux, Chapitre 2: Théorie du phénomène de mirage gravitationnel. Bull. Soc. Roy. Sci. Liège **68**(1–4), 1–305 (1999)
26. Einstein, A.: Über Gravitationswellen. Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), Physikalisch-Mathematische Klasse, pp. 154–167 (1918)
27. Ruggiero, M.L., Tartaglia, A.: Gravitomagnetic effects. Nuovo. Cim. B. **117**(2002), 743–768 (2002)

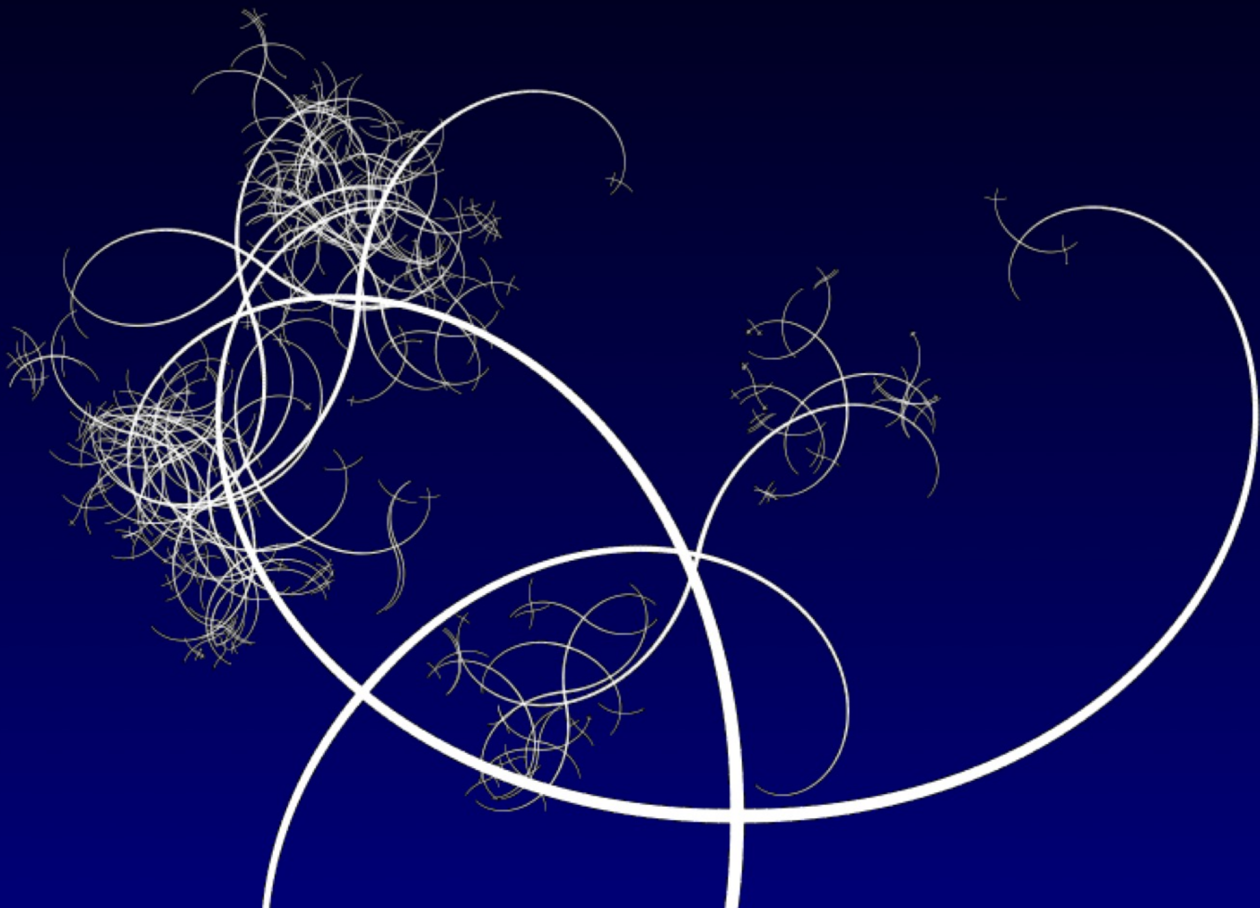
28. Izabel, D.: Mechanical conversion of the gravitational Einstein's constant κ. Pramana J. Phys. **94**, 119 (2020)
29. Izabel D.: What is Space-Time Made of? Les Ulis: EDP Sciences, p. 36 (2021). https://doi.org/10.1051/978-2-7598-2573-8
30. Timoshenko, S.: Strength of materials, part 1 and part 2. D. Van Nostrand Company Inc., New York (1930)
31. Tenev, T.: thesis An Elastic Constitutive Model of Spacetime and its Applications. Univ, Mississippi (2018)
32. Beau, M.: A Time-Dependent Model of Dark Energy Based on Four-Dimensional Continuous Deformation Theory. arXiv preprint, arXiv:1805.03020 (2024). Disponible à : https://arxiv.org/pdf/1805.03020
33. McDonald, K.T.: What is the stiffness of spacetime? Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, April 13, 2018 (updated May 5, 2018) (2018). Disponible à : https://kirkmcd.princeton.edu/examples/stiffness.pdf
34. Carneiro, F.L., Carneiro, B.C.C., Azevedo, D.L., Ulhoa, S.C.: On nanocones as gravitational analog systems. Ann. Phys. (Leipzig) (2025). https://doi.org/10.1002/andp.202400448
35. Antoci, S., Mihich, L.: A four-dimensional Hooke's law can encompass linear elasticity and inertia. Nuovo Cimento B **114**(8), 873–880 (1999). https://doi.org/10.48550/arXiv.gr-qc/9906094
36. Damour, T.: Einstein et la physique du vingtième siècle. Académie des sciences (2005). https://www.academie-sciences.fr/pdf/dossiers/Einstein/Einstein_pdf/Einstein_Damour.pdf
37. Ingold, G.-L., Lambrecht, A.: Casimir effect from a scattering approach. Am. J. Phys. **83**(2), 156–163 (2015). https://doi.org/10.1119/1.4896197. Version préliminaire disponible sur arXiv : arXiv:1404.6919v1 [quant-ph], soumise le 28 avril 2014
38. Bressi, G., Carugno, G., Onofrio, R., Ruoso, G.: Measurement of the Casimir Force between Parallel Metallic Surfaces. Phys. Rev. Lett. **88**(4), 041804 (2002)
39. Nawaz, M.B. Syed, Wiegerink, R.J., Lammerink, T.S.J., Elwenspoek, M.C.: Parallel plate structures for optical modulation and casimir force measurement. In: MME 09 – Proceedings of the 20th Micromechanics Europe Workshop, Toulouse, France, LAAS-CNRS pp. 1–4. Paper ID: 142 (2009)
40. Kolomeisky, E.B., Straley, J.P., Langsjoen, L.S., Zaidi, H.: Casimir effect due to a single boundary as a manifestation of the Weyl problem. J. Phys. A: Math. Theo. **43**(38), 385402 (2010)
41. Denardo, B.C., Puda, J.J., Larraza, A.: A water wave analog of the Casimir effect. Am. J. Phys. **77**(12), 1095–1101 (2009)
42. Tartaglia, A., Radicella, N.: From Elastic Continua to Space-time AIP Conf. Proc. **1241**(1156–1163), 2010 (2010)
43. Beau, M.: Théorie des champs des contraintes et des déformations en relativité générale et expansion cosmologique. Ann. Fond. Louis de Broglie **40** (2015). see also arXiv:1209.0611v2 [gr-qc], p. 4
44. Sakharov, A.D.: Vacuum quantum fluctuations in curved space and the theory of gravitation. Soviet Physics Doklady **12**, 1040–1041 (1968)
45. Landau, L.D., Lifshitz, E.M.: The classical theory of fields, 4th ed., Pergamon Press, Oxford, Chapter 11 (1975)
46. Arnowitt, R., Deser, S., Misner, C.W.: Dynamical Structure and Definition of Energy in General Relativity. Phys. Rev. Lett. **116**, 5 (1959)
47. Arnowitt, R., Deser, S., Misner, C.W.: The Dynamics of General Relativity Gen. Rel. Grav. **40**(2008), 1997–2027 (1962)
48. Komar, A.: Covariant conservation laws in general relativity. Phys. Rev. **113**(3), 934 (1959)
49. Hawking, S.W.: Gravitational radiation in an expanding universe. J. Math. Phys. **9**, 598 (1968)
50. Penrose, R.: Some unsolved problems in classical general relativity. In Seminar on Differential Geometry. Ann. Math. Stud. **102**, 631–668 (1982)
51. Brown, J.D., York, J.W.: Quasilocal energy and conserved charges derived from the gravitational action. Phys. Rev. D. **47**, 1407 (1993)
52. Szabados, L.B.: Quasi-Local Energy-Momentum and Angular Momentum in General Relativity. Living Rev. Relativ. **12**, 4 (2009)
53. Balbus, S.A.: Gravitational wave stress tensor. PNAS **113**(42), 11662–11666 (2016)
54. Balbus, S.A.: Simplified derivation of the gravitational wave stress tensor from the linearized Einstein field equations. Proceed. Natl. Acad. Sci. **113**(42), 11662–11666 (2016)
55. Gauss, C.F.: Disquisitiones generales circa superficies curvas. Commentationes Societatis Regiae Scientiarum Gottingensis Recentiores, Classis Mathematicae, Gottingae (1828), presented to the Royal Society on Oct. 8, 1827
56. Tahim, M.O., Landim, R.R., Almeida, C.A.S.: Spacetime as a deformable solid. Modern. Phys. Lett. A. **24**(15), 1209–1217 (2009)

57. Lobo, F.S.N., Olmo, G.J., Rubiera-Garcia, D.: Crystal clear lessons on the microstructure of space-time and modified gravity. Phys. Rev. D **91**, 124001 (2015)
58. Izabel, D.: Analogy of spacetime as an elastic medium—Can we establish a thermal expansion coefficient of space from the cosmological constant Λ? Int. J. Modern Phys. D. **32**(13), 2350091–435 (2023)
59. Izabel, D.: Analogy of spacetime as an elastic medium — Estimation of a creep coefficient of space from space data via the MOND theory and the gravitational lensing effect — the ball cluster — and via time data from the GPS effect — comparison, discussion and implication of the results for dark matter and Einstein's field equation. Int. J. Modern Phys. D. **34**(02), 2450070 (2025)
60. Izabel, D.: Can we understand the concepts of the General Relativity and of the Quantum Mechanics based on the principles of the strength of materials? Reflections and proposals. HAL Archives Ouvertes, hal-01562660, version 1 (2017). https://hal.science/hal-01562660v1/document
61. Iliopoulos, J.: Aux origines de la masse. EDP Sciences, Paris (2015)
62. Millette, P.A.: Elastodynamics of the Spacetime Continuum. American Research Press, Rehoboth New Mexico USA, STCED (2019)
63. Synge, J.L.: A theory of elasticity in general relativity. Math. Z. **72**, 82–87 (1959)
64. Rayner, C.B.: Elasticity in General Relativity. Proceed. Royal Society A, Math. Phys. Eng. Sci. **272**(1348), 44–53 (1963)
65. Grot, R.A., Eringen, A.: Relativistic Continuum Mechanics. Int. J. Eng. Sci. **4**, 611–670 (1966)
66. Damour, T.: La relativité générale aujourd'hui. Séminaire Poincaré **9**, 1–40 (2006)
67. Weiss, R.: Ligo and the Discovery of Gravitational Waves, Nobel Lecture, December 8, 2017 by Rainer Weiss Massachusetts Institute of Technology (MIT). MA, USA, Cambridge (2017)
68. Adler, R.J., Casey, B., Jacob, O.C.: Vacuum catastrophe: An elementary exposition of the cosmological constant problem. Am. J. Phys. **63**(7), 620–626 (1995)
69. Kleinert, H.: Emerging gravity from defects in world crystal. Braz. J. Phys. **35**(2A), 359–361 (2005)
70. Elizalde, E., Izaurieta, F., Riveros, C., Salgado, G., Valdivia, O.: Gravitational waves in Einstein–Cartan theory: On the effects of dark matter spin tensor. Phys. Dark Universe, **40**, 101197 (2023). Version préliminaire disponible sur arXiv : arXiv:2204.00090 [gr-qc], soumis le 31 mars 2022, révisé le 20 février 2023
71. Carneiro, F.L., Ulhoa, S.C., Maluf, J.W., da Rocha-Neto, J.F.: Non-linear plane gravitational waves as space-time defects. Eur. Phys. J. C. **81**, 67 (2021)
72. Ruggiero, M.L.: Gravitomagnetic induction in the field of a gravitational Wave. General Relativity Gravitation **54**(9), 97 (2022)
73. Masovic, D.: Acoustic analogies with general relativity, quantum fields, and thermodynamics. Technische Universität Berlin, arXiv:1907.02902v3 [physics.gen-ph] (2022). Disponible à : https://arxiv.org/abs/1907.02902. https://doi.org/10.48550/arXiv.1907.02902
74. Katanaev, M.O.: Geometric theory of defects. Phys. Uspekhi. **48**(7), 675–701 (2005)
75. Kolev, B.: Covariance générale et objectivité. Communication présentée à la Réunion du GDR "Géométrie Différentielle et Mécanique", CNRS, La Rochelle (2023). Document disponible en ligne : https://gdr-gdm.univ-lr.fr/la-rochelle-2023/files/29_juin/14h00-15h55/Boris-Kolev.pdf
76. Souriau, J.M.: La relativité variationnelle. Publ. Sci. Univ. Alger. Sér. A **5**, 103–170 (1958)
77. Souriau, J.M.: Géométrie et relativité. Enseignement des Sciences, VI, Hermann, Paris p. 511 (1964). ISBN : 978-2-7056-6180-6 (réimpression 2008) — Disponible via Gallica BnF et Éditions Jacques Gabay
78. Chapon, M., Darondeau, L., Desmorat, R., Ecker, C., Kolev, B.: General covariant relativistic gradient hyperelasticity. HAL preprint, hal-04792877 (2024). Disponible à : https://hal.science/hal-04792877/file/CDDEK2024.pdf
79. Ciufolini, I.: Time Travel, Clock Puzzles and Their Experimental Tests. EPJ Web Conf. **58**, 01005 (2013). https://doi.org/10.1051/epjconf/20135801005. Version préliminaire disponible sur arXiv : arXiv:1306.1826 [gr-qc]
80. Johnston, R.: Calculation on space-time curvature within the Earth and Sun, Jonston's Archives (2008). https://www.johnstonsarchive.net/relativity/stcurve.pdf
81. Witten, E.: A note on complex spacetime metrics. arXiv:2111.06514v2 [hep-th], soumis le 12 novembre 2021, révisé le 11 février 2022 (2022). Disponible à : https://arxiv.org/abs/2111.06514 — 28 pages
82. Wiszniewski, W.R.: Time, quasi-temporal change and imaginary numbers. PhD Thesis, University of New South Wales p. 638 (2006). Disponible à : https://unsworks.unsw.edu.au/handle/1959.4/23401
83. Völgyesi, L., Moser, M.: The inner structure of the earth. Period. Polytech. Chem. Eng. **26**(3–4), 155–204 (1982). Disponible en ligne : https://pp.omikk.bme.hu/ch/article/view/2955
84. Paturel, G.: Cours élémentaire d'astronomie et d'astrophysique. Observatoire de Lyon, publié dans les Cahiers Clairaut, numéros 105 (2004) à 116 (2006), CLEA-ASTRO, France (2006). Disponible en ligne : http://clea-astro.eu/lunap/cours-elementaire-dastronomie/2023%2002%2002%20Cours%20Elementaire%20dastronomie%20%20DEF.pdf

85. Abbott, B.P., et al.: (LIGO scientific collaboration and Virgo collaboration), A Search for Tensor, Vector, and Scalar Polarizations in the Stochastic Gravitational-Wave Background. Phys. Rev. Lett. **120**, 201102 (2018)
86. Hou, S., Fan, X.L., Zhu, T., Zhu, Z.H.: Nontensorial gravitational wave polarizations from the tensorial degrees of freedom: 1 Linearized Lorentz-violating theory of gravity with s tensor. Phys. Rev. D. **109**, 084011 (2024)
87. Abbott, B.P., et al.: (LIGO scientific collaboration and Virgo collaboration), First Search for Nontensorial Gravitational Waves from Known Pulsars. Phys. Rev. Lett. **120**, 031104 (2018)
88. Mathur, S.: Gravitational wave polarizations: A test of general relativity using binary black hole mergers. Senior thesis (major), California Institute of Technology, Division of Physics, Mathematics and Astronomy (2020). Disponible à : https://resolver.caltech.edu/CaltechTHESIS:08062020-222003579 — https://doi.org/10.7907/q9qa-7770
89. Will, C.: The Confrontation between General Relativity and Experiment. Living Rev. Relativ. **17**, 4 (2014)
90. Wettea, K.: Searches for continuous gravitational waves from neutron stars A twenty-year retrospective. Astropart. Phys. **153**, 102880 (2023)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

NISSUNA UMANA INVESTIGAZIONE SI PUO DIMANDARE VERA SCIENZIA
S'ESSA NON PASSA PER LE MATEMATICHE DIMOSTRAZIONI
LEONARDO DA VINCI



vol. 13 no. 2 2025

Mathematics and Mechanics of Complex Systems

David Izabel, Yves Rémond and Matteo Luca Ruggiero

SOME GEOMETRICAL ASPECTS OF GRAVITATIONAL WAVES USING CONTINUUM MECHANICS ANALOGY: STATE OF THE ART AND POTENTIAL CONSEQUENCES

msp

MATHEMATICS AND MECHANICS OF COMPLEX SYSTEMS
Vol. 13 (2025), No. 2, pp. 201–236
DOI: 10.2140/memocs.2025.13.201

SOME GEOMETRICAL ASPECTS OF GRAVITATIONAL WAVES USING CONTINUUM MECHANICS ANALOGY: STATE OF THE ART AND POTENTIAL CONSEQUENCES

DAVID IZABEL, YVES RÉMOND AND MATTEO LUCA RUGGIERO

We use an analogy between continuum mechanics and general relativity to investigate, from the perspective of elasticity and crystal plasticity, the deformations of space measured by LIGO/Virgo interferometers during the passage of gravitational waves over Earth. The results of different innovative or existing mechanical models are compared with each other and compared with the observations in the framework of general relativity and Einstein–Cartan theory. Despite limitations, there is a convergence of results: the polarizations of gravitational waves can be viewed as expressions of an equivalent elastic media deformation tensor. Additionally, an anisotropy of space properties is unavoidable at the measurement point of the gravitational wave if we rely on the current first-order general relativity, which predict that gravitational waves generate deformations only in transverse planes. It is demonstrated that the classical polarizations of general relativity can be associated with a state of pure torsion in the analogous elastic medium and acted upon by the rotation of massive bodies such as black holes. This approach involves a transverse isotropic medium composed of independent sheets that deform perpendicularly to the direction of propagation of these waves. Considering geometric torsion in general relativity, associated with plastic crystallography, allows for the examination of complementary polarizations in the direction of wave propagation. This makes it possible to connect these sheets and reconstruct a complete, coherent 3D environment.

1. Introduction

Einstein's theory of general relativity is over 100 years old and is now widely verified. Thus, according to this theory, space-time could be an elastic, deformable physical object. These distortions disappear when the object that created them disappears. Hence the notion of the elasticity of space. Gravitation is thus a manifestation of the geometric deformation of space-time under the effect of the masses or energy density found therein. The manifestations of the deformations of this space-time are now known and measured with great precision in several very specific situations. From a historical perspective, we can mention the variation in the apparent position of stars placed behind the sun during an eclipse measured by Eddington in 1919 [17], the expansion of the universe where galaxies are "fixed" in a space that expands in an increasingly accelerated manner characterized by Hubble's law established in 1929. More recently, we mention the entrainment of the reference frame of space-time by angular distortion by the

Communicated by Francesco dell'Isola.
MSC2020: 74-XX, 83C25.
PACS2010: 04.50.KD, 46.9.
Keywords: general relativity, gravitational wave, polarization, elasticity, continuum mechanics, torsion, crystal plasticity, gravitoelectromagnetism.

rotation of the Earth (experiment conducted with the satellite gravity probe B carried out with gyroscopes placed in orbit 642 km from the Earth from 2004 to 2010, Lense–Thirring (frame dragging) effects and geodetic precession) [21], the simultaneous deformations of elongation and shortening in each of the arms of the LIGO/Virgo interferometers during the passage of gravitational waves measured for the first time on 15 September 2014 [1; 2]. A thorough overview of 100 years of testing general relativity can be found in [57]. All these manifestations of space-time distortions have led many physicists and mechanics researchers (such as A. Sakharov [48], J. L. Synge [52], C. B. Rayner [44], R. Grot [24], T. G. Tenev and M. F. Horstemeyer [55], P. A. Millette [41], D. Izabel [28; 29] and T. Damour [15]) to consider that the theory of general relativity in the weak field could be considered by analogy as a kind of theory of elasticity, a kind of Hooke's law of a deformable elastic space-time medium.

Thus, either some authors begin with general relativity and attempt to present it within the formalism of continuous mechanics, while other start with 3D continuum mechanics and generalize it to 4D by introducing a "mechanistic" metric that includes the effect of time.

The latest work that, to our knowledge, provides an updated assessment of this topic was giving by [33; 34; 14], based on the seminal works [8; 9; 13; 18; 25; 30; 35; 36; 5; 42; 50; 51; 52].

Summarizing the theoretical context in which our work is situated, we will assume that both general relativity and continuum mechanics are based on the principle of general covariance, which, as B. Kolev aptly summarizes in [34], requires the introduction of three components:

- Lagrangian-type functionals $\mathscr{L}$, depending on the metric g defined on the 4-dimensional universe manifold M, and depending on various fields Ψ;
- Tensors, such as the energy-momentum tensor, dependent on these fields;
- Field equations.

Our approach is therefore placed in the framework of the general covariance of field equations by the group of diffeomorphisms. Or expressed explicitly, a Lagrangian $\mathscr{L}(g, \Psi, ..)$ is general covariant if it satisfies $\mathscr{L}(\varphi^* g, \varphi^* \Psi) = \mathscr{L}(g, \psi)$ for any diffeomorphism φ. Note, still with B. Kolev, that the functional $\mathfrak{H}(g) = \int R_g \mathrm{Vol}_g$, where R_g represents the scalar curvature, is general covariant. Its gradient L^2, the Einstein tensor $G(g)$ is also general covariant. It consequently satisfies $\forall\varphi$, $G_{\varphi^* g} = \varphi^* G(g)$, and $\mathrm{div}^g G(g) = 0$. An energy-momentum tensor $T(g, \Psi)$ satisfying the Einstein equation $G(g) = T(g, \Psi)$ therefore satisfy the mechanics type equation: $\mathrm{div}\, T = 0$. The rest of this work will therefore concern energy-momentum tensors satisfying these two properties. The reader is referred to the work of J. M. Souriau [50; 51] and his successors already cited, for the definitions of the perfect matter field (as a section of a vector bundle) and the conformations, allowing the proposal of relativistic constitutive laws [14].

It is within the framework of this analogy that we mainly place ourselves in this paper. We will not develop further the very theoretical aspects described above. Nor will we address the controversies and debates that these concepts continue to generate, but we will concentrate on the consequences on the mechanical properties of the medium that the experiments induce. Following these preliminary remarks, we can define this "elastic gravitational analogy" based on the three principles of equivalences.

- The perturbations h_{ij} of space in the presence of gravitational waves are linked to the Green–Lagrange covariant tensor: $D = \frac{1}{2}(\varphi^* g - g)$, which will be assimilated in the following, under infinitesimal deformations, as the geometric linearization of the strain tensor $D = \varepsilon = \varepsilon_{ij}$,
- Einstein's equation connects, within the medium, the strain tensor (versus metric perturbation in weak field) to the equivalent stress field, akin to Hooke's law, with the aid of an equivalent compliance matrix.
- The energy density of space-time itself ρc^2 is correlated, through quantum field theory or the Casimir effect, with a nonzero energy density of the vacuum. This vacuum energy density is mirrored in the analogy by the Young's modulus Y of the equivalent elastic medium, given by the equation $Y = \rho c^2$.

In addition to these hypotheses, we must recall the debates on the relativistic equivalent validity of the stress field used in continuum mechanics [22]. Our approach add some further elements to this debate. However, when deformations occur in a vacuum far from the spacetime loading, as is the case with gravitational waves arriving on Earth, the stress-energy tensor is $T = T_{\mu\nu} = 0$.

To maintain a complete Hooke's law, it is necessary to consider an elastic strain energy tensor of the vacuum itself: $T_{e,\mu\nu}$, linked to the deformations correlated with $h_{\mu\nu}$ (the small perturbation of the Minkowski's tensor). This is one way to calculate the gravitational wave energy or to study the spacetime itself as an equivalent elastic medium [54; 7; 53].

We add, however, that continuum mechanics models are based on a primary concept, which is the kinematics of the phenomena. The concept of stress field is a secondary concept which is introduced only after the kinematics choice using the virtual power principle [23; 20], named virtual power theorem if we consider as primary principle, the principle of general covariance. The constitutive laws which link these two concepts, or their time derivatives, are well defined by the local state method [37].

2. Methods

The following methodology was employed to evaluate some geometrical aspects of gravitational waves in linearized general relativity from the perspective of the analogy with continuum mechanics.

(1) Investigation of the discrepancy between the deformations measured during the passage of gravitational waves and the theoretical predictions of linearized general relativity, using deformation measurements made by LIGO/Virgo interferometers.

(2) Analysis of test masses arranged in a circle of the different deformations and associated polarizations according to various versions of general relativity (classical in the first-order, second-order in gravitoelectromagnetism, modified with Einstein–Cartan torsion, and other modified versions).

(3) Study of space deformations during the passage of a gravitational wave by considering either existing or new models, such as interferometer arms, torsional space cylinders, isotropic transverse media of elastic solids.

(4) Comparison of the results from classical first- and second-order or modified theories of general

relativity (Einstein–Cartan and others) with the predictions of various continuum mechanics. Specifically, we study the potential complementary deformations of the equivalent cosmic medium and their consequences on the characteristics of the media. We also explore interactions with the theory of defects in crystalline media regarding the number and types of gravitational wave polarizations.

3. Gravitational waves in relativistic theories of gravity

3.1. *General relativity.* According to Einstein's theory of general relativity, gravitation is the geometry of spacetime: specifically, spacetime is a four-dimensional pseudo-riemannian manifold M, which is a pair $(M, g_{\mu\nu})$, where M is a connected four-dimensional Hausdorff manifold and $g_{\mu\nu}$ is the metric tensor.[1] Due to its Riemannian structure, spacetime is endowed with an affine connection compatible with the metric, known as the Levi-Civita connection. We are interested in gravitational waves, which are particular vacuum solutions of Einstein's equations:

$$G^{\mu\nu} = -\frac{8\pi G}{c^4} T^{\mu\nu}, \tag{1}$$

where $G^{\mu\nu}$ is the Einstein tensor defined in terms of the Ricci tensor $R^{\mu\nu}$ and scalar curvature R, as $G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$ and $T^{\mu\nu}$ is the energy-momentum tensor. To obtain the wave equations, we suppose that the metric tensor in a weak gravitational field can be written in the form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{2}$$

where $h_{\mu\nu}$ is a small perturbation of the Minkowski tensor $\eta_{\mu\nu}$ of flat spacetime ($h^{\mu\nu} \ll \eta^{\mu\nu}$). By setting $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$, where $h = h^{\mu}_{\mu}$, Einstein's equation can be written in the form

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}, \tag{3}$$

where $\Box = \partial_\mu \partial^\mu = \nabla^2 - (1/c^2)\partial/\partial t^2$. In a vacuum ($T_{\mu\nu} = \mathbf{0}$) this becomes $\Box \bar{h}_{\mu\nu} = \mathbf{0}$, whose solutions are gravitational waves propagating in empty space, which can be written in the form

$$\bar{h}_{\mu\nu} = A_{\mu\nu} \cos(k_\sigma x^\sigma), \tag{4}$$

$$A_{\mu\nu} = A_+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + A_\times \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{5}$$

where A_+ and $A_\times$ are the amplitude of the wave in the two polarization states, and k^σ is the four-plane wave vector $k^\sigma = (\omega/c; \vec{k})$, where ω is the frequency and $k = \|\vec{k}\| = \omega/c$ is the wave number, with $k^\sigma k_\sigma = 0$. This solution is given in the so-called TT gauge: the deformation caused by the wave is transverse to the propagation direction, and the amplitude tensor $A_{\mu\nu}$ is traceless. For instance, if we consider test masses positioned on a circle of radius R before the passage of the wave, the deformation

[1] Greek indices run from 0 to 3, while Latin indices run from 1 to 3.

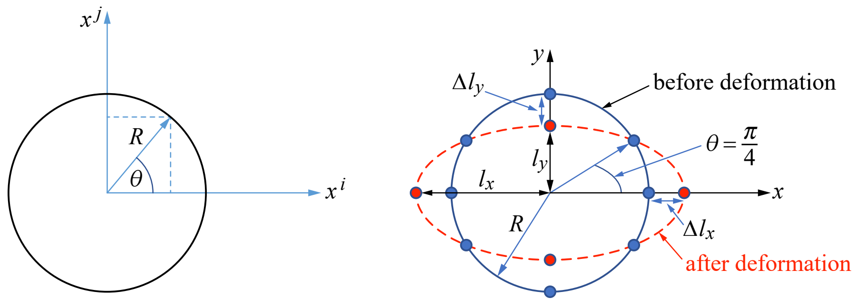


Figure 1. Displacement of test masses in the xy-plane when a gravitational wave propagates in the z-direction.

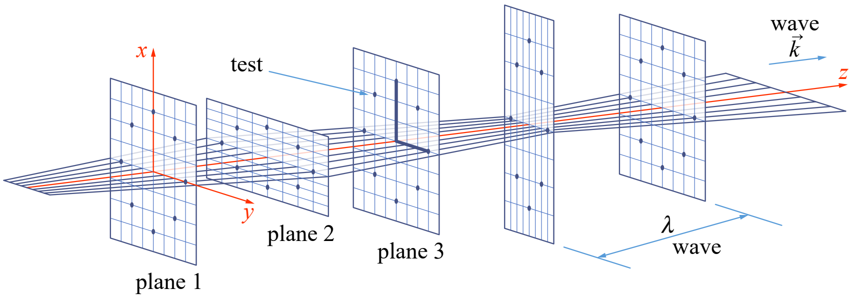


Figure 2. Displacements of test masses when a gravitational wave passes in the direction z following successive transverse planes.

provoked by the A_+ polarization is given by

$$\Delta S = R\left[1 + \tfrac{1}{2} A_+ \cos \omega t \cos 2\theta\right] \tag{6}$$

and it is depicted in Figure 1, for a wave propagating along the z direction. A similar deformation is obtained for polarization $A^\times$, which corresponds to a rotation of Figure 1 by 45 degrees. Additionally, the evolution of the deformation is depicted in Figure 2.

According to this description, the deformations provoked by the wave are in the plane orthogonal to the propagation direction: strictly speaking, this is true if we confine ourselves to the first order with respect to the reference position. Up to second order (see, e.g., D. Baskaran and L. P. Grishchuk [6] and M. L. Ruggiero in 2022 [45]), deformations also occur along the propagation direction. This effect (significantly enlarged) is depicted in Figure 3.

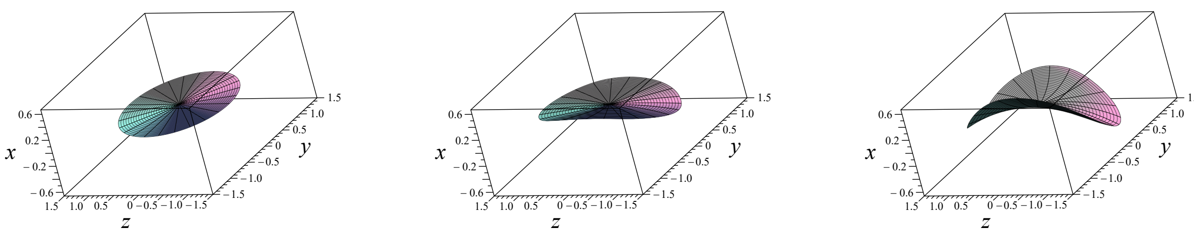


Figure 3. Temporal evolution of test masses due to the effects of A+ polarization with up to second order [45].

3.2. ***Modified theories of gravity.*** Altough general relativity is the most successful model we have for understanding gravitational interactions, there are still unresolved issues regarding the application of GR to large-scale structures, such as the problems of dark matter and dark energy and. Additionally, we do not yet know how to reconcile it with quantum mechanics. Consequently, several proposals have been made to extend Einstein's theory to address these issues. Many of these proposed theories have a richer geometrical structure. For instance, in Einstein–Cartan theory, torsion is present in addition to curvature. Specifically, torsion is related to the spin of the sources of the gravitational field [32]. For our purposes, it is interesting to note that this theory introduces additional polarizations for gravitational waves [19]. Our complementary polarizations appear as shown in Figure 4 and are associated with the polarization matrices

$$P^{(+)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad P^{(\times)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad P^{(b)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{7}$$

$$P^{(l)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad P^{(xz)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad P^{(yz)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{8}$$

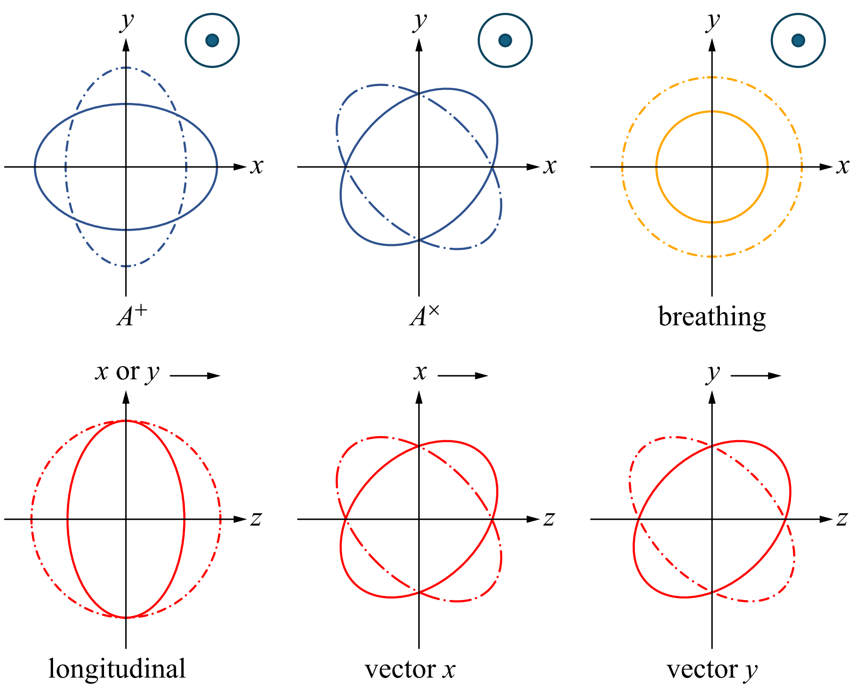


Figure 4. Complementary polarizations that appear in the case of torsional modified general relativity in the case of the Einstein–Cartan–Sciamma–Kibble theory [19].

theory	type of polarization					
	$+$	$\times$	x	y	breathing	longitudinal
general relativity	yes	yes	no	no	no	no
GR in noncompactified 4/6D Minkowski	yes	yes	yes	yes	yes	yes
Einstein/ether	yes	yes	yes	yes	yes	yes
5D Kaluza–Klein	yes	yes	yes	yes	yes	no
Randall–Sundrum braneworld	yes	yes	no	no	no	no
Dvali–Gabadadze–Porrati braneworld	yes	yes	—	—	—	—
Brans–Dicke massive	yes	yes	no	no	yes	yes
Brans–Dicke massless	yes	yes	no	no	yes	no
F(R) metric gravity	yes	yes	no	no	yes	yes
bimetric theory	yes	yes	yes	yes	yes	yes
Palatini gravity	yes	yes	no	no	no	no
scalar tensor theory	yes	yes	no	no	yes	yes

Table 1. Different polarizations associated with different theories of general relativity, according to [40] and [43]. See Figure 4 for the different polarization types. A dash indicates the answer depends on the version of the theory.

The first two matrices in (7) are the classical ones predicted and measured by classical general relativity. The remaining four matrices, $(7)_3$ and (8), result from the introduction of the Einstein–Cartan–Sciamma–Kibble torsion.

More generally, as discussed by L. A. Philippoz [43] and S. Mathur [40], different alternative theories of gravity correspond to specific polarization features for gravitational waves, which are summarized in Table 1. Interestingly, all these modified theories converge on the same idea: that one or more polarizations complementary to A^+ and $A^\times$ should potentially exist. Only advanced measurements of deformations, by coupling various interferometers on Earth or using LISA, are likely to detect such polarizations. We will revisit this at the end of the paper.

Having completed the state of the art of gravitational wave polarizations in classical or modified general relativity, we will now see what the analogy of the continuum mechanics elasticity can help interpreting the results of general relativity (gravitational waves) recalled in the previous paragraphs.

3.3. ***Results of measurements made by LIGO/Virgo interferometers.*** The first direct measurement of a gravitational wave occurred on September 14, 2015 (GW150914) and was presented on February 11, 2016 [1]. Two signals corresponding to the merger of two black holes that occurred 1.3 billion years ago were successively detected by the two LIGO interferometers in the USA.

The nature of these signals is shown in Figure 5, whose lower pane shows the almost perfect superposition of the two signals successively detected by the two interferometers 3000 km apart. Figure 7 shows the superposition of the theoretical curves and measured data after eliminating the noise. General relativity therefore predicts this type of signal remarkably well.

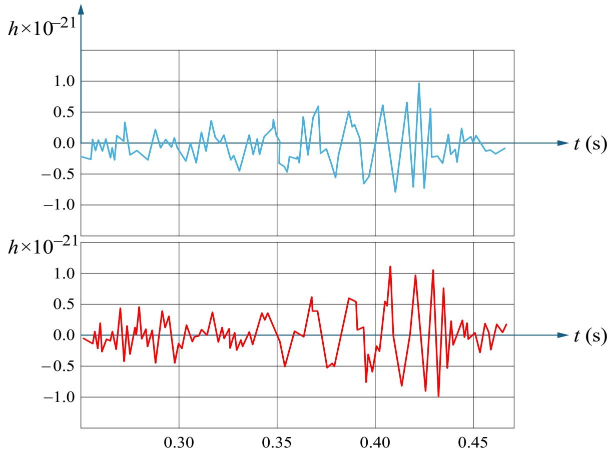


Figure 5. Signal GW150914 picked up by the two LIGO laser interferometers sourced by B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) – First Hanford second Livingstone [1]. The x-axis represents time in seconds, and the y-axis represents the h-deformations of space on the order of $+/-\ 10^{-21}$.

A second interesting signal is GW170817 [2], emanating from the merger of two neutron stars, also known as a kilonova. It was observed simultaneously from both perspectives: gravitational waves and electromagnetic waves. This confirms, if needed, the remarkable efficiency and accuracy of the measurements. There can no longer be any doubt that rigid space deforms extremely little ($h_{xx} = -h_{yy} = 10^{-21}$) when stressed by large masses (black holes, neutron stars, etc.) concentrated in small volumes and rotating relative to each other at high speeds and high acceleration near the final coalescence time.

3.4. ***Discrepancies between LIGO/Virgo interferometer measurements and the classical linearized theory of general relativity.*** By carefully comparing the two GW150914 signals, it is possible to show the deviation in units 10^{-21} between the theory of general relativity and the interferometer measurements (Figure 6). These curves demonstrate the precision of general relativity, and indicate that any theory improving it should explain this few percent difference.

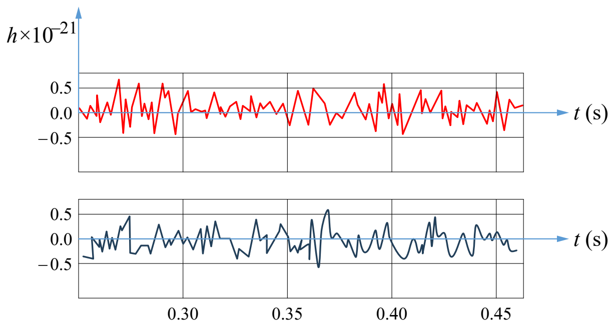


Figure 6. Gap between the theory of general relativity and interferometer measurements for GW150914 source B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) [1].

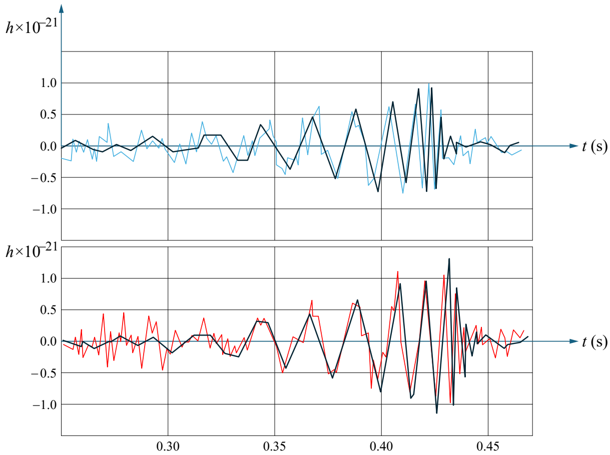


Figure 7. Superposition of the theoretical curves (finite elements) and measured having eliminated the noise and interferometer measurements for GW150914 source B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) [1].

Viewing these results of extremely small deformations of space, physically measured by the two LIGO interferometers, which behave like giant strain gauges — each interferometer arm, traversed by a laser beam reflecting on various mirrors, is 4 km long and operates in a vacuum — one can be led to analogize the vacuum as an elastic medium that deforms under the effect of energy or masses present within it.

Obviously, the continuum mechanics theory of elasticity seems appropriate for modeling these deformations of space by analogy.

In the following sections, we will explore the strengths and limitations of this analogy, its potential contributions to our understanding of general relativity, and whether it can guide us in analyzing classical or modified theories of general relativity that aim to address the few percent difference between measurements and Einstein's version (Figures 6 and 7).

4. Characterization of the deformations of space during the passage of a gravitational wave using different approaches of continuum mechanics

4.1. *Solid-state model for space.*

4.1.1. *Stacking thin sheets in elasticity.* Several important points can be made about this paper:

a- Correspondence between polarizations established in general relativity and deformations established in the mechanics of continuous media.

The hypotheses include, among other things, that in a weak field, the metric is expressed as

$$g^{\mu\nu} = \eta^{\mu\nu} + 2\varepsilon^{\mu\nu}, \tag{9}$$

where $\varepsilon^{\mu\nu}$ is the equivalent of a first-gradient strain tensor comprising spatial and temporal strain components (00, 01, 02, 03).

Comparing expressions (9) and (2), the following equivalence emerges:

$$2\varepsilon^{\mu\nu}(\text{in mechanics}) = h^{\mu\nu}(\text{in GR in weak field}) \tag{10}$$

This correspondence is fundamental for the rest of this paper, as it implies that the components of the polarization matrices of gravitational waves can be interpreted, within the context of the analogy with the elastic medium, as the components of a deformation tensor of the associated elastic cosmic space [55; 28; 29].

b- Relations between longitudinal and transversal strains. Since gravitational waves are transverse, (no longitudinal wave), and the deformations are of equal intensity but opposite sign in each arm of the interferometers, this implies Poisson's ratios of the equivalent anisotropic elastic cosmic medium as:

$$\nu_{xy} = \nu_{yx} = 1. \tag{11}$$

There are two ways to obtain this value. First, measurements are made on the LIGO/Virgo interferometers, where elongation and shortening are measured simultaneously in each of the arms with the same intensity and opposite signs (Figure 5). Alternatively, we can assume an elastic medium and impose the absence of a longitudinal wave (since gravitational waves are transverse according to classical general relativity). Indeed, the velocity of a compression wave in an anisotropic elastic medium can be written as

$$c_{\text{pressure}} = \sqrt{\frac{Y_{xy}(1-\nu_{xy})}{\rho(1+\nu_{xy})(1-2\nu_{xy})}} = 0 \implies \nu_{xy} = 1. \tag{12}$$

The other variation $\nu_{yx} = 1$ is observed in the strains of each arms (Figures 1 and 5). We therefore find that, as interferometers are in the xy-plane, $\nu_{xy} = \nu_{yx} = 1$.

This observation regarding the values of these Poisson's ratios has the following consequences:

- The distortions of space caused in the planes x, y perpendicular to the direction z of propagation of the gravitational waves (Figure 2), associated with polarizations according to the principle described above, are of the same magnitude but of the opposite sign,
- On one hand, the elastic medium associated with space in continuum mechanics is necessarily anisotropic due to the value of these Poisson's ratios. On the other hand, we note the absence of polarization and, thus, complementary deformation in the z direction of wave propagation in classical general relativity.
- According to this result, this equivalent elastic spatial medium would consist of stacking of sheets that deform independently of each other ($\nu_{xz} = \nu_{yz} = 0$). The spatial medium thus consists more in a tailored medium at a very small scale than an equivalent homogeneous medium (Figure 2) [54] where we have strains in three directions of space, not only in two directions (x, y) as in classical general relativity.

Note that in the case of a gravitational wave not perpendicular to the interferometer arms, the angle θ of the observer is taken into account as follows [10]:

$$h_{+(t)} = \frac{4G\mu a^2\omega^2}{Rc^4}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\omega t), \quad h_{\times(t)} = \frac{4G\mu a^2\omega^2}{Rc^4}\cos\theta\sin(2\omega t), \tag{13}$$

where $\omega = \sqrt{GM/a^3}$, $M = m_1 + m_2$, $\mu = m_1 m_2/M$, θ is the angle between the normal to the rotation plane of the two bodies and the direction R of the observer, a is the distance between the two rotating bodies (e.g., black holes), and R is the distance between the observer and the system in rotation ($R \gg a$.)

c- The Young's moduli of the equivalent elastic medium can be expressed in terms of fundamental constants. This model was developed by T. G. Tenev and M. F. Horstemeyer [55], among others. In this paper, cosmic space is assumed to consist of ultrathin sheets whose thickness is that of the Planck length, by denoted l_p.

The consequence is that T. G. Tenev, and M. F. Horstemeyer in [55] arrive at these expressions of Young's moduli of cosmic fabric:

$$Y = Y_x = Y_y = E_x = E_y = \frac{6c^7}{2\pi\hbar G^2} = \frac{24}{l_p^2\kappa} \tag{14}$$

(Y_z is not defined in their model).

The cosmic fabric in this paper is an extension of the cosmic medium as seen by [55] with a transversal isotropic behavior.

The numerical application leads to values of the Young's moduli Y_x and Y_y of the cosmic fabric, (in correlation with the energy of the vacuum according to Sakharov [48]) that are outside the usual standards if we assume a stacking of space sheets of the Planck thickness constituting this cosmic fabric.

$$Y_{x\,(\text{vacuum})} = Y_{y\,(\text{vacuum})} = 4.4 \times 10^{113}\,\text{Pa}.$$

This value is associated with a vacuum density of $\rho_{\text{vacuum}} = 1.3 \times 10^{96}\,\text{kg/m}^3$, which is so extreme and lacks a clear understanding.

Two additional remarks emerge from this analysis:

The Hawking black hole radiation equation is one of the few relations that combine the three fundamental constants c, h and G, in addition to Boltzmann's constant k_B. This can be achieved through the analogy of the equivalent elastic medium via the Young's modulus Y with equation (14). Thus, the elastic analogy provides a way to combine these three fundamental physical constants.

By reversing relation (14), the gravitational constant G becomes dependent on the elastic characteristics of the equivalent cosmic elastic medium. In their approaches, T. G. Tenev and M. F. Horstemeyer did not consider an anisotropic medium with several Young's moduli. Thus they propose:

$$G = \sqrt{\frac{6c^7}{hY_k}} \tag{15}$$

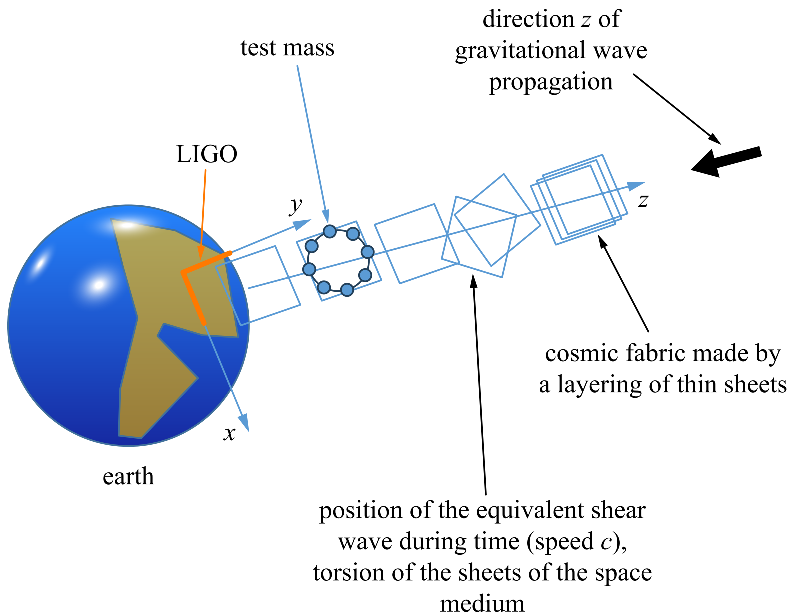


Figure 8. Visualization of the successive deformations of a stack of space sheets during the passage of a gravitational wave.

The elastic medium associated with space in continuum mechanics is necessarily anisotropic due to the values of these Poisson's ratios (transverse isotropic in the plan x, y). It is also anisotropic due to the absence of z-polarization, which is connected with the deformation of space as seen in (10). Y_k is the Young's modulus in the direction k.

4.1.2. *Deformation study of two 90° space tubes containing the interferometer arms.* D. Izabel in [28] and [29] generalized the work of T. G. Tenev, and M. F. Horstemeyer in [55] (Figure 8) regarding a medium consisting of several thin sheets of Planck thickness each characterized by an associated Young's modulus $Y = Y_x = Y_y$ and energy. Izabel sought the analogy with the mechanics of continuous media not by modifying the field equation of general relativity but by introducing mechanical parameters into the constant of proportionality κ, which becomes the flexibility characteristic of the cosmic fabric expressed as a function of Young's modulus. He finds there a formulation analogous to that of a generalized Hooke's law.

Thus, in the case of interferometers positioned in a plane of this cosmic sheet, in each arm he is able to write [28]

$$\frac{1}{L^2}(\varepsilon_{xx})^2 = 4(1+\nu_{xy})\pi \frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{U}{V}, \qquad \frac{1}{L^2}(\varepsilon_{yy})^2 = 4(1+\nu_{yx})\pi \frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{U}{V}. \tag{16}$$

These amount to a mechanical version of the Einstein's field equation (1) in space.

L is the length of the interferometer arm, U is the elastic deformation energy of the volume V of the arm, f is the natural frequency of vibration of the space in the arm in accordance with the gravitational wave, ρ is the density of the vacuum, ε_{xx} and ε_{yy} are the deformations of the space in the arms, with $\nu = \nu_{xy} = \nu_{yx} = 1$, the Poisson's ratios in the xy-plane.

Assuming equivalence in curvature tensor (the radius of curvature R locally tends to $1/L^2$ so, is quasiflat as for the curvature k of the universe) we can write in the plane of the interferometers:

$$R_{ij} = \begin{bmatrix} 1/L^2 & 0 \\ 0 & 1/L^2 \end{bmatrix} \begin{bmatrix} (\varepsilon_{xx})^2 & 0 \\ 0 & (\varepsilon_{yy})^2 \end{bmatrix}. \tag{17}$$

Defining $U/V = T_{xx} = T_{yy}$, the components of the strain energy tensor of the cosmic fabric are, with U as the strain energy and V as the volume of the interferometer arms,

$$T_{ij} = \begin{pmatrix} T_{xx} & 0 \\ 0 & T_{yy} \end{pmatrix} \tag{18}$$

If we use $\upsilon = \nu_{xy} = \nu_{yx} = 1$, as explained in Section 4.1.1, in the expressions

$$4(1+\nu)\pi \frac{\pi f^2}{\rho} \frac{1}{c^4} = 8\pi \frac{G}{c^4}, \quad G = \frac{\pi f^2}{\rho} = \frac{\pi f^2 c^2}{Y}, \tag{19}$$

we lowercased "We" obtain Einstein's expression (20) in weak field general relativity transposed into an equivalent elastic medium in the plane of the interferometer as

$$R_{ij} = \frac{8\pi G}{c^4} T_{ij} = \kappa T_{ij} \tag{20}$$

Remark. It can be shown [28], [29] that the expression (19) for the cosmic fabric flexibility κ is related to this expression of G as a function of the squared frequency of vibration of the space in the tube and the energy density of the vacuum in quantum field theory. Expression (19) developed in [28] and [29] shows that the Young's modulus can be expressed as a function of G, f and c.

The conclusion of this study is that the expression of Einstein's general relativity can be seen in planes xy transverse to the direction of propagation of the gravitational wave as a Hooke's law, with κ playing the role of the flexibility of the equivalent cosmic fabric structure, but with an anisotropy that remains nonstandard.

4.1.3. *Torsionally stressed space cylinder.* In publications [28] and [29], the author uses the analogy between the perturbations of the metric and the associated distortions of space (see formula (10)). Thus, the two classical polarizations A^+ and $A^\times$ of gravitational waves can be read as follows:

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct-z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \varepsilon_{xy(A_+)} = \frac{1}{2} A_+ \cos\left(\frac{\omega}{c}(ct-z)\right) \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{21}$$

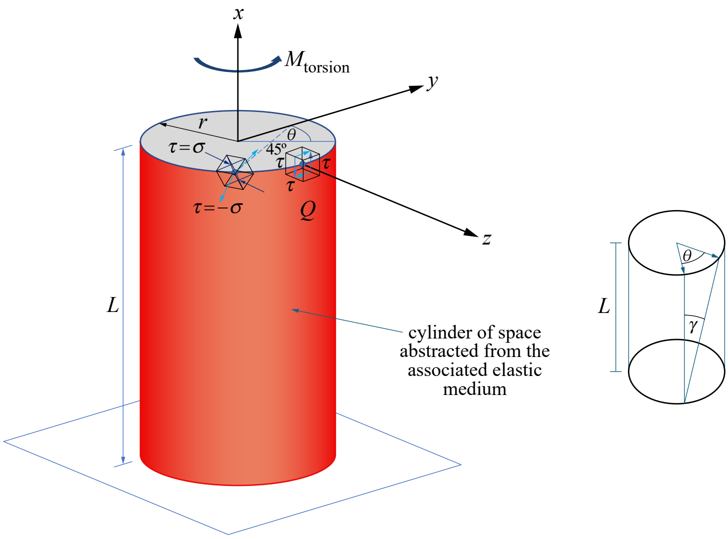


Figure 9. Pure torsion state of an isotropic material in elasticity theory.

These two components of deformations correspond to elongations and shortenings; see formula (10). Moreover,

$$h_{\mu\nu} = A_\times \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \varepsilon_{xy(A_\times)} = \frac{1}{2} A_\times \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (22)$$

These two components of deformations correspond to distortions, which in elasticity correspond to pure torsion (Figure 9).

Gravitational waves are caused by the rotation of binaries (two black holes, two neutron stars, one black hole and one neutron star, etc.) that can be seen as a twist of space due to their rotation relative to each other (Figure 10).

However, if we use the analogy in the other direction by asking the following question: can an elastic reading of the two polarizations obtained in general relativity offer us additional information in the weak field? By examining the two deformation tensors above, we observe elongations ε_{xx} and ε_{yy} shortenings in the xy-plane associated with the deformations and, already widely measured with the LIGO/Virgo interferometers. Depending on the elasticity, angular distortions in 45° planes would also be possible associated with the deformations ε_{xy} and ε_{yx} (Figure 11) and [28]. It is not possible to measure these distortions by current interferometers because they are not designed for this. This should be possible using the LISA interferometer or the future three-arm triangle Einstein telescope or multiple pulsars as was done for the 2023 detection of the stochastic gravitational wave background:

$$\tan \gamma_{ij} \approx \gamma_{ij} = \frac{b}{L} \quad (23)$$

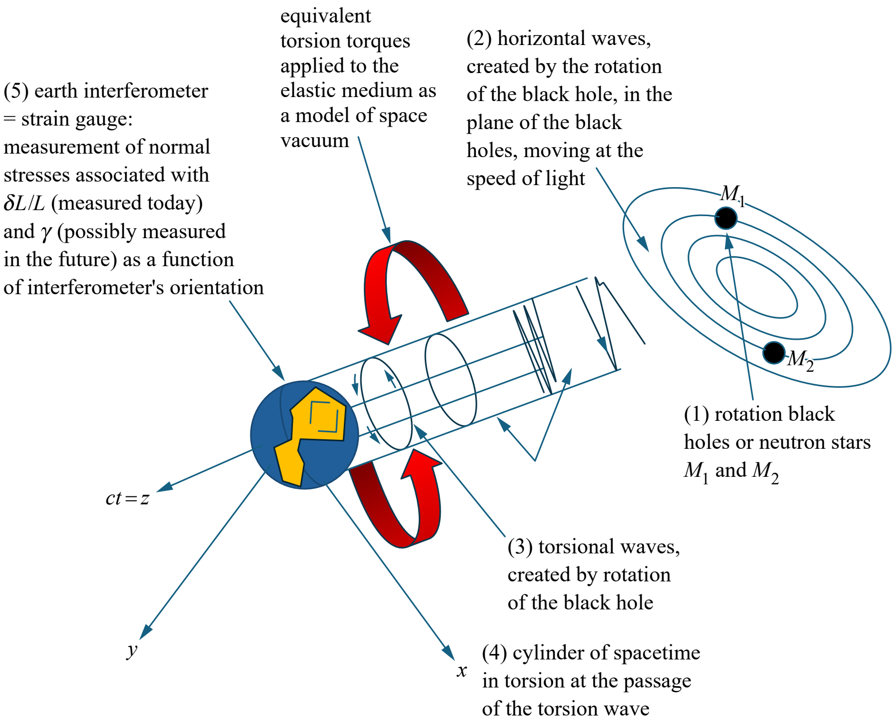


Figure 10. Effect of the rotation of a binary following the analogy of the torsional elastic medium.

Finally, in [28] by studying a cylinder of elastic space twisted, the author finds a mechanical version of general relativity in which it again appears $G = \pi f^2/\rho$. The deformations associated with the curvature are angular distortions, noted as γ in Figure 11, where T is the strain energy U divided by the volume V of the interferometer's arms:

$$U = \frac{1}{2}\int_0^L \frac{M_t^2}{\mu I_t} dx, \quad \frac{1}{L^2}\gamma^2 = 16\pi \frac{G}{c^4} \times T. \tag{24}$$

This can be compared with (3) by replacing $h_{\mu\nu}$ with (10) using this mechanical expression of general relativity in the weak field:

$$\Box\left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu} 2\bar{\varepsilon}\right) = -\frac{16\pi G}{c^4} T_{\mu\nu}. \tag{25}$$

5. Analysis and discussion of the results obtained according to general relativity and the analogy of the elastic medium

5.1. ***Concerning the analogy between the polarizations of gravitational waves and the different forms of the associated deformation tensor of space.*** One of the interesting contributions of the elastic medium analogy is the opportunity to interpret, according to [55; 28; 29], the polarizations of gravitational waves in linearized general relativity as components of a space deformation tensor by applying the analogy of the elastic medium to cosmic space.

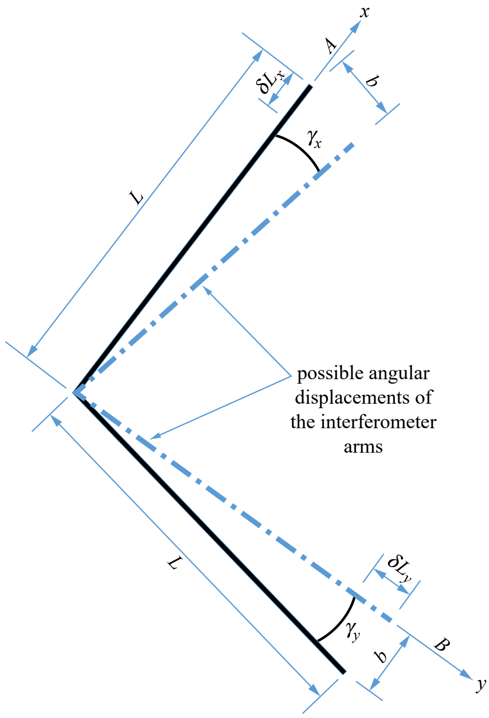


Figure 11. Nonmeasurable angular distortions of the arms interferometers in the xy-plane (top view).

This provides a new illustration of the two polarizations A^{+} and $A^{\times}$. In the analogy of the elastic medium subjected to pure torsion by two massive rotating objects (e.g., GW150914), the strain tensor consists of four components. two diagonal components corresponding to facets stressed in tensile and compressive motion, and two components in the other corresponding diagonal for another facet at 45° of the previous normal stress (Figure 12). Due to the Mohr circle associated with this pure torsion, these shear stresses τ are of the same intensity as the normal stresses σ on the other facet.

5.2. ***Possible lateral deformations of the interferometers.*** According to [28], these potential angular deformations could be easily calculated or measured geometrically using the Mohr circle for two of the arms of the future LISA experiment [26].

The layout is as follows (Figure 12):

1. Plot the deformation ε_{xx} of Arm 1 from the measurement of the elongations of this arm, calculated from the time it takes for the laser beam to travel the distance between Satellite 1 and Satellite 2.
2. Trace the deformation $\varepsilon_{yy} = -\varepsilon_{xx}$.
3. Plot the Mohr circle passing through ε_{xx} and ε_{yy}.
4. The deformation $\varepsilon_{\text{Arm 2}}$ of Arm 2 is reported from the measurement of the shortening of this arm (this is a shortening slightly less than the maximum shortening located at 90° (stated by LIGO and Virgo).

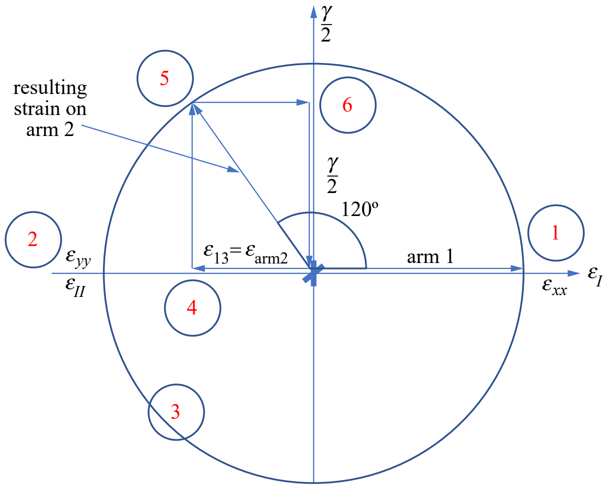


Figure 12. Determination of a potential angular distortion from the future LISA (arms at 60°).

5. Draw the direction of Arm 2 by rotating $-120°$ (due to the angle of $60°$ between the arms of the future LISA interferometer) the direction of the vector associated with the facet of Arm 2.
6. From this, we derive the second component (half distortion $\frac{\gamma}{2}$) for the facet associated with Arm 2 on the vertical axis.

The value of the distortion is intersected by the lengthening and shortening of Arm 3 with the expression α seen between the arms below (Figure 13):

$$cos\alpha_{(t)} = \frac{L_2^2 + L_1^2 - L_3^2}{2L_1L_2} = 60° + \gamma' \tag{26}$$

5.3. ***Concerning the anisotropy of space based solely on current general relativity.*** We have seen that linearized general relativity implies, in the case of gravitational waves, two unique polarizations that can

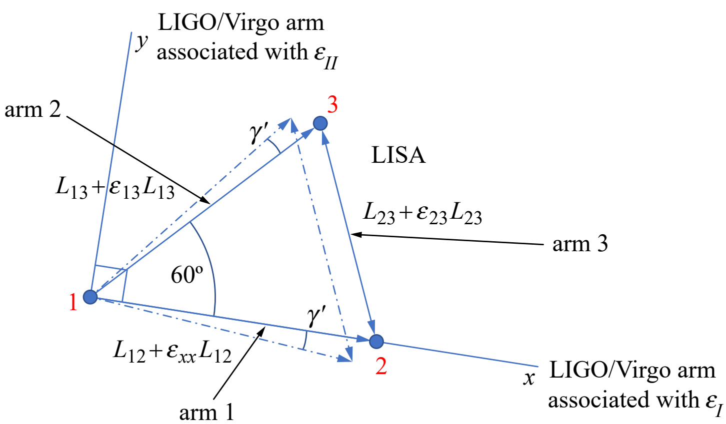


Figure 13. Determination of angles α and γ' from variations in LISA measurements of arms lengths.

be read as two expressions of a strain tensor involving a Poisson's ratio of 1 in these distorted transverse planes [28] and [55]. Since general relativity does not predict a longitudinal wave, the associated space model is found in plane deformations without any deformations in the direction of wave propagation involving zero Poisson's ratios in all planes including the direction of propagation. In this analogy, the elastic medium seems to be made up of transverse plane independent of each others. From a mechanical point of view, it seems clear that, if we adhere to general relativity in its current version, that this blatant anisotropy is contradictory to the hypothesis generally made in physics of a homogeneous and isotropic transverse medium. Clearly, in our analogy, three-dimensional elastic space can no longer be isotropic. We are missing the deformations ε_{xz}, ε_{yz}.

We therefore have, in the case of this local transverse isotropy and the notations of the ASTER code:

For the Young's moduli:

$$Y_L = E_L = Y_T = E_T. \tag{27}$$

For shear moduli:

$$G_{TN} = G_{LN}, \tag{28}$$

$$G_{LT} = \frac{E_L}{2(1+\nu_{LT})}. \tag{29}$$

On the basis of deformations, in the planes perpendicular to the direction of propagation of gravitational waves in weak field general relativity, unmodified, we have

$$\frac{\nu_{LN}}{E_L} = \frac{\nu_{TN}}{E_L} = 0. \tag{30}$$

Therefore

$$\nu_{NT} = \nu_{NL} = 0, \tag{31}$$

$$\nu_{LN} = \nu_{TN} = 0; \tag{32}$$

while in the xy-plane,

$$\nu_{LT} = \nu_{TL} = 1. \tag{33}$$

From the displacements and associated strains imposed at the cosmic fabric by the mass present in space or for vacuum cosmic fabric subjected to the gravitational wave elastic energy [49; 3], it is thus possible to define the equivalent stress field (see introduction) as described in formulas (34), (35), and (36).

Thus, writing the generalized Hooke's law in the frame of reference (L, T, N), where N is he direction of propagation as $\hat{\varepsilon} = K^{-1}\hat{\sigma}$, with:

$\hat{\varepsilon}^T = (\varepsilon_{LL}, \varepsilon_{TT}, \varepsilon_{NN}, 2\varepsilon_{LT}, 2\varepsilon_{LN}, 2\varepsilon_{TN})$ and $\hat{\sigma}^T = (\sigma_{LL}, \sigma_{TT}, \sigma_{NN}, \sigma_{LT}, \sigma_{LN}, \sigma_{TN})$, K^{-1} is the compliance matrix. While the definition of the strain tensor is clear with the space deformation theory, the same cannot be said for the stress tensor $\hat{\sigma}^T$. In our case, we define this stress tensor as the equivalent stress field that induces the observed strain tensor in linearized elasticity under small strain.

The generalized transverse isotropic Hooke's law is

$$\begin{Bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{Bmatrix} = [K^{-1}] \begin{Bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{Bmatrix} \tag{34}$$

where $[K^{-1}]$, the classical compliance matrix at the point $M(L, T, N)$ in the frame of reference (L, T, N), is given by

$$\begin{bmatrix} 1/E_L & -\upsilon_{LT}/E_L & -\upsilon_{LN}/E_L & 0 & 0 & 0 \\ -\upsilon_{TL}/E_T & 1/E_T & -\upsilon_{TN}/E_T & 0 & 0 & 0 \\ -\upsilon_{NL}/E_N & -\upsilon_{NT}/E_N & 1/E_N & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\upsilon_{LT})/E_L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{LN} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{TN} \end{bmatrix}_{M(L,T,N)} \tag{35}$$

In the case of classical gravitational waves, K^{-1} becomes

$$\begin{bmatrix} 1/E_L & -1/E_L & 0 & 0 & 0 & 0 \\ -1/E_L & 1/E_L & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/E_N & 0 & 0 & 0 \\ 0 & 0 & 0 & 4/E_L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{LN} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{LN} \end{bmatrix}_{M(N,L,T)} \tag{36}$$

To find a spatial and local behavior of the medium, the elastic analogy associated to the modified general relativity suggests the existence of deformations following the direction N ($N = z$, the direction of propagation of the gravitational wave) with Poisson's ratios $\nu_{NT} = \nu_{NL} \neq 0$ associated with equivalent normal stresses σ_N, $(L, T, N) = (x, y, z)$.

5.4. ***Concerning the analogy between geometric torsion and crystal plasticity of the equivalent elastic medium.*** This notion of a complementary vector to close a path on a surface, related to geometric torsion in the Einstein–Cartan theory, is also present in crystal plasticity [49] when, at the atomic level, local plasticization occurs [32]. There are two types of defects: screw dislocations (Figure 15) and edge dislocations (Figure 14, left), sliding by shear effect, and disinclination by forced rotations (Figure 14, right).

In both cases, there is a discontinuity in the network. Mathematically, when we make a path through this dislocation, we must use a closure vector called the Burgers vector, which is equivalent to the geometric torsion explained above; see Figure 15.

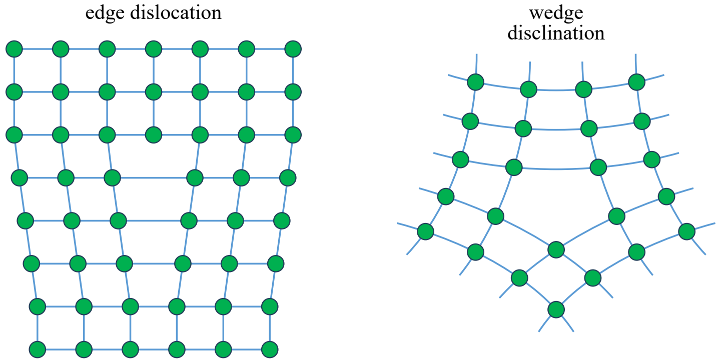


Figure 14. Example defects: edge dislocation and disclination.

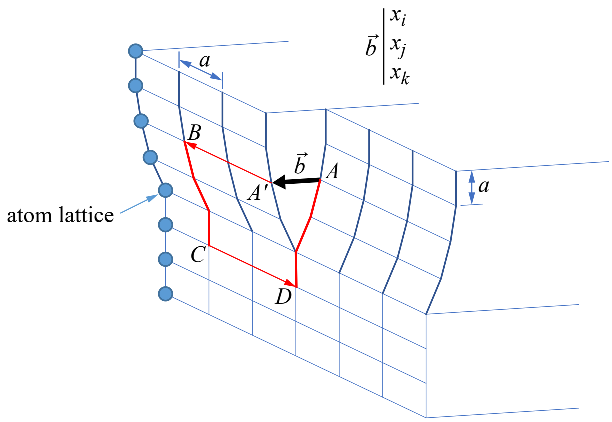


Figure 15. Screw dislocation: the Burgers vector $\vec{b}$, joining the points A and A'.

By writing the path $ABCD$ in Figure 15, we can see that mathematically the equivalent Burgers vector is expressed as follows [32]:

$$db^{\mu} = -\Gamma^{\mu}_{\nu\lambda} dA^{\nu\lambda}. \tag{37}$$

$dA^{\nu\lambda}$ being antisymmetric, the symmetric part of the affine bond vanishes, and only the antisymmetric part exists, which is written $\Gamma^{\mu}_{[\nu\lambda]}$. So, the equivalent Burgers vector is

$$db^{\mu} = -\Gamma^{\mu}_{[\nu\lambda]} dA^{\nu\lambda}. \tag{38}$$

The bridge is then made with geometric twisting

$$-T^{\mu}_{\nu\lambda} = -2\Gamma^{\lambda}_{[\mu\nu]}, \tag{39}$$

by defining

$$-[\nu\lambda] = \tfrac{1}{2}(\lambda\nu - \nu\lambda) \tag{40}$$

The theory of defects in crystal plasticity is therefore a mirror of geometric torsion in general relativity [32]. The analogy with two distinct mechanical concepts — elasticity and perfect plasticity — are not

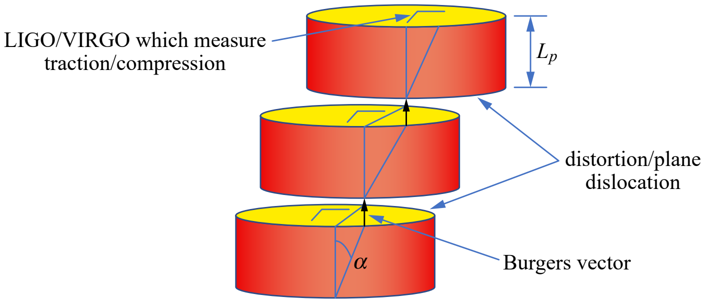


Figure 16. Possible local dislocations between the different layers of the structure of the fabric of spacetime (analogy) during the passage of a gravitational wave.

contradictory here. Indeed, the bridge between the theory of defects and the Einstein–Cartan theory with torsion, as explained above, will imply in the analogy of the elastic medium that planes of space could successively slide locally as a defect during the passage of a gravitational wave, as shown in Figure 16. This is compatible with a possible shear modulus of the medium. There would be no propagation of the torsion as such. Two alternatives are possible: either the space would plasticize locally by shear/distortion, or the analogy would reach its limit in this example.

It is known [11; 47] that the modified Einstein–Cartan theory of general relativity with geometric torsion comprises two field equations, one equivalent to that developed by Einstein (38) and another corresponding to spins of space (39). It has been shown by the authors in [11; 47] that the mathematical formalism associated with this geometric torsion via the Burgers vector (37) to (40) is similar to that in crystal plasticity [38; 47].

It is shown via [19] that polarizations complementary to those predicted in classical general relativity and $(A^+, A^\times)$ appear when we consider this geometric torsion.

We know from [55] that the linearized Einstein equation leads to the squeeze (9) (equivalent to (10)), constituting a bridge that allows us to read polarizations in the elastic domain A^+ and $A^\times$ as components of a strain tensor. This approach has been illustrated in two concrete cases of space twisting in [28] and [29].

However, we have not yet shown that the second spin equation in the Einstein–Cartan theory corresponds to another equation of correspondence between polarizations and deformations in the plastic domain. Is there therefore a transitional formalism equivalent to equation (10) in plasticity associated with defect theory [47] and associated complementary polarizations [11] and [19]? In other words, in plasticity (corresponding to geometric torsion according to [47]), can complementary polarizations also be effectively read as components of a deformation tensor of a four-dimensional space (and no longer only as deformations in successive planes independent of each other as shown in [55] and [28])? The bridge between the components of the polarization tensors of gravitational waves in the case of modified general relativity (Einstein–Cartan) and, by analogy, the deformations linked with an associated elastoplastic medium was made in [12].

The authors thus considered a gravitational wave as a defect (propagation of a Burgers vector) propagating in an equivalent solid medium. The result of their study is again a compression component H in space and shear and distortion components $\pm\sqrt{2}a_i$, as described in expression (41) in four dimensions and (42) in three dimensions.

$$\varepsilon_{\mu\nu} = \frac{1}{2}\begin{pmatrix} H & -\sqrt{2}a_1 & -\sqrt{2}a_2 & H \\ -\sqrt{2}a_1 & 0 & 0 & -\sqrt{2}a_1 \\ -\sqrt{2}a_2 & 0 & 0 & -\sqrt{2}a_2 \\ H & -\sqrt{2}a_1 & -\sqrt{2}a_2 & H \end{pmatrix} \tag{41}$$

They also cite the existence of what we called a torsion wave in [28], which they refer to as a J-dependent gyratonic wave (a spin of space), in which the function J is related to the spinning nature of the gyratons. [12]:

$$\varepsilon^{(i)(j)} = \frac{1}{2}\begin{pmatrix} 0 & 0 & \frac{\sqrt{2}J}{\rho A} sin\Phi \\ 0 & 0 & -\frac{\sqrt{2}J}{\rho A} cos\Phi \\ \frac{\sqrt{2}J}{\rho A} sin\Phi & -\frac{\sqrt{2}J}{\rho A} cos\Phi & H/A \end{pmatrix} \tag{42}$$

Note that in [12], the authors placed themselves within the framework of nonlinear plane gravitational waves, or parallel-propagating plane front waves (P-P waves) [39]. Indeed, as this study considers the parallelism between crystal plasticity, the implementation of which characterizes a plasticization of the medium by rearrangement of atoms (and therefore a nonlinearity between deformations and stresses) [47], and Einstein–Cartan's nonlinear modified general relativity associated with this theory [47; 11; 19], it makes sense to move away from the realm of traditional elastic waves to consider nonlinear plane waves known as PPs.

Note also that the result they obtain (expressions 43 and 44) for a gravitational wave propagating in the direction z is connected to polarizations A^+ and $A^\times$ gravitational waves in classical general relativity via the following expressions from [39]:

$$H = A_+(u)(x^2 - y^2), \quad H = A_\times(u)xy \tag{43}$$

Recently, in [59; 58; 16; 60], the authors consider a shape memory of space (i.e., a certain residual plasticity of this medium).

The null values in the above two tensors (41) and (42) result from the fact that the authors focused on the geometric torsion part associated with the Burgers vector itself, which is associated with crystal plasticity [32; 47], i.e., the second equation of the Einstein–Cartan theory that presents an analogy with this theory. Thus, as in [19] where complementary polarizations appear because of this geometric torsion in the components (zz), (zx), (zy) and (zt) in 4 dimensions, in [12], complementary deformations appear according to these same components and only for them. However, this publication [12] shows that, unlike the classical equation of general relativity where the bridge between polarization and deformation is direct via expression (10), this time, for the torsion component, the correspondence between the components of the polarization tensors and the deformation tensors in four and three dimensions is no longer direct. The publication [12] explains mathematically how to make this transition.

On the other hand, our paradigm for reading the components of polarizations as components of a strain tensor remains the same.

5.5. ***Concerning the convergence of the different models regarding possible polarizations of gravitational waves in the direction of their propagation.*** In order to analyze the convergence between the different approaches resulting from general relativity, we consider the following:

Concerning the first case: (second-order linearization of $h_{\mu\nu}$ [6; 45], the Einstein–Cartan theory of geometric torsion [47; 11], the theories of modified general relativity [43; 40]) and approaches derived from the analogy of elastic space, we find polarizations complementary to the two classics A^+ and $A^\times$. Concerning the second case: (strong anisotropy of the medium [55], crystal plasticity [32; 47] (junction of transverse planes by successive plastic slips)), we can see by virtue of the analogy between the polarizations of gravitational waves and the deformations of the equivalent elastic medium [55; 28; 29], longitudinal spatial deformations complementing the transverse deformations.

It should be stressed that this convergence aims to continue optimizing the performance of interferometric sensors, to add arms to have a triangular measurement system (Future LISA or Einstein telescope) to possibly detect them and thus settle the question.

5.6. ***Concerning the extreme smallness of the deformations/polarizations in the longitudinal direction of gravitational wave propagation (if they really exist).*** If such longitudinal deformations/polarizations exist, the measurement of the gap between theoretical and real gravitational waves (Figure 5) seems to indicate that they are much smaller than those already measured by the current LIGO/Virgo interferometers [1; 2]. The second-order study of $h_{\mu\nu}$ in gravitoelectromagnetism is more nuanced, the intensity of the out-of-plane deformations depends on the frequency of the gravitational wave [45; 46].

5.7. ***Concerning the importance of excellent coordination of interferometers on Earth complemented by future LISA-type interferometers or the future Einstein telescope or multiple pulsars.*** Publications [3; 27; 4; 40; 57; 56] show that many research teams are currently working to theorize and measure these potential complementary polarizations of gravitational waves, including in the direction of gravitational wave propagation. In [40], the author explains how these different possible polarizations could be studied by an even more efficient interconnection of the different interferometers on Earth.

6. Conclusion

Concerning the convergence between the analogy of space as an elastic medium and the results of general relativity

The analogy of continuum mechanics with general relativity in the weak field is interesting because it allows us to better understand certain aspects of the latter. The metric perturbation tensor is assimilated with twice the strain tensor [55], we notice the similarity between the stress tensor and the energy-momentum tensor [55; 28]. It is possible to consider the parametrizable constant κ as a function of the mechanical characteristics of space [28]. Then, Einstein's equation appears similar to Hooke's law [55; 28; 29], and finally, gravitational waves are similar to medium shear waves [55; 28]. It also provides a new illustration of the origin of the two polarizations instead of one, or six (pure twisting of an elastic medium) [28]. Finally, it suggests distortion (lateral displacement of laser beams of interferometers).

Concerning the divergence between the analogy of space as an elastic medium and the results of general relativity. The analogy of elasticity theory with general relativity in a weak field also reaches its limits and raises questions about its real representativeness, given the extremely high intensity of the associated Young's moduli [55; 41; 28], the value of the Poisson's ratios of 1 in the interferometer plane and 0 out of plane, leading to a strong anisotropy of the medium in any point M and during wave propagation, contrary to the fundamental hypothesis of the homogeneity of the cosmic medium.

Concerning the convergence between current research and what the analogy of space as an elastic medium with the results of general relativity suggests. The discrepancy between the measured and theoretical curves of space deformations during the passage of a gravitational wave (Figure 5) suggests that there is still room for improvement in the general relativity. However, this improvement is a priori extremely small, as this gap is very small [1] and [2].

Research avenues to try to complete this general relativity concern, in particular, potential complementary polarizations of gravitational waves [19; 40; 43].

The modified elastic analogy of general relativity with Einstein–Cartan geometric torsion connected to the theory of defects [11; 19; 47], as well as the approach of general relativity developed in the second order [6; 45], involve additional deformations in the direction of propagation of gravitational waves. Such approaches make it possible to find a coherent spatial behavior and therefore a little less anisotropy of the elastic space associated in our analogy. The geometric torsion, as developed for example by Einstein–Cartan theory can be assimilated at a plasticity between each successive planes via the analogy of the crystallography and defect theory. On this basis, a 3D equivalent anisotropic space medium becomes possible (polarization in three directions and not only on transverse plane, coupled by analogy with strains in three dimensions and not only in two dimensions) [46; 31]. These additional corrections are extremely small [32] and are therefore consistent with what is revealed by the measurements of the deviation between the measured deformation curves associated with gravitational waves and the same deformation curves from unmodified general relativity [1; 2; 3]. According to our paradigm associating these deformations with complementary polarizations, this implies detecting them in order to validate or invalidate these deformations associated with these complementary polarizations.

The multiplicity of recent publications on these potential complementary polarizations in relativistic physics on the one hand [3; 27; 4; 40; 57; 56] and on how to measure them on the other hand indicates (future LISA and Einstein telescope) that this topic is a key research point today. Time will tell whether or not this precise point is a point of convergence between the analogy of the elastic medium and general relativity in the weak field.

References

[1] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, L. Aiello, A. Ain, P. Ajith, B. Allen, A. Allocca, P. A. Altin, S. B. Anderson, W. G. Anderson, K. Arai, M. A. Arain, M. C. Araya, C. C. Arceneaux, J. S. Areeda, N. Arnaud, K. G. Arun, S. Ascenzi, G. Ashton, M. Ast, S. M. Aston, P. Astone, P. Aufmuth, C. Aulbert, S. Babak, P. Bacon, M. K. M. Bader, P. T. Baker, F. Baldaccini, G. Ballardin, S. W. Ballmer, J. C. Barayoga, S. E. Barclay, B. C. Barish, D. Barker, F. Barone, B. Barr, L. Barsotti, M. Barsuglia, D. Barta, J. Bartlett, M. A. Barton, I. Bartos, R. Bassiri, A. Basti, J. C. Batch, C. Baune, V. Bavigadda, M. Bazzan, B. Behnke, M. Bejger, C. Belczynski, A. S. Bell, C. J. Bell, B. K. Berger, J. Bergman, G. Bergmann, C. P. L. Berry, D. Bersanetti, A. Bertolini, J. Betzwieser, S. Bhagwat, R. Bhandare, I. A. Bilenko, G. Billingsley, J. Birch, I. A. Birney, O. Birnholtz, S. Biscans, A. Bisht, M. Bitossi, C. Biwer, M. A. Bizouard, J. K. Blackburn, C. D. Blair, D. G. Blair, R. M. Blair, S. Bloemen, O. Bock, T. P. Bodiya, M. Boer, G. Bogaert, C. Bogan, A. Bohe, P. Bojtos, C. Bond, F. Bondu, R. Bonnand, B. A. Boom, R. Bork, V. Boschi, S. Bose, Y. Bouffanais, A. Bozzi, C. Bradaschia, P. R. Brady, V. B. Braginsky, M. Branchesi, J. E. Brau, T. Briant, A. Brillet, M. Brinkmann, V. Brisson, P. Brockill, A. F. Brooks, D. A. Brown, D. D. Brown, N. M. Brown, C. C. Buchanan, A. Buikema, T. Bulik, H. J. Bulten, A. Buonanno, D. Buskulic, C. Buy, R. L. Byer, M. Cabero, L. Cadonati, G. Cagnoli, C. Cahillane, J. C. Bustillo, T. Callister, E. Calloni, J. B. Camp, K. C. Cannon, J. Cao, C. D. Capano, E. Capocasa, F. Carbognani, S. Caride, J. C. Diaz, C. Casentini, S. Caudill, M. Cavaglià, F. Cavalier, R. Cavalieri, G. Cella, C. B. Cepeda, L. C. Baiardi, G. Cerretani, E. Cesarini, R. Chakraborty, T. Chalermsongsak, S. J. Chamberlin, M. Chan, S. Chao, P. Charlton, E. Chassande-Mottin, H. Y. Chen, Y. Chen, C. Cheng, A. Chincarini, A. Chiummo, H. S. Cho, M. Cho, J. H. Chow, N. Christensen, Q. Chu, S. Chua, S. Chung, G. Ciani, F. Clara, J. A. Clark, F. Cleva, E. Coccia, P.-F. Cohadon, A. Colla, C. G. Collette, L. Cominsky, M. Constancio, A. Conte, L. Conti, D. Cook, T. R. Corbitt, N. Cornish, A. Corsi, S. Cortese, C. A. Costa, M. W. Coughlin, S. B. Coughlin, J.-P. Coulon, S. T. Countryman, P. Couvares, E. E. Cowan, D. M. Coward, M. J. Cowart, D. C. Coyne, R. Coyne, K. Craig, J. D. E. Creighton, T. D. Creighton, J. Cripe, S. G. Crowder, A. M. Cruise, A. Cumming, L. Cunningham, E. Cuoco, T. D. Canton, S. L. Danilishin, S. D'Antonio, K. Danzmann, N. S. Darman, C. F. Da Silva Costa, V. Dattilo, I. Dave, H. P. Daveloza, M. Davier, G. S. Davies, E. J. Daw, R. Day, S. De, D. DeBra, G. Debreczeni, J. Degallaix, M. De Laurentis, S. Deléglise, W. Del Pozzo, T. Denker, T. Dent, H. Dereli, V. Dergachev, R. T. DeRosa, R. De Rosa, R. DeSalvo, S. Dhurandhar, M. C. Díaz, L. Di Fiore, M. Di Giovanni, A. Di Lieto, S. Di Pace, I. Di Palma, A. Di Virgilio, G. Dojcinoski, V. Dolique, F. Donovan, K. L. Dooley, S. Doravari, R. Douglas, T. P. Downes, M. Drago, R. W. P. Drever, J. C. Driggers, Z. Du, M. Ducrot, S. E. Dwyer, T. B. Edo, M. C. Edwards, A. Effler, H.-B. Eggenstein, P. Ehrens, J. Eichholz, S. S. Eikenberry, W. Engels, R. C. Essick, T. Etzel, M. Evans, T. M. Evans, R. Everett, M. Factourovich, V. Fafone, H. Fair, S. Fairhurst, X. Fan, Q. Fang, S. Farinon, B. Farr, W. M. Farr, M. Favata, M. Fays, H. Fehrmann, M. M. Fejer, D. Feldbaum, I. Ferrante, E. C. Ferreira, F. Ferrini, F. Fidecaro, L. S. Finn, I. Fiori, D. Fiorucci, R. P. Fisher, R. Flaminio, M. Fletcher, H. Fong, J.-D. Fournier, S. Franco, S. Frasca, F. Frasconi, M. Frede, Z. Frei, A. Freise, R. Frey, V. Frey, T. T. Fricke, P. Fritschel, V. V. Frolov, P. Fulda, M. Fyffe, H. A. G. Gabbard, J. R. Gair, L. Gammaitoni, S. G. Gaonkar, F. Garufi, A. Gatto, G. Gaur, N. Gehrels, G. Gemme, B. Gendre, E. Genin, A. Gennai, J. George, L. Gergely, V. Germain, A. Ghosh, A. Ghosh, S. Ghosh, J. A. Giaime, K. D. Giardina, A. Giazotto, K. Gill, A. Glaefke, J. R. Gleason, E. Goetz, R. Goetz, L. Gondan, G. González, J. M. G. Castro, A. Gopakumar, N. A. Gordon, M. L. Gorodetsky, S. E. Gossan, M. Gosselin, R. Gouaty, C. Graef, P. B. Graff, M. Granata, A. Grant, S. Gras, C. Gray, G. Greco, A. C. Green, R. J. S. Greenhalgh, P. Groot, H. Grote, S. Grunewald, G. M. Guidi, X. Guo, A. Gupta, M. K. Gupta, K. E. Gushwa, E. K. Gustafson, R. Gustafson, J. J. Hacker, B. R. Hall, E. D. Hall, G. Hammond, M. Haney, M. M. Hanke, J. Hanks, C. Hanna, M. D. Hannam, J. Hanson, T. Hardwick, J. Harms, G. M. Harry, I. W. Harry, M. J. Hart, M. T. Hartman, C.-J. Haster, K. Haughian, J. Healy, J. Heefner, A. Heidmann, M. C. Heintze, G. Heinzel, H. Heitmann, P. Hello, G. Hemming, M. Hendry, I. S. Heng, J. Hennig, A. W. Heptonstall, M. Heurs, S. Hild, D. Hoak, K. A. Hodge, D. Hofman, S. E. Hollitt, K. Holt, D. E. Holz, P. Hopkins, D. J. Hosken, J. Hough, E. A. Houston, E. J. Howell, Y. M. Hu, S. Huang, E. A. Huerta, D. Huet, B. Hughey, S. Husa, S. H. Huttner, T. Huynh-Dinh, A. Idrisy, N. Indik, D. R. Ingram, R. Inta, H. N. Isa, J.-M. Isac, M. Isi, G. Islas, T. Isogai, B. R. Iyer, K. Izumi, M. B. Jacobson, T. Jacqmin, H. Jang, K. Jani, P. Jaranowski, S. Jawahar, F. Jiménez-Forteza, W. W. Johnson, N. K. Johnson-McDaniel, D. I. Jones, R. Jones, R. J. G. Jonker, L. Ju, K. Haris, C. V. Kalaghatgi, V. Kalogera, S. Kandhasamy, G. Kang, J. B. Kanner, S. Karki, M.

Kasprzack, E. Katsavounidis, W. Katzman, S. Kaufer, T. Kaur, K. Kawabe, F. Kawazoe, F. Kéfélian, M. S. Kehl, D. Keitel, D. B. Kelley, W. Kells, R. Kennedy, D. G. Keppel, J. S. Key, A. Khalaidovski, F. Y. Khalili, I. Khan, S. Khan, Z. Khan, E. A. Khazanov, N. Kijbunchoo, C. Kim, J. Kim, K. Kim, N.-G. Kim, N. Kim, Y.-M. Kim, E. J. King, P. J. King, D. L. Kinzel, J. S. Kissel, L. Kleybolte, S. Klimenko, S. M. Koehlenbeck, K. Kokeyama, S. Koley, V. Kondrashov, A. Kontos, S. Koranda, M. Korobko, W. Z. Korth, I. Kowalska, D. B. Kozak, V. Kringel, B. Krishnan, A. Królak, C. Krueger, G. Kuehn, P. Kumar, R. Kumar, L. Kuo, A. Kutynia, P. Kwee, B. D. Lackey, M. Landry, J. Lange, B. Lantz, P. D. Lasky, A. Lazzarini, C. Lazzaro, P. Leaci, S. Leavey, E. O. Lebigot, C. H. Lee, H. K. Lee, H. M. Lee, K. Lee, A. Lenon, M. Leonardi, J. R. Leong, N. Leroy, N. Letendre, Y. Levin, B. M. Levine, T. G. F. Li, A. Libson, T. B. Littenberg, N. A. Lockerbie, J. Logue, A. L. Lombardi, L. T. London, J. E. Lord, M. Lorenzini, V. Loriette, M. Lormand, G. Losurdo, J. D. Lough, C. O. Lousto, G. Lovelace, H. Lück, A. P. Lundgren, J. Luo, R. Lynch, Y. Ma, T. MacDonald, B. Machenschalk, M. MacInnis, D. M. Macleod, F. Magaña Sandoval, R. M. Magee, M. Mageswaran, E. Majorana, I. Maksimovic, V. Malvezzi, N. Man, I. Mandel, V. Mandic, V. Mangano, G. L. Mansell, M. Manske, M. Mantovani, F. Marchesoni, F. Marion, S. Márka, Z. Márka, A. S. Markosyan, E. Maros, F. Martelli, L. Martellini, I. W. Martin, R. M. Martin, D. V. Martynov, J. N. Marx, K. Mason, A. Masserot, T. J. Massinger, M. Masso-Reid, F. Matichard, L. Matone, N. Mavalvala, N. Mazumder, G. Mazzolo, R. McCarthy, D. E. McClelland, S. McCormick, S. C. McGuire, G. McIntyre, J. McIver, D. J. McManus, S. T. McWilliams, D. Meacher, G. D. Meadors, J. Meidam, A. Melatos, G. Mendell, D. Mendoza-Gandara, R. A. Mercer, E. Merilh, M. Merzougui, S. Meshkov, C. Messenger, C. Messick, P. M. Meyers, F. Mezzani, H. Miao, C. Michel, H. Middleton, E. E. Mikhailov, L. Milano, J. Miller, M. Millhouse, Y. Minenkov, J. Ming, S. Mirshekari, C. Mishra, S. Mitra, V. P. Mitrofanov, G. Mitselmakher, R. Mittleman, A. Moggi, M. Mohan, S. R. P. Mohapatra, M. Montani, B. C. Moore, C. J. Moore, D. Moraru, G. Moreno, S. R. Morriss, K. Mossavi, B. Mours, C. M. Mow-Lowry, C. L. Mueller, G. Mueller, A. W. Muir, A. Mukherjee, D. Mukherjee, S. Mukherjee, N. Mukund, A. Mullavey, J. Munch, D. J. Murphy, P. G. Murray, A. Mytidis, I. Nardecchia, L. Naticchioni, R. K. Nayak, V. Necula, K. Nedkova, G. Nelemans, M. Neri, A. Neunzert, G. Newton, T. T. Nguyen, A. B. Nielsen, S. Nissanke, A. Nitz, F. Nocera, D. Nolting, M. E. N. Normandin, L. K. Nuttall, J. Oberling, E. Ochsner, J. O'Dell, E. Oelker, G. H. Ogin, J. J. Oh, S. H. Oh, F. Ohme, M. Oliver, P. Oppermann, R. J. Oram, B. O'Reilly, R. O'Shaughnessy, C. D. Ott, D. J. Ottaway, R. S. Ottens, H. Overmier, B. J. Owen, A. Pai, S. A. Pai, J. R. Palamos, O. Palashov, C. Palomba, A. Pal-Singh, H. Pan, Y. Pan, C. Pankow, F. Pannarale, B. C. Pant, F. Paoletti, A. Paoli, M. A. Papa, H. R. Paris, W. Parker, D. Pascucci, A. Pasqualetti, R. Passaquieti, D. Passuello, B. Patricelli, Z. Patrick, B. L. Pearlstone, M. Pedraza, R. Pedurand, L. Pekowsky, A. Pele, S. Penn, A. Perreca, H. P. Pfeiffer, M. Phelps, O. Piccinni, M. Pichot, M. Pickenpack, F. Piergiovanni, V. Pierro, G. Pillant, L. Pinard, I. M. Pinto, M. Pitkin, J. H. Poeld, R. Poggiani, P. Popolizio, A. Post, J. Powell, J. Prasad, V. Predoi, S. S. Premachandra, T. Prestegard, L. R. Price, M. Prijatelj, M. Principe, S. Privitera, R. Prix, G. A. Prodi, L. Prokhorov, O. Puncken, M. Punturo, P. Puppo, M. Pürrer, H. Qi, J. Qin, V. Quetschke, E. A. Quintero, R. Quitzow-James, F. J. Raab, D. S. Rabeling, H. Radkins, P. Raffai, S. Raja, M. Rakhmanov, C. R. Ramet, P. Rapagnani, V. Raymond, M. Razzano, V. Re, J. Read, C. M. Reed, T. Regimbau, L. Rei, S. Reid, D. H. Reitze, H. Rew, S. D. Reyes, F. Ricci, K. Riles, N. A. Robertson, R. Robie, F. Robinet, A. Rocchi, L. Rolland, J. G. Rollins, V. J. Roma, J. D. Romano, R. Romano, G. Romanov, J. H. Romie, D. Rosińska, S. Rowan, A. Rüdiger, P. Ruggi, K. Ryan, S. Sachdev, T. Sadecki, L. Sadeghian, L. Salconi, M. Saleem, F. Salemi, A. Samajdar, L. Sammut, L. M. Sampson, E. J. Sanchez, V. Sandberg, B. Sandeen, G. H. Sanders, J. R. Sanders, B. Sassolas, B. S. Sathyaprakash, P. R. Saulson, O. Sauter, R. L. Savage, A. Sawadsky, P. Schale, R. Schilling, J. Schmidt, P. Schmidt, R. Schnabel, R. M. S. Schofield, A. Schönbeck, E. Schreiber, D. Schuette, B. F. Schutz, J. Scott, S. M. Scott, D. Sellers, A. S. Sengupta, D. Sentenac, V. Sequino, A. Sergeev, G. Serna, Y. Setyawati, A. Sevigny, D. A. Shaddock, T. Shaffer, S. Shah, M. S. Shahriar, M. Shaltev, Z. Shao, B. Shapiro, P. Shawhan, A. Sheperd, D. H. Shoemaker, D. M. Shoemaker, K. Siellez, X. Siemens, D. Sigg, A. D. Silva, D. Simakov, A. Singer, L. P. Singer, A. Singh, R. Singh, A. Singhal, A. M. Sintes, B. J. J. Slagmolen, J. R. Smith, M. R. Smith, N. D. Smith, R. J. E. Smith, E. J. Son, B. Sorazu, F. Sorrentino, T. Souradeep, A. K. Srivastava, A. Staley, M. Steinke, J. Steinlechner, S. Steinlechner, D. Steinmeyer, B. C. Stephens, S. P. Stevenson, R. Stone, K. A. Strain, N. Straniero, G. Stratta, N. A. Strauss, S. Strigin, R. Sturani, A. L. Stuver, T. Z. Summerscales, L. Sun, P. J. Sutton, B. L. Swinkels, M. J. Szczepańczyk, M. Tacca, D. Talukder, D. B. Tanner, M. Tápai, S. P. Tarabrin, A. Taracchini, R. Taylor, T. Theeg, M. P. Thirugnanasambandam, E. G. Thomas, M. Thomas, P. Thomas, K. A. Thorne, K. S. Thorne, E. Thrane, S. Tiwari, V. Tiwari, K. V. Tokmakov, C. Tomlinson, M. Tonelli, C. V. Torres, C. I. Torrie, D. Töyrä, F. Travasso, G. Traylor, D. Trifirò, M. C. Tringali, L. Trozzo, M. Tse, M. Turconi, D. Tuyenbayev, D. Ugolini, C. S. Unnikrishnan, A. L. Urban, S. A. Usman, H. Vahlbruch, G. Vajente, G. Valdes, M. Vallisneri, N. van Bakel, M. van Beuzekom, J. F. J. van den Brand, C. Van Den Broeck, D. C. Vander-Hyde, L. van der Schaaf, J. V. van Heijningen, A. A. van Veggel, M. Vardaro, S. Vass, M. Vasúth, R. Vaulin, A. Vecchio, G. Vedovato, J.

Veitch, P. J. Veitch, K. Venkateswara, D. Verkindt, F. Vetrano, A. Viceré, S. Vinciguerra, D. J. Vine, J.-Y. Vinet, S. Vitale, T. Vo, H. Vocca, C. Vorvick, D. Voss, W. D. Vousden, S. P. Vyatchanin, A. R. Wade, L. E. Wade, M. Wade, S. J. Waldman, M. Walker, L. Wallace, S. Walsh, G. Wang, H. Wang, M. Wang, X. Wang, Y. Wang, H. Ward, R. L. Ward, J. Warner, M. Was, B. Weaver, L.-W. Wei, M. Weinert, A. J. Weinstein, R. Weiss, T. Welborn, L. Wen, P. Weßels, T. Westphal, K. Wette, J. T. Whelan, S. E. Whitcomb, D. J. White, B. F. Whiting, K. Wiesner, C. Wilkinson, P. A. Willems, L. Williams, R. D. Williams, A. R. Williamson, J. L. Willis, B. Willke, M. H. Wimmer, L. Winkelmann, W. Winkler, C. C. Wipf, A. G. Wiseman, H. Wittel, G. Woan, J. Worden, J. L. Wright, G. Wu, J. Yablon, I. Yakushin, W. Yam, H. Yamamoto, C. C. Yancey, M. J. Yap, H. Yu, M. Yvert, A. Zadrożny, L. Zangrando, M. Zanolin, J.-P. Zendri, M. Zevin, F. Zhang, L. Zhang, M. Zhang, Y. Zhang, C. Zhao, M. Zhou, Z. Zhou, X. J. Zhu, M. E. Zucker, S. E. Zuraw, and J. Zweizig, "Observation of gravitational waves from a binary black hole merger", *Phys. Rev. Lett.* **116**:6 (2016), art. id. 061102.

[2] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Afrough, B. Agarwal, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, L. Aiello, A. Ain, P. Ajith, B. Allen, G. Allen, A. Allocca, P. A. Altin, A. Amato, A. Ananyeva, S. B. Anderson, W. G. Anderson, S. V. Angelova, S. Antier, S. Appert, K. Arai, M. C. Araya, J. S. Areeda, N. Arnaud, K. G. Arun, S. Ascenzi, G. Ashton, M. Ast, S. M. Aston, P. Astone, D. V. Atallah, P. Aufmuth, C. Aulbert, K. AultONeal, C. Austin, A. Avila-Alvarez, S. Babak, P. Bacon, M. K. M. Bader, S. Bae, M. Bailes, P. T. Baker, F. Baldaccini, G. Ballardin, S. W. Ballmer, S. Banagiri, J. C. Barayoga, S. E. Barclay, B. C. Barish, D. Barker, K. Barkett, F. Barone, B. Barr, L. Barsotti, M. Barsuglia, D. Barta, S. D. Barthelmy, J. Bartlett, I. Bartos, R. Bassiri, A. Basti, J. C. Batch, M. Bawaj, J. C. Bayley, M. Bazzan, B. Bécsy, C. Beer, M. Bejger, I. Belahcene, A. S. Bell, B. K. Berger, G. Bergmann, S. Bernuzzi, J. J. Bero, C. P. L. Berry, D. Bersanetti, A. Bertolini, J. Betzwieser, S. Bhagwat, R. Bhandare, I. A. Bilenko, G. Billingsley, C. R. Billman, J. Birch, I. A. Birney, O. Birnholtz, S. Biscans, S. Biscoveanu, A. Bisht, M. Bitossi, C. Biwer, M. A. Bizouard, J. K. Blackburn, J. Blackman, C. D. Blair, D. G. Blair, R. M. Blair, S. Bloemen, O. Bock, N. Bode, M. Boer, G. Bogaert, A. Bohe, F. Bondu, E. Bonilla, R. Bonnand, B. A. Boom, R. Bork, V. Boschi, S. Bose, K. Bossie, Y. Bouffanais, A. Bozzi, C. Bradaschia, P. R. Brady, M. Branchesi, J. E. Brau, T. Briant, A. Brillet, M. Brinkmann, V. Brisson, P. Brockill, J. E. Broida, A. F. Brooks, D. A. Brown, D. D. Brown, S. Brunett, C. C. Buchanan, A. Buikema, T. Bulik, H. J. Bulten, A. Buonanno, D. Buskulic, C. Buy, R. L. Byer, M. Cabero, L. Cadonati, G. Cagnoli, C. Cahillane, J. Calderón Bustillo, T. A. Callister, E. Calloni, J. B. Camp, M. Canepa, P. Canizares, K. C. Cannon, H. Cao, J. Cao, C. D. Capano, E. Capocasa, F. Carbognani, S. Caride, M. F. Carney, G. Carullo, J. Casanueva Diaz, C. Casentini, S. Caudill, M. Cavaglià, F. Cavalier, R. Cavalieri, G. Cella, C. B. Cepeda, P. Cerdá-Durán, G. Cerretani, E. Cesarini, S. J. Chamberlin, M. Chan, S. Chao, P. Charlton, E. Chase, E. Chassande-Mottin, D. Chatterjee, K. Chatziioannou, B. D. Cheeseboro, H. Y. Chen, X. Chen, Y. Chen, H.-P. Cheng, H. Chia, A. Chincarini, A. Chiummo, T. Chmiel, H. S. Cho, M. Cho, J. H. Chow, N. Christensen, Q. Chu, A. J. K. Chua, S. Chua, A. K. W. Chung, S. Chung, G. Ciani, R. Ciolfi, C. E. Cirelli, A. Cirone, F. Clara, J. A. Clark, P. Clearwater, F. Cleva, C. Cocchieri, E. Coccia, P.-F. Cohadon, D. Cohen, A. Colla, C. G. Collette, L. R. Cominsky, M. Constancio, L. Conti, S. J. Cooper, P. Corban, T. R. Corbitt, I. Cordero-Carrión, K. R. Corley, N. Cornish, A. Corsi, S. Cortese, C. A. Costa, M. W. Coughlin, S. B. Coughlin, J.-P. Coulon, S. T. Countryman, P. Couvares, P. B. Covas, E. E. Cowan, D. M. Coward, M. J. Cowart, D. C. Coyne, R. Coyne, J. D. E. Creighton, T. D. Creighton, J. Cripe, S. G. Crowder, T. J. Cullen, A. Cumming, L. Cunningham, E. Cuoco, T. Dal Canton, G. Dálya, S. L. Danilishin, S. D'Antonio, K. Danzmann, A. Dasgupta, C. F. Da Silva Costa, V. Dattilo, I. Dave, M. Davier, D. Davis, E. J. Daw, B. Day, S. De, D. DeBra, J. Degallaix, M. De Laurentis, S. Deléglise, W. Del Pozzo, N. Demos, T. Denker, T. Dent, R. De Pietri, V. Dergachev, R. De Rosa, R. T. DeRosa, C. De Rossi, R. DeSalvo, O. de Varona, J. Devenson, S. Dhurandhar, M. C. Díaz, T. Dietrich, L. Di Fiore, M. Di Giovanni, T. Di Girolamo, A. Di Lieto, S. Di Pace, I. Di Palma, F. Di Renzo, Z. Doctor, V. Dolique, F. Donovan, K. L. Dooley, S. Doravari, I. Dorrington, R. Douglas, M. Dovale Álvarez, T. P. Downes, M. Drago, C. Dreissigacker, J. C. Driggers, Z. Du, M. Ducrot, R. Dudi, P. Dupej, S. E. Dwyer, T. B. Edo, M. C. Edwards, A. Effler, H.-B. Eggenstein, P. Ehrens, J. Eichholz, S. S. Eikenberry, R. A. Eisenstein, R. C. Essick, D. Estevez, Z. B. Etienne, T. Etzel, M. Evans, T. M. Evans, M. Factourovich, V. Fafone, H. Fair, S. Fairhurst, X. Fan, S. Farinon, B. Farr, W. M. Farr, E. J. Fauchon-Jones, M. Favata, M. Fays, C. Fee, H. Fehrmann, J. Feicht, M. M. Fejer, A. Fernandez-Galiana, I. Ferrante, E. C. Ferreira, F. Ferrini, F. Fidecaro, D. Finstad, I. Fiori, D. Fiorucci, M. Fishbach, R. P. Fisher, M. Fitz-Axen, R. Flaminio, M. Fletcher, H. Fong, J. A. Font, P. W. F. Forsyth, S. S. Forsyth, J.-D. Fournier, S. Frasca, F. Frasconi, Z. Frei, A. Freise, R. Frey, V. Frey, E. M. Fries, P. Fritschel, V. V. Frolov, P. Fulda, M. Fyffe, H. Gabbard, B. U. Gadre, S. M. Gaebel, J. R. Gair, L. Gammaitoni, M. R. Ganija, S. G. Gaonkar, C. Garcia-Quiros, F. Garufi, B. Gateley, S. Gaudio, G. Gaur, V. Gayathri, N. Gehrels, G. Gemme, E. Genin, A. Gennai, D. George, J. George, L. Gergely, V. Germain, S. Ghonge, A. Ghosh, A. Ghosh, S. Ghosh, J. A. Giaime, K. D. Giardina, A. Giazotto, K. Gill, L. Glover, E. Goetz, R. Goetz, S. Gomes, B. Goncharov, G.

González, J. M. Gonzalez Castro, A. Gopakumar, M. L. Gorodetsky, S. E. Gossan, M. Gosselin, R. Gouaty, A. Grado, C. Graef, M. Granata, A. Grant, S. Gras, C. Gray, G. Greco, A. C. Green, E. M. Gretarsson, P. Groot, H. Grote, S. Grunewald, P. Gruning, G. M. Guidi, X. Guo, A. Gupta, M. K. Gupta, K. E. Gushwa, E. K. Gustafson, R. Gustafson, O. Halim, B. R. Hall, E. D. Hall, E. Z. Hamilton, G. Hammond, M. Haney, M. M. Hanke, J. Hanks, C. Hanna, M. D. Hannam, O. A. Hannuksela, J. Hanson, T. Hardwick, J. Harms, G. M. Harry, I. W. Harry, M. J. Hart, C.-J. Haster, K. Haughian, J. Healy, A. Heidmann, M. C. Heintze, H. Heitmann, P. Hello, G. Hemming, M. Hendry, I. S. Heng, J. Hennig, A. W. Heptonstall, M. Heurs, S. Hild, T. Hinderer, W. C. G. Ho, D. Hoak, D. Hofman, K. Holt, D. E. Holz, P. Hopkins, C. Horst, J. Hough, E. A. Houston, E. J. Howell, A. Hreibi, Y. M. Hu, E. A. Huerta, D. Huet, B. Hughey, S. Husa, S. H. Huttner, T. Huynh-Dinh, N. Indik, R. Inta, G. Intini, H. N. Isa, J.-M. Isac, M. Isi, B. R. Iyer, K. Izumi, T. Jacqmin, K. Jani, P. Jaranowski, S. Jawahar, F. Jiménez-Forteza, W. W. Johnson, N. K. Johnson-McDaniel, D. I. Jones, R. Jones, R. J. G. Jonker, L. Ju, J. Junker, C. V. Kalaghatgi, V. Kalogera, B. Kamai, S. Kandhasamy, G. Kang, J. B. Kanner, S. J. Kapadia, S. Karki, K. S. Karvinen, M. Kasprzack, W. Kastaun, M. Katolik, E. Katsavounidis, W. Katzman, S. Kaufer, K. Kawabe, F. Kéfélian, D. Keitel, A. J. Kemball, R. Kennedy, C. Kent, J. S. Key, F. Y. Khalili, I. Khan, S. Khan, Z. Khan, E. A. Khazanov, N. Kijbunchoo, C. Kim, J. C. Kim, K. Kim, W. Kim, W. S. Kim, Y.-M. Kim, S. J. Kimbrell, E. J. King, P. J. King, M. Kinley-Hanlon, R. Kirchhoff, J. S. Kissel, L. Kleybolte, S. Klimenko, T. D. Knowles, P. Koch, S. M. Koehlenbeck, S. Koley, V. Kondrashov, A. Kontos, M. Korobko, W. Z. Korth, I. Kowalska, D. B. Kozak, C. Krämer, V. Kringel, B. Krishnan, A. Królak, G. Kuehn, P. Kumar, R. Kumar, S. Kumar, L. Kuo, A. Kutynia, S. Kwang, B. D. Lackey, K. H. Lai, M. Landry, R. N. Lang, J. Lange, B. Lantz, R. K. Lanza, S. L. Larson, A. Lartaux-Vollard, P. D. Lasky, M. Laxen, A. Lazzarini, C. Lazzaro, P. Leaci, S. Leavey, C. H. Lee, H. K. Lee, H. M. Lee, H. W. Lee, K. Lee, J. Lehmann, A. Lenon, E. Leon, M. Leonardi, N. Leroy, N. Letendre, Y. Levin, T. G. F. Li, S. D. Linker, T. B. Littenberg, J. Liu, X. Liu, R. K. L. Lo, N. A. Lockerbie, L. T. London, J. E. Lord, M. Lorenzini, V. Loriette, M. Lormand, G. Losurdo, J. D. Lough, C. O. Lousto, G. Lovelace, H. Lück, D. Lumaca, A. P. Lundgren, R. Lynch, Y. Ma, R. Macas, S. Macfoy, B. Machenschalk, M. MacInnis, D. M. Macleod, I. Magaña Hernandez, F. Magaña Sandoval, L. Magaña Zertuche, R. M. Magee, E. Majorana, I. Maksimovic, N. Man, V. Mandic, V. Mangano, G. L. Mansell, M. Manske, M. Mantovani, F. Marchesoni, F. Marion, S. Márka, Z. Márka, C. Markakis, A. S. Markosyan, A. Markowitz, E. Maros, A. Marquina, P. Marsh, F. Martelli, L. Martellini, I. W. Martin, R. M. Martin, D. V. Martynov, J. N. Marx, K. Mason, E. Massera, A. Masserot, T. J. Massinger, M. Masso-Reid, S. Mastrogiovanni, A. Matas, F. Matichard, L. Matone, N. Mavalvala, N. Mazumder, R. McCarthy, D. E. McClelland, S. McCormick, L. McCuller, S. C. McGuire, G. McIntyre, J. McIver, D. J. McManus, L. McNeill, T. McRae, S. T. McWilliams, D. Meacher, G. D. Meadors, M. Mehmet, J. Meidam, E. Mejuto-Villa, A. Melatos, G. Mendell, R. A. Mercer, E. L. Merilh, M. Merzougui, S. Meshkov, C. Messenger, C. Messick, R. Metzdorff, P. M. Meyers, H. Miao, C. Michel, H. Middleton, E. E. Mikhailov, L. Milano, A. L. Miller, B. B. Miller, J. Miller, M. Millhouse, M. C. Milovich-Goff, O. Minazzoli, Y. Minenkov, J. Ming, C. Mishra, S. Mitra, V. P. Mitrofanov, G. Mitselmakher, R. Mittleman, D. Moffa, A. Moggi, K. Mogushi, M. Mohan, S. R. P. Mohapatra, I. Molina, M. Montani, C. J. Moore, D. Moraru, G. Moreno, S. Morisaki, S. R. Morriss, B. Mours, C. M. Mow-Lowry, G. Mueller, A. W. Muir, A. Mukherjee, D. Mukherjee, S. Mukherjee, N. Mukund, A. Mullavey, J. Munch, E. A. Muñiz, M. Muratore, P. G. Murray, A. Nagar, K. Napier, I. Nardecchia, L. Naticchioni, R. K. Nayak, J. Neilson, G. Nelemans, T. J. N. Nelson, M. Nery, A. Neunzert, L. Nevin, J. M. Newport, G. Newton, K. K. Y. Ng, P. Nguyen, T. T. Nguyen, D. Nichols, A. B. Nielsen, S. Nissanke, A. Nitz, A. Noack, F. Nocera, D. Nolting, C. North, L. K. Nuttall, J. Oberling, G. D. O'Dea, G. H. Ogin, J. J. Oh, S. H. Oh, F. Ohme, M. A. Okada, M. Oliver, P. Oppermann, R. J. Oram, B. O'Reilly, R. Ormiston, L. F. Ortega, R. O'Shaughnessy, S. Ossokine, D. J. Ottaway, H. Overmier, B. J. Owen, A. E. Pace, J. Page, M. A. Page, A. Pai, S. A. Pai, J. R. Palamos, O. Palashov, C. Palomba, A. Pal-Singh, H. Pan, H.-W. Pan, B. Pang, P. T. H. Pang, C. Pankow, F. Pannarale, B. C. Pant, F. Paoletti, A. Paoli, M. A. Papa, A. Parida, W. Parker, D. Pascucci, A. Pasqualetti, R. Passaquieti, D. Passuello, M. Patil, B. Patricelli, B. L. Pearlstone, M. Pedraza, R. Pedurand, L. Pekowsky, A. Pele, S. Penn, C. J. Perez, A. Perreca, L. M. Perri, H. P. Pfeiffer, M. Phelps, O. J. Piccinni, M. Pichot, F. Piergiovanni, V. Pierro, G. Pillant, L. Pinard, I. M. Pinto, M. Pirello, M. Pitkin, M. Poe, R. Poggiani, P. Popolizio, E. K. Porter, A. Post, J. Powell, J. Prasad, J. W. W. Pratt, G. Pratten, V. Predoi, T. Prestegard, M. Prijatelj, M. Principe, S. Privitera, R. Prix, G. A. Prodi, L. G. Prokhorov, O. Puncken, M. Punturo, P. Puppo, M. Pürrer, H. Qi, V. Quetschke, E. A. Quintero, R. Quitzow-James, F. J. Raab, D. S. Rabeling, H. Radkins, P. Raffai, S. Raja, C. Rajan, B. Rajbhandari, M. Rakhmanov, K. E. Ramirez, A. Ramos-Buades, P. Rapagnani, V. Raymond, M. Razzano, J. Read, T. Regimbau, L. Rei, S. Reid, D. H. Reitze, W. Ren, S. D. Reyes, F. Ricci, P. M. Ricker, S. Rieger, K. Riles, M. Rizzo, N. A. Robertson, R. Robie, F. Robinet, A. Rocchi, L. Rolland, J. G. Rollins, V. J. Roma, J. D. Romano, R. Romano, C. L. Romel, J. H. Romie, D. Rosińska, M. P. Ross, S. Rowan, A. Rüdiger, P. Ruggi, G. Rutins, K. Ryan, S. Sachdev, T. Sadecki, L. Sadeghian, M. Sakellariadou, L. Salconi, M. Saleem, F. Salemi,

A. Samajdar, L. Sammut, L. M. Sampson, E. J. Sanchez, L. E. Sanchez, N. Sanchis-Gual, V. Sandberg, J. R. Sanders, B. Sassolas, B. S. Sathyaprakash, P. R. Saulson, O. Sauter, R. L. Savage, A. Sawadsky, P. Schale, M. Scheel, J. Scheuer, J. Schmidt, P. Schmidt, R. Schnabel, R. M. S. Schofield, A. Schönbeck, E. Schreiber, D. Schuette, B. W. Schulte, B. F. Schutz, S. G. Schwalbe, J. Scott, S. M. Scott, E. Seidel, D. Sellers, A. S. Sengupta, D. Sentenac, V. Sequino, A. Sergeev, D. A. Shaddock, T. J. Shaffer, A. A. Shah, M. S. Shahriar, M. B. Shaner, L. Shao, B. Shapiro, P. Shawhan, A. Sheperd, D. H. Shoemaker, D. M. Shoemaker, K. Siellez, X. Siemens, M. Sieniawska, D. Sigg, A. D. Silva, L. P. Singer, A. Singh, A. Singhal, A. M. Sintes, B. J. J. Slagmolen, B. Smith, J. R. Smith, R. J. E. Smith, S. Somala, E. J. Son, J. A. Sonnenberg, B. Sorazu, F. Sorrentino, T. Souradeep, A. P. Spencer, A. K. Srivastava, K. Staats, A. Staley, M. Steinke, J. Steinlechner, S. Steinlechner, D. Steinmeyer, S. P. Stevenson, R. Stone, D. J. Stops, K. A. Strain, G. Stratta, S. E. Strigin, A. Strunk, R. Sturani, A. L. Stuver, T. Z. Summerscales, L. Sun, S. Sunil, J. Suresh, P. J. Sutton, B. L. Swinkels, M. J. Szczepańczyk, M. Tacca, S. C. Tait, C. Talbot, D. Talukder, D. B. Tanner, M. Tápai, A. Taracchini, J. D. Tasson, J. A. Taylor, R. Taylor, S. V. Tewari, T. Theeg, F. Thies, E. G. Thomas, M. Thomas, P. Thomas, K. A. Thorne, K. S. Thorne, E. Thrane, S. Tiwari, V. Tiwari, K. V. Tokmakov, K. Toland, M. Tonelli, Z. Tornasi, A. Torres-Forné, C. I. Torrie, D. Töyrä, F. Travasso, G. Traylor, J. Trinastic, M. C. Tringali, L. Trozzo, K. W. Tsang, M. Tse, R. Tso, L. Tsukada, D. Tsuna, D. Tuyenbayev, K. Ueno, D. Ugolini, C. S. Unnikrishnan, A. L. Urban, S. A. Usman, H. Vahlbruch, G. Vajente, G. Valdes, M. Vallisneri, N. van Bakel, M. van Beuzekom, J. F. J. van den Brand, C. Van Den Broeck, D. C. Vander-Hyde, L. van der Schaaf, J. V. van Heijningen, A. A. van Veggel, M. Vardaro, V. Varma, S. Vass, M. Vasúth, A. Vecchio, G. Vedovato, J. Veitch, P. J. Veitch, K. Venkateswara, G. Venugopalan, D. Verkindt, F. Vetrano, A. Viceré, A. D. Viets, S. Vinciguerra, D. J. Vine, J.-Y. Vinet, S. Vitale, T. Vo, H. Vocca, C. Vorvick, S. P. Vyatchanin, A. R. Wade, L. E. Wade, M. Wade, R. Walet, M. Walker, L. Wallace, S. Walsh, G. Wang, H. Wang, J. Z. Wang, W. H. Wang, Y. F. Wang, R. L. Ward, J. Warner, M. Was, J. Watchi, B. Weaver, L.-W. Wei, M. Weinert, A. J. Weinstein, R. Weiss, L. Wen, E. K. Wessel, P. Weßels, J. Westerweck, T. Westphal, K. Wette, J. T. Whelan, S. E. Whitcomb, B. F. Whiting, C. Whittle, D. Wilken, D. Williams, R. D. Williams, A. R. Williamson, J. L. Willis, B. Willke, M. H. Wimmer, W. Winkler, C. C. Wipf, H. Wittel, G. Woan, J. Woehler, J. Wofford, K. W. K. Wong, J. Worden, J. L. Wright, D. S. Wu, D. M. Wysocki, S. Xiao, H. Yamamoto, C. C. Yancey, L. Yang, M. J. Yap, M. Yazback, H. Yu, H. Yu, M. Yvert, A. Zadrożny, M. Zanolin, T. Zelenova, J.-P. Zendri, M. Zevin, L. Zhang, M. Zhang, T. Zhang, Y.-H. Zhang, C. Zhao, M. Zhou, Z. Zhou, S. J. Zhu, X. J. Zhu, A. B. Zimmerman, M. E. Zucker, and J. Zweizig, "GW170817: observation of gravitational waves from a binary neutron star inspiral", *Phys. Rev. Lett.* **119**:16 (2017), art. id. 161101.

[3] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Afrough, B. Agarwal, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, L. Aiello, A. Ain, P. Ajith, B. Allen, G. Allen, A. Allocca, P. A. Altin, A. Amato, A. Ananyeva, S. B. Anderson, W. G. Anderson, S. V. Angelova, S. Antier, S. Appert, K. Arai, M. C. Araya, J. S. Areeda, N. Arnaud, S. Ascenzi, G. Ashton, M. Ast, S. M. Aston, P. Astone, D. V. Atallah, P. Aufmuth, C. Aulbert, K. AultONeal, C. Austin, A. Avila-Alvarez, S. Babak, P. Bacon, M. K. M. Bader, S. Bae, P. T. Baker, F. Baldaccini, G. Ballardin, S. W. Ballmer, S. Banagiri, J. C. Barayoga, S. E. Barclay, B. C. Barish, D. Barker, K. Barkett, F. Barone, B. Barr, L. Barsotti, M. Barsuglia, D. Barta, J. Bartlett, I. Bartos, R. Bassiri, A. Basti, J. C. Batch, M. Bawaj, J. C. Bayley, M. Bazzan, B. Bécsy, C. Beer, M. Bejger, I. Belahcene, A. S. Bell, B. K. Berger, G. Bergmann, J. J. Bero, C. P. L. Berry, D. Bersanetti, A. Bertolini, J. Betzwieser, S. Bhagwat, R. Bhandare, I. A. Bilenko, G. Billingsley, C. R. Billman, J. Birch, I. A. Birney, O. Birnholtz, S. Biscans, S. Biscoveanu, A. Bisht, M. Bitossi, C. Biwer, M. A. Bizouard, J. K. Blackburn, J. Blackman, C. D. Blair, D. G. Blair, R. M. Blair, S. Bloemen, O. Bock, N. Bode, M. Boer, G. Bogaert, A. Bohe, F. Bondu, E. Bonilla, R. Bonnand, B. A. Boom, R. Bork, V. Boschi, S. Bose, K. Bossie, Y. Bouffanais, A. Bozzi, C. Bradaschia, P. R. Brady, M. Branchesi, J. E. Brau, T. Briant, A. Brillet, M. Brinkmann, V. Brisson, P. Brockill, J. E. Broida, A. F. Brooks, D. A. Brown, D. D. Brown, S. Brunett, C. C. Buchanan, A. Buikema, T. Bulik, H. J. Bulten, A. Buonanno, D. Buskulic, C. Buy, R. L. Byer, M. Cabero, L. Cadonati, G. Cagnoli, C. Cahillane, J. Calderón Bustillo, T. A. Callister, E. Calloni, J. B. Camp, M. Canepa, P. Canizares, K. C. Cannon, H. Cao, J. Cao, C. D. Capano, E. Capocasa, F. Carbognani, S. Caride, M. F. Carney, J. C. Diaz, C. Casentini, S. Caudill, M. Cavaglià, F. Cavalier, R. Cavalieri, G. Cella, C. B. Cepeda, P. Cerdá-Durán, G. Cerretani, E. Cesarini, S. J. Chamberlin, M. Chan, S. Chao, P. Charlton, E. Chase, E. Chassande-Mottin, D. Chatterjee, B. D. Cheeseboro, H. Y. Chen, X. Chen, Y. Chen, H.-P. Cheng, H. Chia, A. Chincarini, A. Chiummo, T. Chmiel, H. S. Cho, M. Cho, J. H. Chow, N. Christensen, Q. Chu, A. J. K. Chua, S. Chua, A. K. W. Chung, S. Chung, G. Ciani, R. Ciolfi, C. E. Cirelli, A. Cirone, F. Clara, J. A. Clark, P. Clearwater, F. Cleva, C. Cocchieri, E. Coccia, P.-F. Cohadon, D. Cohen, A. Colla, C. G. Collette, L. R. Cominsky, M. Constancio, L. Conti, S. J. Cooper, P. Corban, T. R. Corbitt, I. Cordero-Carrión, K. R. Corley, N. Cornish, A. Corsi, S. Cortese, C. A. Costa, E. Coughlin, M. W. Coughlin, S. B. Coughlin, J.-P. Coulon, S. T. Countryman, P. Couvares, P. B. Covas, E. E. Cowan, D. M. Coward, M. J. Cowart, D. C.

Coyne, R. Coyne, J. D. E. Creighton, T. D. Creighton, J. Cripe, S. G. Crowder, T. J. Cullen, A. Cumming, L. Cunningham, E. Cuoco, T. D. Canton, G. Dálya, S. L. Danilishin, S. D'Antonio, K. Danzmann, A. Dasgupta, C. F. Da Silva Costa, V. Dattilo, I. Dave, M. Davier, D. Davis, E. J. Daw, B. Day, S. De, D. DeBra, J. Degallaix, M. De Laurentis, S. Deléglise, W. Del Pozzo, N. Demos, T. Denker, T. Dent, R. De Pietri, V. Dergachev, R. De Rosa, R. T. DeRosa, C. De Rossi, R. DeSalvo, O. de Varona, J. Devenson, S. Dhurandhar, M. C. Díaz, L. Di Fiore, M. Di Giovanni, T. Di Girolamo, A. Di Lieto, S. Di Pace, I. Di Palma, F. Di Renzo, Z. Doctor, V. Dolique, F. Donovan, K. L. Dooley, S. Doravari, I. Dorrington, R. Douglas, M. Dovale Álvarez, T. P. Downes, M. Drago, C. Dreissigacker, J. C. Driggers, Z. Du, M. Ducrot, P. Dupej, S. E. Dwyer, T. B. Edo, M. C. Edwards, A. Effler, H.-B. Eggenstein, P. Ehrens, J. Eichholz, S. S. Eikenberry, R. A. Eisenstein, R. C. Essick, D. Estevez, Z. B. Etienne, T. Etzel, M. Evans, T. M. Evans, M. Factourovich, V. Fafone, H. Fair, S. Fairhurst, X. Fan, S. Farinon, B. Farr, W. M. Farr, E. J. Fauchon-Jones, M. Favata, M. Fays, C. Fee, H. Fehrmann, J. Feicht, M. M. Fejer, A. Fernandez-Galiana, I. Ferrante, E. C. Ferreira, F. Ferrini, F. Fidecaro, D. Finstad, I. Fiori, D. Fiorucci, M. Fishbach, R. P. Fisher, M. Fitz-Axen, R. Flaminio, M. Fletcher, H. Fong, J. A. Font, P. W. F. Forsyth, S. S. Forsyth, J.-D. Fournier, S. Frasca, F. Frasconi, Z. Frei, A. Freise, R. Frey, V. Frey, E. M. Fries, P. Fritschel, V. V. Frolov, P. Fulda, M. Fyffe, H. Gabbard, B. U. Gadre, S. M. Gaebel, J. R. Gair, L. Gammaitoni, M. R. Ganija, S. G. Gaonkar, C. Garcia-Quiros, F. Garufi, B. Gateley, S. Gaudio, G. Gaur, V. Gayathri, N. Gehrels, G. Gemme, E. Genin, A. Gennai, D. George, J. George, L. Gergely, V. Germain, S. Ghonge, A. Ghosh, A. Ghosh, S. Ghosh, J. A. Giaime, K. D. Giardina, A. Giazotto, K. Gill, L. Glover, E. Goetz, R. Goetz, S. Gomes, B. Goncharov, G. González, J. M. Gonzalez Castro, A. Gopakumar, M. L. Gorodetsky, S. E. Gossan, M. Gosselin, R. Gouaty, A. Grado, C. Graef, M. Granata, A. Grant, S. Gras, C. Gray, G. Greco, A. C. Green, E. M. Gretarsson, P. Groot, H. Grote, S. Grunewald, P. Gruning, G. M. Guidi, X. Guo, A. Gupta, M. K. Gupta, K. E. Gushwa, E. K. Gustafson, R. Gustafson, O. Halim, B. R. Hall, E. D. Hall, E. Z. Hamilton, G. Hammond, M. Haney, M. M. Hanke, J. Hanks, C. Hanna, M. D. Hannam, O. A. Hannuksela, J. Hanson, T. Hardwick, J. Harms, G. M. Harry, I. W. Harry, M. J. Hart, C.-J. Haster, K. Haughian, J. Healy, A. Heidmann, M. C. Heintze, H. Heitmann, P. Hello, G. Hemming, M. Hendry, I. S. Heng, J. Hennig, A. W. Heptonstall, M. Heurs, S. Hild, T. Hinderer, D. Hoak, D. Hofman, K. Holt, D. E. Holz, P. Hopkins, C. Horst, J. Hough, E. A. Houston, E. J. Howell, A. Hreibi, Y. M. Hu, E. A. Huerta, D. Huet, B. Hughey, S. Husa, S. H. Huttner, T. Huynh-Dinh, N. Indik, R. Inta, G. Intini, H. N. Isa, J.-M. Isac, M. Isi, B. R. Iyer, K. Izumi, T. Jacqmin, K. Jani, P. Jaranowski, S. Jawahar, F. Jiménez-Forteza, W. W. Johnson, D. I. Jones, R. Jones, R. J. G. Jonker, L. Ju, J. Junker, C. V. Kalaghatgi, V. Kalogera, B. Kamai, S. Kandhasamy, G. Kang, J. B. Kanner, S. J. Kapadia, S. Karki, K. S. Karvinen, M. Kasprzack, M. Katolik, E. Katsavounidis, W. Katzman, S. Kaufer, K. Kawabe, F. Kéfélian, D. Keitel, A. J. Kemball, R. Kennedy, C. Kent, J. S. Key, F. Y. Khalili, I. Khan, S. Khan, Z. Khan, E. A. Khazanov, N. Kijbunchoo, C. Kim, J. C. Kim, K. Kim, W. Kim, W. S. Kim, Y.-M. Kim, S. J. Kimbrell, E. J. King, P. J. King, M. Kinley-Hanlon, R. Kirchhoff, J. S. Kissel, L. Kleybolte, S. Klimenko, T. D. Knowles, P. Koch, S. M. Koehlenbeck, S. Koley, V. Kondrashov, A. Kontos, M. Korobko, W. Z. Korth, I. Kowalska, D. B. Kozak, C. Krämer, V. Kringel, A. Królak, G. Kuehn, P. Kumar, R. Kumar, S. Kumar, L. Kuo, A. Kutynia, S. Kwang, B. D. Lackey, K. H. Lai, M. Landry, R. N. Lang, J. Lange, B. Lantz, R. K. Lanza, A. Lartaux-Vollard, P. D. Lasky, M. Laxen, A. Lazzarini, C. Lazzaro, P. Leaci, S. Leavey, C. H. Lee, H. K. Lee, H. M. Lee, H. W. Lee, K. Lee, J. Lehmann, A. Lenon, M. Leonardi, N. Leroy, N. Letendre, Y. Levin, T. G. F. Li, S. D. Linker, T. B. Littenberg, J. Liu, R. K. L. Lo, N. A. Lockerbie, L. T. London, J. E. Lord, M. Lorenzini, V. Loriette, M. Lormand, G. Losurdo, J. D. Lough, C. O. Lousto, G. Lovelace, H. Lück, D. Lumaca, A. P. Lundgren, R. Lynch, Y. Ma, R. Macas, S. Macfoy, B. Machenschalk, M. MacInnis, D. M. Macleod, I. Magaña Hernandez, F. Magaña Sandoval, L. Magaña Zertuche, R. M. Magee, E. Majorana, I. Maksimovic, N. Man, V. Mandic, V. Mangano, G. L. Mansell, M. Manske, M. Mantovani, F. Marchesoni, F. Marion, S. Márka, Z. Márka, C. Markakis, A. S. Markosyan, A. Markowitz, E. Maros, A. Marquina, F. Martelli, L. Martellini, I. W. Martin, R. M. Martin, D. V. Martynov, K. Mason, E. Massera, A. Masserot, T. J. Massinger, M. Masso-Reid, S. Mastrogiovanni, A. Matas, F. Matichard, L. Matone, N. Mavalvala, N. Mazumder, R. McCarthy, D. E. McClelland, S. McCormick, L. McCuller, S. C. McGuire, G. McIntyre, J. McIver, D. J. McManus, L. McNeill, T. McRae, S. T. McWilliams, D. Meacher, G. D. Meadors, M. Mehmet, J. Meidam, E. Mejuto-Villa, A. Melatos, G. Mendell, R. A. Mercer, E. L. Merilh, M. Merzougui, S. Meshkov, C. Messenger, C. Messick, R. Metzdorff, P. M. Meyers, H. Miao, C. Michel, H. Middleton, E. E. Mikhailov, L. Milano, A. L. Miller, B. B. Miller, J. Miller, M. Millhouse, M. C. Milovich-Goff, O. Minazzoli, Y. Minenkov, J. Ming, C. Mishra, S. Mitra, V. P. Mitrofanov, G. Mitselmakher, R. Mittleman, D. Moffa, A. Moggi, K. Mogushi, M. Mohan, S. R. P. Mohapatra, M. Montani, C. J. Moore, D. Moraru, G. Moreno, S. R. Morriss, B. Mours, C. M. Mow-Lowry, G. Mueller, A. W. Muir, A. Mukherjee, D. Mukherjee, S. Mukherjee, N. Mukund, A. Mullavey, J. Munch, E. A. Muñiz, M. Muratore, P. G. Murray, K. Napier, I. Nardecchia, L. Naticchioni, R. K. Nayak, J. Neilson, G. Nelemans, T. J. N. Nelson, M. Nery, A. Neunzert, L. Nevin, J. M. Newport, G. Newton, K. K. Y. Ng, T. T. Nguyen,

D. Nichols, A. B. Nielsen, S. Nissanke, A. Nitz, A. Noack, F. Nocera, D. Nolting, C. North, L. K. Nuttall, J. Oberling, G. D. O'Dea, G. H. Ogin, J. J. Oh, S. H. Oh, F. Ohme, M. A. Okada, M. Oliver, P. Oppermann, R. J. Oram, B. O'Reilly, R. Ormiston, L. F. Ortega, R. O'Shaughnessy, S. Ossokine, D. J. Ottaway, H. Overmier, B. J. Owen, A. E. Pace, J. Page, M. A. Page, A. Pai, S. A. Pai, J. R. Palamos, O. Palashov, C. Palomba, A. Pal-Singh, H. Pan, H.-W. Pan, B. Pang, P. T. H. Pang, C. Pankow, F. Pannarale, B. C. Pant, F. Paoletti, A. Paoli, M. A. Papa, A. Parida, W. Parker, D. Pascucci, A. Pasqualetti, R. Passaquieti, D. Passuello, M. Patil, B. Patricelli, B. L. Pearlstone, M. Pedraza, R. Pedurand, L. Pekowsky, A. Pele, S. Penn, C. J. Perez, A. Perreca, L. M. Perri, H. P. Pfeiffer, M. Phelps, O. J. Piccinni, M. Pichot, F. Piergiovanni, V. Pierro, G. Pillant, L. Pinard, I. M. Pinto, M. Pirello, M. Pitkin, M. Poe, R. Poggiani, P. Popolizio, E. K. Porter, A. Post, J. Powell, J. Prasad, J. W. W. Pratt, G. Pratten, V. Predoi, T. Prestegard, M. Prijatelj, M. Principe, S. Privitera, G. A. Prodi, L. G. Prokhorov, O. Puncken, M. Punturo, P. Puppo, M. Pürrer, H. Qi, V. Quetschke, E. A. Quintero, R. Quitzow-James, F. J. Raab, D. S. Rabeling, H. Radkins, P. Raffai, S. Raja, C. Rajan, B. Rajbhandari, M. Rakhmanov, K. E. Ramirez, A. Ramos-Buades, P. Rapagnani, V. Raymond, M. Razzano, J. Read, T. Regimbau, L. Rei, S. Reid, D. H. Reitze, W. Ren, S. D. Reyes, F. Ricci, P. M. Ricker, S. Rieger, K. Riles, M. Rizzo, N. A. Robertson, R. Robie, F. Robinet, A. Rocchi, L. Rolland, J. G. Rollins, V. J. Roma, J. D. Romano, R. Romano, C. L. Romel, J. H. Romie, D. Rosińska, M. P. Ross, S. Rowan, A. Rüdiger, P. Ruggi, G. Rutins, K. Ryan, S. Sachdev, T. Sadecki, L. Sadeghian, M. Sakellariadou, L. Salconi, M. Saleem, F. Salemi, A. Samajdar, L. Sammut, L. M. Sampson, E. J. Sanchez, L. E. Sanchez, N. Sanchis-Gual, V. Sandberg, J. R. Sanders, B. Sassolas, P. R. Saulson, O. Sauter, R. L. Savage, A. Sawadsky, P. Schale, M. Scheel, J. Scheuer, J. Schmidt, P. Schmidt, R. Schnabel, R. M. S. Schofield, A. Schönbeck, E. Schreiber, D. Schuette, B. W. Schulte, B. F. Schutz, S. G. Schwalbe, J. Scott, S. M. Scott, E. Seidel, D. Sellers, A. S. Sengupta, D. Sentenac, V. Sequino, A. Sergeev, D. A. Shaddock, T. J. Shaffer, A. A. Shah, M. S. Shahriar, M. B. Shaner, L. Shao, B. Shapiro, P. Shawhan, A. Sheperd, D. H. Shoemaker, D. M. Shoemaker, K. Siellez, X. Siemens, M. Sieniawska, D. Sigg, A. D. Silva, L. P. Singer, A. Singh, A. Singhal, A. M. Sintes, B. J. J. Slagmolen, B. Smith, J. R. Smith, R. J. E. Smith, S. Somala, E. J. Son, J. A. Sonnenberg, B. Sorazu, F. Sorrentino, T. Souradeep, A. P. Spencer, A. K. Srivastava, K. Staats, A. Staley, M. Steinke, J. Steinlechner, S. Steinlechner, D. Steinmeyer, S. P. Stevenson, R. Stone, D. J. Stops, K. A. Strain, G. Stratta, S. E. Strigin, A. Strunk, R. Sturani, A. L. Stuver, T. Z. Summerscales, L. Sun, S. Sunil, J. Suresh, P. J. Sutton, B. L. Swinkels, M. J. Szczepańczyk, M. Tacca, S. C. Tait, C. Talbot, D. Talukder, D. B. Tanner, D. Tao, M. Tápai, A. Taracchini, J. D. Tasson, J. A. Taylor, R. Taylor, S. V. Tewari, T. Theeg, F. Thies, E. G. Thomas, M. Thomas, P. Thomas, K. A. Thorne, E. Thrane, S. Tiwari, V. Tiwari, K. V. Tokmakov, K. Toland, M. Tonelli, Z. Tornasi, A. Torres-Forné, C. I. Torrie, D. Töyrä, F. Travasso, G. Traylor, J. Trinastic, M. C. Tringali, L. Trozzo, K. W. Tsang, M. Tse, R. Tso, L. Tsukada, D. Tsuna, D. Tuyenbayev, K. Ueno, D. Ugolini, C. S. Unnikrishnan, A. L. Urban, S. A. Usman, H. Vahlbruch, G. Vajente, G. Valdes, N. van Bakel, M. van Beuzekom, J. F. J. van den Brand, C. Van Den Broeck, D. C. Vander-Hyde, L. van der Schaaf, J. V. van Heijningen, A. A. van Veggel, M. Vardaro, V. Varma, S. Vass, M. Vasúth, A. Vecchio, G. Vedovato, J. Veitch, P. J. Veitch, K. Venkateswara, G. Venugopalan, D. Verkindt, F. Vetrano, A. Viceré, A. D. Viets, S. Vinciguerra, D. J. Vine, J.-Y. Vinet, S. Vitale, T. Vo, H. Vocca, C. Vorvick, S. P. Vyatchanin, A. R. Wade, L. E. Wade, M. Wade, R. Walet, M. Walker, L. Wallace, S. Walsh, G. Wang, H. Wang, J. Z. Wang, W. H. Wang, Y. F. Wang, R. L. Ward, J. Warner, M. Was, J. Watchi, B. Weaver, L.-W. Wei, M. Weinert, A. J. Weinstein, R. Weiss, L. Wen, E. K. Wessel, P. Weßels, J. Westerweck, T. Westphal, K. Wette, J. T. Whelan, B. F. Whiting, C. Whittle, D. Wilken, D. Williams, R. D. Williams, A. R. Williamson, J. L. Willis, B. Willke, M. H. Wimmer, W. Winkler, C. C. Wipf, H. Wittel, G. Woan, J. Woehler, J. Wofford, K. W. K. Wong, J. Worden, J. L. Wright, D. S. Wu, D. M. Wysocki, S. Xiao, H. Yamamoto, C. C. Yancey, L. Yang, M. J. Yap, M. Yazback, H. Yu, H. Yu, M. Yvert, A. Zadrożny, M. Zanolin, T. Zelenova, J.-P. Zendri, M. Zevin, L. Zhang, M. Zhang, T. Zhang, Y.-H. Zhang, C. Zhao, M. Zhou, Z. Zhou, S. J. Zhu, X. J. Zhu, M. E. Zucker, and J. Zweizig, "Search for tensor, vector, and scalar polarizations in the stochastic gravitational-wave background", *Phys. Rev. Lett.* **120**:20 (2018), art. id. 201102.

[4] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, C. Affeldt, M. Afrough, B. Agarwal, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, L. Aiello, A. Ain, P. Ajith, G. Allen, A. Allocca, P. A. Altin, A. Amato, A. Ananyeva, S. B. Anderson, W. G. Anderson, S. Antier, S. Appert, K. Arai, M. C. Araya, J. S. Areeda, N. Arnaud, K. G. Arun, S. Ascenzi, G. Ashton, M. Ast, S. M. Aston, P. Astone, P. Aufmuth, C. Aulbert, K. AultONeal, A. Avila-Alvarez, S. Babak, P. Bacon, M. K. M. Bader, S. Bae, P. T. Baker, F. Baldaccini, G. Ballardin, S. W. Ballmer, S. Banagiri, J. C. Barayoga, S. E. Barclay, B. C. Barish, D. Barker, F. Barone, B. Barr, L. Barsotti, M. Barsuglia, D. Barta, J. Bartlett, I. Bartos, R. Bassiri, A. Basti, J. C. Batch, C. Baune, M. Bawaj, M. Bazzan, B. Bécsy, C. Beer, M. Bejger, I. Belahcene, A. S. Bell, B. K. Berger, G. Bergmann, C. P. L. Berry, D. Bersanetti, A. Bertolini, J. Betzwieser, S. Bhagwat, R. Bhandare, I. A. Bilenko, G. Billingsley, C. R. Billman, J. Birch, I. A. Birney, O. Birnholtz, S.

Biscans, A. Bisht, M. Bitossi, C. Biwer, M. A. Bizouard, J. K. Blackburn, J. Blackman, C. D. Blair, D. G. Blair, R. M. Blair, S. Bloemen, O. Bock, N. Bode, M. Boer, G. Bogaert, A. Bohe, F. Bondu, R. Bonnand, B. A. Boom, R. Bork, V. Boschi, S. Bose, Y. Bouffanais, A. Bozzi, C. Bradaschia, P. R. Brady, V. B. Braginsky, M. Branchesi, J. E. Brau, T. Briant, A. Brillet, M. Brinkmann, V. Brisson, P. Brockill, J. E. Broida, A. F. Brooks, D. A. Brown, D. D. Brown, N. M. Brown, S. Brunett, C. C. Buchanan, A. Buikema, T. Bulik, H. J. Bulten, A. Buonanno, D. Buskulic, C. Buy, R. L. Byer, M. Cabero, L. Cadonati, G. Cagnoli, C. Cahillane, J. Calderón Bustillo, T. A. Callister, E. Calloni, J. B. Camp, M. Canepa, P. Canizares, K. C. Cannon, H. Cao, J. Cao, C. D. Capano, E. Capocasa, F. Carbognani, S. Caride, M. F. Carney, J. Casanueva Diaz, C. Casentini, S. Caudill, M. Cavaglià, F. Cavalier, R. Cavalieri, G. Cella, C. B. Cepeda, L. Cerboni Baiardi, G. Cerretani, E. Cesarini, S. J. Chamberlin, M. Chan, S. Chao, P. Charlton, E. Chassande-Mottin, D. Chatterjee, B. D. Cheeseboro, H. Y. Chen, Y. Chen, H.-P. Cheng, A. Chincarini, A. Chiummo, T. Chmiel, H. S. Cho, M. Cho, J. H. Chow, N. Christensen, Q. Chu, A. J. K. Chua, S. Chua, A. K. W. Chung, S. Chung, G. Ciani, R. Ciolfi, C. E. Cirelli, A. Cirone, F. Clara, J. A. Clark, F. Cleva, C. Cocchieri, E. Coccia, P.-F. Cohadon, A. Colla, C. G. Collette, L. R. Cominsky, M. Constancio, L. Conti, S. J. Cooper, P. Corban, T. R. Corbitt, K. R. Corley, N. Cornish, A. Corsi, S. Cortese, C. A. Costa, M. W. Coughlin, S. B. Coughlin, J.-P. Coulon, S. T. Countryman, P. Couvares, P. B. Covas, E. E. Cowan, D. M. Coward, M. J. Cowart, D. C. Coyne, R. Coyne, J. D. E. Creighton, T. D. Creighton, J. Cripe, S. G. Crowder, T. J. Cullen, A. Cumming, L. Cunningham, E. Cuoco, T. D. Canton, S. L. Danilishin, S. D'Antonio, K. Danzmann, A. Dasgupta, C. F. Da Silva Costa, V. Dattilo, I. Dave, M. Davier, D. Davis, E. J. Daw, B. Day, S. De, D. DeBra, J. Degallaix, M. De Laurentis, S. Deléglise, W. Del Pozzo, T. Denker, T. Dent, V. Dergachev, R. De Rosa, R. T. DeRosa, R. DeSalvo, J. Devenson, R. C. Devine, S. Dhurandhar, M. C. Díaz, L. Di Fiore, M. Di Giovanni, T. Di Girolamo, A. Di Lieto, S. Di Pace, I. Di Palma, F. Di Renzo, Z. Doctor, V. Dolique, F. Donovan, K. L. Dooley, S. Doravari, I. Dorrington, R. Douglas, M. Dovale Álvarez, T. P. Downes, M. Drago, R. W. P. Drever, J. C. Driggers, Z. Du, M. Ducrot, J. Duncan, S. E. Dwyer, T. B. Edo, M. C. Edwards, A. Effler, H.-B. Eggenstein, P. Ehrens, J. Eichholz, S. S. Eikenberry, R. A. Eisenstein, R. C. Essick, Z. B. Etienne, T. Etzel, M. Evans, T. M. Evans, M. Factourovich, V. Fafone, H. Fair, S. Fairhurst, X. Fan, S. Farinon, B. Farr, W. M. Farr, E. J. Fauchon-Jones, M. Favata, M. Fays, H. Fehrmann, J. Feicht, M. M. Fejer, A. Fernandez-Galiana, I. Ferrante, E. C. Ferreira, F. Ferrini, F. Fidecaro, I. Fiori, D. Fiorucci, R. P. Fisher, R. Flaminio, M. Fletcher, H. Fong, P. W. F. Forsyth, S. S. Forsyth, J.-D. Fournier, S. Frasca, F. Frasconi, Z. Frei, A. Freise, R. Frey, V. Frey, E. M. Fries, P. Fritschel, V. V. Frolov, P. Fulda, M. Fyffe, H. Gabbard, M. Gabel, B. U. Gadre, S. M. Gaebel, J. R. Gair, L. Gammaitoni, M. R. Ganija, S. G. Gaonkar, F. Garufi, S. Gaudio, G. Gaur, V. Gayathri, N. Gehrels, G. Gemme, E. Genin, A. Gennai, D. George, J. George, L. Gergely, V. Germain, S. Ghonge, A. Ghosh, A. Ghosh, S. Ghosh, J. A. Giaime, K. D. Giardina, A. Giazotto, K. Gill, L. Glover, E. Goetz, R. Goetz, S. Gomes, G. González, J. M. Gonzalez Castro, A. Gopakumar, M. L. Gorodetsky, S. E. Gossan, M. Gosselin, R. Gouaty, A. Grado, C. Graef, M. Granata, A. Grant, S. Gras, C. Gray, G. Greco, A. C. Green, P. Groot, H. Grote, S. Grunewald, P. Gruning, G. M. Guidi, X. Guo, A. Gupta, M. K. Gupta, K. E. Gushwa, E. K. Gustafson, R. Gustafson, B. R. Hall, E. D. Hall, G. Hammond, M. Haney, M. M. Hanke, J. Hanks, C. Hanna, O. A. Hannuksela, J. Hanson, T. Hardwick, J. Harms, G. M. Harry, I. W. Harry, M. J. Hart, C.-J. Haster, K. Haughian, J. Healy, A. Heidmann, M. C. Heintze, H. Heitmann, P. Hello, G. Hemming, M. Hendry, I. S. Heng, J. Hennig, J. Henry, A. W. Heptonstall, M. Heurs, S. Hild, D. Hoak, D. Hofman, K. Holt, D. E. Holz, P. Hopkins, C. Horst, J. Hough, E. A. Houston, E. J. Howell, Y. M. Hu, E. A. Huerta, D. Huet, B. Hughey, S. Husa, S. H. Huttner, T. Huynh-Dinh, N. Indik, D. R. Ingram, R. Inta, G. Intini, H. N. Isa, J.-M. Isac, M. Isi, B. R. Iyer, K. Izumi, T. Jacqmin, K. Jani, P. Jaranowski, S. Jawahar, F. Jiménez-Forteza, W. W. Johnson, D. I. Jones, R. Jones, R. J. G. Jonker, L. Ju, J. Junker, C. V. Kalaghatgi, V. Kalogera, S. Kandhasamy, G. Kang, J. B. Kanner, S. Karki, K. S. Karvinen, M. Kasprzack, M. Katolik, E. Katsavounidis, W. Katzman, S. Kaufer, K. Kawabe, F. Kéfélian, D. Keitel, A. J. Kemball, R. Kennedy, C. Kent, J. S. Key, F. Y. Khalili, I. Khan, S. Khan, Z. Khan, E. A. Khazanov, N. Kijbunchoo, C. Kim, J. C. Kim, W. Kim, W. S. Kim, Y.-M. Kim, S. J. Kimbrell, E. J. King, P. J. King, R. Kirchhoff, J. S. Kissel, L. Kleybolte, S. Klimenko, P. Koch, S. M. Koehlenbeck, S. Koley, V. Kondrashov, A. Kontos, M. Korobko, W. Z. Korth, I. Kowalska, D. B. Kozak, C. Krämer, V. Kringel, B. Krishnan, A. Królak, G. Kuehn, P. Kumar, R. Kumar, S. Kumar, L. Kuo, A. Kutynia, S. Kwang, B. D. Lackey, K. H. Lai, M. Landry, R. N. Lang, J. Lange, B. Lantz, R. K. Lanza, A. Lartaux-Vollard, P. D. Lasky, M. Laxen, A. Lazzarini, C. Lazzaro, P. Leaci, S. Leavey, C. H. Lee, H. K. Lee, H. M. Lee, H. W. Lee, K. Lee, J. Lehmann, A. Lenon, M. Leonardi, N. Leroy, N. Letendre, Y. Levin, T. G. F. Li, A. Libson, T. B. Littenberg, J. Liu, R. K. L. Lo, N. A. Lockerbie, L. T. London, J. E. Lord, M. Lorenzini, V. Loriette, M. Lormand, G. Losurdo, J. D. Lough, C. O. Lousto, G. Lovelace, H. Lück, D. Lumaca, A. P. Lundgren, R. Lynch, Y. Ma, S. Macfoy, B. Machenschalk, M. MacInnis, D. M. Macleod, I. Magaña Hernandez, F. Magaña Sandoval, L. Magaña Zertuche, R. M. Magee, E. Majorana, I. Maksimovic, N. Man, V. Mandic, V. Mangano, G. L. Mansell, M. Manske, M. Mantovani, F. Marchesoni, F. Marion, S. Márka, Z. Márka, C.

Markakis, A. S. Markosyan, E. Maros, F. Martelli, L. Martellini, I. W. Martin, D. V. Martynov, K. Mason, A. Masserot, T. J. Massinger, M. Masso-Reid, S. Mastrogiovanni, A. Matas, F. Matichard, L. Matone, N. Mavalvala, N. Mazumder, R. McCarthy, D. E. McClelland, S. McCormick, L. McCuller, S. C. McGuire, G. McIntyre, J. McIver, D. J. McManus, T. McRae, S. T. McWilliams, D. Meacher, G. D. Meadors, J. Meidam, E. Mejuto-Villa, A. Melatos, G. Mendell, R. A. Mercer, E. L. Merilh, M. Merzougui, S. Meshkov, C. Messenger, C. Messick, R. Metzdorff, P. M. Meyers, F. Mezzani, H. Miao, C. Michel, H. Middleton, E. E. Mikhailov, L. Milano, A. L. Miller, A. Miller, B. B. Miller, J. Miller, M. Millhouse, O. Minazzoli, Y. Minenkov, J. Ming, C. Mishra, S. Mitra, V. P. Mitrofanov, G. Mitselmakher, R. Mittleman, A. Moggi, M. Mohan, S. R. P. Mohapatra, M. Montani, B. C. Moore, C. J. Moore, D. Moraru, G. Moreno, S. R. Morriss, B. Mours, C. M. Mow-Lowry, G. Mueller, A. W. Muir, A. Mukherjee, D. Mukherjee, S. Mukherjee, N. Mukund, A. Mullavey, J. Munch, E. A. M. Muniz, P. G. Murray, K. Napier, I. Nardecchia, L. Naticchioni, R. K. Nayak, G. Nelemans, T. J. N. Nelson, M. Neri, M. Nery, A. Neunzert, J. M. Newport, G. Newton, K. K. Y. Ng, T. T. Nguyen, D. Nichols, A. B. Nielsen, S. Nissanke, A. Nitz, A. Noack, F. Nocera, D. Nolting, M. E. N. Normandin, L. K. Nuttall, J. Oberling, E. Ochsner, E. Oelker, G. H. Ogin, J. J. Oh, S. H. Oh, F. Ohme, M. Oliver, P. Oppermann, R. J. Oram, B. O'Reilly, R. Ormiston, L. F. Ortega, R. O'Shaughnessy, D. J. Ottaway, H. Overmier, B. J. Owen, A. E. Pace, J. Page, M. A. Page, A. Pai, S. A. Pai, J. R. Palamos, O. Palashov, C. Palomba, A. Pal-Singh, H. Pan, B. Pang, P. T. H. Pang, C. Pankow, F. Pannarale, B. C. Pant, F. Paoletti, A. Paoli, M. A. Papa, H. R. Paris, W. Parker, D. Pascucci, A. Pasqualetti, R. Passaquieti, D. Passuello, B. Patricelli, B. L. Pearlstone, M. Pedraza, R. Pedurand, L. Pekowsky, A. Pele, S. Penn, C. J. Perez, A. Perreca, L. M. Perri, H. P. Pfeiffer, M. Phelps, O. J. Piccinni, M. Pichot, F. Piergiovanni, V. Pierro, G. Pillant, L. Pinard, I. M. Pinto, M. Pitkin, R. Poggiani, P. Popolizio, E. K. Porter, A. Post, J. Powell, J. Prasad, J. W. W. Pratt, V. Predoi, T. Prestegard, M. Prijatelj, M. Principe, S. Privitera, R. Prix, G. A. Prodi, L. G. Prokhorov, O. Puncken, M. Punturo, P. Puppo, M. Pürrer, H. Qi, J. Qin, S. Qiu, V. Quetschke, E. A. Quintero, R. Quitzow-James, F. J. Raab, D. S. Rabeling, H. Radkins, P. Raffai, S. Raja, C. Rajan, M. Rakhmanov, K. E. Ramirez, P. Rapagnani, V. Raymond, M. Razzano, J. Read, T. Regimbau, L. Rei, S. Reid, D. H. Reitze, H. Rew, S. D. Reyes, F. Ricci, P. M. Ricker, S. Rieger, K. Riles, M. Rizzo, N. A. Robertson, R. Robie, F. Robinet, A. Rocchi, L. Rolland, J. G. Rollins, V. J. Roma, R. Romano, C. L. Romel, J. H. Romie, D. Rosińska, M. P. Ross, S. Rowan, A. Rüdiger, P. Ruggi, K. Ryan, S. Sachdev, T. Sadecki, L. Sadeghian, M. Sakellariadou, L. Salconi, M. Saleem, F. Salemi, A. Samajdar, L. Sammut, L. M. Sampson, E. J. Sanchez, V. Sandberg, B. Sandeen, J. R. Sanders, B. Sassolas, B. S. Sathyaprakash, P. R. Saulson, O. Sauter, R. L. Savage, A. Sawadsky, P. Schale, J. Scheuer, E. Schmidt, J. Schmidt, P. Schmidt, R. Schnabel, R. M. S. Schofield, A. Schönbeck, E. Schreiber, D. Schuette, B. W. Schulte, B. F. Schutz, S. G. Schwalbe, J. Scott, S. M. Scott, E. Seidel, D. Sellers, A. S. Sengupta, D. Sentenac, V. Sequino, A. Sergeev, D. A. Shaddock, T. J. Shaffer, A. A. Shah, M. S. Shahriar, L. Shao, B. Shapiro, P. Shawhan, A. Sheperd, D. H. Shoemaker, D. M. Shoemaker, K. Siellez, X. Siemens, M. Sieniawska, D. Sigg, A. D. Silva, A. Singer, L. P. Singer, A. Singh, R. Singh, A. Singhal, A. M. Sintes, B. J. J. Slagmolen, B. Smith, J. R. Smith, R. J. E. Smith, E. J. Son, J. A. Sonnenberg, B. Sorazu, F. Sorrentino, T. Souradeep, A. P. Spencer, A. K. Srivastava, A. Staley, M. Steinke, J. Steinlechner, S. Steinlechner, D. Steinmeyer, B. C. Stephens, R. Stone, K. A. Strain, G. Stratta, S. E. Strigin, R. Sturani, A. L. Stuver, T. Z. Summerscales, L. Sun, S. Sunil, P. J. Sutton, B. L. Swinkels, M. J. Szczepańczyk, M. Tacca, D. Talukder, D. B. Tanner, M. Tápai, A. Taracchini, J. A. Taylor, R. Taylor, T. Theeg, E. G. Thomas, M. Thomas, P. Thomas, K. A. Thorne, K. S. Thorne, E. Thrane, S. Tiwari, V. Tiwari, K. V. Tokmakov, K. Toland, M. Tonelli, Z. Tornasi, C. I. Torrie, D. Töyrä, F. Travasso, G. Traylor, D. Trifirò, J. Trinastic, M. C. Tringali, L. Trozzo, K. W. Tsang, M. Tse, R. Tso, D. Tuyenbayev, K. Ueno, D. Ugolini, C. S. Unnikrishnan, A. L. Urban, S. A. Usman, H. Vahlbruch, G. Vajente, G. Valdes, M. Vallisneri, N. van Bakel, M. van Beuzekom, J. F. J. van den Brand, C. Van Den Broeck, D. C. Vander-Hyde, L. van der Schaaf, J. V. van Heijningen, A. A. van Veggel, M. Vardaro, V. Varma, S. Vass, M. Vasúth, A. Vecchio, G. Vedovato, J. Veitch, P. J. Veitch, K. Venkateswara, G. Venugopalan, D. Verkindt, F. Vetrano, A. Viceré, A. D. Viets, S. Vinciguerra, D. J. Vine, J.-Y. Vinet, S. Vitale, T. Vo, H. Vocca, C. Vorvick, D. V. Voss, W. D. Vousden, S. P. Vyatchanin, A. R. Wade, L. E. Wade, M. Wade, R. Walet, M. Walker, L. Wallace, S. Walsh, G. Wang, H. Wang, J. Z. Wang, M. Wang, Y.-F. Wang, Y. Wang, R. L. Ward, J. Warner, M. Was, J. Watchi, B. Weaver, L.-W. Wei, M. Weinert, A. J. Weinstein, R. Weiss, L. Wen, E. K. Wessel, P. Weßels, T. Westphal, K. Wette, J. T. Whelan, B. F. Whiting, C. Whittle, D. Williams, R. D. Williams, A. R. Williamson, J. L. Willis, B. Willke, M. H. Wimmer, W. Winkler, C. C. Wipf, H. Wittel, G. Woan, J. Woehler, J. Wofford, K. W. K. Wong, J. Worden, J. L. Wright, D. S. Wu, G. Wu, W. Yam, H. Yamamoto, C. C. Yancey, M. J. Yap, H. Yu, H. Yu, M. Yvert, A. Zadrożny, M. Zanolin, T. Zelenova, J.-P. Zendri, M. Zevin, L. Zhang, M. Zhang, T. Zhang, Y.-H. Zhang, C. Zhao, M. Zhou, Z. Zhou, S. J. Zhu, X. J. Zhu, M. E. Zucker, J. Zweizig, S. Buchner, I. Cognard, A. Corongiu, P. C. C. Freire, L. Guillemot, G. B. Hobbs, M. Kerr, A. G. Lyne, A. Possenti, A. Ridolfi, R. M.

Shannon, B. W. Stappers, and P. Weltevrede, "First search for nontensorial gravitational waves from known pulsars", *Phys. Rev. Lett.* **120**:3 (2018), art. id. 031104.

[5] R. Al Nahas, M. Wang, B. Panicaud, E. Rouhaud, A. Charles, and R. Kerner, "Covariant spacetime formalism for applications to thermo-hyperelasticity", *Acta Mech.* **233**:6 (2022), 2309–2334.

[6] D. Baskaran and L. P. Grishchuk, "Components of the gravitational force in the field of a gravitational wave", *Classical and Quantum Gravity* **21**:17 (2004), 4041–4061.

[7] M. R. Beau, "Théorie des champs des contraintes et des déformations en relativité générale et expansion cosmologique", *Ann. Fond. Louis de Broglie* **40** (2015). arXiv 1209.0611v2

[8] J.-F. Bennoun, "Étude des milieux continus élastiques et thermodynamiques en relativité générale", *Ann. Inst. Henri Poincaré* **III**:1 (1965), 41–110.

[9] J. D. Brown, "Elasticity theory in general relativity", *Classical and Quantum Gravity* **38**:8 (2021), art. id. 085017.

[10] D. Buskulic, "Ondes gravitationnelles, aspects théoriques et expérimentaux", pp. 231–289 in *Gravitation, theory and experiments: Proceedings of the* 3rd *School of Theoretical Physics* (Jigel, Algeria, 2009), Herman, 2013.

[11] S. Capozziello, G. Lambiase, and C. StornaioloI, "Geometric classification of the torsion tensor in space-time", *Ann. Physik* **10**:8 (2001), 713–727.

[12] F. L. Carneiro, S. C. Ulhoa, J. W. Maluf, and J. F. da Rocha-Neto, "Non-linear plane gravitational waves as space-time defects", *Eur. Phys. J. C* **81** (2021), 67.

[13] B. Carter and H. Quintana, "Foundations of general relativistic high-pressure elasticity theory", *Proc. R. Soc. Lond. A* **331**:1584 (1972), 57–83.

[14] M. Chapon, L. Darondeau, R. Desmorat, C. Ecker, and B. Kolev, "General covariant relativistic gradient hyperelasticity", 2024, available at https://hal.science/hal-04792877. Ffhal-04792877f.

[15] T. Damour, "La relativité générale aujourd'hui", *Sém. Poincaré* **IX** (2006), 1–40.

[16] C. Duval, G. W. Gibbons, P. A. Hovarthy, and P. M. Zhang, "Carroll symmetry of plane gravitational waves", *Classical and Quantum Gravity* **34** (2017), art. id. 175003.

[17] F. W. Dyson, A. S. Eddington, and C. Davidson, "A determination of the deflection of light by the sun's gravitational field, from observations made at the total eclipse of May 29, 1919", *Phil. Trans. R. Soc. A* **220** (1920), 291–333.

[18] A. Einstein, *The meaning of relativity*, Princeton University Press, Princeton, NJ, 1988. Reprint of the 1956 edition.

[19] E. Elizalde, F. Izaurieta, C. Riveros, G. Salgado, and O. Valdivia, "Gravitational waves in ECSK theory: robustness of mergers as standard sirens and nonvanishing torsion", preprint, 2022. arXiv 2204.00090

[20] M. Epstein and R. E. Smelser, "An appreciation and discussion of Paul Germain's "The method of virtual power in the mechanics of continuous media, I: Second-gradient theory"", *Math. Mech. Complex Syst.* **8**:2 (2020), 191–199.

[21] C. W. F. Everitt, D. B. DeBra, B. W. Parkinson, J. P. Turneaure, J. W. Conklin, M. I. Heifetz, G. M. Keiser, A. S. Silbergleit, T. Holmes, J. Kolodziejczak, M. Al-Meshari, J. C. Mester, B. Muhlfelder, V. Solomonik, K. Stahl, P. Worden, W. Bencze, S. Buchman, B. Clarke, A. Al-Jadaan, H. Al-Jibreen, J. Li, J. A. Lipa, J. M. Lockhart, B. Al-Suwaidan, M. Taber, and S. Wang, "Gravity Probe B: final results of a space experiment to test general relativity", *Phys. Rev. Lett.* **106**:22 (2011), art. id. 221101.

[22] GDR, Collective discussions: Conférence du GDR Géométrie différentielle et mécanique, CNRS, La Rochelle, 2023.

[23] P. Germain, "The method of virtual power in continuum mechanics, II: Application to continuous media with microstructure", *SIAM J. Appl. Math.* **25** (1973), 556–575.

[24] R. A. Grot and A. Eringen, "Relativistic continuum mechanics", *Int. J. Eng. Sci.* **4** (1966), 611–670.

[25] R. A. Grot and A. Eringen, "Relativistic continuum mechanics, part I: Mechanics and thermodynamics", *Int. J. Eng. Sci.* **4**:6 (1966), 611–638.

[26] K. Heiduschke, "On tensor projections, stress or stretch vectors and their relations to Mohr's three circles", *Math. Mech. Complex Syst.* **14**:2 (2024), 173–216.

[27] S. Hou, X. L. Fan, T. Zhu, and Z. H. Zhu, "Nontensorial gravitational wave polarizations from the tensorial degrees of freedom, 1: Linearized Lorentz-violating theory of gravity with s tensor", *Phys. Rev. D* **109** (2024), art. id. 084011.

[28] D. Izabel, "Mechanical conversion of the gravitational Einstein's constant κ", *Pramana J. Phys.* **94** (2020), art. id. 119.

[29] D. Izabel, *What is space time made of*?, EDP Science, 2021.

[30] J. Kijowski and G. Magli, "Relativistic elastomechanics as a Lagrangian field theory", *J. Geom. Phys.* **9**:3 (1992), 207–223.

[31] H. Kleinert, *Gauge fields in condensed matter*, *vol. II*: *Stresses and defects*, World Scientific, Singapore, 1989.

[32] H. Kleinert, "Emerging gravity from defects in world crystal", *Brazil. J. Phys.* **35**:2a (2005), 359–361.

[33] B. Kolev, "Éléments de géométrie différentielle à l'usage des mécaniciens", Cours, 5$^{\text{ème}}$ école d'été de mécanique théorique, CNRS, Quiberon, 2016, available at https://hal.science/hal-03330418.

[34] B. Kolev, "Covariance générale et objectivité", Conférence du GDR Géométrie différentielle et mécanique, CNRS, La Rochelle, 2023.

[35] B. Kolev and R. Desmorat, "Souriau's general covariant formulation of relativistic hyperelasticity revisited", *J. Mech. Phys. Solids* **181** (2023), art. id. 105463.

[36] B. Kolev and R. Desmorat, "Objective rates as covariant derivatives on the manifold of Riemannian metrics", *Arch. Ration. Mech. Anal.* **248**:4 (2024), 66.

[37] J. Lemaitre, J.-L. Chaboche, A. Benallal, and R. Desmorat, *Mécanique des matériaux solides*, Dunod, 2020.

[38] F. S. N. Lobo, G. J. Olmo, and D. Rubiera-Garcia, "Crystal clear lessons on the microstructure of space-time and modified gravity", *Phys. Rev. D* **91**:12 (2015), art. id. 124001.

[39] J. W. Maluf, J. F. da Rocha-Neto, S. C. Ulhoa, and F. L. Carneiro, "Variations of the energy of free particles in the pp-wave spacetimes", *Universe* **4**:7 (2018), 74.

[40] S. Mathur, *Gravitational wave polarizations*: *a test of general relativity using binary black hole mergers*, Ph.D. thesis, California Institute of Technology, 2020.

[41] P. A. Millette, *Elastodynamics of the spacetime continuum*: *STCED*, American Research Press, Rehoboth, New Mexico, USA, 2019.

[42] J. D. Norton, "General covariance and the foundations of general relativity: Eight decades of dispute", *Rep. Prog. Phys.* **56**:7 (1993), 791–858.

[43] L. A. Philippoz, *On the polarization of gravitational waves*, Ph.D. thesis, Universität Zürich, 2018.

[44] C. B. Rayner, "Elasticity in general relativity", *Proc. R. Soc. A* **272**:1348 (1963), 44–53.

[45] M. L. Ruggiero, "Gravitomagnetic induction in the field of a gravitational wave", *Gen. Relat. Gravit.* **54**:9 (2022), 97.

[46] M. L. Ruggiero and D. Astesiano, "Low energy limits of general relativity and galactic dynamics", *Math. Mech. Complex Syst.* **11**:2 (2023), 271–286.

[47] M. L. Ruggiero and A. Tartaglia, "Einstein–Cartan theory as a theory of defects in space-time", *Am. J. Phys.* **71** (2003), 1303–1313.

[48] A. D. Sakharov, "Vacuum quantum fluctuations in curved space and the theory of gravitation", *Soviet Phys. Dokl.* **12** (1968), 1040–1041.

[49] S. Shrikanth, K. M. Knowles, S. Neelakantan, and R. Prasad, "Planes of isotropic poisson's ratio in anisotropic crystalline solids", *Int. J. Solids Struct.* **191-192** (2020), 628–645.

[50] J.-M. Souriau, "La relativité variationnelle", *Publ. Sci. Univ. Alger. Sér. A* **5** (1958), 103–170.

[51] J.-M. Souriau, *Géométrie et relativité*, Enseignement des Sciences, VI, Hermann, Paris, 1964.

[52] J. L. Synge, "A theory of elasticity in general relativity", *Math. Z.* **72**:1 (1959), 82–87.

[53] M. O. Tahim, R. R. Landim, and C. A. S. Almeida, "Spacetime as a deformable solid", *Modern Phys. Lett. A* **24**:15 (2009), 1209–1217.

[54] A. Tartaglia and N. Radicella, "From elastic continua to space-time", *AIP Conference Proceedings* **1241** (2010), 1156–1163.

[55] T. G. Tenev and M. F. Horstemeyer, "Mechanics of spacetime: a solid mechanics perspective on the theory of general relativity", *Int. J. Modern Phys. D* **27**:8 (2018), art. id. 1850083.

[56] K. Wettea, "Searches for continuous gravitational waves from neutron stars: a twenty-year retrospective", *Astropart. Phys.* **153** (2023), art. id. 102880.

[57] C. Will, "The confrontation between general relativity and experiment", *Living Rev. Relat.* **17** (2014), 4.

[58] P. M. Zhang, C. Duval, G. W. Gibbons, and P. A. Hovarthy, "Soft gravitons and the memory effect for plane gravitational waves", *Phys. Rev. D* **96** (2017), art. id. 064013.

[59] P. M. Zhang, C. Duval, W. Gibbons, and P. A. Hovarthy, "The memory effects for plane gravitational waves", *Physics Letters B* **772** (2017), 743–746.

[60] P. M. Zhang, M. Cariglia, C. Duval, M. Elbistan, G. W. Gibbons, and P. A. Horvathy, "Ion traps and the memory effect for periodic gravitational waves", *Phys. Rev. D* **98** (2018), art. id. 089901.

Received 24 Apr 2024. Revised 20 Mar 2025. Accepted 26 Apr 2025.

DAVID IZABEL: d.izabel@aliceadsl.fr
ICube Laboratory, University of Strasbourg / CNRS, 67000 Strasbourg, France

and

University of L'Aquila, Italy

YVES RÉMOND: remond@unistra.fr
ICube Laboratory, University of Strasbourg / CNRS, 67000 Strasbourg, France

MATTEO LUCA RUGGIERO: matteoluca.ruggiero@unito.it
Dipartimento di Matematica "G. Peano", Università degli studi di Torino, 10123 Torino, Italy

MATHEMATICS AND MECHANICS OF COMPLEX SYSTEMS

msp.org/memocs

MEMOCS is a journal of the International Research Center for the Mathematics and Mechanics of Complex Systems at the Università dell’Aquila, Italy.

See inside back cover or msp.org/memocs for submission instructions.

The subscription price for 2025 is US $225/year for the electronic version, and $300/year (+$25, if shipping outside the US) for print and electronic. Subscriptions, requests for back issues and changes of subscriber address should be sent to MSP.

Mathematics and Mechanics of Complex Systems (ISSN 2325-3444 electronic, 2326-7186 printed) at Mathematical Sciences Publishers, 798 Evans Hall #3840, c/o University of California, Berkeley, CA 94720-3840 is published continuously online.

MEMOCS peer review and production are managed by EditFlow® from MSP.

PUBLISHED BY

mathematical sciences publishers

nonprofit scientific publishing

http://msp.org/

MATHEMATICS AND MECHANICS OF COMPLEX SYSTEMS
vol. 13 no. 2 2025

MEMOCS is a journal of the International Research Center for the Mathematics and Mechanics of Complex Systems at the Università dell'Aquila, Italy.

International Journal of Modern Physics D
(2023) 2350091 (30 pages)

DOI: 10.1142/S0218271823500918

Analogy of spacetime as an elastic medium—Can we establish a thermal expansion coefficient of space from the cosmological constant Λ?

Izabel David*
d.izabel@aliceadsl.fr

Received 2 August 2023
Revised 22 September 2023
Accepted 24 September 2023
Published

This paper advances the state-of-the-art by extending the study of the analogy between the fabric of spacetime and elasticity. As no prior work exists about a potential spacetime thermal expansion coefficient α, we explore the analogy of general relativity with the theory of elasticity by considering the cosmological constant Λ as an additional space curvature of the structure of space due to a thermal gradient coming from the cosmic web and the cold vacuum and we propose $(\frac{\alpha_S \Delta T}{e})^2 = (\frac{1}{R_0})^2 = \Lambda$ with R_0 being the curvature radius of the space fabric. It follows from this analogy and from the supposed space model consisting of thin sheets of Planck thickness l_p curved by this thermal gradient ΔT a possible thermal expansion coefficient of the equivalent elastic medium modeling the space $\alpha_S = \frac{l_p\sqrt{\Lambda}}{\Delta T}$ of the order of $\alpha_{\text{space-QFT}} = 1.16 \times 10^{-6}\,\text{K}^{-1}$. As spacetime and not only space must be considered in general relativity, this paper also proposes an innovative approach which consists in introducing into the interval ds^2 of special relativity a temperature effect $T : d_s^2 = (1 \pm \alpha_t T)^2 c^2 dt^2 - (1 \pm \alpha_s T)^2 [dx^2 + dy^2 + dz^2]$ (entropy variations correlated with time laps, based on temperature variations affecting always physically the clocks) based on different thermal expansion coefficients for space and time with for the flow of time $t : \frac{ct}{cn\tau} = \frac{k_B t}{nh} \times \Delta T = \alpha_t \Delta T$. With $T \approx 10^6\,\text{K}$, $n = 1$, the associate time interval is $4.8 \times 10^{-17}\,\text{s}$ and $\alpha_t = 1.0 \times 10^{-6}\,K^{-1}$. The consequence of this hypothesis is that dark energy potentially becomes a thermal spacetime curvature $(\frac{\alpha_f T}{l_p})^2$ with f equal to s or t depending of the temperature, the thermal entropy variation of the universe, the Planck thickness and time, that increases since the Big bang, depending on thermal expansion coefficients for spacetime α_s and α_t as a function, respectively, of Λ, $\frac{k_B}{h} \times t$, in opposition to spacetime curvature gravity due to mass/energy density as described in general relativity.

Keywords: Spacetime fabric; general relativity; elasticity theory; quantum mechanics; expansion coefficient; cosmological constant; dark energy; time.

PACS Number(s): 04.50.Kd, 46.90.+s

*No affiliation.

1. Introduction

1.1. *Measured strains of space and analogy of the general relativity theory with the elasticity theory*

Einstein's theory of general relativity is over 100 years old and is now widely verified. Thus, spacetime is according to this theory a deformable elastic physical object. Gravitation is thus a manifestation of the geometric deformation of spacetime under the effect of the masses or energy density therein. The manifestations of the deformations of this spacetime are now known and measured with great precision. We can quote the apparent position variation of stars placed behind the sun during an eclipse measured by Edington,[1] the frame dragging of spacetime by angular distortion by the rotation of the earth (experiment prob B, Lense-Thirring and frame dragging effects),[2] the simultaneous lengthening and shortening deformations in each of the arms of Ligo/Virgo type interferometers during the passage of gravitational waves,[3,4] gravitational lenses or substantial masses located between a galaxy and our field sighting on earth distorts space to the point of making it appear to us in the shape of a circle (a bit like a candle placed behind the flat circle of a stemmed wine glass appears circular by its transparency reflection), and finally the expansion of the universe where the galaxies are "fixed" in a space which expands in an increasingly accelerated way characterized by Hubble's law. All these manifestations of the deformations of spacetime have led many physicists like Sakharov,[5] Synge,[6] Rayner,[7] Grot,[8] Vasilev and Fedorov,[9,10] Brown,[11] Tenev and Horstemeyer,[12] Millette[13] and many others as Damour in its conferences and books consider that the theory of general relativity is a kind of theory of the elasticity of a deformable elastic spacetime medium. We then speak of "elastic metric" or "elastic theory of gravitation". It is within the framework of this analogy that we place ourselves in this paper.

If by analogy, therefore, spacetime is considered as an equivalent four-dimensional deformable elastic medium, where a simplistic two-dimensional image is a heavy ball put on a rubber sheet deformed by this ball, they are two consequences. We will study it in the next two paragraphs.

1.2. *The mechanical characteristics of the equivalent elastic medium in the field of the analogy of the elasticity theory with the General Relativity—review of the state-of-the-art*

This equivalent elastic medium must therefore be characterized with the usual parameters linked to all elastic mediums and to the elasticity theory (Young's modulus $Y = E$, Poisson's ratio ν, density ρ, etc). Thus various authors have sought to establish an equivalent Young's modulus of the space noted Y_{space}. We can quote R.Weiss during his nobel prize speech about gravitationnal waves I quote "*In other words, it takes enormous amounts of energy to distort space. One way to say it is,* *the stifness (Young's modulus) of space at a distortion frequency of* 100 Hz *is* 10^{20} *larger than steel*".

Tenev and Horstemeyer[12] who propose by considering the spacetime made up of thin elastic sheets of the thickness of Planck the following formulation (1) giving $Y_{\text{space}} = 4.4 \times 10^{113}\,\text{N/m}^2$:

$$Y_{\text{space}} = \frac{6c^7}{2\pi\hbar G^2} = \frac{24}{l_p^2 \kappa}. \tag{1}$$

In this expression, c is the speed of light, G is the gravitational constant, $\hbar$ is the reduced Planck's constant, l_p is Planck's length (thicknesses of the thin sheets supposed to constitute the space fabric in Ref. 12) and κ is Einstein's gravitational constant ($\kappa = \frac{8\pi G}{c^4}$).

Beau proposes a space bulk modulus[14] and arrives at $K_{\text{space}} = 1.64 \times 10^{109}\,\text{N/m}^2$.

McDonald[15] proposes another expression of the Young's modulus (2) based on dimensional equations and obtain:

$$Y_{\text{space}} = \frac{c^2 f^2}{G}, \tag{2}$$

where f is the frequency of the gravitational wave. He thus obtains for a gravitational wave of 100 Hz $Y_{\text{space}} = 10^{20}\ \text{Y}_{\text{steel}}$ so, $Y_{\text{space}} = 4.5 \times 10^{31}$ Pa as R. Weiss.

Izabel in Ref. 16 arrives at an expression similar to Mc Donald by studying the elastic deformations in space dynamics located in the arms of the Ligo/Virgo laser interferometers and by studying the elastic deformations of a space cylinder twisted by the rotation of two black holes. He thus obtains the expression (3) of Young's modulus of space similar to McDonald at the factor π close:

$$Y_{\text{space}} = \frac{\pi f^2 c^2}{G}. \tag{3}$$

This leads by considering a density ρ from quantum field theory (hypothesis similar to Tenev and Horstemeyer[12]) at $Y_{\text{space}} = 1$ at 4.0×10^{113} Pa.

Melissinos[17] considers vibrating plates in the planes of the arms of the interferometers and arrives at the following expression (4) of Young's modulus of space:

$$Y_{\text{space}} < \frac{\pi c^2 f^2}{4G} \times \frac{c\Delta\tau}{\Delta z}. \tag{4}$$

In this expression, $\Delta\tau$ is the length of the gravitational wave burst and the total path length traversed by the GW is designated by Δz. The numerical application is done with $\Delta\tau \approx 1\,\text{s}$ and $\Delta z \approx 400\,\text{Mpc}$ and leads it to a next value of Young's modulus $Y_{\text{space}} < 2.5 \times 10^{-17}(c^2 f^2/G)$.

Finally, let us quote Hwang in Ref. 18 which by comparing the energy of gravitational waves for different frequencies (35 to 100 Hz and different GW) with the deformation energy of a spring modeling the lengthening and shortening of the arms of the interferometers arrives at values of the Young's modulus of space time $Y_{\text{space}} = 1.0 \times 10^{36}$ at 1.0×10^{54} Pa.

Another key parameter of any elastic medium is of course the coefficient of transverse deformation, the space Poisson's ratio ν. Again, many authors have proposed values. From the simultaneous deformations of the arms of the Ligo/Virgo interferometers (while one arm is shortened by a deformation by shortening of the order of -10^{-21} the other lengthens by the same amount of $+10^{-21}$) Tenev and Horstemeyer[12] propose $\nu = 1$ which presupposes a certain anisotropy of space which behaves like a kind of thousand leaves during the passage of a gravitational wave, each plane deforming successively according to the polarizations A^{+} and $A^{\times}$. The Poisson's ratio being close to zero in the direction of propagation of the wave and being equal to 1 in the plane perpendicular to the direction of propagation.

Izabel in Ref. 16 arrives at the same conclusion on this Poisson's ratio.

Concerning the equivalent density of the medium space ρ, Sakharov[5] shows that quantum considerations of space (quantum field theory) make it possible to go back to an elastic theory of space (5). We quote it below:

"In Einstein's theory of gravitation, it is postulated that the action of spacetime depends on the curvature (R is the invariant of the Ricci tensor):

$$S(R) = -\frac{1}{16\pi G}\int (dx)\sqrt{-g}R. \tag{5}$$

The presence of action (1) results in a "metric elasticity" of space, that is, the generalized forces that oppose the curve of space."

Tenev and Horstemeyer[12] and Izabel[16] also follow this path which leads them by considering the minimum nonzero energy of the vacuum to the following expression (6):

$$\rho = \frac{Y_{\text{space}}}{4c^2} = 1.3 \times 10^{96}\,\frac{\text{kg}}{\text{m}^3}. \tag{6}$$

Millette in Ref. 13 (formula (19.36)) proposes the following expression (7) for the density of space:

$$\bar{\rho}_0 = \frac{32c^5}{\hbar G^2} = 1.7 \times \frac{10^{98}\,\text{kg}}{\text{m}^3}. \tag{7}$$

Finally concerning the shear modulus of the middle space Millette always in Ref. 13 (formulae (19.14) and (19.22)) proposes the following expression (8a):

$$\bar{\mu}_0 = \mu = G = \frac{Y}{2(1+\nu)} = \frac{32c^7}{\hbar G^2}1.5 \times 10^{115}\,\text{N/m}^2. \tag{8a}$$

For bulk modulus, (formula (19.21) of Ref. 13) the following equation can be seen:

$$\bar{\kappa}_0 = K = \frac{\bar{\mu}_0}{32} = \frac{c^7}{\hbar G^2} = \lambda + \frac{2}{3}\mu = \frac{Y}{3(1-2\nu)} = 0.046875 \times 10^{115}\,\text{N/m}^2. \tag{8b}$$

That concludes this review of the characteristics of space following the analogy of it as an equivalent elastic medium. Two conclusions emerge from this state-of-the-art.

First, the mechanical parameters of the equivalent elastic material constituting in our analogy the comic fabric are not of the same magnitude as those found on Earth given the orders of magnitude extremely small of the strains of (10^{-21}), extremely large of Young's modulus (10^{113} Pa), the Poisson's ratio outside the usual standards ($\nu = 1$), the density ρ of the medium also extremly large (10^{98} kg/m^3) and its associated anisotropy linked to Poisson's ratio if it is 1 in a direction perpendicular to the propagation of the wave.

Second, and this is partly the subject of this paper, no publication to our knowledge deals with a possible expansion coefficient α of this space medium. We are therefore going to determine an original and innovative approach in this paper to try to propose a mechanical expression and a numerical value of this possible expansion coefficient α of the equivalent spatial fabric on the one hand and to study the consequences on time on the other hand.

1.3. *Analogy about the behavior law of the space following the general relativity with the Hooke's law in elasticity without and with cosmological constant* $\boldsymbol{\Lambda}$*—thermal gradient implication about the different curvatures that have to be considered*

The second aspect about this analogy is the law governing this equivalent elastic medium, this very special space fabric. Indeed, it is well known that Einstein's equation of general relativity without cosmological constant Λ relates the curvature of space $G_{\mu\nu}$ to the density of energy which deforms it $T_{\mu\nu}$ see Eq. (9).

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}. \tag{9}$$

In this expression $R_{\mu\nu}$ is the Ricci tensor resulting from the contraction of the Riemann tensor, R is the scalar curvature resulting from the contraction of the Ricci tensor and $T_{\mu\nu}$ the momentum energy tensor. $\mu\nu$ varying from 0 for time to 3 for the three dimensions of space.

Izabel showed in Ref. 16 that the general relativity equation in four dimensions presents an analogy with, respectively, the expressions in one and two dimensions of the curvature of beam (10), (11) and plate (12) in pure bending under two moments applied at each extremity (see Fig. 1) according to Timoshenko's strength of materials theory issued of the elasticity theory.

$$\frac{1}{R^2} = \frac{2}{EI} \left(\frac{W_{\text{ext(total)}}}{L} \right) = K \left(\frac{W_{\text{ext(total)}}}{L} \right). \tag{10}$$

In analogy with:

$$G^{\mu\nu} = -\frac{8\pi G}{c^4} (T^{\mu\nu}) = -\kappa (T^{\mu\nu}). \tag{11}$$

In these expressions, R is the radius of curvature of a beam of span L, moment of inertia I and Young's modulus $E = Y$ associated with a work of the external forces W. K is the mechanical coupling constant between the curvature and the strain energy U of the beam which is equal to the work applied external forces W.

Or in theory of the plates of thickness h following[19] with R_x, R_y, R_{xy}, the curvature radii of the plate in bending in the different directions, $\Delta x \cdot \Delta y \cdot t$ an elementary volume of the plate of thikness t, ν the Poisson's ratio and E Young's modulus of the material constituting the plate.

$$\left[\left(\frac{1}{R_x}\right)^2 + \left(\frac{1}{R_y}\right)^{.2} + 2(1-\nu)\left\{\left(\frac{1}{R_{xy}}\right)^2\right\} + 2\nu\left\{\frac{1}{R_x}\frac{1}{R_y}\right\}\right] = \frac{24(1-\nu^2)}{Et^2} \times \frac{\Delta U}{t\Delta x \Delta y}. \quad (12)$$

Tenev and Horstemeyer[12] but also Damour in his book "if Einstien was told to me", showed this analogy from a tensor point of view in terms of Hooke's law (13) and (14) with σ^{kl} the stress tensor, ε^{ij} the strain tensor, Y is Young's modulus and g^{kl} a metric.

$$\sigma^{kl} = \frac{Y}{1+\nu}\left(\frac{\nu}{1-2\nu} g^{ij} g^{kl} + g^{ik} g^{il}\right)\varepsilon_{ij}, \quad (13)$$

$$T_{\mu\nu} = \frac{1}{\kappa}\left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}\right). \quad (14)$$

In the space fabric model of Tenev and Horstemeyer,[12] space is assumed to be made up of ultra-thin sheets of Planck thickness, having an elastic behavior. It is in this space model that we will place ourselves in the rest of this publication.

But the tensorial equation of Einstein (9) can be also written by considering this time the cosmological constant Λ as a materialization of a certain repulsive dark energy. It is written as follows (see formula (15) and (16)):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu}. \quad (15)$$

Or by factoring Einstein's constant κ:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4}\left[T_{\mu\nu} - \frac{c^4 \Lambda}{8\pi G} g_{\mu\nu}\right]. \quad (16)$$

But Einstein's tensorial equation (9) considering the cosmological constant Λ can be also written as an additional curvature present in all space (frame of this paper). It is written as follows formula (17):

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}. \quad (17)$$

If we follow completely the analogy of the beam or the plate in elasticity describes before it exists not one but two sources of curvature (see Chap. 3): one under the applied masses developed in Refs. 12 and 16 and one under the temperature gradient (so with 0 mass), hence our idea of associating the second with the cosmological constant if our analogy is correct, the first one having already been partially demonstrated in Ref. 16.

So, we come to the subject of this publication. In the same way that we have shown that Einstein's constant κ, by analogy with an elastic medium made of thin sheets of thiknesses l_p, (plate theory) could be expressed in terms of mechanical constants[12,16] see expression (12) ($\frac{24(1-\nu^2)}{Et^2} \rightarrow \kappa \rightarrow Y = \frac{24}{l_p^2\kappa}$), the cosmological constant Λ which is generally associated with a dark energy[20] opposed to gravitation (15) and (16), can it not also be expressed in terms of mechanical parameters as an additional curvature present in all space (17) due at a thermal gradient applied at these thin sheets of thicknesses l_p?

This constant Λ generally associated with an expansion of space[21] "Dark energy as a curvature of spacetime induced by quantum vacuum fluctuations", can it not be correlated with a hidden mechanical behavior of space via a parameter missing from the state-of-the-art cited above in the analogy of space as a elastic medium, namely a thermal expansion coefficient α of it? Does not the phenomenal temperature difference between the cosmic web that fills the entire universe and the icy vacuum at $-2.73\,\mathrm{K}$ constitute such a thermal gradient sufficient to impose on the sheets[12] constituting the space of a thickness of Planck in the framework of our model an additional source curvature of this cosmological constant? It is these hypothesis that we will study in this paper, thus placing ourselves in the continuity of the publications and theoretical models of Tenev and Horstemeyer[12] and Izabel.[16]

2. Methods

The following methodology has been implemented to estimate, within the framework of the analogy of space as an equivalent elastic medium, the value of a possible thermal expansion coefficient α of space in line with the cosmology constant Λ associated with an additional thermal space curvature in the framework of a Planck's thickness sheet space model as considered by Tenev and Horstemeyer.[12]

(1) Within the framework of the analogy of an elastic medium to model the deformations of space in the presence of mass energy, restructure the simplified and appropriate mechanical models among those already developed to evaluate a possible generalized thermal curvature of this one.
(2) Search for certain scientific data that can feed this mechanical model of curved space under the effect of a thermal gradient between the cosmic web and the space vacuum.
(3) Consider the cosmological constant Λ not as a gravitationally repulsive dark energy (if placed to the right of Eqs. (15) and (16) but as an additional curvature present in all space (if placed to the left of Eq. (17)) and analysis of the consequences of this approach.
(4) Extract from the mechanical model of curvature of space under thermal gradient and from the scientific data available in connection with this model by

considering the cosmological constant Λ, a possible thermal expansion coefficient α of the space medium.

(5) Discuss the representativeness of this model by varying the assumptions (values of Λ) and seeing the consequences on the value of the thermal expansion coefficient of the associated elastic space fabric.
(6) Discuss the additional verifications necessary to confirm this approach.
(7) Investigate by the state-of-the-art analysis, the potential effect of temperature on time via entropy variation and its profound implication on the interval $\mathbf{ds^2}$ in special relativity, value of Λ in general relativity expressed in terms of curvature of space and time.
(8) Presentation of the possible consequences of this model concerning the interpretation of dark energy.
(9) Proposition of a testing way to verify this theory.

3. Search for a Simplified Mechanical Model of Space Allowing to Represent a Thermal Curvature of this One

3.1. *Analogy of the thermal curvature of an elastic medium with a simplified approach of resistance of materials in one dimension—beam under thermal gradient*

According to the strength of materials theory, two phenomena and only two can create a curvature of a physical object (beam, plate, shell, 3 dimensions structure)

- masses supported by the object,
- a thermal gradient applied to the object.

We studied the first type of curvature in Ref. 16 and showed that masses placed on a beam create curvature (18). Thus we have shown in Ref. 16 that the curvature of a beam in pure bending (solicited by two moments M at each end) (18) takes a form similar to Einstein's equation from the point of view of the analogy of space as an elastic medium in the form curvature = K × a linear strain energy density U/L:

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{U}{L}\right), \tag{18}$$

where U is the strain energy of the beam, L is the span, e is the height of the beam, E is Young's modulus of the material, $I = \frac{be^3}{12}$, is the moment of inertia, $(b \times e)$ is the section and R is the curvature radius (see Fig. 1).

The second type of curvature, we study it in this paper, is the curvature created by the difference in temperature between the two extreme fibers of the beam. The beam is then subjected to a thermal gradient ΔT which leads to an elongation of the heated fibers and a shortening of the cooled fibers. These differences in elongation create a curvature without internal forces of the beam if this one is not constrained in displacement somewhere along its surface.

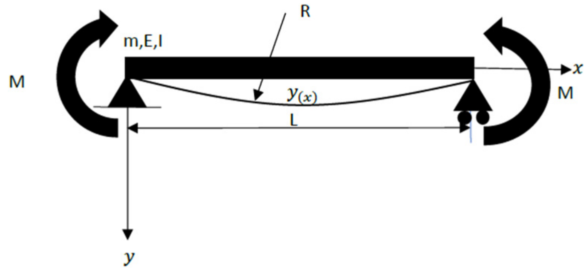


Fig. 1. Definition of a Timoshenko's beam in pure bending.

Damour tells us in his various conferences and publications that curvature in the Einstein sense has the dimension of an angle divided by a surface (see also the first definition of curvature and differential geometry in Gauss work[22]). We will show that is indeed the case of a beam loaded by a thermal gradient (temperature difference T between the lower and upper fibers of the beam $\Delta_T = T_{\text{ext}} - T_{\text{int}}$) with T_{ext} and T_{int} the temperature of each side of the sheet considered.

Let us now consider the case of an identical beam in pure bending undergoing a thermal gradient (see Fig. 2):

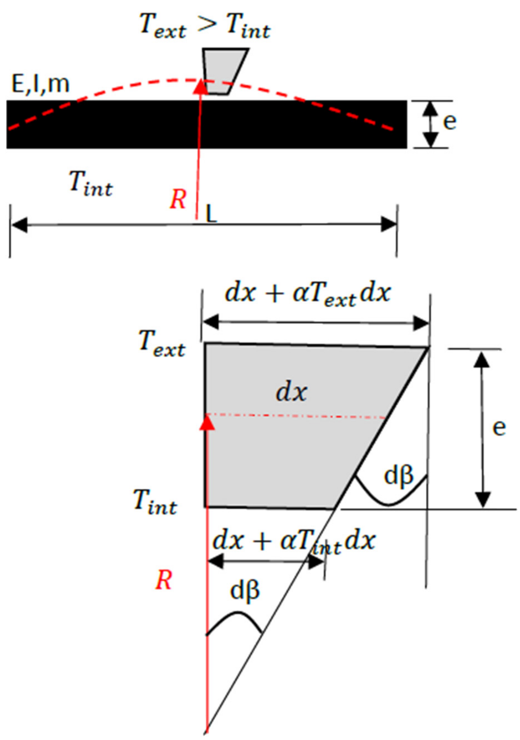


Fig. 2. Strain of a beam in bending under a thermal gradient ΔT.

The strain energy U (19) is always with M a bending moment for a beam of span L, thickness e and rigidity EI:

$$U = \frac{1}{2}\int_0^L \frac{M^2}{EI}dx. \tag{19}$$

Figure 2 and formula (20) allow us to write the following geometric relations between the angle β (radiant) and the curvature $(1/R)$ of the beam with e the height of the beam, α its expansion coefficient and ΔT the thermal gradient applied to this beam:

$$\begin{aligned}\tan(d\beta) &= \frac{dx}{R} = \frac{(dx + \alpha T_{\text{ext}}dx) - (dx + \alpha T_{\text{int}}dx)}{e} \\ &= \frac{(\alpha T_{\text{ext}} - \alpha T_{\text{int}})dx}{e} = \frac{\alpha \Delta T dx}{e}.\end{aligned} \tag{20}$$

Given Fig. 2, from the relation between the curvature of a beam under a thermal gradient and the second derivative of its displacement equation $y(x)$, we obtain the expression (21):

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{1}{R} = \frac{\tan(d\beta)}{dx} = \frac{d\beta}{dx} = \frac{\alpha\Delta T}{e}. \tag{21}$$

Considering the new expression of the moment M from (21), we have

$$M = EI\frac{d\beta}{dx}. \tag{22}$$

By replacing the moment M by its expression above (22) in the expression of the strain energy U of the beam (19), we then obtain (23):

$$U = \frac{1}{2}\int_0^L \frac{\left(EI\frac{d\beta}{dx}\right)^2}{EI}dx. \tag{23}$$

So, after simplification (EI = constant allong the beam) we obtain (24):

$$U = \frac{EI}{2}\left(\frac{d\beta}{dx}\right)^2 L. \tag{24}$$

After some mathematical calculations, the final result is Eq. (25):

$$\boldsymbol{\left(\frac{d\beta}{dx}\right)^2 = \frac{2}{EI}\frac{U}{L}}. \tag{25}$$

That can be compared with the expression recalled above in the case of the beam in pure bending (18).

We therefore have an angle divided by an area as the definition of the curvature $(1/R)$ squared of the beam (26).

$$\boldsymbol{\left(\frac{d\beta}{dx}\right)^2 = \frac{d\beta^2}{dx\times dx} = \left(\frac{1}{R}\right)^2}. \tag{26}$$

This corroborates the expression of Damour from the differential geometry of Gauss developed by Riemann[22] and given again in the formula (27):

$$\text{Curvature} = \frac{\alpha + \beta + \gamma - 180^\circ}{\text{Area}}. \tag{27}$$

The strain energy of a beam of span L, rigidity EI=YI for a constant thermal gradient ΔT is given by formula (28):

$$U_{\Delta T} = \frac{EI}{2}\left(\frac{d\beta}{dx}\right)^2 L = \frac{EI}{2}\left(\frac{\alpha\Delta T}{e}\right)^2 L. \tag{28}$$

So :

$$\left(\boldsymbol{\frac{\alpha\Delta T}{e}}\right)^{\mathbf{2}} = \boldsymbol{\frac{2}{EI}\frac{U_{\Delta T}}{L}}. \tag{29}$$

As our analogy is based on the elasticity theory, we can superimpose the load cases and thus superimpose the case of the beam in pure bending (due to the two moments M due at masses at each extremity of the beam, see Fig. 1) with the case of the beam under thermal gradient uniform constant ΔT (see Fig. 2), we then obtain the formula (30) of the generalized curvature of a beam under load and thermal gradient with $W_{\text{ext,total}(M+\Delta T)}$ the total external work of the external forces under moment and thermal gradient:

$$\frac{1}{R^2} + \left(\boldsymbol{\frac{\alpha\Delta T}{e}}\right)^{\mathbf{2}} = \frac{2}{EI}\left(\frac{U_M}{L}\right) + \boldsymbol{\frac{2}{EI}}\left(\boldsymbol{\frac{U_{\Delta T}}{L}}\right) = \frac{2}{EI}\left(\frac{W_{\text{ext,total}(M+\Delta T)}}{L}\right). \tag{30}$$

This expression is compatible from the point of view of the elastic analogy with the expression (17) of Einstein's field equation with cosmological constant Λ according to the correspondences (31) and (32) if one assumes (hypothesis of this article) that the cosmological constant Λ is by analogy correlated with a thermal gradient applied in all space:

- For the masses curvature the analogy between general relativity and strength of material/elasticity is:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \rightarrow \frac{1}{R^2}. \tag{31}$$

- For the thermal curvature the analogy between general relativity and strength of material/elasticity is:

$$\Lambda g_{\mu\nu} \rightarrow \left(\boldsymbol{\frac{\alpha\Delta T}{e}}\right)^{\mathbf{2}}. \tag{32}$$

About the formula (31), indeed, for memory, it can be proved that the Ricci tensor for a classical 2sphere is $2/R^2$ (see book what is space time made of? of Izabel).

3.2. *Analogy of space as an elastic medium with the thermal curvature of a thin plate associated according to Timoshenko's theory*

Considering space as a fabric made up of thin sheets has already been explored as we have said above by many authors. Let us quote Melissinos,[17] Tenev and Horstemeyer,[12] Izabel,[16] and Perko.[23]

It is therefore quite natural that we take up this hypothesis on the structure of the fabric of space.

Moreover, if we take a piece of the universe locally, its surface will be considered almost flat according to the value of the cosmological curvature $k = 0$ obtained by measurements from the Planck satellite.[24] In these publications, it is proven that the joint constraint with BAO measurements on space curvature is consistent with a flat universe

$$\Omega k = -kc^2(r_0 H_0)^{-2} = -0.0010 + 0.0018/ - 0.0019$$

As a reminder, the space expansion scale factor is written $R_{(t)} = a_{(t)} = \frac{r_{(t)}}{r_0}$. r_0 is the radius of reference and $r_{(t)}$,the radius of the 3 sphere in the metric of Friedmann–Lemaitre–Robertson–Walker.

The Hubble constant squared ($1/\mathrm{s}^2$) is :$H_0^2 = \frac{8\pi G}{3}\rho_c$ where ρ_c is the critical density of the medium.

So, the curvature k (unit $1/\mathrm{m}^2$) of the univers can be considered as flat (33).

$$\Omega_k = -kc^2(r_0 H_0)^{-2} = -\frac{3kc^2}{8\pi G\rho_C a^2}, \tag{33}$$

where Ω_k is therefore the dimensionless curvature parameter of space (34):

$$\frac{\frac{1}{m^2} \times \frac{m^2}{s^2}}{\frac{m^3}{kgs^2} \times \frac{kg}{m^3} \times 1} = 1. \tag{34}$$

It is therefore not absurd to consider space as an infinitely long thin plates superposition given the very large radius of curvature of the universe associated with its gigantic size.[25]

In this case the curvature of a thin plate under a thermal gradient is well known and is given in Timoshenko in his book[19] in Chap. 14, formula (50). We give the expression below (formula (35)) similar to that obtained in the case of a beam :

$$\frac{\alpha \Delta T}{e} = \frac{1}{R}. \tag{35a}$$

Or squared to stay consistent with the previous paragraph regarding beam theory (35b):

$$\left(\frac{\alpha \Delta T}{e}\right)^2 = \left(\frac{1}{R}\right)^2. \tag{35b}$$

In these two expressions, the curvature squared of the plate $(\frac{1}{R})^2$ is therefore linked to the thickness of the considered plate e, to the thermal expansion coefficient α associated with the elastic material and to the thermal temperature gradient ΔT between the extreme fibers of the plate.

It is therefore this model that we will consider later in the continuity of the authors cited above.[12,16] We must therefore establish the different parameters involved in this model. Namely, what thickness e of the sheets? what intensity of the thermal gradient ΔT? what value of the curvature $1/R$? This is what we will study in the next section.

4. Search for Certain Scientific Data that can Feed this Mechanical Model of Thermal Curvature of the Space Fabric

4.1. *What plate thickness consider?*

We know from Sakharov[5] that the potential quantum nature of vacuum can generate an elastic metric of space. Tenev and Horstemeyer in Ref. 12 consider Planck thickness sheets. Izabel in Ref. 16 manages to find Young's moduli of the different authors by considering also a Planck length. Millette[13] does the same.

Consequently, we will consider as in Ref. 12 within the framework of this publication, a thickness e of plate or fiber of the elastic fabric of space equal to l_p the length of Planck (36).

$$e = e_p = l_p = \sqrt{\frac{\hbar G}{c^3}}. \tag{36}$$

Remark 1. This model of elastic fabric of thickness l_p has been studied in the thesis of Tenev to modelize the sun gravity. There is a perfect accordance between theory and model.[12,26]

4.2. *Which thermal gradient consider?*

New data show a temperature gradient between the absolute vacuum at 2.73 K and the cosmic web see Ref. 27, "The Cosmic Thermal History Probed by Sunyaev–Zeldovich Effect Tomography". In this paper, we can read:

"We estimate Te, the density-weighted electron temperature of the universe, which goes from 7×10^5 K at $z = 1$ to 2×10^6 K today" The cosmic thermal history probed by Sunyaev-Zeldovich effect tomography YI-KUAN CHIANG , RYU MAKIYA, BRICE MÉNARD ET EIICHIRO KOMATSU".[27]

In this paper, the authors therefore highlight a certain thermal gradient between the cold zones of the universe and the very hot zones at the level of the cosmic web. So, this recent paper suggests that the average temperature of the gas present in the large structures of the observable Universe has been multiplied by 10 during the last 10 billion years to reach about two million Kelvin today.

We will therefore retain this hypothesis: $\Delta T = 2,000,000°K$.

4.3. *What curvature of space associated with this thermal gradient consider?*

Peebles in his paper[20] review the different approaches for linking the cosmological constant and dark energy.

Santos in Ref. 21 study the dark energy induced by the curvature of spacetime by quantum vacuum fluctuations.

On the basis of these two papers in particular, we will therefore postulate that the cosmological constant is the source of a curvature of space linked to a thermal gradient acting on the thin sheets of space of quantum thickness equal to the length of Planck.

To be consistent with the approaches of Sakharov[5] and Tenev and Horstemeyer,[12] we consider in this study the cosmological constant resulting from vacuum fluctuations (quantum field theory). We will see in Chap. 6 how to explore all the scientific options, what gives other values of the cosmological constant Λ- resulting from cosmological observations.

We make this hypothesis based on quantum field theory taking into account all of the previous paragraphs:

(1) the analogy of the space fabric as an elastic medium works well with the general relativity equation for mass/energy,[5–17]
(2) the elastic analogy suggests two possible curvatures and only two: one resulting from the masses/energy acting in the medium and one resulting from the thermal gradient acting on the medium,[16]
(3) there is obviously a thermal gradient present in space following Ref. 27,
(4) many authors[12,16,23] and measurements of gravitational waves[3,4] suggest the presence of plane deformations of elastic media (spatial fabric),
(5) Sakharov[5] showed that general relativity is correlated with an elastic metric resulting from quantum fluctuation of space.

4.4. *Final model retained to estimate a coefficient of thermal expansion of the fabric of the equivalent space*

Considering the previous paragraphs, we therefore arrive at the simplified model given in Fig. 3.

Thus, the almost flat thin sheets of space ($k = 0, T$ of space curvature tending towards infinity[12,24,25]) of Planck's thickness (l_p) according to the reasoning by Sakharov[5] and Tenev and Horstemeyer[12] undergo a thermal gradient resulting from the differences in average temperature between the cosmic web and the space vacuum.[27] This causes their curvature. It is this curvature that the cosmological constant Λ[20,21] can represent. By placing ourselves in the analogy of space functioning as a medium, an elastic fabric, the mechanics of continuous media by

Timoshenko's plate theory[19] allows us to model its behavior in a simplified way and to extract a thermal expansion coefficient α of the space fabric.

Remark 2. The thermal conductivity λ of the vacuum is by definition $0\,\mathrm{W/m^2 \cdot K}$. This allows us to have a thermal gradient between the two faces of thickness $e = l_p$.

5. Consequence of Considering the Cosmological Constant as a Generalized Thermal Curvature

5.1. *Determination of the space fabric expansion coefficient*

From Timoshenko's expression (36) (see also Ref. 5) of a spatial plate curvature ($k = 0$) see Refs. 24 and 25 in pure bending loaded by a thermal gradient ΔT[27] and in considering the cosmological constant Λ as an additional curvature (and not a dark energy) associated with the thermal gradient acting in space[20,21] we postulate Eq. (37):

$$\left(\frac{\alpha_S \Delta T}{e}\right)^2 = \left(\frac{1}{R_0}\right)^2 = \Lambda, \tag{37}$$

where R_0 is the curvature radius of one of the thin sheet supposed constitute the space fabric[12] (see Fig. 3).

We deduce from the above expression the thermal expansion coefficient α_S of the spatial fabric:

$$\alpha_S = \frac{\boldsymbol{e}\sqrt{\Lambda}}{\boldsymbol{\Delta T}}. \tag{38}$$

By definition of the comological constant with ρ_{vacuum} the vacuum density and c the speed of light we obtain (39):

$$\frac{c^4 \Lambda}{8\pi G} = \rho_{\text{vacuum}} c^2. \tag{39}$$

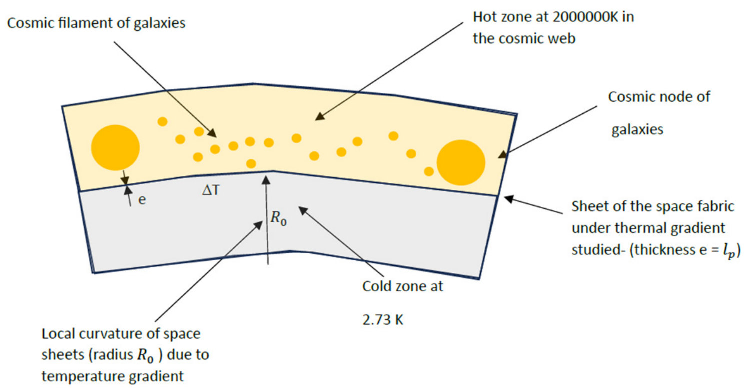


Fig. 3. Simplified mechanical model of elastic space undergoing a curvature by a thermal gradient between a very hot zone and a very cold zone of the universe.

So, after some mathematic calculations (40):

$$\Lambda = \frac{8\pi G\rho_{\text{vacuum}}}{c^2}. \tag{40}$$

By transferring this expression to the formula of the thermal expansion coefficient of the space fabric (38), we obtain the expression for the space thermal expansion coefficient α_S from the density of the vacuum (41):

$$\alpha_S = \frac{\boldsymbol{e}\sqrt{\frac{\boldsymbol{8\pi G\rho_{\text{vacuum}}}}{\boldsymbol{c^2}}}}{\boldsymbol{\Delta T}} = \frac{\boldsymbol{e}\sqrt{\Lambda}}{\boldsymbol{\Delta T}}. \tag{41}$$

If we consider a space plate thickness e of Planck dimension (35) like Tenev and Horstemeyer[12] and the associated vacuum energy, we obtain (42) a first approximation of this thermal expansion coefficient α_S.

$$\alpha_S = \frac{\sqrt{\frac{hG}{2\pi c^3}}\sqrt{\frac{8\pi G\rho_{\text{vacuum,QFT}}}{c^2}}}{\Delta T}. \tag{42}$$

So, all calculations performed give (43):

$$\alpha_S = \frac{\boldsymbol{2G}\sqrt{\frac{\boldsymbol{h\rho_{\text{vacuum,QFT}}}}{\boldsymbol{c^5}}}}{\boldsymbol{\Delta T}}, \tag{43}$$

where $\hbar$ is the reduced Planck's constant $(h/2\pi)$, ρ is the quantum vacuum density according to quantum field theory, G is the gravitational constant, c is the speed of light and ΔT is the thermal gradient applied to these sheets of space.

Consider the expression of the thermal expansion coefficient α_S:

We check that the dimensional equation (44) is satisfied:

$$\alpha_S = \frac{\frac{m^3}{kgs^2}\sqrt{\frac{\frac{kgm^2}{s}\times\frac{kg}{m^3}}{\frac{m^5}{s^5}}}}{K} = K^{-1}. \tag{44}$$

Numerical application using the constants of physics and using the same assumptions as Millette,[13] and Tenev and Horstemeyer[12] gives:

$$G = 6.6743015 \times 10^{-11}\,\text{m}^3/\text{kg}\cdot\text{s}^2,$$

$$h = 6.62607004 \times 10^{-34}\,\text{m}^2\cdot\text{kg/s},$$

$$\rho_{\text{vacuum,QFT}} = 1.11 \times 10^{96}\,\text{kg/m}^3,$$

$$\Delta T = 2000000\,K,$$

$$c = 299,792,458\,\text{m/s}.$$

We obtain for the thermal expansion coefficient of the space fabric:

$$\alpha_{S,QFT} = 1.16317 \times 10^{-6}\,\text{K}^{-1}.$$

For memory, the expansion coefficient of steel is worth $12.0 \times 10^{-6}\,\text{K}^{-1}$.

We therefore obtain a result that seems realistic since the space is rather rigid if we refer to the value of κ which represents the flexibility (1/rigidity) of the space and which is equal to $2.0766\times10^{-43}\,\text{N}^{-1}$.

We find this result directly from Eq. (41) with the following data by considering Λ from quantum field theory:

$$\Lambda_{\text{QFT}} = 2.0717 \times 10^{70}\,\text{m}^{-2},$$
$$e = l_p = 1.61626 \times 10^{-35}\,\text{m},$$
$$\Delta T = 2000000\,\text{K}.$$

6. Discussion of the Representativeness of the Mechanical Model of Thermal Curvature of Space

It is well known that one of the greatest challenges of physics today is this problem of the cosmological constant Λ which, depending on the hypothesis adopted to establish it (vacuum energy resulting from quantum fluctuations in the ground state provided by quantum field theory or cosmological observations via the Planck satellite in particular) leads to a ratio of 10^{120} between the two values of Λ! We do not claim in this paper to solve this problem, simply from a scientific point of view it is important to explore what these two values of the cosmological constant imply on the coefficient of potential thermal expansion of the fabric of space.

In this case, the application of formula (41) with the following numerical values:

$$l_p = e = 1.61626 \times 10^{-35}\,\text{m},$$
$$\Lambda = 1.088 \times 10^{-52}\,\text{m}^{-2},$$
$$\Delta T = 2000000\,\text{K}.$$

It leads to an extremely very small coefficient of thermal expansion...

$$\alpha_{S,\text{astrophysics}} = 8.42936 \times 10^{-68}\,\text{K}^{-1}.$$

7. Discussion of the additional Verifications Necessary to Validate or not this Model

To really validate our model, it would of course be necessary to solve this problem of the possible variation of values of the cosmological constant Λ. A model of the universe reproducing the construction of the cosmic web from general relativity and observations exists, however these models do not integrate any mechanical behavior such as the analogy of space as a deformable elastic medium.

It would undoubtedly be necessary to model the space, containing and contained in a sheet structure, to set up the cosmic web and impose the thermal gradient between the hot and cold zones to check whether or not a curvature of thermal origin appears on average. On the entire universe, find out what its intensity is and therefore take advantage of it to evaluate the true value of the cosmological constant and therefore the true value of the thermal expansion coefficient of the space fabric.

8. Consequence of a Thermal Expansion Coefficient on Time for Special Relativity, General Relativity and Quantum Gravity

8.1. *Necessary transition from space to spacetime in the consideration of temperature*

We have seen in the previous chapters that the analogy with an elastic continuous medium proposes a kind of curvature of zero mass of space related to a temperature effect. But since general relativity is aimed at the curvature of spacetime, and not only space, it is necessary to specify how this temperature effect could act on time.

8.2. *Effect of temperature on time*

8.2.1. *Generalities on clocks and their physical sensitivities to temperature*

It is well known that temperature variations can affect clocks in different ways, especially in mechanical and electronic clocks.

In mechanical clocks, metal parts can expand or contract depending on temperature. This can result in minimal variations in component dimensions, which in turn can affect the accuracy of movement. The materials used in the mechanisms may react differently to temperature changes, which can cause shifts in the clock rhythm. So we have a spatial effect.

In electronic clocks, temperature can influence the frequency of quartz oscillators used to measure time. Quartz crystals have an electromechanical resonance that is sensitive to temperature. Temperature variations can slightly alter the frequency at which the quartz oscillates, which can lead to deviations in the time count. If the equivalent of quartz is very small, see the case of atomic clocks where it is the frequency of an atom that intervenes, we have a quantum effect.

When it comes to electronic processors, temperature variations can also impact their performance. When a processor heats up, its components can expand slightly, which could potentially affect the speed of electrical signals through the circuit. However, modern processor designs usually incorporate temperature control mechanisms to minimize this effect.

So, by thinking about it, the temperature actually influences time by playing on dimensional variations of space or on the vibrations of quartz, so more deeply at the quantum level via $E = h\nu$.

So, to measure time, we need clocks which, whatever the physics and the technology used, will be systematically sensitive to the temperature of the environment in which they are immersed. This is the conclusion of this paragraph.

The following question then arises: while it is clear that temperature affects time measuring devices, does it also fundamentally affect time, which is an abstract entity in physics independent from any measuring instruments?

We will see in the following section by reviewing the state-of-the-art on this question, how different authors have approached this question by focusing on time as a change in entropy, that is to say at a increased disorder, which in turn is linked to a temperature effect.

8.2.2. *State-of-the-art on the link between time, a clock, the entropy variation and the temperature in a black body*

In Refs. 28 and 29, the authors study what is a clock? what kind of measurements can be done? how temperature can affect the time fundamentaly and they explain how time passes according to the increase in entropy and therefore temperature.

In these papers, it is clear that the time itself cannot be defined without a physical process to measure it. I quote[28]:

"Einstein pointed out that the definition of time must be based on the clock measure, but it has been pointed out the need of physical meaning for time coordinates. In the analysis of clocks, a timeclock relation has been introduced; it states that there is a conceptually necessary relation between time and a physical process which functions as the core of a clock. This time-clock relation implies that a physical process must exist as the basis of a clock. Time and the physical process cannot be defined independently. The time-clock relation involves also a reference to a physical process in conformity with physical laws. Consequently, a well-defined use of time requires that time has aphysical basis."

So, from this reflection, time is inseparable from a physical process and as in Sec. 8.2.1 we recalled that all physical processes are subject to temperature effects, so time should also be affected by temperature in addition to what Einstein showed in special relativity, namely that time depends on the speed of the observer.

Therefore, the authors in Ref. 28 and formula (1) in Ref. 29 introduce the definition of the time interval in relation to the local entropy S and the local entropy rate $\dot{S}$, as follows:

$$t = \frac{S}{\dot{S}}. \tag{45}$$

From the entropy variation the authors obtain in Ref. 28 formula (12) and Ref. 29 a variation τ function of the frequency ν of the physical object used to construct a physical clock as required by Einstein. However this time this clock depends on the temperature T, the Planck constant h and the Botzman constant k_B. They obtain then the fundamental expression (46):

$$\tau_{(T)} = \frac{1}{\nu} = \frac{h}{k_B} \times \frac{1}{T} = \frac{4.799243 \times 10^{-11}[\mathrm{K} \cdot \mathrm{s}]}{T[K]}. \tag{46}$$

An finally according to Ref. 28, formula (13) and Ref. 29 in the field of a black body, there is a relation (47) giving the variation of the flow of time t according to the temperature T, the periodic time of oscillation τ and n a natural number.

$$t_{(T)} = n\tau = n\frac{h}{k_B} \times \frac{1}{T} = n \cdot \frac{4.799243 \times 10^{-11}[K.s]}{T} \approx n\frac{4.80 \times 10^{-11}[Ks]}{T}. \quad (47)$$

Remark 3. In Ref. 28, Table 1, we can see that the effect of the temperature on time is very small.

For $T = 10^6$ K, n =1 the time variation interval linked to this temperature is 4.8×10^{-17} s.

Remark 4. If we take the inverse of the expression (47) and multiply by the time t each side of the equation we obtain (48) that allows to define a time-dependant thermal expansion coefficient:

$$\frac{t}{n\tau} = \frac{k_B t}{nh} \times T = \alpha_t T. \quad (48)$$

Thus, the term $\frac{k_B t}{nh}$ has the dimension of an expansion coefficient (K^{-1}) depending on time which will note α_t:

So, the time should be influenced at a quantum level function of the temperature T, the Planck constant h and the Boltzmann constant k_B and thus by the entropy of the medium S. The time is quantified ($n \times h$).

Thus, associated with $T = 1 \times 10^6$ K (see order of magnitude of the temperature of the cosmic web[27]), $n = 1$ and considering a time interval of 4.8×10^{-17} s (following Table 1 of Ref. 28), we obtain with (48) an evaluation of the amplitude of this time-dependent thermal expansion coefficient:

$$\alpha_t = 1.0{\times}10^{-6}\mathrm{K}^{-1}.$$

So, an order of magnitude for α_t similar to that which we obtained for α_s in the first part of our article based on the comological constant Λ as a value of the thermal curvature of the fabric of space.

Remark 5. In the Cosmic fabric model,[12] Tenev and Horstemeyer related time lapse to the speed of signal propagation within the fabric.

Thus the variation of the time lapse (49) is written as follows:

$$\frac{d\tau}{dt} = \frac{1}{(1 + \varepsilon^{3D})}. \quad (49)$$

With the formula (50), we have

$$\varepsilon^{3D}_{,kk} = c^2\kappa\rho, \quad (50)$$

where $\varepsilon^{3D}_{,kk} \equiv \nabla^2\varepsilon^{3D}$ is the Laplacian of the volumetric strain, c is the speed of light, κ is the Einstein constant, and ρ is the density of matter-energy.

In this expression, ε^{3D} is a scalar field that represents the fractional increase of the fabric's mid-hypersurface volume (51):

$$\varepsilon^{3D} \equiv \varepsilon_i^i. \tag{51}$$

The habitual strain tensor is as follows:

$$\varepsilon_{ij} = \frac{1}{2}(g_{ij} - \delta_{ij}). \tag{52}$$

Thus the time depends on a variation of volume due to the stress energy tensor that generate strain on the elastic medium (53):

$$\frac{dV}{d\bar{V}} = (1 + \varepsilon^{3D}). \tag{53}$$

But to come back at the complete analogy with the elastic medium, this varaition of volume can also come from temperature (54):

$$\frac{dV}{d\bar{V}} = \alpha_S \Delta T. \tag{54}$$

Finally in Eq. (48) we can transpose this to the time as a ratio of length of the space fabric (we see thus the importance of the speed of light as a intrisic characteristic of the space fabric) see (55):

$$\frac{ct}{cn\tau} = \frac{k_B t}{nh} \times \Delta T. \tag{55}$$

8.2.3. *State-of-the-art about cosmological time, coordinate time linked to entropy and the dynamics of the expansion of the universe*

In Ref. 30, "Cosmological Time, Entropy and Infinity" the authors go in the same direction as Refs. 28 and 29 by linking time to entropy but go further by associating this variation of entropy with the dynamic temporal evolution of the universe in its entirety from the big bang until now.

I quote "Time is a parameter playing a central role in our most fundamental modelling of natural laws. Relativity theory shows that the comparison of times measured by different clocks depends on their relative motion and on the strength of the gravitational field in which they are embedded. In standard cosmology, the time parameter is the one measured by fundamental clocks (i.e. clocks at rest with respect to the expanding space). This proper time is assumed to flow at a constant rate throughout the whole history of the universe. We make the alternative hypothesis that the rate at which the cosmological time flows depends on the dynamical state of the universe. In thermodynamics, the arrow of time is strongly related to the second law, which states that the entropy of an isolated system will always increase with time or, at best, stay constant. Hence, we assume that the time measured by fundamental clocks is proportional to the entropy of the region of the universe that is causally connected to them. Under that simple assumption, we find it possible to build toy cosmological models that present an acceleration of their expansion without any need for dark energy while being spatially closed and finite, avoiding the need to deal with infinite values."

In this paper, the following hypothesis is made, I quote again "the cosmological time t measured by such observers is proportional to the entropy of the region of the universe that is causally connected to them."

The authors propose so the following expressions of the time function of the entropy at the univers level (see the 4 points below described in Ref. 30):

- First the link between the universe entropy and the space temperature T:

Based on the CMB photon gas which the authors[31] assume is very close to thermodynamic equilibrium, the entropy S is written as follows (see (56)):

$$S = \frac{4\pi^2 k_B^4}{45c^3\hbar^3} V T^3, \tag{56}$$

where V is the volume considered (e.g. horizon V_{horiz}), $\hbar$ is the reduced Planck's constant, c is the speed of light, k_B is the Boltzmann's constant and T is the temperature associed.

- Second, the link between the temporal variation of horizon entropy S_{horiz} (57) and universe dynamic expansion:

$$\frac{dS_{\text{horiz}}}{dt} = \frac{64\pi^3 k_B^4}{45\hbar^3} T_0^3 \frac{1}{H_{0,t}^2 \Omega_{0,t}^2} (\Omega_{0,t} + R(1 - \Omega_{0,t})), \tag{57}$$

where T_0 is the present CMB temperature ($T \times R = T_0 \times R_0$), R is the classical scale factor (sometime noted a also), k is the curvature, the Robertson walker metric, t is the classical time variable, $H_{0,t}$ (58) is the Hubble constant, $\Omega_{0,t}$ (59) is the matter density paremeter.

$$H_{0,t} = \frac{1}{R_0} \left.\frac{dR}{dt}\right|_{t=t_0}, \tag{58}$$

$$\Omega_{0,t} = \frac{8\pi G \rho_0}{3H_{0,t}^2}. \tag{59}$$

- Thirdly the link between the entropy variation and the cosmic time dτ (see (60)):

$$\frac{d\tau}{dt} = \frac{dS_{\text{horiz}}/dt}{dS_{\text{horiz}}/dt|_{BB}} = 1 + R\left(\frac{1}{\Omega_{0,t}} - 1\right). \tag{60}$$

With $dS_{\text{horiz}}/dt|_{BB}$ for the temporal variation of the entropy in the causally connected volume at the BigBang (the coordinate time t (s) that flow at constant rate is equal to τ the cosmological time at the Big Bang of which the unit is the varying length).

- Fourth, the curvature of space which is linked to the interaction of variation of entropy, itself linked to time and temperature:

If $\frac{d\tau}{dt} = 1$ for a flat universe ($\Omega_{0,t} = 1$) the cosmological time flow at a constant rate. It is not the case in the curved space we have the following:

$$\Omega_{0,\tau} = \frac{1}{\Omega_0, t}. \tag{61}$$

The curvature k is for $R_0 = 1$ given in the following equation:

$$k = \frac{H_{0,t}^2(\Omega_{0,t} - 1)}{c^2}. \tag{62}$$

In conclusion of this state-of-the-art, we find with Refs. 30 and 31, equation by equation, a direct link between temperature, the evolution of entropy S, the evolution of time t and the associated curvatures k of the universe. We will focus in the following paragraphs on this direct link between temperature and time via an equivalent thermal expansion coefficient of time to be inline with our mechanical analogy of the spacetime. Of course, based on Refs. 28–31 this link between time and temperature passes in general relativity by the notion of spacetime associated this time with curvature of time but also of associated space therefore to a temperature effect. This therefore goes somewhere in the direction of a confirmation of our initial idea of a cosmological constant linked to the thermal curvature of space.

8.3. *Generalization of the special relativity interval taking into account temperature*

Basing of the precendent section and formula (48) or (55), this invites us to go far and to postulate a rewriting of the interval ds^2 of special relativity to introduce a mechanical thermal effect which gives Eq. (63) with α_t a potentiel dilatation coefficient of the time and α_s a potential expansion coefficient of space supposed homogenous:

$$d_s^2 = c^2(dt \pm \alpha_t T dt)^2 - (dx \pm \alpha_s T dx)^2 - (dy \pm \alpha_s T dy)^2 - (dz \pm \alpha_s T dz)^2. \tag{63}$$

We postulate that α_s is connected with the cosmological constant Λ of space by the expression $\alpha_s = \frac{\boldsymbol{l_p}\sqrt{\Lambda}}{\boldsymbol{\Delta T}}$.

α_t is connected with the quantum mechanic and thermodynamic and time following Refs. 28 and 29 by the expression $\alpha_t \approx \frac{k_B t}{nh}$.

Indeed, given the special character and the unknown nature of time, there is no reason in first approach for a coefficient of time expansion to be identical to that of space.

Thus, the expression (63) becomes expression (64) if we separate the spatial part of the time part as

$$d_s^2 = (1 \pm \alpha_t T)^2 c^2 dt^2 - (1 \pm \alpha_s T)^2 [dx^2 + dy^2 + dz^2]. \tag{64}$$

Some important points arises from this expression.

(a) The two quantities in Eq. (64) above have an opposite sign so a potential contradictory effect.

(b) According to the Big Bang theory, space and time are created at the same time. According to general relativity we arrive at a sungularity at the beginning of the universe where the energy (and therefore the associated mass) is infinitely large and therefore the curvature is infinitely large in an infinitely small volume which requires the introduction of quantum mechanics and therefore the creation of a theory of quantum gravity. But we also often forget, at this moment in the universe the temperature is also infinite and therefore constitutes from the beginning a variable of the problem potentially influencing spacetime, therefore space and time in our fabric model cosmic are influenced by temperature. So, it's logical for us to introduce it in the interval formula.

(c) Of course, if there is no thermal effect, the interval reverts to the classical interval in special relativity.

8.4. *Can curvature, time and temperature be related in general relativity?*

The theory of general relativity in its current version does not directly take into account a slowing effect of time due to heat or temperature, nor does it specifically propose a "thermal curvature of time". This is what we deduce from our previous reflections in Sec. 8.3 based on Refs. 26,29–31. Indeed, from (64) we have rather a simple dilatation effect and not a curvature effect in potentially all the directions.

However, under certain circumstances, heat can indirectly have an effect on time by altering the curvature of spacetime. For example, in very massive and hot astrophysical objects, such as neutron stars or black holes, the effects of heat and pressure can influence the curvature of spacetime and therefore the trajectories of moving objects but also the time (time and space become as reversed) (see Ref. 32). The Hawking's temperature of black hole, that is the first quantum gravity equation depending on c, h, k_B and G and M the black hole Mass, goes in this direction.

There are also related concepts in theoretical physics, such as string theory and quantum gravity, that attempt to unify general relativity with quantum mechanics. In these theories, it is possible that temperature-related phenomena could have implications for the nature of spacetime, but this remains an ongoing area of research and exact specifics are not yet well established.

Thus, based on (64) for the new definition of the interval, taking into account[28] "Time & clocks: A thermodynamic approach" and[30] "Time and Thermodynamics Extended Discussion on Time & clocks : A thermodynamic approach" see https://www.sciencedirect.com/science/article/pii/S2211379720311013 for the temporal variation as a function of temperature and the associated temporal expansion coefficient α_t, from Eq. (37) for the relationship between the cosmological constant Λ and the space expansion coefficient α_s, from Ref. 12 for the thin sheet structure of thickness l_p, from Refs. 30 and 31 for the relationship between the temperature, the temporal variation, the entropic variation linked to the expansion of the universe and the associated curvature, we therefore have the following modification of Einstein's equation (17) expressed in terms of mechanical curvature:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \left(\frac{\alpha_f T}{l_p}\right)^2 g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}. \tag{65}$$

Thus the cosmological constant Λ introduced by Einstein, considered as potentially the source of dark energy, becomes in our paper an additional thermal curvature $(\alpha_f T/l_p)^2$, connected for space and time to two complementary mechanical parameters linked to the spacetime fabric, namely its coefficient of thermal expansion of space $(f = s) \to \alpha_s$ and its coefficient of thermal expansion of time $(f = t) \to \alpha_t$ as well as the sandwich texture of it (multilayer of thickness l_p) all depending on the temperature $T(\to \Delta T)$ acting in connection with the variation of entropy during the temporal evolution of the universe.[30]

Remark 6. We are not certain today that the dark energy translated into our elastic model in the form of thermal curvature must be strictly constant or not.

Thus, in Einstein's equation the cosmological constant is constant, but in our approach it can potentially vary depending on the temperature or the temperature gradient.

Remark 7. The thermal expansion coefficient of time depends on t if we start from the approaches of Refs. 28 and 29.

8.5. *Synthesis on spatio-temporal approach to consider the temperature on the curvature of spacetime fabric*

First of all we recall our hypothesis resulting from the analysis of gravitational waves. The deformations of the space seem to be decoupled between the transverse deformations in the (xy) plans and the distance z of the waves propagation. Thus (ct and z) act at a same level (in the same direction) in a TT gauge in the expression of the deformations of gravitational waves. What leads Tenev and Horstemeyer assumed the space consisting of thin sheets thick of Planck. I quote.[12]

"The thickness must be very small so that the fabric can behave as an essentially 3D object at ordinary lengthscales and be an appropriate analogy of 3D physical space. The thickness itself defines a microscopic lengthscale at which the behavior of the physical world would have to differ significantly from our ordinary experience. A value equal or comparable to Planck's length l_p meets this criteria. However, the exact value of the thickness is not essential to the model as long as it is small but notvanishingly so".

We place ourselves at a distance from the cosmic web so as to consider on a given sheet of space a constant average temperature applied to said sheet (it is actually necessary to dissociate the quantum, local and global thermal effect associated with the scale factor, great thermal curvature of space) which allows us to guard against the inevitable thermal variations at smaller scales associated with the cosmic web.

These two postulates in place we have:

A first effect associated with our space thermal expansion coefficient which implies a thermal expansion of the space in the plane xy (see Fig. 4). The curvature of the space k being almost zero, it appears to us as a leaf that expands in part because of the average temperature of the cosmic web.

A second effect associated with our time thermal expansion coefficient involves this time a following thermal curvature linked this time between the temperature difference between two layers of Planck thickness space folowing the z direction.

Figure 4 gives an idea of the concepts developed in this paper.

So in our model illustrated in Fig. 4 in a way near to Ref. 30:

(1) Spacetime expands as time progresses every moment since the Big Bang.
(2) The engine of time, the fact that each present moment is renewed indefinitely is correlated with entropy variation which can only increase over time: $t = \frac{S}{\dot{S}}$.
(3) Time function of temperature because linked to entropy variation itself is quantified by: $t_{(T)} = n\tau = n\frac{h}{k_B} \times \frac{1}{T}$.
(4) The entropy S is linked to a temperature effect by the Bolzmann constant and by Ω different microstates by $(S = k_B \log \Omega)$.

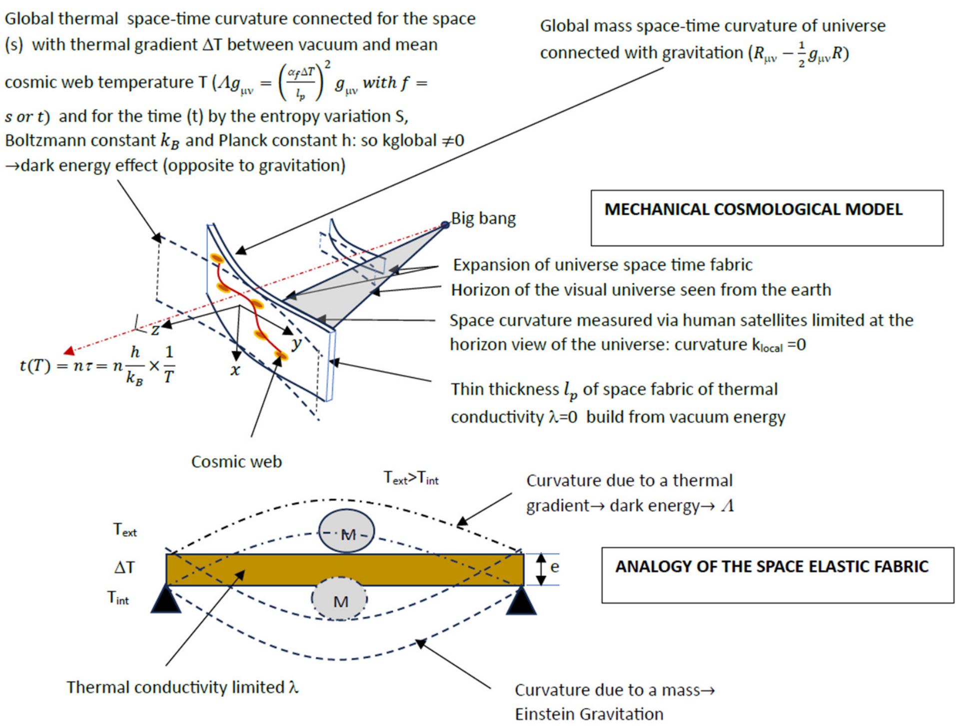


Fig. 4. Overview of the principles and concepts developped in this paper (thermal curvature of the spacetime connected to Λ, time, entropy variation and temperature).

(5) Space expands and via the interval ds^2 curves not only in the presence of mass (by general relativity) but also as a function of temperature or temperature delta (contribution of the analogy of general relativity with the theory of elasticity) which we propose to correlate with the cosmological constant Λ as thermal curvature of quantum space sheets of thickness l_p.
(6) By general relativity not only is space curved, but so is spacetime, so seeing what precedes time ($\times c$) expands and curves by the effect of variation in entropy itself linked to an evolution of temperature.[28–30]
(7) The curvature of spacetime due to mass is opposed to the curvature of spacetime due to temperature namely the curvature contribution that is identified by the cosmological constant Λ.

9. Presentation of the Possible Consequences of this Model Concerning the Interpretation of Dark Energy

The publications of Refs. 20 and 21 in particular and many others seem to indicate that the dark energy source of expansion of the universe is connected to the cosmological constant in particular when this one is positioned on the right of Eqs. (15) and (16). We are interested in this paper in a cosmological constant placed on the left of Eq. (17) that is to say in an additional curvature present in all spacetime. This more mechanistic approach makes it possible to no longer having recourse to an energy of unknown origin and is closer to a functioning of the universe as a structure "charged" within it by the cosmic web and undergoing the thermal gradient of this one.

But this approach raises other questions: which thermal gradient considered? does space really have a sheets structure deforming relative to each other as the deformations associated with the polarizations $\mathbf{A}^+$ and $\mathbf{A}^\times$ that seem to suggest the gravitational waves? What is the real thickness of these sheets?how to integrate the anisotropy of space suggested by a Poisson's ratio of 1?The quantum field theory approach seems more coherent because it leads to a coeffcient of thermal expansion compatible with materials on Earth, nevertheless the difference with the astrophysical value of the cosmological constant raises a real fundamental question.Moreover the values of the Young's moduli that we recalled at the beginning of the paper of on the one hand and the vacuum energy densities on the other hand being so outside the usual values of elastic materials on Earth that it is advisable to be particularly careful with respect to this "quasi-normal" value of the coefficient of expansion thermal that we propose in this study.

Another consequence of our study is that given that general relativity applies to space time, introducing a link between the cosmological constant and a temperature effect implies that temperature influences also time.

We therefore have by interpreting the expression (64), on the right a spatial part which only varies as a function of the temperature T and the coefficient of spatial thermal expansion $\alpha_{\boldsymbol{S}}$ which nowadays is that of the vacuum (cold) in interaction with the temperature of the cosmic web (warm) which creates thermal gradient and curvature (so rather stable to day in mean), while the left part linked to time and to the coefficient of time thermal expansion α_t has only increased since the Big Bang. Thus the curvature term in (65) $-(\alpha_f T/l_p)^2$ represents the thermal curvature which is opposed at the gravitational curvature due to mass.

The part of the thermal curvature associated with time could have an apparent effect of accelerating the expansion of space. In (65) the global thermal space time curvature is so in our model in the opposite direction of mass curvature to play the game of the dark energy.

The temperature of the cosmic web also reflected an expansion of space in its **xy** plane (see Fig. 4) with (k apparent $= 0$).

So, our approach is similar to Ref. 30 but using a mechanical formalism based on thermal expansion coefficient of the spacetime fabric.

Following our reflection, we could have a general curvature of spacetime linked to temperature which is broken down into two parts. A first spatial part due to a thermal gradient applied to the thin sheets of space fabric which could be linked to the cosmological constant Λ (expansion of the universe and positive curvature k). A second resulting from a time/entropy/temperature effect. Both creating a total negative thermal curvature k opposite to the curvature linked to mass and therefore to gravitation.

10. How to Verify this Effect of the Temperature on the Time

Measuring the variation of time as a function of temperature on atomic clocks is a good way to test the ideas of this paper.

The idea is to put two strictly identical atomic clocks under the same height conditions in the earth's gravity field and to vary the temperature for one of them in order to measure the possible time lag with respect to the remaining one at the initial temperature.

11. Conclusions

We explore in this study the analogy of space as an elastic medium by focusing on the mechanical parameters associated with any elastic four-dimensional fabric, its coefficients of thermal expansion α_S for space and α_t for the time. The state-of-the-art is rather poor or even nonexistent on their definitions and their intensities within the framework of an elastic model of space. Some research exists for the time dilatation function of temperature. By taking inspiration from models of space fabric in the form of thin sheets, placing ourselves within the framework of quantum field theory both on the value of the cosmological constant Λ and on the thickness of these sheets of the order of the Planck's length, considering the recent measurements making it possible to establish the orders of magnitude of the space thermal gradient between the hot and cold zones of the universe and finally considering that these sheets bend under this thermal gradient by analogy with mechanics plates, we propose a formulation and a value of the expansion coefficients of the space fabric.

Since general relativity is built on spacetime, this thermal curvature approach implies that not only space but also time could vary with temperature. We therefore propose an expression for the temporal expansion coefficient $\alpha_{\mathbf{t}}$ based on the ratio between Botzmann's constant and Planck's constant in connection with a variation in entropy of the universe and therefore its temperature. Since general relativity is itself based on special relativity, this results in the need to take into account variations in distance and time as a function of temperature. We therefore propose a way to modify the definition of the spacetime invariant $\mathbf{ds^2}$ by introducing both a spacetime coefficient and a temporal expansion coefficient.

When we return to the simplistic analogy cited at the beginning of the stretched fabric supporting a heavy ball, it indeed appears logical that the fabric supporting this ball elongates more or less depending on its temperature in addition to spatial flexibility and mass/energy which distorts it.

Thus gravitation influences space and time but would also be linked to temperature. Temperature would influence spacetime curvature at the quantum scale (possible nature of the time expansion coefficient) and at large scale (possible cosmological constant effect in link with the cosmic web). The answer in our elastic medium analogy would be of two types. A first on a lengthening and shortening of spacetime, a second on a thermal curvature of spacetime connected with the dark energy.

This study remains a initial pproach which somehow clears this path given the great variability of the intensity of the cosmological constant depending on whether one considers its value from quantum field theory or its value from cosmological measurements. Other modelings are certainly necessary to fine-tune the value of these thermal expansion coefficients of the spacetime fabric and this study remains only a first approach.

Einstein said that God is not playing dice when he talks about quantum mechanics, but perhaps God is playing structural engineer with a cosmic structure driven by a thermal gradient manifesting as dark energy acting throughout the universe—at least that's the question this paper poses.

Acknowledgments

Finally, we would like to thank late R Gregoire, a great mechanician, who through his teaching based on the research of "how it works" guided me in my reflection. We warmly thank the reviewer for all their suggestions and corrections, in particular on the need to distribute this thermal effect both spatially and temporally.

ORCID

Izabel David https://orcid.org/0000-0003-2428-3749

References

1. F. W. Dyson, A. S. Eddington and C. Davidson, *Philos. Trans. R. Soc. Lond.* **220** (1929) 291.
2. C. W. F. Everitt, *Phys. Rev. Lett.* **106** (2011) 221101.
3. Collective and al, *Phys. Rev. Lett.* **116** (2016) 061102.
4. Collective and al, *Phys. Rev. Lett.* **119** (2017) 161101.
5. A. D. Sakharov, *Sov. Phys. Dokl.* **12** (1968) 1040.
6. J. L. Synge, *Math. Zeitschr.* **72** (1959) 82, https://link.springer.com/article/10.1007/BF01162939.
7. C. B. Rayner and R. Proc, *Soc. A. Math. Phys. Eng.* **272** (1963) 44.
8. R. Grot and A. Eringen, *Int. J. Eng. Sci.* **2** (1966) 1.
9. V. V. Vasiliev and L. V. Fedorov, *Mech. Sol.* **53** (2018) 256.
10. V. V. Vasilev and L.V. Fedorov, *Mech. Sol.* **56** (2021) 404.
11. J. D. Brown, *Class. Quantum Grav.* **38** (2021) 085017.
12. T. G. Tenev and M. F. Horstemeyer, *Int. J. Mod. Phys. D* **27** (2018) 1850083.
13. P. A. Millette, *Elastodynamic of the Space Time Continuum* (American, Research, Press, New York, 2019).
14. M. R. Beau, *On the Nature of Space-Time, Cosmological Inflation, and Expansion of the Universe* (Massachusetts, Boston, 2018).
15. K. McDonald, *What is the Stiffness of Spacetime* (Princeton University, 2018).
16. D. Izabel, *Pram. J. Phys.* **94** (2020) 119.
17. A. C. Melissinos, *Upper Limit on the Stiffness of Space-time* (Rochester, 2018), arXiv:1806.01133 [physics.gen-ph].
18. S. R. Hwang, *Estimation of Spacetime Stiffness Based on LIGO Observations* (Keelung, Taiwan, Sep. 27, 2020).
19. S. Timoshenko and J. N. Goodier, *Theory of Elasticity* (McGraw Hill, New York, 1951).
20. P. J. E. Peebles and B. Ratra, *Rev. Mod. Phys.* **75** (2003) 559.
21. E. Santos, *Astron. Space Sci.* **332** (2011) 423–435.
22. C. F. Gauss, *Disquisitiones Generales Circa Superficies Curvas* (Societate, Gottingae, 1828).
23. H. A. Perko, *J. Phy. Conf. Ser.* **1956** (2021) 1.
24. Collectif, *Astron. Astrophys. Rev.* **571** (2014) 1.
25. D. Izabel, *What is space time made of?* (Paris, 2021), https://laboutique.edpsciences.fr/produit/1190/9782759825745/what-is-space-time-made-of.
26. T. Tenev, *An Elastic Constitutive Model of Spacetime and its Applications* (Mississippi State University, Mississippi, 2018).
27. Y. K. Chiang, R. Makiya, B. Ménard and E. Komatsu, *Astron. J.* **902** (2020) 56.
28. U. Lucia and G. Grisolia, *Res. Phys.* **16** (2020) 102977.
29. A. Chatterjee and G. Iannacchione, arXiv:2007.09398v1.
30. C. Hauret, P. Magain and J. Biernaux, *MDPI Entropy* **19** (2017) 357.
31. C. A. Eganc and H. Lineweaver, *Astron. J.* **710** (2010) 1825.
32. H. Hadi, K. Atazadeh and F. Darabi, *Phys. Lett. B.* **834** (2022) 137471.

Pramana – J. Phys. (2020) 94:119
https://doi.org/10.1007/s12043-020-01954-5

Mechanical conversion of the gravitational Einstein's constant κ

IZABEL DAVID

Institut National des Sciences Appliquées de Rennes 20, Avenue des Buttes de Coësmes, CS 70839 35708, Rennes Cedex 7, France
E-mail: d.izabel@aliceadsl.fr

MS received 3 May 2019; revised 20 December 2019; accepted 18 February 2020

Abstract. This study attempts to answer the question of what space is made of and explores in this objective the analogy between the Einstein's gravitational geometrical theory in one- and two-dimensional linear deformations and a possible space material based on strain measures done on the Ligo or Virgo interferometers. It draws an analogy between the Einstein's gravitational constant κ and the Young's modulus and Poisson's ratio of an elastic material that can constitute the space fabric, in the context of propagation of weak gravitational waves. In this paper, the space is proposed to have an elastic microstructure of 1.566×10^{-35} m grain size as proposed in string theory, with an associated characteristic frequency f. The gravitational constant G is the macroscopic manifestation of the said frequency via the formula $G = \pi f^2/\rho$, where ρ is the density of the space material.

Keywords. Space–time fabric; general relativity; quantum mechanics; Young's modulus; strength of the materials; gravitational waves; gravity Probe B; Hubble's law; space–time curvature; Einstein's constant; dark matter; string theory; graviton.

PACS Nos 04.50.Kd; 46.90.+s

1. Introduction

Quantum mechanics and general relativity are the twin pillars of modern physics, but while they have coexisted they have remained broadly irreconcilable.

In order to solve this dilemma, we must go back to the foundations of these two theories to see if something must not be changed at a fundamental level so as to bring them closer.

To date, the general relativity [1–3] clearly dethroned the gravitation according to Newton. It is clear that the concept of Newton's gravitational force is in fact an illusion. General relativity shows indeed that two masses fall against each other not because they attract each other but because they follow the curvature, the deformation of the space–time. But have we really drawn all the consequences of this conceptual error in Newton's gravitation?

In fact, Newton's formula is the basis of the definition of the gravitational constant since the Newtonian gravitational force F is proportional to the gravitational constant G, to the product of the masses M and m, and is inversely proportional to the square of the radius r which separates these two masses (1).

$$F = G \times \frac{M \times m}{r^2}. \tag{1}$$

Considering that this Newton's mathematical expression of the gravitation is only a weak field simplification of general gravitation, that an illusion of force, should we not also consider that the constant G is also an illusion disappearing with the Newton's formula who created it?

Just as it is necessary to abandon Newton's formulation in strong gravitational field, should we not also abandon G as an indivisible universal constant because it is at the basis of this Newtonian formula (1)?

But in this case, is it possible to reconstruct the Einstein's constant κ without going through G but by going through a different theory? Can we separate G from the more fundamental parameters?

To answer these questions it is interesting to compare the strong and weak points of the gravitation according to Newton and Einstein.

Published online: 13 August 2020

The strong points of Newton's gravitation are

(a) It allowed the discovery, mathematically, of the planet Neptune by Urban Le Verrier. Therefore it works obviously well.
(b) It explains all the effects of gravity on Earth and in the solar system except the Mercury perihelion delay (effect of strong gravitational field near the Sun).

The weak points of Newton's gravitation are:

(a) It only works if the objects have masses. It is not, therefore, possible to predict the curvature of a ray of light tangent at the Sun by the effect of gravitation, though predicted by general relativity and verified by Arthur Eddington on May 29, 1919 [4].
(b) The forces are instantaneous (therefore applied faster than the speed of light) and their mode of transmission is as yet unexplained while the special relativity shows that no phenomenon can be faster than the speed of light.
(c) Its results are imprecise for the action of gravity in strong field. Indeed, the delay in the perihelion of the Mercury planet is defined exactly by the general relativity at 43 arcsec [5] while it is much weaker using the Newton's gravitational approach.
(d) The formulation depends on the parameter r, and if the objects of mass M and m rotate with respect to each other at speeds tending towards the speed of light, the effects of the special relativity change the notion of distance for each observer. Which value of r should be used in calculations in this case?
(e) In this formulation, space–time is a rigid non-deformable object, whereas in general relativity it is precisely the deformation of space–time that generates gravitation giving us the illusion that forces act and attract objects between them.

In view of all these points, it is therefore clear that Newton's concept of force is meaningless. It is an illusion. This formulation is only a simplification of a larger theory, the general relativity. This formulation works in low gravity fields but is false in strong fields.

We can also ask ourselve, why the Universe would depends on a constant G with strange dimensions: inverse of a density by a frequency squared (see (2a) and (2b)). Some authors propose, based on this dimensional equation, that G depends effectively on density and frequency [6].

$$G = 1/(\mathrm{kg/m^3}) \times 1/\mathrm{s}^2 \tag{2a}$$

$$G(\rho, f^2) = cte \times \frac{1}{\rho} f^2. \tag{2b}$$

In general relativity [1], G is introduced because it comes from the use of Poisson's equation (3a) to calibrate the constant κ (eqs (4a) and (4b)) (analysis at the component 00, time t of the different tensors of the Einstein's gravitation formula): see (3b).

Indeed, the Laplacian of the gravitational field, $\Delta\phi$, follows the Poisson's equation

$$\Delta\phi = 4\pi G\rho \tag{3a}$$

and the 00 component of the metric $g_{\mu\nu}$ is then

$$g_{00} \approx 1 + \frac{2\phi}{c^2}. \tag{3b}$$

It is also surprising, in hindsight, to see that Einstein precisely calibrates his equations on the time component of his tensors (3b), while precisely the formulae of Newton (1) and Poisson (3a) that follow are independent of time!

κ (4a) connects the Ricci tensor $R_{\mu\nu}$ (issued of the tensor contraction of the curvature tensor, a function of the metric $g_{\mu\nu}$ and of its second partial derivatives), with the stress energy tensor $T_{\mu\nu}$ (external mass/energy applied at the space–time fabric) in the Einstein's gravitational field equation (5a).

$$\kappa = \frac{8\pi G}{c^4} \tag{4a}$$

$$\kappa = \frac{\frac{\mathrm{m}^3}{\mathrm{kg\ s}^2}}{\frac{\mathrm{m}^4}{\mathrm{s}^4}} = \frac{\mathrm{s}^2}{\mathrm{kg\ m}} = \frac{1}{\mathrm{Newton}} = N^{-1}. \tag{4b}$$

The Einstein's gravitational field equation is

$$G^{\mu\nu} = \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right) = -\frac{8\pi G}{c^4} T^{\mu\nu} = -\kappa T^{\mu\nu} \tag{5a}$$

$$\frac{1}{\mathrm{m}^2} = \frac{1}{\mathrm{N}} \times \frac{\mathrm{N\ m}}{\mathrm{m}^3}. \tag{5b}$$

We do not show here the cosmological constant Λ as the possible source of dark energy [7]. In addition R is a tensorial contraction of $R_{\mu\nu}$.

Additionally, as the Newton's expression (1) is false for the concept of force (it is the deformation of space–time which gives the illusion of an attractive force between two objects of mass M and m and so there is no attractive force), and as the constant of gravitation is directly related to this concept of force (see Newton's gravitation formulation), is it not necessary to abandon the proportionality factor G associated with this force in general relativity?

Therefore, the question which needs to be asked is the following: If Newton's gravitation did not exist, could Einstein have been able to calibrate κ without going through the Newton's limit, without using the Poisson's equation and the time component (00) of its tensor, but by using directly the spatial components (1,2,3) of its tensors indicated in bold in (5c)?

$$\begin{bmatrix} G^{00} & G^{01} & G^{02} & G^{03} \\ G^{10} & \mathbf{G^{11}} & \mathbf{G^{12}} & \mathbf{G^{13}} \\ G^{20} & \mathbf{G^{21}} & \mathbf{G^{22}} & \mathbf{G^{23}} \\ G^{30} & \mathbf{G^{31}} & \mathbf{G^{32}} & \mathbf{G^{33}} \end{bmatrix} = \kappa \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & \mathbf{T^{11}} & \mathbf{T^{12}} & \mathbf{T^{13}} \\ T^{20} & \mathbf{T^{21}} & \mathbf{T^{22}} & \mathbf{T^{23}} \\ T^{30} & \mathbf{T^{31}} & \mathbf{T^{32}} & \mathbf{T^{33}} \end{bmatrix}. \tag{5c}$$

Or is it possible to find the Einstein constant κ by going through a different theory using the spatial components of the gravitational field tensors?

To define which theory to use, we must now explore the strong and weak points of general relativity. The strong points of the general relativity are:

(a) It addresses all the gaps of Newton's formulation with an extraordinary precision.
(b) It predicts the curvature of a ray of light passing near the Sun during an eclipse.
(c) It introduces time in tensorial writing in four dimensions. The phenomena are no longer instantaneous and respect the special relativity.
(d) It predicts exactly the delay of the perihelion of the Mercury planet, in a strong gravitational field.
(f) It no longer considers space–time as a rigid physical object but as a deformable object, compatible with observations made more than a century ago.

The weak points of the general relativity are:

(a) The Einstein's field equation is a mathematical description of the space–time deformation due to energy density present in this space. Gravitation is therefore mathematically described by the metric of the space–time $g_{\mu\nu}$. This metric is a mathematical description in four dimensions of the space–time deformation.

But this mathematical (geometrical) description of the space said nothing about the physical nature and the possible mechanical properties of this deformed elastic medium constituting the space fabric [8,9].

If we take the analogy of a simple beam in pure bending in Timoshenko's strength of the material, a simplification of the elasticity theory, it is possible to understand the fundamental concept used in Einstein's general relativity (space–time curvature = κ energy density) on the one hand and to assimilate the Mohr's stresses circle in elasticity (see figure 12) at the spin of 2 of the graviton in quantum field theory on the other hand. Indeed, when the facet carrying the stresses makes a complete rotation of one turn on the elastic model in reality, the facet carrying the stresses on the Mohr's circle makes two turns (see Feynman lectures on gravitation – lecture 3, paragraph 3.4, figures 3.3 and 3.4 [10]).

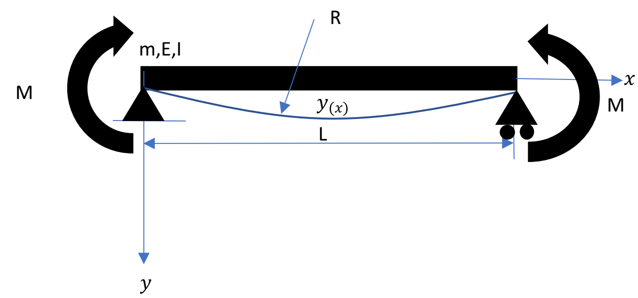


Figure 1. Timoshenko beam, with radius of curvature R, deflection y, loaded by two equal bending moments M.

So, this beam (see figure 1) has a span L (unit m), is made of an elastic material of Young's modulus $Y = E$ (unit MPa $= \mathrm{MN/m^2}$), has a section $S = bh$ (unit $\mathrm{m^2}$), an inertia $I = bh^3/12$ (unit $\mathrm{m^4}$), a mass m (unit kg/m) and a radius of curvature R (unit m).

The equivalent of the curvature equation of this beam analogy, is the equation which defines in elasticity (strength of the material is a simplification of this theory) the deflection of the beam $y_{(x)}$ and the rotation $\theta_{(x)}$ of the beam sections under the influence of two external bending moments M applied at each end which act as the outer mass curving the beam.

The relation to find the equivalent geodesic $y_{(x)}$, depending on the curvature is

$$\frac{\mathrm{d}^2 y_{(x)}}{\mathrm{d}x^2} = \frac{\mathrm{d}\left(\frac{\mathrm{d}y_{(x)}}{\mathrm{d}x}\right)}{\mathrm{d}x} = \frac{\mathrm{d}\theta_{(x)}}{\mathrm{d}x} = \frac{1}{R}. \tag{6}$$

As we are in pure bending, we have ($M_{(x)} = cte$):

$$M_{(x)} = M = -\frac{EI}{R}. \tag{7}$$

The strain energy U (= work done by internal forces) of the beam in pure bending is

$$U = \frac{1}{2}\int_0^L \frac{(M)^2}{EI}\mathrm{d}x = \frac{M^2 L}{2EI}. \tag{8a}$$

In expression (8a), we see the link between the external work, function of M on the right and the strain energy U due to the inner work on the left. By substituting M in

(8a) for its value given in (7), we obtain for the internal forces or internal strain energy of the beam:

$$U = \frac{1}{2}\frac{EIL}{R^2}. \tag{8b}$$

We can reformulate eq. (8b) to extract the curvature term

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{U}{L}\right) = \frac{M^2}{(EI)^2}. \tag{8c}$$

Or to have a similar presentation to Einstein's formalism, we replace $2/EI$ by K and we obtain

$$\frac{1}{R^2} = K\left(\frac{U}{L}\right) = \frac{M^2}{(EI)^2}. \tag{8d}$$

When $K = 2/EI$, the coupling constant (8d) is between the curvature, $(\frac{1}{R^2})$, on the left and the linear strain energy density, $(\frac{U}{L})$, on the right. The constant K is therefore identical to flexibility (8c). In addition, we can write the expression of the external work created by the two moments on the beam. The expression of the external work of moment M is

$$W_{\text{ext(1 moment)}} = \frac{1}{2}M\theta. \tag{9a}$$

The rotation θ on each support of our beam under two constant moments is:

$$\theta = \frac{ML}{2EI}. \tag{9b}$$

By referring (9b) to (9a) we obtain for the moment, a new expression of the external work applied on the beam:

$$W_{\text{ext(1 moment)}} = \frac{M^2L}{4EI}. \tag{9c}$$

We can express M^2 according to the total external work of the beam under two constant moments M at each end:

$$W_{\text{ext(total)}} = \frac{M^2L}{2EI}. \tag{9d}$$

So we have

$$M^2 = \frac{2EI}{L}W_{\text{ext(total)}} \tag{9e}$$

That we can substitute in the expression of the inner work U (8c):

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{U}{L}\right) = \frac{M^2}{(EI)^2} = \frac{2}{EI}\left(\frac{W_{\text{ext(total)}}}{L}\right). \tag{10a}$$

Thus, the equation of curvature of a beam in pure bending (10a) can be considered as a one-dimensional analogy of the equation of the Einstein field in four dimensions (5a) (see table 1):

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{W_{\text{ext(total)}}}{L}\right) = K\left(\frac{W_{\text{ext(total)}}}{L}\right) \tag{10b}$$

$$G^{\mu\nu} = -\frac{8\pi G}{c^4}\left(T^{\mu\nu}\right) = -\kappa\left(T^{\mu\nu}\right). \tag{10c}$$

For memory, the scalar curvature R of a sphere of radius r, is $2/r^2$. And this parallelism also teaches us that it must exist as an internal work of the fabric of space–time taking into account expression (10d); tensor not developed by Einstein. We shall call $M^{\mu\nu}$ this tensor (see (10e) and (10h)):

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{W_{\text{ext(total)}}}{L}\right) = K\left(\frac{W_{\text{ext(total)}}}{L}\right) = \frac{2}{EI}\left(\frac{U}{L}\right) \tag{10d}$$

$$G^{\mu\nu} = -\frac{8\pi G}{c^4}T^{\mu\nu} = -\kappa T^{\mu\nu} = -\kappa M^{\mu\nu}. \tag{10e}$$

Indeed, the masses/energies applied within the structure of the space bend the fabric which constitutes it. As a result, its fibres compress, stretch, shear and bend. These actions generate an internal work, a strain energy.

The equation to the dimensions of the fundamental principle linking curvature and strain energy density is (10f)

$$\frac{1}{\text{m}^2} = \frac{\text{s}^2}{\text{kg m}^3} \times \frac{\left(\frac{\text{kgm}^2}{\text{s}^2}\right)}{\text{m}}. \tag{10f}$$

What we can introduce again

$$\frac{1}{\text{m}^2} = \frac{\text{s}^2}{\text{kg m}} \times \frac{\text{U}}{\text{m}^3} = \frac{1}{\text{m}^2} = \frac{1}{\text{N}} \times \frac{\text{U}}{\text{m}^3}. \tag{10g}$$

But the deflection line $y_{(x)}$ of the beam is not the beam itself. There is a physical object in the form of the beam at the start! So if there is a parallelism between the concepts of strength of the material (and by extension elasticity theory) and general relativity (see table 1), it must exist as an elastic substance in the space, a fabric, a sort of frame in correspondence with the beam.

In addition, the beam has a characteristic frequency f (eigenvalue) and a material of density ρ, and so if space is like a beam in each of its three dimensions, it must also intrinsically have these two fundamental characteristics.

In this parallelism, there are only two major differences:

1. The constant K depends on the rigidity of the medium and is expressed from the bending inertia of the beam I and the Young's modulus E characterising the material of the beam which is not the case for κ (in appearance) which is defined definitively according to G and c (non-variable constant value as a function of the space–time fabric characteristics).
2. The strain energy U depends on the beam itself. In general relativity, the stress energy tensor is

Table 1. Parallelism between strength of the material in bending (1D) and general relativity (4D).

Parameters	General relativity (four dimensions)	Unit (s)	Strength of the material (one dimension)	Unit (s)
Principle	$G^{\mu\nu} = \kappa T^{\mu\nu}$	$\frac{1}{\mathrm{m}^2}$	$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{U}{L}\right) = \frac{2}{EI}\left(\frac{W_{\mathrm{ext(total)}}}{L}\right)$	$\frac{1}{\mathrm{m}^2}$
Curvature	$G^{\mu\nu}$	$\frac{1}{\mathrm{m}^2}$	$\frac{1}{R^2}$	$\frac{1}{\mathrm{m}^2}$
Strain energy density of the space fabric	Have to be built $M^{\mu\nu}$	$\frac{\mathrm{N\ m}}{\mathrm{m}^3}$	$\left(\frac{U}{L}\right)$	$\frac{\mathrm{N\ m}}{\mathrm{m}}$
External work	$T^{\mu\nu}$	$\frac{\mathrm{N\ m}}{\mathrm{m}^3}$	$\frac{W_{\mathrm{ext(total)}}}{L}$	$\frac{\mathrm{N\ m}}{\mathrm{m}}$
Proportionality factor	$\kappa = \frac{8\pi G}{c^4}$	$\frac{1}{\mathrm{N}}$	$K = \frac{2}{EI}$	$\frac{1}{\mathrm{N\ m}^2}$

associated with the charge (mass) applied to the space, not with the space itself. According to the mechanical principle, external work = internal work (see 10d), we can reconstruct the bridge between the two approaches. Indeed, if the analogy with the beam in 1D (or plate theory in 2D) is exact, it must exist as a tensor of stresses/(normal efforts, bending and torsion moments and shear loads) associated with the curvature of the space. In this case, expression (5a) will become, with a mechanical stress tensor acting within the space–time fabric (interior work) (10h) and κ a coupling constant:

$$\begin{bmatrix} G^{00} & G^{01} & G^{02} & G^{03} \\ G^{10} & G^{11} & G^{12} & G^{13} \\ G^{20} & G^{21} & G^{22} & G^{23} \\ G^{30} & G^{31} & G^{32} & G^{33} \end{bmatrix} = -\kappa \begin{bmatrix} \sigma^{tt} & \tau^{tx} & \tau^{ty} & \tau^{tz} \\ \tau^{xt} & \sigma^{xx} & \tau^{xy} & \tau^{xz} \\ \tau^{yt} & \tau^{yx} & \sigma^{yy} & \tau^{yz} \\ \tau^{zt} & \tau^{zx} & \tau^{zy} & \sigma^{zz} \end{bmatrix}, \tag{10h}$$

where $\sigma^{\mu\nu}$ are the normal stresses in the space–time fabric and $\tau^{\mu\nu}$ are the shear stresses in the space–time fabric both created by the curvature of the space fabric under the external masses.

The constant of proportionality κ is the same as we consider the work of the internal forces ($W_{\mathrm{int}} = U$) or the work of the external forces (W_{ext}), that is to say (K, κ). Later in our reasoning, we shall progress by considering U and therefor the reader must keep in mind expression (10d) connecting the external work to the inner work. We shall therefore make the shortcut associating T with U.

(b) There is a fracture in the quantum field theory that allows to describe the standard model, the vacuum energy, the Casimir force and general relativity. The general relativity wonderfully explains the mechanics of the infinitely large and the quantum mechanics that of the infinitely small. The problem is that these two theories do not overlap. Gravitation has not yet been quantified (independent of h) and is deterministic (not probabilistic as in quantum mechanics)! the graviton, boson vector of the force of gravity and keystone of quantum gravitation in connection with the quantum field theory has not yet been measured.

Nevertheless, we can quote string theory with strings open or closed of 1.0×10^{-35} m length and quantum gravity that try to solve this problem.

Based on the analogy of the beam in pure bending in one dimension, it seems natural to consider an elastic substance to associate with the space fabric. Consequently, a special elastic material should constitute the space described in the Einstein's field equation [8,9].

But, do we have solid proofs of this elastic material and of this space behaviour in weak field which would allow us to study the space following the elasticity theory? The answer is yes and it will be studied in more detail in §3.

Our aim is to explore these issues in five different ways simultaneously by first considering the time and space separately and then reassembling them at the end of the article as a second step:

- Consider that space is made of an elastic substance/material,
- calibrate κ using the spatial components rather than the temporal components of the Einstein's tensor. Thus, the temporal part is studied separately from the spacial part of the gravitation field tensors (see §10): use the elasticity theory, the strains measured on the interferometers Ligo and Virgo, the concept curvature = K energy density, the gravitational waves in weak field,
- replace G by mechanical and physical parameters,
- build a link with the quantum field theory by using the vacuum energy property to physically calibrate G and κ,
- conclude on the possible new characteristics of the space material.

Moving on, we shall propose a new mechanical and physical interpretation of the coupling constant κ in order to provide a different perspective on the understanding of the vacuum and of the potential material constituting the space elastic medium. By handling the nature of time separately from the nature of the space material it is hoped to contribute to the building of a bridge between the general relativity, the quantum field theory and the string theory.

2. Methods

The following methodology has been used focussing on the space part of the gravitational field equation with time being treated separately (see [11]):

(1) Analysis of the behaviour of space from the measurements made for more than 100 years in general relativity and quantum mechanics (vacuum energy) deducing an elastic behaviour.
(2) Research of the elastic theory which capture this behaviour: Hooke's law, Timoshenko's bending theory, beam, plates. Use analogy between the spiral form of the galaxies, the whirlwinds forming during cyclones or whirlpools on the sea as seen during the tsunami of 11 march 2011 in Japan and the potential presence of a space-material fluid at low speed, behaving for objects (gravitational waves) moving at the speed of light like an equivalent solid elastic material (see [8,9]).
(3) Demonstration, in weak gravity field, of the parallelism between the expression of the Einstein's gravity field (curvature $= \kappa \times$ energy density) in four dimensions and the expression obtained in elasticity/strength of the materials in two dimensions (curvature $= K \times$ the elastic strain energy density of two linear elements as the interferometer arms): use elasticity theory and assimilate each void volume of space inside the interferometer arm as a harmonic elastic oscillator made of vacuum full of an equivalent virtual elastic material.
(4) Application of the analogy obtained to the space material included in the two orthogonal tubes of the Ligo/Virgo interferometers: demonstration of the parallelism between the Einstein's field equation (tensorial equation in 4D) and the strain field of these two tubes like that of a strain gauge (tensorial equation in 2D) by using the work principle, $W_{\text{external}} = W_{\text{internal}}$.
(5) Deduction of this parallelism, via the elastic behaviour of the space, of a new mechanical expression of the gravitational constant G and of the Einstein constant κ based on longitudinal, torsional and shear waves in the medium [12,13].
(6) Proposition of numerical values of different parameters of these new expressions of the gravitational constant G and κ from the calculated vacuum energy.
(7) Checking the orders of magnitude of different parameters of these new expressions from the constants of physics.
(8) Proposition of the Young's modulus value $E = Y$ and Poisson's ratio ν for the space medium.
(9) Proposition of a method for measuring the Young's modulus of the space material from the Casimir effect and the possible shear effect in the plane of the interferometers.
(10) Possible reformulation of the Einstein's gravity field equation from this new approach to the constant κ.
(11) Consequences of the nature and characteristics of the space material constituting the microscopic fabric of the Universe (quantification of space and time).

3. Analysis of the space behaviour from the measurements made for more than 100 years in general relativity and quantum mechanics (vacuum energy)

The Einstein's gravitational field equation (5a) has been well verified experimentally. Indeed, all the gravitation tests and measurements carried out for more than 100 years constitute proofs which confirm that space is an elastic deformable physical object, which is not made of 'nothing' but that it is filled with 'something', a field, a material that is elastic, [8,9]. Indeed

(a) The acceleration of the expansion of the Universe has been observed [14]: the galaxies move away from us faster when they are away from us (Hubble's law (11)). In other words, the galaxies are 'motionless' and it is the space between them that dilates like dots on a balloon, which is inflated. So, the space fabric is an elastic body that stretches.

$$v = H_0 d, \tag{11}$$

where v is the recessional velocity of the galaxy, H_0 is the Hubble constant and d is the distance of the galaxy from the observator.

(b) The rays of light tangent at the Sun are curved by the deformation of the space around it. The light rays coming from stars follow the curvatures of an elastic body deformed by the present mass ([4] Eddington in 1919 measured the deflection angle of the stars around the Sun during an eclipse of 1.75‴).

(c) The space is elastic. As the Sun moves, the space curvature in its wake disappears and becomes flat. The rays of light become straight again.

(d) The rotation of the Earth twists the space fabric around it, indeed gyroscopes placed in orbit at 400 km around the Earth are deflected by this space rotation/torsion/curvature (see Probe B experiment [15]. Thus, a horizontal angle of 0.000010833° has been measured from the gyroscopes placed in vacuum in accordance with the prediction of general relativity).

(e) The space deformations produced by the coalescence, for example, of two black holes in the form of gravitational waves [16–18] were measured for the first time by the Ligo interferometers in 2015 (official announcement of GW150914 detected by Ligo on February 11, 2016). Other observations followed, including a fusion of neutron stars (pulsars) GW170817 also observed in electromagnetic radiation. So the interferometers have effectively measured signals which are moving at the speed of light c inside the frame of the vacuum space medium. So, if there are strains, then there is an elastic physical body that is deformed and we measure these deformations. Consequently, following the elasticity theory, the material of the physical body can be characterised by the Young's modulus E and Poisson's ratio ν. A strain tensor ε_{ij} can thus be established from the space strains $\delta L/L$ measured in the arms of the interferometers. The magnitude of these strains is 10^{-21}. Mechanically speaking, these strains measured are in the principal direction. They correspond to the transverse elastic waves of the medium with a propagation direction perpendicular to the plane build by the two arms of the interferometers. The formulas below describe the space metric g_{ij} built from h_{ij} (12b) a disturbance of the flat metric η_{ij} (12a) in weak gravitational field and the link with the strain tensor ε_{ij}:

$$g_{ij} = \eta_{ij} + h_{ij} = \eta_{ij} + 2\varepsilon_{ij} \tag{12a}$$

$$h_{ij} = 2\varepsilon_{ij} \tag{12b}$$

$$\frac{\delta L}{L} = \frac{\delta_i}{L} = 10^{-21} = \varepsilon_{ij} = \frac{1}{2}h_{ij}. \tag{12c}$$

(f) And finally, we can consider that the space rings like any elastic material according to the study conducted by Ringermacher and Mead [19].

Based on all these points, it seems logical to consider that space is made of a strange elastic material, a new type of Ether as explained by Einstein himself [20,21].

From the aforementioned observations, it seems that:

(a) The space fabric appears to be physically made up of 'Something' since the galaxies are trained by this 'Something' that is expanding accelerated.

(b) The space bends (analogy with the notion of curvature in elasticity theory see (6)) in the presence of energy density, mass. The sum of the angles of a triangle in the presence of mass is no longer 180° (see [8,9]).

(c) The space fabric is deformed and transmits gravitational waves by elongations and shortenings. So, small strains (interferometers) or angles (Gravity Probe B) are measured when we place it sufficiently far from the said mass/energy (weak field principle).

(d) The space is elastic, the curvature disappears when the mass that created it disappears.

(e) A black hole is a type of yielding of space loaded to the extreme [12,13].

(f) In the application of elasticity theory, space is proposed to have a quantified microstructure with an associated characteristic frequency f. G is the macroscopic manifestation of the said frequency (see eqs (2a) and (2b))

4. Research of the elastic theory which approaches the space behaviour: Hooke's law, Timoshenko's theory

4.1 *Hooke's law*

All these elements suggest that

(a) An elastic material constitutes the space (E, ν).

(b) Space is proposed to have a quantified microstructure of radius r with an associated characteristic frequency f.

(c) The Hooke's law applies to space medium.

See T Damour's book "if Einstein was told to me, chapter 3, the elastic space–time", where Einstein's equation is simplified by $D(g) = K{\cdot}T$ where D is a deformation tensor, T is a tensile tensor and K is a factor of proportionality

Consequently, it seems logical to apply the elasticity parameters to the deformable and elastic medium that constitutes it. This is reflected mathematically by the two expressions below relating to the normal stresses σ (13) and tangential stresses τ (14) connected respectively to the strain ε by the elasticity modulus E (the so-called Young's modulus) and to the shear strain (angle γ) by the shear modulus μ (some times noted as G) (see [22,23]).

$$\sigma = \varepsilon E = \frac{\delta l}{L}E \tag{13}$$

$$\tau = \gamma\mu = \gamma\frac{E}{2(1+\nu)}. \tag{14}$$

It is important to see that

(a) at strain, $\left(\frac{\delta L}{L}\right)$, is associated with Young's modulus E and normal stress σ,
(b) at the shear strain, (angle γ) is associated with a shear modulus μ, Poisson's ratio ν and shear stress τ.

This approach of general relativity by the elasticity theory is not new and has been studied by many researchers (see [11,24–48]). We review now the main points concerning, in particular, the relationship between the stress tensor and the stress–energy tensor (§4.2) on the one hand and between the metric tensor and the strain tensor (§4.3) on the other hand.

4.2 *Similarity between the stress tensor in elasticity and the stress–energy tensor in general relativity*

The Einstein's gravitational field formula (5a) can also be expressed in the following form:

$$T^{\mu\nu} = -\frac{c^4}{8\pi G}\times\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right). \tag{15a}$$

The equation to dimensions is

$$\frac{\frac{\text{kg m}^2}{\text{s}^2}}{\text{m}^3} = \frac{\text{N m}}{\text{m}^3} = \frac{\text{kg}}{\text{m s}^2} = \frac{\left(\frac{\text{m}}{\text{s}}\right)^4}{\frac{\text{m}^3}{\text{kg s}^2}}\times\left(\frac{1}{\text{m}^2}\right) = \frac{\text{kg m}}{\text{s}^2}\times\left(\frac{1}{\text{m}^2}\right). \tag{15b}$$

Or in a compact form, it becomes

$$\frac{\text{N m}}{\text{m}^3} = \frac{\text{N}}{\text{m}^2} = \text{N}\times\frac{1}{\text{m}^2}. \tag{15c}$$

We then notice that the energy density has the same dimension as the stress in Newton's classical mechanics. Moreover, it is possible to demonstrate that the stress–energy tensor (16a) is like stress tensor (16b) by replacing in it velocities v_i with four velocities u_μ and moving from a three-dimensional space to a four-dimensional space–time of density ρ (see [49] and Appendix A).

$$T_{\mu\nu} = \rho u_\mu u_\nu \tag{16a}$$

$$\sigma_{ij} = \rho v_i v_j. \tag{16b}$$

The stress tensor σ_{ij} in three-dimensional elasticity is written depending on the strain tensor ε_{ij} and two constants according to the Young's modulus of the medium $Y = E$ and the Poisson's ratio ν:

$$\sigma_{ij} = \frac{E}{(1+\nu)}\left(\varepsilon_{ij} + \frac{\nu}{(1-2\nu)}\varepsilon_{kk}\delta_{ij}\right) \tag{16c}$$

or

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij}. \tag{16d}$$

With the Lame coefficients:

$$\mu = \frac{E}{2(1+\nu)} \tag{17}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}. \tag{18}$$

In this expression, ε_{kk} is the trace of the strain tensor, δ_{ij} is the Kronecker symbol and ν is the Poisson's ratio.

4.3 *Relationship between the elastic strain tensor and the metric tensor in weak gravitational fields*

4.3.1 *Case of elasticity in four dimensions.*

Following [11], Chapter 2.6, formula 2.9, we have in four dimensions:

$$g_{\mu\nu} = (\eta_{\mu\nu} + h_{\mu\nu}) = (\eta_{\mu\nu} + 2\varepsilon_{\mu\nu}). \tag{19}$$

The metric tensor $g_{\mu\nu}$ in a weak field is therefore equivalent to the metric in flat field to which is added a perturbation which is only twice the strain tensor $\varepsilon_{\mu\nu}$. The general structure of the strain tensor ε_{ij} in the theory of three-dimensional elasticity is given in (20a) and (20b).

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}. \tag{20a}$$

With the displacements u_i and u_j we have

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{20b}$$

4.3.2 *Case of gravitational waves*

4.3.2.1 Theoretical aspects. The linearised form of the Einstein's equation in weak gravitational fields is, see (21a) and [50]:

$$\partial^\lambda\partial_\lambda\overline{h}_{\mu\nu} = \Box\overline{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu}. \tag{21a}$$

In vacuum (case of the gravitational waves), we have

$$\partial^\lambda\partial_\lambda\overline{h}_{\mu\nu} = \Box\overline{h}_{\mu\nu} = 0. \tag{21b}$$

The d'Alembertian wave operator,

$$\overline{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{h} \tag{22a}$$

$$\overline{h}_{\mu\nu} = A_{\mu\nu}\cos\left(k_\sigma x^\sigma\right) \tag{22b}$$

$k^\sigma = \left(\frac{\omega}{c};\vec{k}\right)$ quadri vector wave of the plane wave,

$$x^\sigma = (ct, x, y, z) \tag{23}$$

$$\left\|\vec{k}\right\|^2 = \frac{\omega^2}{c^2} \tag{24}$$

ω is the circular frequency of the wave.

$$A_{\mu\nu} = A_+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + A_\times \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{25}$$

$$\eta_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{26}$$

h is the trace of $h_{\mu\nu}$ and

$$\overline{h} = -h. \tag{27}$$

Using (22a), to have a new expression of (21a) function of ε_{ij}, we obtain a new expression of Einstein's gravitational field (28a)

$$\partial^\lambda \partial_\lambda \left(h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{h}\right) = \Box\left(h_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{h}\right) = -\frac{16\pi G}{c^4} T_{\mu\nu}. \tag{28a}$$

By replacing $h_{\mu\nu}$ by $2\varepsilon_{\mu\nu}$ (see (12c)) in (28a) we obtain:

$$\partial^\lambda \partial_\lambda \left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\overline{\varepsilon}\right) = \Box\left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\overline{\varepsilon}\right) = -\frac{16\pi G}{c^4} T_{\mu\nu}. \tag{28b}$$

After simplification by 2, we obtain the same constant κ in weak field:

$$\partial^\lambda \partial_\lambda \left(\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{\varepsilon}\right) = \Box\left(\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overline{\varepsilon}\right) = -\frac{8\pi G}{c^4} T_{\mu\nu}. \tag{28c}$$

The formulation of particle position variations arranged along a circle during the passage of a gravitational wave perpendicular to the plane xy, allows to find expression (29) by considering, for example, polarisation A_+ (see figure 2).

The movements of these particles correspond always to pure compression or pure traction of the space medium inside the interferometric tube. The deformation of the circle containing the particles is identical but rotates by 45° according to the type of polarisation (A_+ or $A_\times$).

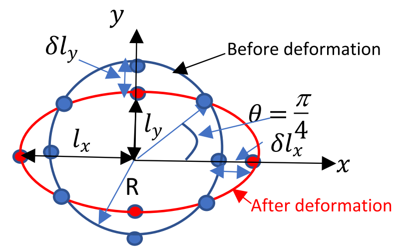


Figure 2. Example of particle coordinates subjected to a gravitational wave polarised A_+ particles propagating perpendicular to the plane xy.

The particle position measured from the centre of the circle is

$$l^2 = R^2 - R^2 A_+(\cos(2\theta)) \cos\left(\frac{\omega}{c}(ct - z)\right), \tag{29}$$

where l is the final length after deformation, z is the direction of the wave propagation, t is the time, θ is the angle between the abcissa x and R, the radius of the circle where the particles are positioned, ω is the circular frequency, c is the speed of light, A_+ is the first wave polarisation and $A_\times$ is the second wave polarisation. With the metric $g_{\mu\nu}$ (19) and dimensionless perturbation $h_{\mu\nu}$:

For a polarised wave A_+:

$$h_{\mu\nu} = A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{+1} & \mathbf{0} & \mathbf{0} \\ 0 & \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ 0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{30a}$$

For a polarised wave $A_\times$:

$$h_{\mu\nu} = A_\times \cos\left(\frac{\omega}{c}(ct - z)\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{0} & \mathbf{+1} & \mathbf{0} \\ 0 & \mathbf{+1} & \mathbf{0} & \mathbf{0} \\ 0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{30b}$$

The spatial part of $h_{\mu\nu}$ is indicated in bold. l^2 is the final length squared and R^2 is the initial length squared.

We can calculate the variation in length due to the displacement of the particle.

$$\frac{l^2 - R^2}{R^2} = \frac{(\text{final length})^2 - (\text{initial length})^2}{(\text{initial length})^2} = -A_+(\cos(2\theta)) \cos\left(\frac{\omega}{c}(ct - z)\right), \tag{31a}$$

where $L_F = L_i + \delta_i$ is the final length, L_i is the initial length and δ_i is the length variation.

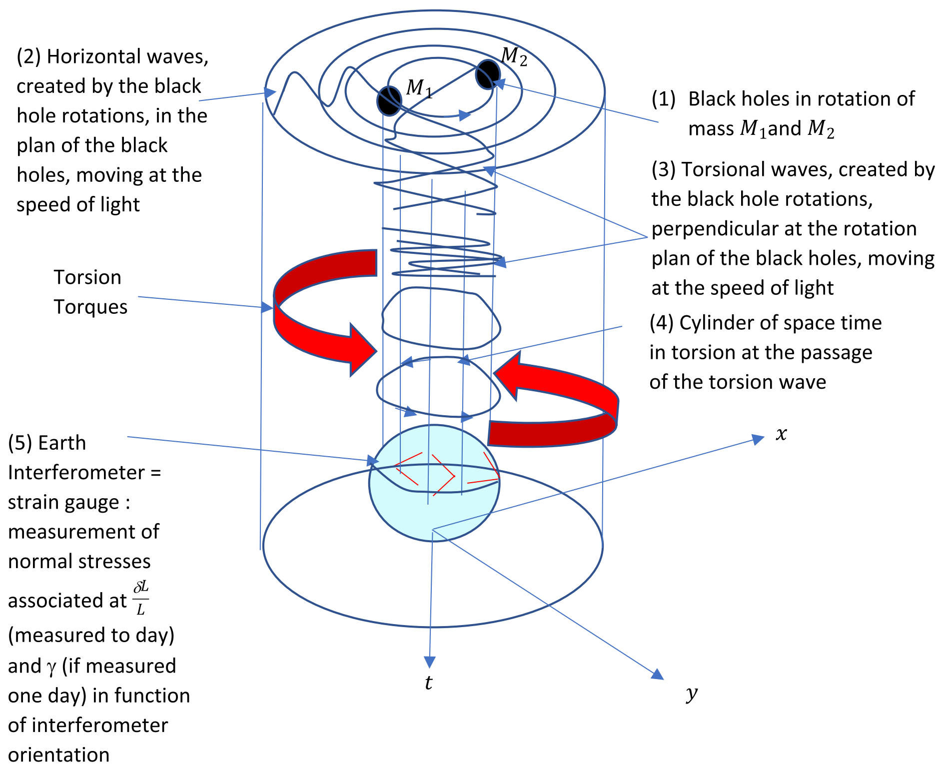


Figure 3. Torsional waves created by a binary system in rotation.

With $\delta_i \ll L_i$

$$\frac{l^2 - R^2}{R^2} = \frac{L_F^2 - L_i^2}{L_i^2} = \frac{L_i^2 + 2L_i\delta_i + \delta_i^2 - L_i^2}{L_i^2}. \quad (31b)$$

With $\delta_i^2 \ll \delta_i$

$$\frac{l^2 - R^2}{R^2} \cong \frac{2\delta_i}{L_i} \quad (31c)$$

and with the strain definition

$$\varepsilon = \frac{L_F - L_i}{L_i} = \frac{L_i + \delta_i - L_i}{L_i} = \frac{\delta_i}{L_i}. \quad (31d)$$

So finally we obtain

$$\frac{l^2 - R^2}{R^2} = \frac{L_F^2 - L_i^2}{L_i^2} \cong \frac{2\delta_i}{L_i} = 2\varepsilon. \quad (31e)$$

To correlate with the Hooke‘s law (13)

$$\frac{l^2 - R^2}{R^2} \cong -A_+(\cos(2\theta))\cos\left(\frac{\omega}{c}(ct - z)\right) \cong 2\varepsilon. \quad (31f)$$

So, the perturbation $h_{\mu\nu}$ and consequently the metric $g_{\mu\nu}$ are close to $2\varepsilon_{\mu\nu}$. So, in weak gravity field we demonstrate eq. (19) and the relationship between the metric tensor and the strain tensor well. Thus, the metric tensor in the general relativity approach in weak field is assimilated into elasticity to the flat metric tensor to which the strain tensor is added twice (12a).

At the end of this chapter, with the links between the tensors T_{ij} and σ_{ij} on the one hand and between g_{ij}, h_{ij} and ε_{ij} on the other hand, and taking into account the common principle curvature = $K \times$ the energy density, we have all the bridges necessary to interpret general relativity according to the elasticity theory.

4.3.2.2 Interpretation of the results of the calculation of general relativity on gravitational waves in weak field from the angle of elasticity theory. We consider that the rotation of a binary system (like two black holes, for example) creates a sort of 'torsional waves', in the space fabric (see figure 3 and [3,12,58]).

Indeed, this point was presented and demonstrated by Professor Kip Thorne (Nobel Prize 2017) during his conference on March 6, 2018 at UCI USA [12] "exploring the universe with gravitational waves from big bang to black holes". At this conference accessible on the net, he expains with numerical simulations based on the general relativity, the behaviour of two black holes during their coalescence. In fact there is the creation of two types of vortices (turning to right, turning to left) emerging from rotating black holes, dragging the space medium around them, which merge to create a ring which makes these swirl mixtures with compression and tensile tendencies measured by Ligo and Virgo. The consequences are that gravitational waves are not a classical shear wave but a mixture of vortices whose

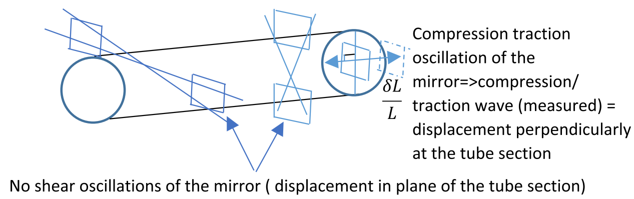


Figure 4. Movements of the laser mirror inside the interferometric tubes formed by the gravitational transversal wave.

results are compression/tensile tendencies measured in the interferometers.

If we compare the results of the general relativity in weak field ($h_{\mu\nu}$) with the elastic strain tensor (ε_{ij}) (see eq. (19) and [10]) we can conclude on the deformation states of the elastic medium in the xy plane of the arms of the interferometer during the passage of a gravitational wave coming from the z direction. Kip Thorne explains also in [13] the geometrodynamics of the space–time made of warped space.

First point:

Therefore, based on expressions (30a) and (30b), it is known that there are two clearly separated types of polarisation of the gravitational wave produced by the coalescence of two massive objects rotating relative to one another: A_+ and $A_\times$ (see [10]).

Second point:

By the relation between $h_{\mu\nu}$ and $\varepsilon_{\mu\nu}$ (see (12c) and (31f) and [11]), there exists for each polarisation of $h_{\mu\nu}$ the equivalent of an associated strain tensor (see [10]):

(a) Based on (30a), (31f) and (12c) associated with the polarisation A_+ we have

$$\varepsilon_{xy(A_+)} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & -\varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{32a}$$

This state of deformations is obtained in the tubes of the interferometer by analysing the forward and backward motions (δL) or strains ($\delta L/L$) of the laser mirrors in the two tube sections (see figure 4).

(b) Based on (30b), (31f) and (12c) associated with the polarisation $A_\times$ we have:

$$\varepsilon_{xy(A_\times)} = \begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{32b}$$

This state of deformations should be obtained in the tubes of the interferometer by analysing the lateral movements of the laser mirrors in the two tube sections (see figures 4 and 5).

These two deformation states (32a) and (32b) certainly prove that $h_{\mu\nu}$ corresponds to a pure shear

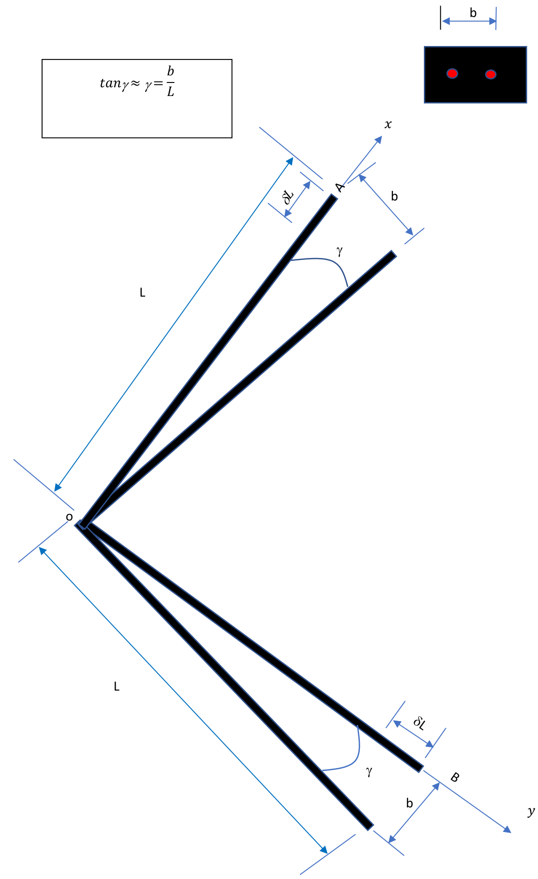


Figure 5. Strains $\frac{\delta L}{L}$ measured and shear strain (angle γ) not measured on the interferometers Ligo and Virgo to this day.

deformation state of the space layers (multisandwich) perpendicular to the direction z of the gravitational wave (see figure 6). It also proves the elastic behaviour of space in perfect correlation with the elastic theory and consequently the existence of an equivalent elastic material in the space vacuum. This fundamental research also proves that it is possible to unify the theory of elasticity with general relativity and thus that it is possible to define an elastic material constituting space.

Third point:

By the theory of elasticity, therefore, there exists, for each deformation tensor, a stress tensor (see [10]):

(a) On the basis of (32a) and (30a), associated with the polarisation A_+, the corresponding stress tensor of pure compression/traction of the space medium in the xy plane is

$$\sigma_{xy(A_+)} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{33a}$$

(b) On the basis of (32b) and (30b), associated with the polarisation $A_\times$, the corresponding stress tensor of

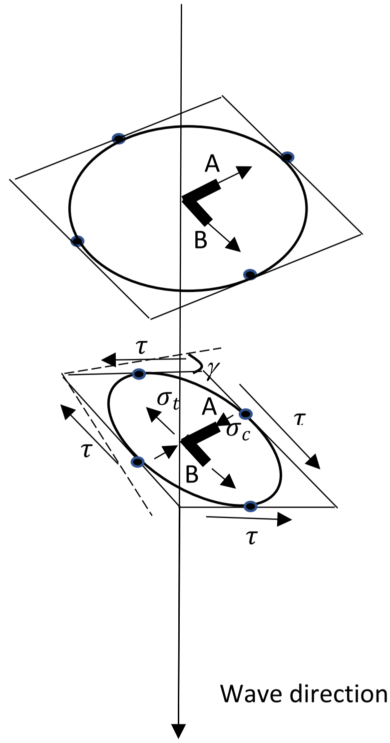


Figure 6. Plane deformations formed by the transverse gravitational wave.

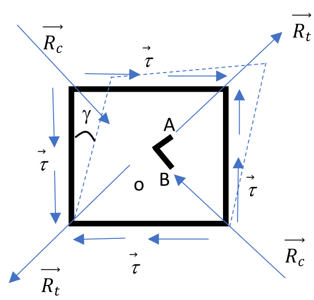


Figure 7. Creation of normal stresses by the combination of shear stresses.

pure shear of the space medium in the xy plane is

$$\sigma_{xy(A_\times)} = \begin{bmatrix} 0 & \tau_{xy} = \sigma_{xx} & 0 \\ \tau_{yx} = \sigma_{yy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{33b}$$

The two stress tensors above, according to the orientation of the facet considered (see figure 8), are characteristic of a pure shear associated with a pure torsion of space following the rotation of two massive objects which merge. Figure 6 shows the plane deformation of the space created perpendicular at the wave direction with the gravitational wave as a transverse wave.

Figure 7 shows how a combination of shear stresses τ create normal stresses σ.

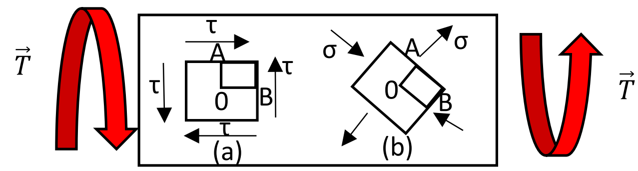


Figure 8. Normal stresses and shear stresses measured on interferometers as a function of their orientations on a space cylinder in torsion.

According to their orientations, the interferometers do not measure anything (Case a) or strains $\frac{\delta L}{L}$ (Case b) as seen in figure 8 and [54].

Fourth point:

Each of its stress states corresponds to a wave velocity characteristic of the elastic medium measured in the interferometric plane by the longitudinal oscillation of the laser mirror.

(a) Associated with the polarisation A_+, a pure longitudinal tensile compression wave velocity in each x or y direction is measured by the laser mirror of the Ligo and Virgo interferometers connected to a tensor according to the expression (33a) (see [53]).

$$\frac{\partial^2 u_{(x,t)}}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u_{(x,t)}}{\partial t^2} = 0. \tag{34a}$$

The Alembert equation is

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\rho}{E} \times \frac{\partial^2 u(x,t)}{\partial t^2} = 0. \tag{34b}$$

Comparing (34a) and (34b), we have the well-known equation

$$\frac{1}{c^2} = \frac{\rho}{E} \tag{35a}$$

$$c = \sqrt{\frac{E}{\rho}}. \tag{35b}$$

(b) Associated with the polarisation $A_\times$, a pure shear wave velocity, not yet measured by the laser mirror of interferometers Ligo and Virgo (lateral movements of the mirrors (see figures 4 and 5), connected to a tensor according to expression (33b) and a torsion torque [54].

Indeed, if we consider that the fabric which constitutes space has a dynamic behaviour similar to the one we have on Earth in the event of an earthquake but with only transverse waves, we only need to consider shear wave S instead of S and P (pressure) waves. Indeed, in line with what we measure, only strains in the plane xy perpendicular to the direction of the gravitational wave are seen (see figures 4–6).

So, the following expression (36) of elasticity theory applies, with u_i as the component i of the displacement vector $\vec{u}$, t as the time, σ_{ij} as the stress tensor and ρ as the mass density (see formula (3.10), and (3.11), chapter 3.3 of [11]):

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial j}. \tag{36}$$

With for the strain tensor ε_{ij}:

$$\varepsilon_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right) \tag{37}$$

The classical elastic wave motion is so with (16c) in (36):

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + 2\mu)\vec{\nabla}(\text{div}(\vec{u})) - \mu \overrightarrow{\text{rot}}\left(\overrightarrow{\text{rot}}(\vec{u})\right) + f_{\text{external}} \tag{38a}$$

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + 2\mu)\vec{\nabla}(\text{div}(\vec{u})) - \mu \vec{\nabla}(\text{div}(\vec{u})) + \mu \vec{\Delta}(\vec{u}) + f_{\text{external}}. \tag{38b}$$

After calculation

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + \mu)\vec{\nabla}(\text{div}(\vec{u})) + \mu \vec{\Delta}(\vec{u}) + f_{\text{external}} \tag{38c}$$

or again

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + \mu)\vec{\nabla}\left(\vec{\nabla} \bullet \vec{u}\right) + \mu \vec{\nabla}^2 \vec{u} + f_{\text{external}}, \tag{38d}$$

where

$$\vec{\nabla}\vec{u} = \overrightarrow{\text{grad}}(\vec{u}) \tag{38e}$$

$$\vec{\nabla}^2 \vec{u} = \vec{\Delta}(\vec{u}). \tag{38f}$$

When $f_{\text{external}} = 0$, the solution of the equation follows the Helmholtz's decomposition that gives two waves that propagate in the elastic medium:

$$\vec{u} = \vec{u}_{\text{pressure,longitudinal}} + \vec{u}_{\text{shear,transversal}}. \tag{39}$$

A pressure with a longitudinal wave of velocity c_{pressure}:

$$c_{\text{pressure}} = \sqrt{\frac{\lambda + 2\mu}{\rho}}. \tag{40}$$

This hypothesis of compression waves in the direction z is not acceptable because in this case the strains are in the same direction as the propagation of the waves. Of course this is not the case for gravitational waves where the strains are in a plane perpendicular to the wave direction of propagation (z). A shear with a transversal wave of velocity c_{shear}:

$$c_{\text{shear}} = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{2(1+\nu)\rho}}. \tag{41}$$

This hypothesis of shear waves is also strictly speaking not acceptable, because the strains measured on the interferometer are not shear strains (angles γ) linked at shear waves but, $\frac{\delta L}{L}$, strains that are always linked at eventual compression waves; but we shall consider it because it is possible that shear strain exists and is not already measured.

So strictly speaking, the gravitational waves cannot be assimilated to classical elastic waves in an elastic medium. They have the particularity to create strains perpendicular to the wave propagation direction (transversal shear wave characteristics) but with elongation and shortening (compression/traction wave characteristics); see [12,13]. The proposition of a space medium made up of a multisandwich of thin sheets sheared perpendicularly to the direction of propagation of the gravitational wave therefore seems a reasonable hypothesis (see figure 11).

4.4 *Determination of the Poisson's ratio intensity*

We can conclude from these wave speed equations, on a potential value of the Poisson's ratio ν.

First approach: Analysis of the particle movements on a circle under a gravitational wave (see figure 2)

The analysis of figure 2 from the calculation of general relativity shows that an object on the xy plane positioned perpendicular to the z direction of the propagation of gravitational transverse waves, is simultaneously compressed in one direction and stretched in the perpendicular direction. The strains are equal but of opposite sign: $\varepsilon_{xx} = -\nu\varepsilon_{yy}$.

With the definition of Poisson's ratio we have

$$\nu = \frac{\text{relative transverse shrinkage}}{\text{relative longitudinal elongation}} = 1.$$

Second approach: In the z direction, the gravitational wave is a transverse wave and not a compression wave

There is no compression wave perpendicular to the plane of the interferometer. So eq. (40) must be equal to 0. With eqs (17) and (18) we obtain $\nu = 1$ again.

Third approach: Based on current data (see [11])

Based on the results of ref. [11] and following eq. (31) we have, the Young's modulus $E = Y = 4.4 \times 10^{113}$ Pa (see §3.4, formula 3.13 [11]) and density $\rho = 1.30 \times 10^{96}$ kg/m^3 (see §3.4, formula 3.14 [11]).

$$\nu = \frac{E}{2c^2\rho} - 1. \tag{42}$$

With eq. (42) and the results of [11] we get

$$\nu_z = 0.8829.$$

This last value of the Poisson's ratio remains acceptable taking into account the uncertainty on the intensity of the vacuum energy (very important according to the quantum field theory and very low according to the value measured in the vacuum space) which is the object of many discussions within the international scientific community.

Conclusion: We retain for the Poisson's ratio: $\nu = 1$.

5. Highlighting the parallelism and differences between the strain energy density in elasticity and the mass energy density in general relativity

5.1 *The strain energy density in elasticity*

5.1.1 *Strain energy in general.* The strain energy density in elasticity is [22]

$$U_{ij} = \frac{1}{2}\sigma_{ij}\varepsilon^{ij}. \tag{43a}$$

By introducing in (43a) the expression of the stress tensor (16c), we obtain the following expression of strain energy density of an elastic body:

$$U_{ij} = \frac{E}{2(1+\nu)}\left[\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right)\right]\varepsilon^{ij} \tag{43b}$$

or

$$\left[\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right)\right]\varepsilon^{ij} = \frac{2(1+\nu)}{E}U_{ij} = \frac{(1+\nu)}{E}\sigma_{ij}\varepsilon^{ij}. \tag{43c}$$

In plate theory there are relations between the strain tensor and the curvature tensor (see eqs (78)–(81)) which brings us closer to Einstein's formalism (see 5a) if we consider that the external work produced by the external masses applied on the space fabric is equal to the internal work of the space fabric curved by these masses. A tensorial space approach is also developed in [42].

5.1.2 *Consequences of the parallelism between the elasticity theory and the general relativity.* Thus, to build a bridge between the elasticity theory and general relativity formalism, we have studied the parallelism between the strength of the material and the general relativity about the principle: curvature $=$ K energy density on the one hand (see Introduction) and the transversality of the physical terms (curvature, metric, strain energy, external work, stresses, strains, coefficient of proportionality K and κ) between the general relativity and the elasticity theory on the other hand (see §4.2 and §4.3).

The parallelism between the general relativity and the strength of material formulas on the principle curvature $=$ K energy density is as follows (see "Introduction"):

$$\frac{1}{R^2} = \frac{2}{EI}\left(\frac{W_{ext(total)}}{L}\right) = K\left(\frac{W_{ext(total)}}{L}\right) = \frac{2}{EI}\left(\frac{U}{L}\right) \tag{10d}$$

$$G^{\mu\nu} = -\frac{8\pi G}{c^4}T^{\mu\nu} = -\kappa\ T^{\mu\nu} = -\kappa\, M^{\mu\nu} \tag{10e}$$

The transversality between general relativity and the strength of the material on the key parameters is as follows: (link between ε_{ij}, h_{ij} and g_{ij} (see (31f), (12c) and (19)) and between $T_{\mu\nu}$ and σ_{ij} (see (16a) and (16b) and Appendix A):

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}\mathrm{R} = -\frac{8\pi G}{c^4}\quad T^{\mu\nu} \tag{5a}$$

$$\partial^{\lambda}\partial_{\lambda}\bar{h}_{\mu\nu} = \Box\,\bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}\quad T_{\mu\nu} \tag{21}$$

$$\left[\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right)\right]\varepsilon^{ij} = \frac{2(1+\nu)}{E}\quad U_{ij} = \frac{(1+\nu)}{E}\sigma_{ij}\varepsilon^{ij} \tag{43c}$$

From this analysis, it is clear that the constant κ in this case, $\frac{8\pi G}{c^4}$ would be closer to mechanical constant of the space medium function of the Young's modulus E and the Poisson's ratio ν. In other terms κ should be proportional to $\frac{(1+\nu)}{E}$.

To finalise the construction of parallelism between general relativity and the theory of elasticity (or the strength of materials which results from it) and to find a mechanical transposition of the Einstein's constant κ, we must now try to find the expression of κ from the curvature $=$ K energy density principle expressed from simple equations of strength of the materials. The comparison of the two formulas (5a) and (21a) with the formula (43c), terms to terms, allows to compare K and κ and to identify the mechanical correspondences between $(1+\nu)/E$ and the parameters G and c).

The examination of (43c) shows that these equations will be of the form

$$f_{(\varepsilon_{ij^2})} = K_{\left(\frac{1+\nu}{E}\right)}U. \tag{43d}$$

5.2 *Proposition of a tensor equation between curvature and space strain energy based on the strains measured on the interferometers Ligo and Virgo*

5.2.1 *Study of a horizontal space cylinder in one direction solicited by a gravitational wave – without the effect of Poisson's ratio – use of the longitudinal velocity of the correlated compression/traction wave with the* *compression/traction stress tensor in the interferometric tube.* We assume in this section that the coalescence of two black holes, for example, creates by their rotations, a torsion of the space as shown in figure 3 and ref. [12,13].

This twisting of the space sheets creates, in the successive xy planes, tensiles and compressions of the space material sheets (multisandwich) that finally arrive on the Earth in each arm xy of the interferometers (see figures 6 and 11). In weak field, for space only, the metric is given by (12a). Einstein's gravitational equation in a weak field for space is (21b). The result is for a polarised wave A_+, the tensor given in (30a).

Note: We choose this polarisation because it corresponds to the displacements measured in the interferometric arms (compression /traction of the volume which create the advances and retreats of the laser mirrors).

With formula (31f) demonstrated in §4.3.2.1, we have a link between the general relativity (disturbance of the spatial part of the metric h_{ij}) and the theory of elasticity (the strain ε_{ij} of the elastic medium) (see eq. (12c)). So, based on (31f) and (12c), at the spatial perturbation of the metric, $(h_{\mu\nu(A_+)})$ corresponds to the strain tensor ε_{ij} (32a). Thanks to the Hooke's and elasticity formula, it corresponds to the strain tensor ε_{ij} (32a) a stress tensor σ_{ij} (33a). This type of stress tensor (33a) represents a longitudinal pure compression/traction wave (35a) associated with the normal effort N (see figure 9 and [10]).

This section considers therefore a tube of a Ligo/Virgo type interferometer of length L and section S loaded with the normal force N as defined in figure 9. Inside the tube, it is considered that the vacuum consists of a space elastic substance made from very small particles to constitute a granular substance, fluid whose granularity of quantum dimension r.

Note: In this simplified approach we deliberately separate the correlation between the x and y directions of the tubes seen in figure 2. This was done in order to see already if the basic principle curvature $= K \times$ the energy density is respected on the one hand and with the eventual stresses on the perpendicular direction of the transverse wave to be in correlation with the stress plane state on the other hand. The consequence is that in this section we do not take into account the Poisson's ratio to be in agreement with the information of the tensor considered here (see eq. (33a)).

Since there are displacements of the laser mirror in the direction of the tube corresponding to the compression/traction of the space medium (see figure 4 and [53]), there are strains and stresses in this framework and therefore a dynamic normal force N and a pure compression/traction wave inside the tube.

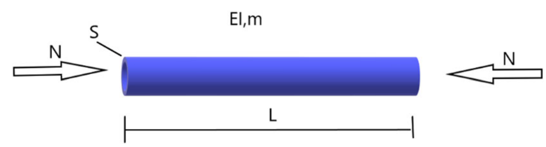


Figure 9. Tube loaded by a normal force N.

The Hooke's law (13) can be written as a function of the displacement $u_{(x)}$:

$$\sigma_{xx} = \varepsilon E = \frac{N}{S} = \left(\frac{u_{(x+dx)} - u_{(x)}}{dx}\right) E = \frac{\delta L}{L} E, \tag{44a}$$

where σ is the normal stress in N/m^2, ε is the strain in %, ($\varepsilon = \frac{N}{ES}$) is measured by Ligo and Virgo, N is the normal force in Newton, S is the tube section in m^2, L is the tube length in m, $V = S \times L$ is the tube volume in m^3, $E = Y$ is the Young's modulus of the material constituting the tube in N/m^2, δL is the length variation in m under the normal force N, $u_{(x)}$ is the longitudinal displacement in m along the longitudinal axis of x.

The stress–displacement relation as a function of rigidity $K = ES/L$, is written as follows:

$$N = \frac{ES}{L}\delta L = K\delta L. \tag{44b}$$

The strain energy U (N m) of the tube in static, when N is constant and taking into account (44b), is

$$U = \frac{1}{2}\int_0^L \frac{N^2}{ES}\mathrm{d}x = \frac{1}{2}\frac{N^2}{K}. \tag{45a}$$

The strain energy of the stiffness spring K (N/m) can be written by substituting N by expression (44b) in (45a):

$$U = \frac{1}{2}K(\delta L)^2. \tag{45b}$$

Moreover, from the strain definition, the displacement variation δL is

$$\varepsilon \times L = \delta L. \tag{46}$$

By introducing (46) in expression (45b) of the strain energy, we obtain

$$(\varepsilon)^2 = \frac{2}{K}\frac{U}{L^2}. \tag{47a}$$

Starting from the rigidity K depending on length L, the Young's modulus E, the tube section S and substituting the expression of L by K in (47a), we obtain

$$(\varepsilon)^2 = 2\frac{K}{E^2}\frac{U}{S^2}. \tag{47b}$$

Considering the tube volume V we extract the section S:

$$\frac{V}{L} = S. \tag{48}$$

By substituting the expression of S (48) in (47b), we obtain the strain energy density U/V:

$$\frac{1}{L^2}(\varepsilon)^2 = 2\frac{K}{V}\frac{1}{E^2} \times \frac{U}{V} \tag{49a}$$

or

$$\frac{U}{V} = \frac{\frac{1}{L^2}(\varepsilon)^2}{2\frac{K}{V}\frac{1}{E^2}}. \tag{49b}$$

We now consider the tube under a dynamic behaviour and with a pure longitudinal wave compression/traction following x in the arm as a consequence of polarised gravitational waves A_+. We note the density ρ as

$$\rho = \frac{m}{V}. \tag{50}$$

The fundamental dynamic equation allows us to find the eigencircular frequency (harmonic oscillator) of the tube made of the space material:

$$U_c + U = \frac{1}{2}m\dot{x}^2_{(t)} + \frac{1}{2}Kx^2_{(t)} = E_0 = 0, \tag{51a}$$

where U_c is the kinetic energy.

By a derivative with respect to t we have

$$\ddot{x}_{(t)} + \frac{K}{m}x_{(t)} = 0 \tag{51b}$$

and of course following the Newton's force definition

$$\text{Force} = m\ddot{x} = -Kx \tag{52}$$

which allows us to express the circular frequency ω according to the tube rigidity K and its mass m:

$$\omega^2 = \frac{K}{m} = (2\pi f)^2 \tag{53a}$$

$$\omega = 2\pi f. \tag{53b}$$

By substituting (53a) in (50) we obtain a new expression for the volume V as

$$V = \frac{m}{\rho} = \frac{K}{\omega^2\rho}, \tag{53c}$$

The total energy density of the system mass-spring U_T is a function of the kinetic energy density U_c and strain energy density U:

$$\frac{U_c}{V} + \frac{U}{V} = \frac{U_T}{V}. \tag{53d}$$

The strain energy density is

$$\frac{U}{V} = \frac{U_T}{V} - \frac{U_c}{V} = T. \tag{53e}$$

By substituting (49b) in (53e) we obtain

$$\frac{\frac{1}{L^2}(\varepsilon)^2}{2\frac{K}{V}\frac{1}{E^2}} = T \tag{54a}$$

or in an equivalent form:

$$\frac{1}{L^2}(\varepsilon)^2 = 2\frac{K}{V}\frac{1}{E^2}T. \tag{54b}$$

By substituting V (53c) in expression (54b) we get

$$\frac{1}{L^2}(\varepsilon)^2 = 2\rho\frac{\omega^2}{E^2} \times T. \tag{54c}$$

By substituting the circular frequency ω (53b) by frequency f in (54c), we obtain

$$\frac{1}{L^2}(\varepsilon)^2 = 8\pi^2\rho\frac{f^2}{E^2} \times T. \tag{54d}$$

In the plane of the interferometer, the movements of the mirror are perpendicular to the tube section (see figure 3). Thus, in the plane of the interferometer, the mirror follows the compression/traction movements of the spatial material inside the volume of the tube (, $\frac{\delta L_x}{L_x}$, in connection with , $u_{(x)}$,). These oscillations in the plane of the interferometer are therefore pure compression/traction longitudinal waves correlated to the transversal waves perpendicular to the plane of the interferometers (see [12] and [53]). So in this section, following the type of tensor considered (33a), the velocity of the wave is given in formula (35a) and the Young's modulus $E = Y = \rho c^2$.

The speed is limited because it propagates in an elastic medium of density ρ. A similar experiment would be to pull more or less quickly a metal ball in the middle of a pile of sand. Depending on the density and intensity of sand, the ball will move more or less quickly. In our case, the bullets should be the photons and the medium would consist of a material of extremely fine granulometry (1×10^{-35} m). We come back now at the tube under a gravitational wave. We multiply and divide by ρ the expression (54d):

$$\frac{1}{L^2}(\varepsilon)^2 = 8\pi^2 f^2\left(\frac{\rho}{E}\right)^2\frac{1}{\rho} \times T. \tag{55}$$

Finally, we have the relation between the dynamic curvature and the strain in the tube:

$$\frac{1}{L^2}(\varepsilon)^2 = 8\pi\left(\frac{\pi f^2}{\rho}\right)\left(\frac{\rho}{E}\right)^2 \times \frac{U}{V}. \tag{56}$$

Taking into account eq. (35a) we obtain

$$\frac{1}{L^2}(\varepsilon)^2 = 8\pi\left(\frac{\pi f^2}{\rho}\right)\frac{1}{c^4} \times \frac{U}{V}. \tag{57}$$

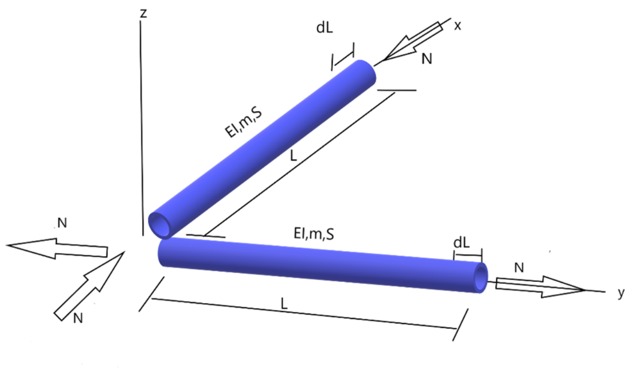


Figure 10. Double perpendicular tube loaded by a normal force.

We notice that the term $\frac{\pi f^2}{\rho}$ has the dimension of the gravitational constant G ($\mathrm{m}^3/(\mathrm{kg{\cdot}s}^2)$), the term $\frac{1}{L^2}(\varepsilon)^2$ has the dimension of a curvature ($1/\mathrm{m}^2$) (see eqs (78) and (81)), the term (U/V) has the dimension of the energy density ($\mathrm{N\,m/m^3}$), the term $(\frac{\pi f^2}{\rho})\frac{1}{c^4}$ has the dimension of the inverse of the load (N^{-1}).

The formula (57) thus satisfies the principle curvature $= K$ energy density. Consequently, we learn that the term $(\frac{\pi f^2}{\rho})$ can be identified with the Einstein's constant κ if G

$$G = \frac{\pi f^2}{\rho}, \tag{58}$$

where f is the natural frequency of the spatial material inside the tube and ρ is the density of the spatial material.

In the next section, we take into account the Poisson's ratio.

5.2.2 *Study of two perpendicular horizontal space cylinders solicited by a gravitational wave – effect of the Poisson's ratio – use of the longitudinal velocity of the correlated compression/traction wave with the compression/traction stress tensor in the interferometric tubes.* We assume the same hypothesis on the strain and stress tensors as in §5.2.1. We consider now the two perpendicular tubes undergoing (z direction) a gravitational wave perpendicular to their plane compressing and dilating them simultaneously as shown in figure 10. Therefore, a Poisson's ratio has to be taken into account in this section.

In this case, the two tubes behave like a gigantic stress/strain gauge. Each arm constitutes a principal direction in the xy plane. We can therefore write a tensor expression in two dimensions and consider a strain tensor and a strain energy tensor. We also assume that the xy plane is deconnected from the z direction (elasticity theory: plane stress problem). The consequences is that the spatial material is considered as multisandwiched thin sheets of thickness $2r$ (see [10–13]) successively twisted during the passage of the shear wave (see figure 11).

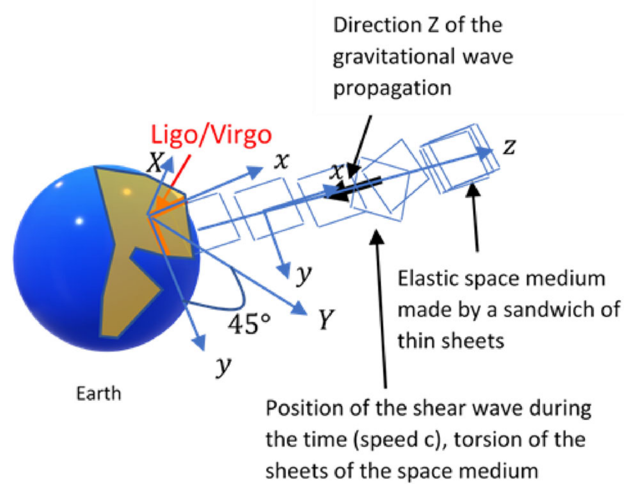


Figure 11. Sandwich structure of the space medium under a gravitational wave.

With the actual measurements made at Ligo and Virgo, we know that when one of the arm is in compression the other is in tension simultaneousely.

First step: Determination of the strains and stresses in the sheet space in torsion

Following the axis $\vec{x}$; $\vec{y}$, we have $\varepsilon_{xx} = -\nu\varepsilon_{yy}$. These two deformations are correlated via the general relativity data by $h_{\mu\nu} = 2\varepsilon_{\mu\nu}$ [8,9,11]. The strain tensor according to the axes system $\vec{x}$; $\vec{y}$ is given in (32a). So, the relations between the strains and the stresses are

$$\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy}) \tag{59}$$

$$\varepsilon_{yy} = \frac{1}{E}(\sigma_{yy} - \nu\sigma_{xx}). \tag{60}$$

Using the stress tensor definition (16c), we get

$$\sigma_{xx} = \frac{E}{(1+\nu)}\left\{\varepsilon_{xx} + \frac{\nu}{(1-2\nu)}(\varepsilon_{xx} + \varepsilon_{yy})\right\} \tag{61a}$$

$$\sigma_{yy} = \frac{E}{(1+\nu)}\left\{\varepsilon_{yy} + \frac{\nu}{(1-2\nu)}(\varepsilon_{xx} + \varepsilon_{yy})\right\}. \tag{61b}$$

Taking into account that, $\varepsilon_{xx} = -\nu\varepsilon_{yy}$ (with $\nu = 1$, see 4.4), expressions (61a) and (61b) become

$$\sigma_{xx} = \frac{E}{(1+\nu)}\{\varepsilon_{xx}\} \tag{62a}$$

$$\sigma_{yy} = -\frac{E}{(1+\nu)}\{\varepsilon_{yy}\}. \tag{62b}$$

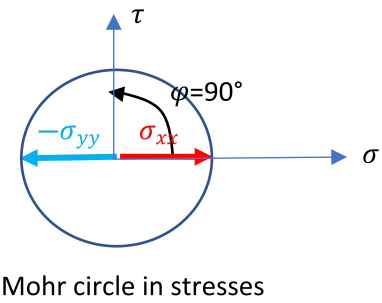


Figure 12. Mohr's circle of the stress state [54].

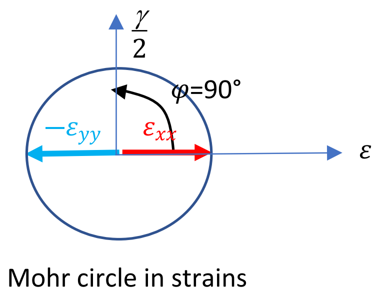


Figure 13. Morh's circle of the strain state [54].

Note: If one of the tube section is in compression ($\vec{x}$ direction) the other one in $\vec{y}$ is in traction (origin of the minus sign).

According to the system of axes $\vec{x}$; $\vec{y}$, the stress tensor is as shown below:

$$\sigma_{xy} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & -\sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{63}$$

Using Mohr's circle (see figure 12) we can confirm the global shear behaviour along a 45° facet (axes system $\vec{X}$; $\vec{Y}$) (see figure 11) and [10].

When we turn 90° on the Mohr's circle, we turn 45° on the real facet (image of the spin 2 of the graviton, see [10]). So on a 45° facet we are in pure shear as shown in figure 7. The strain tensor according to the axes system $\vec{X}$; $\vec{Y}$ is (see figure 13) and [10]:

$$\varepsilon_{XY} = \begin{bmatrix} 0 & \varepsilon_{XY} & 0 \\ \varepsilon_{XY} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{64}$$

The stress tensor according to the axes system $\vec{X}$; $\vec{Y}$ is

$$\sigma_{XY} = \begin{bmatrix} 0 & \tau_{XY} = \sigma_{XX} = \sigma & 0 \\ \tau_{YX} = \sigma_{YY} = \sigma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{65}$$

The shear stress is defined in eq. (11):

$$\tau_{XY} = \frac{E}{2(1+\nu)}\gamma_{XY} = \mu\gamma_{XY}. \tag{66}$$

In addition, we have in elasticity:

$$\varepsilon_{XY} = \frac{1}{2}\gamma_{XY}. \tag{67a}$$

So from (66) and (67a) we get

$$\tau_{XY} = \frac{E}{(1+\nu)}\varepsilon_{XY}. \tag{67b}$$

On the Morh's circle (figure 13) we see that

$$\frac{\gamma_{XY}}{2} = \varepsilon_{XX}. \tag{68}$$

According to this equation, it seems that depending on the orientation of the interferometer in the plane ($\vec{x}$, $\vec{y}$) and ($\vec{X}$, $\vec{Y}$) with respect to the direction of propagation of the gravitational wave ($\vec{z}$), lateral movements of the laser mirrors are possible with the same shape and same wave intensity as the conventional one in compression/traction (see figure 4). It should be interesting to measure these movements especially when the traditional compression/traction motions are not measured because of the position of the interferometer with respect to the direction of propagation of the gravitational wave. From (67a) and (68) we obtain

$$\varepsilon_{XX} = \varepsilon_{XY.} \tag{69}$$

So the deformation of the circle containing the particles is the same (traction in one direction and compression in the other) but the circle rotates by an angle of 45°, see [10].

Second step: Determination of the strain energy of the two connected tubes in the main system of axes x⃗ y⃗

We now consider the strain energy of the two tubes connected in traction/compression according to the axes system $\vec{x}\vec{y}$:

Note: The results will be the same if we consider the axes system $\vec{X}$; $\vec{Y}$ because of the equivalence between shear stresses and normal stresses on the one hand (see (65)) and the equivalence between the strains and angles on the other hand (see (64), (68) and (69) and [11]).

The energy density or energy per unit volume is

$$U_{ij} = \frac{U}{V} = \frac{1}{2}\sigma_{ij}\varepsilon^{ij}. \tag{70}$$

In the axes system $\vec{x}$; $\vec{y}$, we are in pure compression/traction in the tube, and the total strain energy per unit of volume is

$$\frac{U}{V} = \frac{1}{2}\sigma_{xx}\varepsilon_{xx} + \frac{1}{2}\sigma_{yy}\varepsilon_{yy}. \tag{71}$$

As the section S of the tube is constant and as the stresses and strains are constant on S, for a fixed section S of abcissa (x) or (y), we have

$$\frac{U}{S} = \frac{1}{2}\int_0^L \sigma_{xx}\varepsilon_{xx}\mathrm{d}x + \frac{1}{2}\int_0^L \sigma_{yy}\varepsilon_{yy}\mathrm{d}x. \tag{72}$$

When one arm is in compression the other is in traction and so

$$\varepsilon_{xx} = -\varepsilon_{yy} = \frac{N}{ES} \tag{73}$$

and from the Hooke's law:

$$N_x = N_y = N = \frac{ES}{L}\delta L = K\delta L. \tag{74}$$

The expressions of the stresses are given in (62a) and (62b). We can introduce the expressions of theses stresses in eq. (72):

$$\frac{U}{S} = \frac{E}{2(1+\nu)}\int_0^L \{\varepsilon_{xx}\}^2\mathrm{d}x + \frac{E}{2(1+\nu)}\int_0^L \{\varepsilon_{yy}\}^2\mathrm{d}x. \tag{75a}$$

By replacing the strains by their expressions (73), we obtain

$$U = \frac{E}{2(1+\nu)}\int_0^L \left\{\frac{N}{ES}\right\}^2 S\mathrm{d}x + \frac{E}{2(1+\nu)}\int_0^L \left\{\frac{N}{ES}\right\}^2 S\mathrm{d}x. \tag{75b}$$

We obtain the generalisation of expression (45a) in two dimensions:

$$U = \frac{1}{2}(1+\nu)\int_0^L \frac{N^2}{ES}\mathrm{d}x + \frac{1}{2(1+\nu)}\int_0^L \frac{N^2}{ES}\mathrm{d}x \tag{75c}$$

or after simplification

$$U = \frac{1}{(1+\nu)}\frac{N^2 L}{ES}. \tag{75d}$$

The strain energy of a stiffness spring K (N/m) can be written by substituting N by expression (44b) in (75d):

$$U = \frac{1}{(1+\nu)}K(\delta L)^2. \tag{75e}$$

Moreover, according to the strain definition, the displacement variation δL is with $\nu = 1$:

$$\varepsilon_{xx}L = -\varepsilon_{yy}L = \pm\varepsilon \times L = \pm\varepsilon_{ii} \times L = \pm\delta L \tag{75f}$$

with $i = x$ or y. Introducing (75f) into (75e), we get

$$U = \frac{1}{(1+\nu)}K(\varepsilon_{ii})^2 L^2 \tag{75g}$$

or

$$(\varepsilon_{ii})^2 = (1+\nu)\frac{1}{K}\frac{U}{L^2}. \tag{76a}$$

Starting from the rigidity K depending on the length L, the Young's modulus E, the tube section S and substituting the expression of L resulting from K in (76a), we obtain

$$(\varepsilon_{ii})^2 = (1+\nu)K\frac{U}{E^2S^2}. \tag{76b}$$

Considering the tube volume V, the section S is extracted (see (48)). Substitute the expression of S (48) in (76b) gives the U/V strain energy density:

$$\frac{1}{L^2}(\varepsilon_{ii})^2 = (1+\nu)\frac{K}{V}\frac{1}{E^2} \times \frac{U}{V} \tag{76c}$$

or

$$\frac{U}{V} = \frac{\frac{1}{L^2}(\varepsilon_{ii})^2}{(1+\nu)\frac{K}{V}\frac{1}{E^2}}. \tag{76d}$$

We now consider the tubes under dynamic behaviour and with a compression/traction wave following x and y in both arms simultaneousely due to gravitational transversal waves. By proceeding as in §5.2.1 in one dimension we obtain:

$$\frac{1}{L^2}(\varepsilon_{ii})^2 = (1+\nu)\rho\frac{\omega^2}{E^2} \times T. \tag{76e}$$

By substituting the circular frequency ω (53b) by frequency f in (76e), we obtain

$$\frac{1}{L^2}(\varepsilon_{ii})^2 = 4\pi^2(1+\nu)\rho\frac{f^2}{E^2} \times T. \tag{76f}$$

We have a pure compression/traction in the interferometer (not in 3D with shear wave), and so the compression/traction wave equation is (see §4.3.2.1 and figure 4) the same as in §5.2.1 (see (35a) and (35b)). We obtain

$$\frac{1}{L^2}(\varepsilon_{ii})^2 = 4\pi^2(1+\nu)\rho\frac{f^2}{c^4\rho^2} \times T. \tag{76g}$$

On the basis of (75f), we can write this equation according to each strain in each arm of the interferometer:

$$\frac{1}{L^2}(\varepsilon_{xx})^2 = 4(1+\nu) \times \pi \times \frac{\pi f^2}{\rho} \times \frac{1}{c^4} \times \frac{U}{V} \tag{76h}$$

$$\frac{1}{L^2}(\varepsilon_{yy})^2 = 4(1+\nu) \times \pi \times \frac{\pi f^2}{\rho} \times \frac{1}{c^4} \times \frac{U}{V} \tag{76i}$$

and it is interesting to see that with $\nu = 1$ (see 4.4) and $G = \frac{\pi f^2}{\rho}$ we obtain

$$\frac{1}{L^2}(\varepsilon_{ii})^2 = 8\pi \frac{G}{c^4} \times \frac{U}{V} \tag{76j}$$

or with (75f):

$$\frac{1}{L^2}(\varepsilon_{xx})^2 = \frac{8\pi G}{c^4} \times \frac{U}{V} \tag{76k}$$

$$\frac{1}{L^2}(\varepsilon_{yy})^2 = \frac{8\pi G}{c^4} \times \frac{U}{V}. \tag{76l}$$

So we can construct the tensorial expression (76m) on the basis of (76h) and (76i) and following the principle curvature $= K$ strain energy density which is also equal to K times the work of external forces.

$$\begin{bmatrix} \frac{1}{L^2} & 0 \\ 0 & \frac{1}{L^2} \end{bmatrix} \begin{bmatrix} (\varepsilon_{xx})^2 & 0 \\ 0 & (\varepsilon_{yy})^2 \end{bmatrix} = \frac{8\pi G}{c^4} \begin{bmatrix} T_{xx} & 0 \\ 0 & T_{yy} \end{bmatrix}. \tag{76m}$$

Note: If complementary measurements are carried out on an interferometer and if we can confirm that there are also shear strains (angles γ), then the hypothesis of plate behaviour is confirmed, eq. (76m) will be built with 3×3 matrix completed by Poisson's ratio ν, strains ε_{xy} and ε_{yx} and strain energies T_{xy} and T_{yx}.

By opposition, if measurements are carried out and if there are no shear strains (angles γ), it is necessary to consider the hypothesis of torsional wave cited above. In this case, the mechanical model could not be a plate but a space cylinder in torsion due to the rotation of black holes' binary system (see §5.2.3). But all these elements do not change the κ coefficient subject of this article. We continue so on the basis of the actual data, the $\frac{\delta L}{L}$ measured in each direction of the interferometer arms.

It must be remembered that in weak field there is a relationship between the metric g_{ij} and the strain tensor ε_{ij} (see (12a)) and a relationship between the stress energy tensor T^{ij} and the mechanical stress tensor σ^{ij} (see (16a), (16b) and Appendix A). The parallelism between expression (76m) and the plate bending expression is noted for the diagonal terms of (77c) but not the terms concerning the Poisson's ratio (see above). What is important here, is that the term , $\frac{\varepsilon_{ij}}{L^2}$, can be interpretated as a curvature by comparison with the terms, $\frac{\varepsilon_{ij}}{z^2}$, that we have in a plate case ((77a), (78) and (79)). Indeed, in the case of the plate hypothesis [51,52], the strain energy is with z, the thickness of the plate perpendicular to the plane xy:

$$\frac{1}{z^2}\left[(\varepsilon_{xx})^2+(\varepsilon_{yy})^2+2(1-\nu)\frac{1}{4}\{\varepsilon_{xy}\}^2+2\,\nu\{\varepsilon_{xx}\varepsilon_{yy}\}\right]$$

$$= \frac{24(1-\nu^2)}{Eh^3} \times \frac{\mathrm{d}U}{\mathrm{d}x\mathrm{d}y}. \tag{77a}$$

With the plate bending rigidity

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{77b}$$

or

$$\frac{1}{z^2}\left\{\varepsilon_{xx};\varepsilon_{yy};\frac{\varepsilon_{xy}}{2}\right\}\begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 2(1-\nu) \end{pmatrix}\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \frac{\varepsilon_{xy}}{2} \end{pmatrix} = \frac{24(1-\nu^2)}{Eh^2} \times \frac{\mathrm{d}U}{\mathrm{d}x\mathrm{d}yh} \tag{77c}$$

$$\frac{1}{\mathrm{m}^2} = \frac{1}{\mathrm{N}} \times \frac{\mathrm{N\,m}}{\mathrm{m}^3}. \tag{77d}$$

We note that the right term in N m$/m^3$ is like an energy density and the left term in m^{-2} is like a curvature.

We note that $\frac{24(1-\nu^2)}{Eh^2}$ has the same dimension as $\kappa = \frac{8\pi G}{c^4}$ (N^{-1}). We can concider κ as an equivalent flexibility of the space fabric and $1/\kappa$ as an equivalent rigidity of this frame. With the relationships between the curvatures and the second derivatives of z displacements (w) we have

$$\frac{1}{R_x} = \frac{\partial^2 w_{(x,y)}}{\partial x^2},\quad \frac{1}{R_y} = \frac{\partial^2 w_{(x,y)}}{\partial y^2},\quad \frac{1}{R_{xy}} = \frac{\partial^2 w_{(x,y)}}{\partial x \partial y} \tag{78}$$

and the relations between strains and curvatures are

$$\varepsilon_{xx} = -\frac{z}{R_x},\qquad \varepsilon_{yy} = -\frac{z}{R_y},$$

$$\gamma = \varepsilon_{xy} = -2z\frac{\partial^2 w_{(x,y)}}{\partial x \partial y} = -2z\frac{1}{R_{xy}}. \tag{79}$$

The strain tensor ε_{ij} can then be expressed in terms of curvature tensor R_{ij} of the thin plate:

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix} = -z\begin{bmatrix} \frac{1}{R_x} & 2\frac{1}{R_{xy}} \\ 2\frac{1}{R_{xy}} & \frac{1}{R_y} \end{bmatrix} = -zR_{ij} \tag{80}$$

or

$$\left\{\frac{1}{R_x};\frac{1}{R_y};\frac{1}{R_{xy}}\right\}\begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 2(1-\nu) \end{pmatrix}\begin{pmatrix} \frac{1}{R_x} \\ \frac{1}{R_y} \\ \frac{1}{R_{xy}} \end{pmatrix} = \frac{24(1-\nu^2)}{Eh^2} \times \frac{\mathrm{d}U}{\mathrm{d}x\mathrm{d}yh}. \tag{81}$$

Therefore, expression (82) from (76m), can be considered as equivalent at a curvature tensor whose radii of curvature in each direction are infinite ($R \to L$).

$$\begin{bmatrix} \frac{1}{L^2} & 0 \\ 0 & \frac{1}{L^2} \end{bmatrix}\begin{bmatrix} (\varepsilon_{xx})^2 & 0 \\ 0 & (\varepsilon_{yy})^2 \end{bmatrix}. \tag{82}$$

So, we define

$$R_{ij} = \begin{bmatrix} \frac{1}{L^2} & 0 \\ 0 & \frac{1}{L^2} \end{bmatrix}\begin{bmatrix} (\varepsilon_{xx})^2 & 0 \\ 0 & (\varepsilon_{yy})^2 \end{bmatrix} \tag{83a}$$

and we have tensor of strain energy density as

$$T_{ij} = \begin{bmatrix} T_{xx} & 0 \\ 0 & T_{yy} \end{bmatrix}. \tag{83b}$$

Expression (76k) can therefore be written as

$$R_{ij} = \frac{8\pi G}{c^4} T_{ij}. \tag{84}$$

We consider into this expression (76m) an infinite radius of curvature which therefore implies a zero scalar curvature. Indeed, we assume $\varepsilon_{xx} = -\varepsilon_{yy}$ and after rising the second index of the Ricci tensor we get the diagonal part $\left(\varepsilon_{xx}; -\varepsilon_{yy}\right)$, for which the trace is zero.

$$\frac{1}{\mathrm{m}^2} = \frac{1}{\mathrm{N}} \times \frac{\mathrm{N\,m}}{\mathrm{m}^3}. \tag{85}$$

In this expression (eq. (84)) R_{ij} is the equivalent curvature tensor of the space and T_{ij} is the strain energy tensor of the deformed space.

To reach the parallelism with the Einstein's field equation where $G_{\mu\nu}$ cover the curvature tensor of the space and $T_{\mu\nu}$ cover the energy density out of the space (e.g., the Sun curved the space but is not the space itself), we have to consider in our analogy that the work of the external forces is equal to the work of the internal forces created by the strain energy (see (8d), (5a) and (10e)).

$$T_{\text{external}} = T_{\text{internal}}. \tag{86}$$

In this case, our analogy in 2D (analogy (76m)) is close to the Einstein's field equation in four dimensions (5a).

We thus managed to find κ (see §6), by passing through the mechanical components of the stress tensor (and not via the temporal component of the tensor as Einstein did it to be correlated with the Newton's approach in weak gravitational field) corresponding to the internal work of the space fabric which is equal to the external work of the applied masses.

$$U = W_{\text{int}} = W_{\text{ext}}. \tag{87}$$

5.2.3 *Study of a vertical cylinder in pure torsional space (see figures 5 and 14) – use of the shear velocity of the correlated shear wave with the shear stress tensor* [54]. We assume in this section that the coalescence of two black holes for example, creates by their rotations, a torsion of the space as shown in figure 3 and refs [12,13]. In weak field, for space only, the metric is given in (12a). Einstein's gravitational equation in a weak field for space is

$$\partial^\lambda \partial_\lambda \overline{h}_{ij} = \Box \overline{h}_{ij} = -\frac{16\pi G}{c^4} T_{ij}\ . \tag{88}$$

The result for a polarised wave $A_\times$ is eq. (30b). With formula (31f) demonstrated in §4.3.2.1, we have a link between the general relativity (disturbance of the spatial part of the metric h_{ij}) and the theory of elasticity (the strain ε_{ij} of the elastic medium) (see 12c). So, based on (30b) and (12c), the spatial perturbation of the metric $h_{\mu\nu(A_\times)}$ is in correspondence with the following strain tensor [10,54]:

$$\varepsilon_{xy} = \begin{bmatrix} 0 & \varepsilon_{xy} = \varepsilon & 0 \\ \varepsilon_{yx} = \varepsilon & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{89}$$

Thanks to the Hooke's and elasticity formula, there is a correspondence with the strain tensor ε_{xy} (89) and the stress tensor σ_{xy} (90):

$$\sigma_{xy} = \begin{bmatrix} 0 & \tau_{yx} & 0 \\ \tau_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{90}$$

This type of stress tensor is representative of a cylinder in pure torsion, twisted by a torque M_t (see figure 14). For this section, we consider the cube of normal to the facet $\vec{z}$ at a point Q (τ stresses distribution).

Note: The attentive reader will, however, not have failed to notice that the facets subjected to traction and compression are not in the shear plane perpendicular to the direction of the wave (figures 6, 8 and 11), as is the case with gravitational waves, but inclined to 45° (figure 14). This approach is therefore only a simplified model to check the possible value of κ in accordance with the stress tensors (33b) connected to the pure torsion studied according to the elasticity theory [54].

The torsional strain energy is

$$U = \frac{1}{2}\int_0^L \frac{M_t^2}{\mu I_t} \mathrm{d}x = \frac{1}{2}\frac{M_t^2 L}{\mu I_t}. \tag{91a}$$

With θ the angular displacement of the point Q located on the outer surface of the cylinder, we have:

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right) = \frac{M_t}{\mu I_t}. \tag{91b}$$

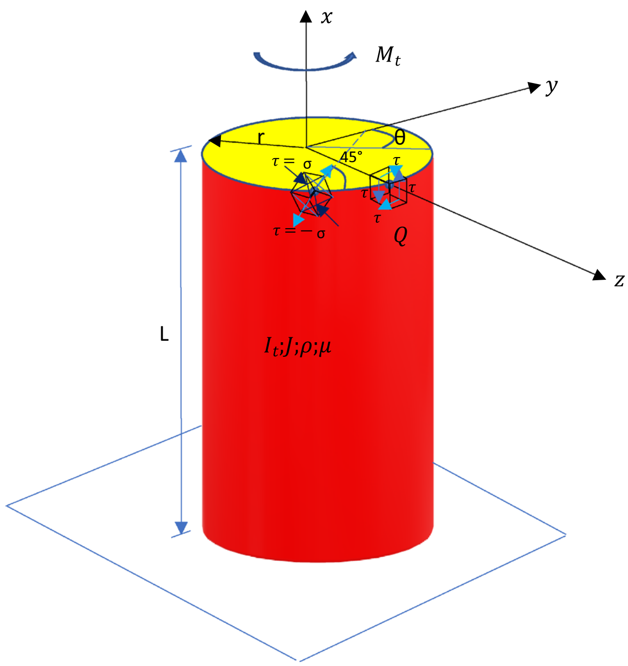


Figure 14. Cylinder clamped in pure torsion [54].

We assume that the torsion torque is constant. Introducing (91b) in (91a) we get

$$U = \frac{1}{2}\mu I_t \left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)^2 L. \tag{92}$$

From expression (92), the equivalent torsional curvature is extracted:

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)^2 = \frac{2}{\mu I_t}\frac{U}{L}. \tag{93}$$

The relationship between the torque M_t and the rotation θ gives the torsional stiffness k according to expression (94a):

$$M_t = k\theta. \tag{94a}$$

By integrating (91b) with respect to x, we can extract a new expression for θ:

$$\theta = \frac{M_t L}{\mu I_t} \tag{94b}$$

or

$$M_t = \frac{\mu I_t}{L}\theta. \tag{94c}$$

Comparing (94c) with (94a), the expression of the torsional stiffness is therefore:

$$k = \frac{\mu I_t}{L}. \tag{95}$$

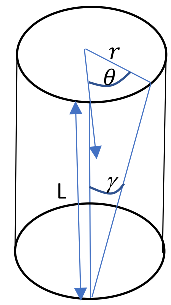


Figure 15. Definition of the shear strain γ.

Introducing (95) into (91a) yields

$$U = \frac{1}{2}\frac{M_t^2 L}{\mu I_t} = \frac{1}{2}\frac{(M_t)^2}{k}. \tag{96a}$$

The introduction of (94a) in (96a) gives

$$U = \frac{1}{2}k\theta^2. \tag{96b}$$

By the Hooke's law (14) we have a relationship between the stress τ and the shear stain γ. Using figure 15, we determine the relationship between the shear strain γ and the angular displacement θ.

We have geometrically:

$$\mathrm{tang}\gamma \approx \gamma = \frac{r\theta}{L} \tag{97a}$$

and the relationship between the stress and the strain (14) is as follows:

$$\tau = G\gamma = G\frac{r\theta}{L}. \tag{97b}$$

By introducing θ from expression (97a) into (96b) we obtain a new formula of the torsional strain energy of the cylinder:

$$U = \frac{1}{2}k\frac{\gamma^2 L^2}{r^2}. \tag{98}$$

That we can rewrite as

$$\frac{\gamma^2}{r^2} = \frac{2U}{kL^2}. \tag{99}$$

We can now replace the torsional stiffness k by its value (95). We extract the length of the cylinder and we obtain

$$L = \frac{\mu I_t}{k}. \tag{100}$$

The introduction of (100) into (99) yields

$$\frac{\gamma^2}{r^2} = \frac{2kU}{\mu^2 I_t^2}. \tag{101}$$

The torsional inertia I_t of a cylinder expressed as a function of S, the surface of the cylinder, is written as

$$I_t = \frac{\pi d^4}{32} = \frac{\pi r^4}{2} = S\frac{r^2}{2}. \tag{102}$$

By introducing I_t in (101) we obtain

$$\gamma^2 r^2 = \frac{8kU}{\mu^2 S^2}. \tag{103}$$

Introducing (104) into (103), we obtain

$$S_c = \frac{V}{L} \tag{104}$$

$$\frac{1}{L^2}\gamma^2 = 8\frac{k}{r^2\mu^2 V} \times \frac{U}{V}\ . \tag{105}$$

We now consider the dynamic behaviour of the cylinder consisting of an elastic substance of density ρ, Young's modulus E, Poisson's ratio ν obtained from of the vacuum energy. The fundamental equation of dynamics is derived from the sum of the kinetic energy and torsional strain energy:

$$U_C + U = \frac{1}{2}J\omega^2 + \frac{1}{2}k\theta^2. \tag{106}$$

For the mechanical characteristics of the cylinder made of a space equivalent material, the following definitions are given:

The moment of inertia of a rotating cylinder is

$$J = \frac{1}{2}\rho\pi r^4 L. \tag{107}$$

The circular frequency is

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} = 2\pi f \tag{108}$$

or

$$\omega^2 = \frac{\mu I_t}{JL}. \tag{109}$$

By introducing (108) into (106) we get

$$U_C + U = \frac{1}{2}J\left(\frac{\mathrm{d}\theta_{(t)}}{\mathrm{d}t}\right)^2 + \frac{1}{2}k\theta_{(t)}^2. \tag{110a}$$

By derivative with respect to t (110a) we obtain, looking for the natural frequency of the cylinder:

$$J\frac{\mathrm{d}^2\theta_{(t)}}{\mathrm{d}t^2} + k\theta_{(t)} = 0. \tag{110b}$$

By introducing the formula of k (95) and J (107) in (110b) we obtain

$$\frac{1}{2}\rho\pi r^4 L\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{\mu I_t}{L}\theta = 0. \tag{110c}$$

Introducing the expression of I_t (102) in (110c) gives

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{2\mu\frac{\pi r^4}{2}}{L\rho\pi r^4 L}\theta = 0 \tag{110d}$$

and after simplification we get

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{\mu}{\rho L^2}\theta = 0. \tag{110e}$$

We verify the dimensional equation of the term before θ:

$$\omega^2 = \frac{k}{J} = \frac{\mu I_t}{JL} = \frac{\mu\frac{\pi r^4}{2}}{\frac{1}{2}\rho\pi r^4 L} = \frac{\mu}{\rho L^2}$$

$$= \frac{\frac{\mathrm{kg\,m}}{\mathrm{s^2 m^2}}}{\frac{\mathrm{kg}}{\mathrm{m^3}}\mathrm{m}^2} = \frac{1}{s^2}. \tag{111a}$$

By replacing the shear modulus μ as a function of k (95) in expression (111a) we obtain

$$\omega^2 = \frac{k}{\rho L I_t}. \tag{111b}$$

In expression (111b), L is extracted and multiplied each side by the surface S of the tube to obtain a new expression of the volume V of the cylinder:

$$L\pi r^2 = V = \frac{k}{\omega^2\rho I_t}\pi r^2. \tag{112a}$$

By introducing V (112a) in (105) we get

$$\frac{1}{L^2}\gamma^2 = 8\frac{k}{r^2\mu^2\left(\frac{k}{\omega^2\rho I_t}\pi r^2\right)} \times \frac{U}{V} \tag{112b}$$

or after simplification

$$\frac{1}{L^2}\gamma^2 = 8\frac{\omega^2\rho I_t}{\pi r^4\mu^2} \times \frac{U}{V}\ . \tag{112c}$$

Introducing the expression of I_t (102) into (112c) we get

$$\frac{1}{L^2}\gamma^2 = 4\rho\left(\frac{\omega}{\mu}\right)^2 \times \frac{U}{V} = 4\rho\left(\frac{\omega}{\mu}\right)^2 \times T. \tag{112d}$$

By replacing ω and the shear modulus μ by their values, we obtain

$$\frac{1}{L^2}\gamma^2 = 4\frac{4\pi^2 f^2\rho}{\left(\frac{E}{2(1+\nu)}\right)^2} \times \frac{U}{V} \tag{112e}$$

or after simplification

$$\frac{1}{L^2}\gamma^2 = 4(1+\nu)^2\frac{16\pi^2 f^2\rho}{E^2} \times \frac{U}{V}. \tag{112f}$$

By multiplying and dividing expression (112f) by ρ we get

$$\frac{1}{L^2}\gamma^2 = 4(1+\nu)^2\frac{16\pi^2 f^2\rho^2}{E^2\rho} \times \frac{U}{V} \tag{112g}$$

or

$$\frac{1}{L^2}\gamma^2 = 64\pi(1+\nu)^2\frac{\pi f^2}{\rho}\left(\frac{\rho}{E}\right)^2 \times \frac{U}{V}. \tag{112h}$$

This time we use the definition of the speed of a transverse wave (torsion wave) acting by shear (see 41) and we obtain

$$\left(\frac{\rho}{E}\right)^2 = \frac{1}{4c^4(1+\nu)^2}. \tag{112i}$$

We introduce (112i) in (112h) and we obtain

$$\frac{1}{L^2}\gamma^2 = 64\pi(1+\nu)^2\frac{\pi f^2}{\rho}\frac{1}{4c^4(1+\nu)^2} \times \frac{U}{V} \tag{112j}$$

or after simplification we get the final result

$$\frac{\gamma^2}{L^2} = 16\pi\frac{\pi f^2}{\rho}\frac{1}{c^4} \times \frac{U}{V}. \tag{112k}$$

Assuming that the external work T is equal to the internal work U, and comparing with (88), we confirm that

$$G = \frac{\pi f^2}{\rho} \tag{113}$$

and

$$\frac{1}{L^2}\gamma^2 = 16\pi\frac{G}{c^4} \times T. \tag{114}$$

That we have to compare (see 28b) with

$$\Box\left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\overline{\varepsilon}\right) = -\frac{16\pi G}{c^4}T_{\mu\nu}. \tag{115}$$

We therefore have a factor 2 on the left term of (115) (dimension $1/\mathrm{m}^2$) which allows us to find the usual value of κ with a factor 8.

$$\frac{1}{L^2}\gamma^2 => \Box\left(2\varepsilon_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}2\overline{\varepsilon}\right), \tag{116}$$

with $\Box$ the d'Alembertian.

So, via (112d) and (116), it will be necessary to divide by 2 the factor before U/V in (112d) to obtain κ.

6. Deduction of the parallelism of a new mechanical expression of the gravitational constant G and Einstein's constant κ based on the Ligo and Virgo measurements

6.1 *General*

The parallelism between the Timoshenko approaches (see (57), (76h), (76i) or (114)) and the Einstein's approach (5a) is therefore demonstrated three times (see §5.2). We note that this parallelism is obtained by using the wave propagation in the medium as the transverse gravitational waves in eq. (5a) is a tensor expression (tensor of the curvatures R_{ij} connected to the strain tensor ε_{ij} and strain energy density tensor T_{ij}) (see (84)) and is based on the Hooke's law (13), the spring elastic strain energy and the eigen circular frequency of the spring/mass system assimilating the tube to a stiffness spring k and mass m. In all the calculations carried out, we thus obtain that the gravitational constant G can be expressed as a function of the density ρ and of the natural characteristic frequency f of an elastic microstructure constituting the vacuum space.

$$G = \frac{\pi f^2}{\rho}.$$

On the basis of this equation of G, we can re-express now the Einstein's constant κ (4a) in mechanical terms. As the displacements of the laser mirrors in the interferometers are limited to forward or backward motions, we only consider the associated traction/compression waves (§5.2.1 and §5.2.2) as representatives of that actual data.

6.2 *In the case of §5.2.1: One arm of the interferometer*

Taking into account the following points:

(a) With compression, tensile wave velocities (35a) correlated with the stress tensor (33a) and the polarised wave A_+ [53],
(b) with the definition of the circular frequency ω of the elastic medium, we obtain based on (54c),

we obtain

$$\kappa = \frac{8\pi G}{c^4} = 2\rho\left(\frac{\omega}{E}\right)^2. \tag{117a}$$

We verify the dimensional equation (117b) of κ, indeed:

$$\rho\left(\frac{\omega}{E}\right)^2 => \frac{\mathrm{kg}}{\mathrm{m}^3}\left(\frac{1}{\mathrm{s}\frac{\mathrm{kg}}{\mathrm{ms}^2}}\right)^2 = \frac{\mathrm{s}^2}{\mathrm{kgm}} = \frac{1}{N}. \tag{117b}$$

6.3 *In the case of §5.2.2 – two arms of the interferometer correlated via the Poisson's ratio* ν

Taking into account the following points:

(a) With compression, tensile wave velocities (35a) are correlated with the stress tensor (33a) and the polarised wave A_+ [53],
(b) with (76f) or (76h) and (76i),
(c) with the definition of the circular frequency ω of the elastic medium,

we obtain

$$\kappa = \frac{8\pi G}{c^4} = (1+\nu)\rho\left(\frac{\omega}{E}\right)^2. \tag{118}$$

Note: We confirm well the parallelism previously demonstrated in §5.1.2. Indeed κ is well proportional to $(1+\nu)$ in the expression, see (43c). In addition, with $\nu = 1$ we obtain (117a).

6.4 *In the case of §5.2.3 – Pure shear and torsion approach*

Taking into account the following points:

(a) With shear wave velocities (41) correlated with the stress tensor (33b) and the polarised wave $A_\times$ [54].
(b) With eqs (112d) and (28b) and taking into account that we have a coefficient 2 on the left term which must be taken into account to reach κ.

Following these hypotheses, we thus obtain from (112d), (115), (116)

$$\kappa = \frac{8\pi G}{c^4} = 2\rho\left(\frac{\omega}{\mu}\right)^2. \tag{119}$$

We obtain a logical result for this case of pure torsion. The formula is the same as that obtained in eqs (117a) and (117b), but this time the shear modulus μ plays the role of Young's modulus E.

7. Physical approach or simple mathematical artefact?

Does all this have such a physical reality, or is it just a mathematical coincidence? Let us take a look at this about the vacuum. The Casimir force [55] in connection with quantum field theory shows that the vacuum has a non-zero ground state, a non-zero energy and therefore, according to the special relativity, a non-zero equivalent mass. The space is therefore a physical object and so is not empty. The measurement of Casimir's force in the vacuum [55] confirms that

1. Particles and virtual antiparticles are created and annihilated spontaneously, generating a force that brings together two parallel plates placed in the vacuum. There is energy in the vacuum.
2. There is a fundamental state different from 0 from the quantified vacuum (QED).
3. There is a scalar field (Brout, Englert, Higgs field) in vacuum space.
4. Emptiness is not a void.

In addition, the vacuum energy has been measured and calculated:

- According to the cosmological constant measurements: $\rho = 1\times10^{-29}$ g/cm^3 (1×10^{-26} kg/m^3),
- According to quantum field theory: $\rho = 1.11\times10^{93}$ g/cm^3,
- According to the cosmological constant measurements: $T_{\text{vacuum}} = 8.987551787\times10^{-10}$ kg m^2/s^2/m^3,
- According to the quantum field theory: $E_{\text{vacuum}} = 1\times10^{113}$ kg m^2/s^2/m^3.

Even if the values are so different, the vacuum energy is not null. That is the fundamental point here.

The best way to know which value is the right one is to measure the Young's modulus via the Casimir test (see §9.4) and to determine the good value of the energy by the formula given in tables 2 and 3. So, following the quantum field theory, the vacuum energy E_v is quantified at the fundamental state:

$$E_v = \frac{1}{2}hf = \frac{1}{2}h\frac{\omega}{2\pi} = \frac{1}{2}\hbar\omega, \tag{120}$$

where E_v is the vacuum energy, h is the Planck's constant and ω is the circular frequency.

In addition, the special relativity laws apply to the vacuum. The energy of emptiness is

$$E_v = mc^2. \tag{121}$$

Using (120) and (121), we deduce the equivalent mass expression m present in vacuum:

$$\frac{1}{2}\frac{\hbar\omega}{c^2} = m \tag{122}$$

and the expression of the vacuum density ρ is

$$\frac{1}{2}\frac{\hbar\omega}{c^2V} = \frac{m}{V} = \rho. \tag{123}$$

It seems therefore coherent that formula (123) is related to the density ρ and to the specific circular frequency ω (via f) that can be, in this case, those of the vacuum (via vacuum energy). We note also that eqs (113), (117a), (118), (119) are related to ω and ρ.

8. Results

8.1 *Numerical application at the vacuum energy – longitudinal wave in the interferometric tubes*

8.1.1 *Theoretical development.* We note T as a vacuum energy density measured in $\mathrm{J/m^3}$ with V a volume of space elastic material that have to be determined.

$$T \times V = E_v = mc^2 = \rho \times V \times c^2. \tag{124a}$$

So we can extract the density ρ of (124a):

$$\rho = \frac{E_v}{Vc^2}. \tag{124b}$$

Considering eq. (120) we extract the frequency f:

$$f = \frac{2E_v}{h}. \tag{124c}$$

From (120), the energy density of the vacuum E_v/V is therefore written as a function of the circular frequency ω and the Planck's constant h:

$$\frac{E_v}{V} = T = \frac{h\omega}{4\pi V}. \tag{125}$$

We extract from (125) the circular frequency:

$$\omega = \frac{4\pi E_v}{h}. \tag{126}$$

By substituting expressions (124b) and (126) in expression (117a), we obtain the Young's modulus E:

$$E = Y = 2E_v^{3/2}\frac{c}{h}\sqrt{\frac{\pi}{GV}}. \tag{127a}$$

By substituting expressions (124b) and (127a) in (35a), we obtain the vacuum energy E_v which depends on volume V:

$$E_v = \rho V c^2 = \frac{Gh^2}{4\pi V c^2} \tag{127b}$$

and from the expression (127b) we determine the vacuum density ρ:

$$\rho = \frac{Gh^2}{4\pi V^2 c^4}. \tag{128}$$

By expressions (35a) and (128) we deduce a new expression of the Young's modulus E:

$$E = \rho c^2 = \frac{G}{4\pi}\left(\frac{h}{cV}\right)^2. \tag{129}$$

Expressions (124c) and (127b) draw the expression of f:

$$f = \frac{Gh}{2\pi V c^2} \tag{130}$$

and expressions (130) and (53b) give the circular frequency:

$$\omega = \frac{Gh}{Vc^2}. \tag{131}$$

We must therefore check the small volume hypothesis (quantum approach). According to the quantum field theory, vacuum density $\rho = 1.11 \times 10^{93}\ \mathrm{g/cm^3}$ and the vacuum energy density $T = 1.00 \times 10^{113}\ \mathrm{J/m^3}$. In addition, the fundamental physic constants are:

$$G = 6.67408 \times 10^{-11}\ \mathrm{m^3/(kg\ s^2)}$$
$$h = 6.62607004 \times 10^{-34}\ \mathrm{m^2\ kg/s}$$
$$c = 299\ 792\ 458\ \mathrm{m/s}$$
$$\kappa = 2.0766 \times 10^{-43}\mathrm{N^{-1}}.$$

As all the formulae are functions of volume V, now we are looking for the value of this volume V that allows to satisfy all the physic constants. From this volume V, it will then be possible to determine the r dimension of the space fabric fibres and the Young's modulus E of the space material.

8.1.2 *Magnitude obtained for the new parameters of G based on vacuum energy.* So, the question is therefore what volume V does to simultaneously satisfy expressions (4a), (35a) and (113). Once all iterative calculations have been carried out, we obtain the results given in table 2.

8.2 *Numerical application at the vacuum energy – global approach by a shear torsion wave*

8.2.1 *Theoretical development.* The approach is the same as in §8.1 but the Young's modulus E is replaced throughout the equation by the shear modulus μ. The energy density T follows eqs (124a) and (125). The density ρ follows eq. (124b). The frequency f is defined in eq. (124c). The circular frequency is equal to (126). By substituting expressions (124b) and (126) in expression (119), we obtain the Young's modulus E.

$$\mu = 2E_v^{3/2}\frac{c}{h}\sqrt{\frac{\pi}{GV}}\ . \tag{132}$$

With the definition of the shear modulus μ we obtain the Young's modulus E:

$$E = Y = 4(1+\nu)E_v^{\frac{3}{2}}\frac{c}{h}\sqrt{\frac{\pi}{GV}}. \tag{133}$$

By substituting expressions (124b) and (127a) in (41), eq. (127b) is again obtained on the vacuum energy E_v as a function of the volume V considered:

$$E_v = \rho V c^2 = \mu V = \frac{Gh^2}{4\pi V c^2} \tag{134}$$

Table 2. Numerical application (case of Young's modulus approach)

Parameters	Physical objects With $v = 1$ Case of longitudinal waves (pure compression/traction)	Units	Values with $E_{\text{vacuum}} = 1\times10^{113}$ J/m^3
Volume	V	m^3	1.61×10^{-104}
Radius of V (link with the string theory and Planck's length)	r if $V = \frac{4}{3}\pi r^3$	m	1.566×10^{-35}
Vacuum energy	$E_v = \rho V c^2 = \frac{Gh^2}{4\pi V c^2}$	$\frac{\text{kg m}^2}{\text{s}^2} = \text{J}$	1.61×10^9
Vacuum energy density (link with the quantum field heory)	$T = \frac{E_v}{V}$	$\frac{\text{kg}}{\text{s}^2\ \text{m}} = \frac{\text{J}}{\text{m}^3}$	1.00×10^{113}
Density	$\rho = \frac{Gh^2}{4\pi V^2 c^4}$	$\frac{\text{kg}}{\text{m}^3}$	1.11×10^{96}
Young's modulus (link with the elasticity theory)	$E = Y = \rho c^2 = \frac{G}{4\pi}\left(\frac{h}{cV}\right)^2$ $E = Y = 2E_v^{3/2}\frac{c}{h}\sqrt{\frac{\pi}{GV}}$	$\frac{\text{kg}}{\text{s}^2\ \text{m}} = \text{Pa}$	1.00×10^{113}
Speed of light (link with the special relativity)	$c = \sqrt{\frac{E}{\rho}}$	$\frac{\text{m}}{\text{s}}$	299792458
Frequency	$f = \frac{Gh}{2\pi V c^2}$	1/s	4.861×10^{42}
Period (link with the Planck's time)	$T = \frac{1}{f}$	s	2.05684×10^{-43}
Gravitation constant (link with the Newton's gravitation)	$G{=}\pi f^2 \frac{1}{\rho}$	$\frac{\text{m}^3}{\text{kg s}^2}$	6.67408×10^{-11}
Circular frequency	$\omega = \frac{Gh}{Vc^2} = 2\pi f = \frac{4\pi E_v}{h}$	1/s	3.05×10^{43}
Einstein's constant (link with the general relativity)	$\kappa{=}2\rho\left(\frac{\omega}{E}\right)^2$	$\frac{1}{\text{Newton}}$	2.07658×10^{-43}

and from expression (127b) the vacuum density ρ (128) is again obtained. By expressions (41) and (128) a new expression of the shear modulus μ is deduced:

$$\mu = \frac{G}{4\pi}\left(\frac{h}{cV}\right)^2 \tag{135}$$

and of the Young's modulus:

$$E = \frac{(1+\nu)G}{2\pi}\left(\frac{h}{cV}\right)^2. \tag{136}$$

Expressions (124c) and (134) restore again expression of f (130) and the expressions (130) and (53b) give the circular frequency (131). The space is thus quantified via radius r of the space fibres:

$$r = \left(\frac{9}{64\pi^3} \times \frac{G}{\mu c^2}\right)^{1/6} h^{1/3} \tag{137}$$

or with the new definition of G:

$$r = \left(\frac{9}{64\pi^2} \times \frac{f^2}{\rho\mu c^2}\right)^{1/6} h^{1/3} \tag{138}$$

and the diameter d is

$$d = \left(\frac{9}{\pi^2} \times \frac{f^2}{\rho\mu c^2}\right)^{1/6} h^{1/3} = \left(\frac{9}{\pi^2\rho\mu}\right)^{1/6}\left(\frac{fh}{c}\right)^{1/3}. \tag{139}$$

8.2.2 *Magnitude obtained for the new parameters of G based on the vacuum energy*

So, the question is therefore what volume V does to simultaneously satisfy expressions (4a), (41) and (113). Once all iterative calculations have been carried out, we obtain the results given in table 3.

Table 3. Numerical application (case of shear modulus approach).

Parameters	Physical objects With $\upsilon = 1$ Case of shear waves (torsion)	Units	Values with $E_{\text{vacuum}} = 1\times10^{113}$ J/m^3
Volume	V	m^3	1.61×10^{-104}
Radius of V (link with the string theory and Planck's length)	r if $V = \frac{4}{3}\pi r^3$	m	1.566×10^{-35}
Vacuum energy	$E_v = \rho V c^2 = \mu V = \frac{Gh^2}{4\pi V c^2}$	$\frac{\text{kg m}^2}{\text{s}^2} = \text{J}$	1.61×10^9
Vacuum energy density (link with the quantum field theory)	$T = \frac{E_v}{V}$	$\frac{\text{kg}}{\text{s}^2\text{m}} = \frac{\text{J}}{\text{m}^3}$	1.00×10^{113}
Density	$\rho = \frac{Gh^2}{4\pi V^2 c^4}$	$\frac{\text{kg}}{\text{m}^3}$	1.11×10^{96}
Young's modulus/shear modulus (link with the elasticity theory)	$E = Y = 2\rho c^2(1+\nu)$ $= \frac{(1+\nu)G}{2\pi}\left(\frac{h}{cV}\right)^2$ $\mu = \frac{G}{4\pi}\left(\frac{h}{cV}\right)^2$ $E = Y = 4(1+\nu)E_v^{3/2}\frac{c}{h}\sqrt{\frac{\pi}{GV}}$	$\frac{\text{kg}}{\text{s}^2\text{ m}} = \text{Pa}$	4.00×10^{113}
Speed of light (link with the special relativity)	$c = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{2(1+\nu)\rho}}$ $c = \frac{1}{f}\sqrt{\frac{GE}{2\pi(1+\nu)}}$	$\frac{\text{m}}{\text{s}}$	299792458
Frequency	$f = \frac{Gh}{2\pi V c^2}$	1/s	4.861×10^{42}
Period (link with the Planck's time)	$T = \frac{1}{f}$	s	2.05684×10^{-43}
Gravitation constant (link with the Newton's gravitation)	$G = \pi f^2 \frac{1}{\rho}$	$\frac{\text{m}^3}{\text{kg s}^2}$	6.67408×10^{-11}
Circular frequency	$\omega = \frac{Gh}{Vc^2} = 2\pi f = \frac{4\pi E_v}{h}$	1/s	3.05×10^{43}
Einstein's constant (link with the general relativity)	$\kappa = 2\rho(\frac{\omega}{\mu})^2$	$\frac{1}{\text{Newton}}$	2.07658×10^{-43}

The results are identical to table 2 except for the Young's modulus that is multiplied by 4.

9. Discussion

9.1 *About the numerical values of the results obtained*

Are our numerical results physically acceptable? We obtain radius r of volume V in correlation with the string dimension defined in the string theory: 1.566×10^{-35} m. We obtain Young's modulus as 4.00×10^{113} Pa, which is compatible with the results of ref. [11] (4.4×10^{113} Pa, see §3.4, formula 3.13).

We obtain $\rho = 1.11\times10^{96}$ kg/m^3 compatible with the results of ref. [11] (1.30×10^{96} kg/m^3, see §3.4, formula 3.14). We find a period equal to 2.05684×10^{-43} s near the Planck's time that is logical because at this time all the interactions have to merge. So the magnitude obtained is correct. In addition, with this volume V we satisfy all the constant values of physics: The Planck's time where all the forces merge, G, c, κ. With expression (58) of G and taking into account eqs (129) and (130) we obtain

$$G = \frac{64}{9}\pi^3\frac{c^4\rho_v}{h^2}r^6 \tag{140}$$

and the numerical application gives

$$G = \frac{64}{9}\pi^3 \times \frac{(299792458)^4 \times 1.11 \times 10^{96} \times \left(1.566 \times 10^{-35}\right)^6}{\left(6.62607004 \times 10^{-34}\right)^2}.$$

$G = 6.67408000 \times 10^{-11}\ \mathrm{m}^3/\mathrm{kg\ s}^2$.

Thus this formulation unites all the theories of the current physics:

(a) quantum mechanics with the Planck constant h,
(b) special relativity with the speed of light c,
(c) quantum field theory with the vacuum energy (vacuum density) ρ_v,
(d) string theory with the size of the medium grain size r (string size),
(e) The gravitation with G.

9.2 *About the two approaches: Longitudinal and shear waves*

Irrespective of whether we adopt the approach following the longitudinal waves of pure compression/traction or pure shear waves (pure torsion) we obtain identical numerical values for all the parameters except Young's modulus recalibrated by the effect of the Poisson's ratio (see tables 2 and 3).

9.3 *Quid de Michelson–Morley and the existence of a certain Ether*

Considering an elastic body for the space medium immediately raises the question of Ether. We recall first the conclusions of Michelson and Morley's [56] which are:

"From all of the above, it seems reasonably certain that if there is a relative movement between the Earth and the luminous Ether, it must be small, small enough to refute the explanation of Fresnel's aberration. ..."

Few comments:

(a) It is important not to confuse the vibration of a medium supposed to be a luminescent Ether that does not exist (light spreads alone without need of medium) with a fabric of curved space whose light follows curvatures (e.g. around the Sun).
(b) Contrary to what is often said, Michelson and Morley did not write that there was no Ether but that it had to be small enough not to be detected,
(c) It is well known that the Higgs field does not interfere with the photon or with the light. It is then obvious that it is impossible to detect this field with light. It is therefore possible that the experience of Michelson and Morley with the light is not the right experience to detect the material that constitutes the elastic space fabric.
(d) We can see the shear strain of space created by the rotation of the Earth measured by the satellite probe B, as the expression of the torsion of space is akin to [15] a certain Ether characterised by $g_{\mu\nu}$. This experience can be considered as the right one to measure the twist of the space and the dynamic state of this new relativistic Ether.

The light follows the curvature of the space but does not need a luminescence medium to spread.

If the Ether of luminescence is dead [56], the relativistic Ether [20,21] appears to be alive.... In Einstein's letter to Lorentz of June 17, 1916 [20], we can read:

"I agree with you that the general theory of relativity is closer to the Ether hypothesis than the special theory. This new theory of Ether, however, would not violate the principle of relativity, because the state of this g_{mn} = The Ether would not be that of the rigid body in an independent state of motion, but each state of motion would depend on the position determined by the material processes".

9.4 *Experimental checking of the Young's modulus of the space medium*

An experimental way to establish the Young's modulus E of vacuum is to perform the Casimir test. This test consists of taking two metal plates spaced at a very short distance L_0, and to position them in the vacuum by simultaneously measuring force F and the horizontal displacements (see figure 16). Generally, the force alone is measured, and the displacement calculated in the Casimir experiment. The result of the test is to draw the stress/strain curve. Of course, the slope of the stress/strain curve corresponds to the Young's modulus E of the space medium according to eq. (142).

9.5 *Experimental checking of the shear of the space*

It will be interesting in the interferometer Ligo/Virgo to measure the eventual lateral movements of the laser beam (or the variation of angles between the two laser beams) to determine the eventual shear strain γ created on the space by the passage of gravitational waves (see figure 5).

10. Consequence on the time of this research

10.1 *Behaviour of time as an elastic material*

In this article, we focussed on the spatial part of Einstein's tensor. Based on the new definition of G, we can now return to the temporal part of the Einstein's tensor (see also [11], chapters 2.4.3 and 2.6). Einstein has demonstrated that the stress energy tensor $T_{\mu\nu}$ curves space but also the time at the location of this mass via a proportionality factor κ. This space–time deformation, mathematically characterised by the metric variation $g_{\mu\nu}$ is the gravitation. Near a black hole the curvature of time is such that time expands endlessly giving the impression that it stops. The question is therefore the following: can we consider time as elastic? For memory, some measured facts are:

(a) At high speed v, time expands: A clock placed in a rapidly moving airplane (t') slowed down compared to a clock stayed on Earth (t) (special relativity principle):

$$\Delta t' = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\Delta t. \tag{141}$$

(b) At high speed the distance contracts:

$$\Delta z' = \sqrt{1-\left(\frac{v}{c}\right)^2}\Delta z. \tag{142}$$

Therefore, time has an elastic behaviour, it lengthens or shortens and time has a behaviour opposite to that of space (dilation = negative contraction) (see (149)). The metric is connected to the interval, $\mathrm{d}s^2$. So in a non-inertial frame of reference for example (if the coordinates according to x, y, z do not vary), we have:

$$\mathrm{d}s^2 = g_{00}c^2\mathrm{d}t^2 \tag{143a}$$

or in the general case, the unknowns of the Einstein's equation are the 10 components of the metric tensor $g_{\mu\nu}$:

$$\begin{aligned}\mathrm{d}s^2 = {} & g_{00}c^2\mathrm{d}t^2 + 2g_{10}\mathrm{d}xc\mathrm{d}t \\ & +2g_{20}\mathrm{d}yc\mathrm{d}t + 2g_{30}\mathrm{d}zc\mathrm{d}t \\ & +g_{11}\mathrm{d}x^2 + 2g_{12}\mathrm{d}x\mathrm{d}y + 2g_{13}\mathrm{d}x\mathrm{d}z \\ & +g_{22}\mathrm{d}y^2 + 2g_{23}\mathrm{d}y\mathrm{d}z \\ & +g_{33}\mathrm{d}z^2.\end{aligned} \tag{143b}$$

For this we have to focus on the time component of the metric (3b):

$$g_{00} = \eta_{00} + h_{00} = 1 + 2\varepsilon_{00} \approx 1 + \frac{2\phi}{c^2}. \tag{144}$$

Thus, the expression of the perturbation $h_{\mu\nu}$ for the time component (00 index) is:

$$h_{00} = 2\varepsilon_{00} \approx \frac{2\phi}{c^2}. \tag{145a}$$

The equivalent strain of the time is therefore

$$\varepsilon_{00} \approx \frac{\phi}{c^2}. \tag{145b}$$

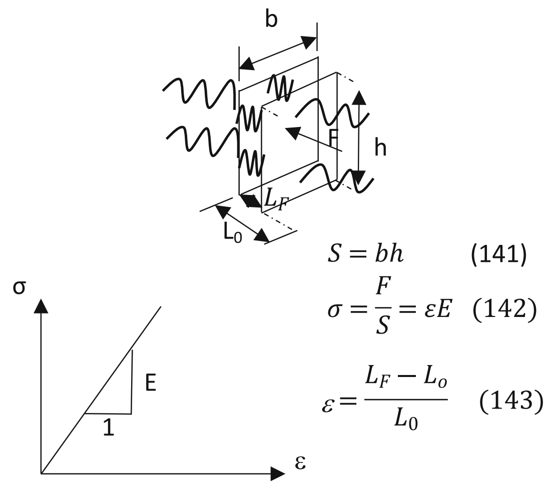


Figure 16. Test to measure the equivalent Young's modulus $E = Y$ of the space medium.

The gravitational potential of a sphere of radius R and mass M, is written as

$$\phi = \frac{GM}{R} = \frac{\frac{\mathrm{m}^3}{\mathrm{kg\ s}^2}\mathrm{kg}}{\mathrm{m}}. \tag{146}$$

By introducing (144) into (143b) we obtain

$$\varepsilon_{00} \approx \frac{GM}{Rc^2}. \tag{147a}$$

Taking into account the new definition of the function G (143) of the density ρ and the frequency f of the space material:

$$\varepsilon_{00} \approx \frac{\pi f^2 M}{\rho R c^2}. \tag{147b}$$

We square the expression (145b):

$$\varepsilon_{00}{}^2 \approx \frac{\pi^2 f^4 M^2}{\rho^2 R^2 c^4}. \tag{148a}$$

Taking into account eqs (76a) and (43c), the strain squared is proportional at $(1+\nu)$:

$$\varepsilon_{00}{}^2 \approx (1+\nu)\frac{\pi^2 f^4 M^2}{\rho^2 R^2 c^4}. \tag{148b}$$

Given the new definition of G we obtain

$$\varepsilon_{00}{}^2 \approx (1+\nu)\pi\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{f^2 M^2}{R^2\rho}. \tag{148c}$$

We multiply each side of eq. (148c) by $1/R^2$:

$$\frac{1}{R^2}\varepsilon_{00}{}^2 \approx (1+\nu)\pi\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{f^2 M^2}{R^3\rho R}. \tag{148d}$$

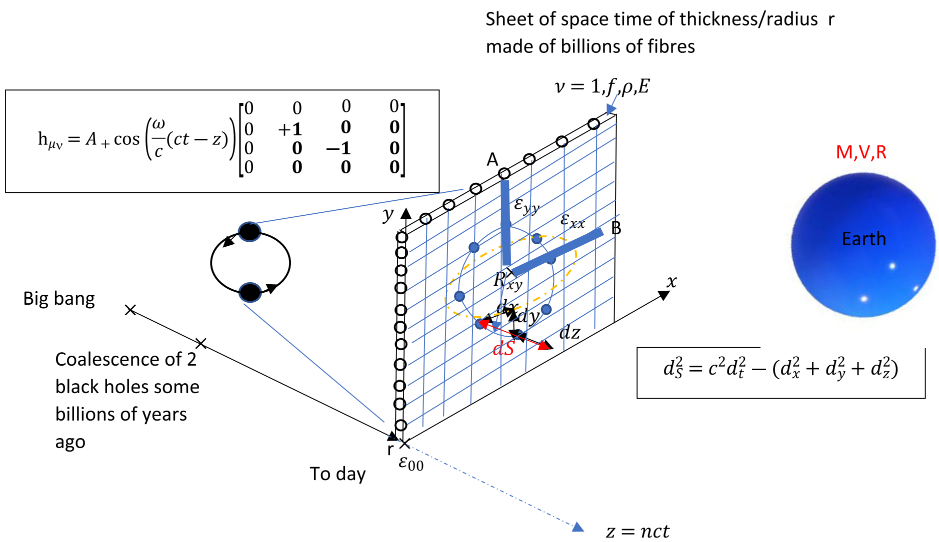


Figure 17. An instantaneous photo (taken at the speed of light) of the xy plane space deformed by the gravitational wave propagating along z direction.

For memory the volume V of a sphere is

$$V = \frac{4}{3}\pi R^3. \tag{148e}$$

We introduce this volume into eq. (147a) via R^3:

$$\frac{1}{R^2}{\varepsilon_{00}}^2 \approx (1+\nu)\pi\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{f^2M^2}{\frac{3V}{4\pi}\rho R} \tag{148f}$$

and we get after some calculations:

$$\frac{1}{R^2}{\varepsilon_{00}}^2 \approx \frac{4}{3}\pi^2(1+\nu)\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{f^2M^2}{V\rho R}\ . \tag{148g}$$

Noting that $\rho V = M$ and the last term of the equation as dimension of energy density U/V (see (148b)):

$$\frac{f^2M}{R} = \frac{\mathrm{kg}}{\mathrm{m\ s^2}} = \frac{\frac{\mathrm{kg\ m^2}}{\mathrm{s^2}}}{\mathrm{m^3}} = \frac{U}{V} \tag{148h}$$

and we obtain

$$\frac{1}{R^2}{\varepsilon_{00}}^2 \approx \frac{4}{3}\pi\pi(1+\nu)\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{U}{V}. \tag{148i}$$

Taking into account that $3.1 \approx 3$, we obtain

$$\frac{1}{R^2}{\varepsilon_{00}}^2 \approx 4\pi(1+\nu)\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{U}{V}. \tag{148j}$$

With ($\nu = 1$) (see §4.4):

$$\frac{1}{R^2}{\varepsilon_{00}}^2 \approx 8\pi\frac{\pi f^2}{\rho}\frac{1}{c^4}\frac{U}{V}. \tag{148k}$$

We obtain, for the temporal component of the disturbance of the metric, an expression similar to that which one obtains by considering a beam in pure compression/traction (see (57)). So, time behaves like an elastic material.

10.2 *Relates time lapses to the thickness of the space fibres*

For memory, some measured facts are

(a) Time slows down when it is immersed in a gravitational field (see general relativity). Therefore,

 (1) Gravitation curves space–time. Time is no longer absolute and become an illusion.
 (2) The more there is gravitation, the more there is curvature and the more there is tension in the material and the more the slow down time expands.

(b) The time acts with a different sign of the space,

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2. \tag{149}$$

For a gravitational wave, the passage of time is a succession of instantaneous pictures following z direction. Figure 17 shows an instantaneous photo of space–time taken during the passage of a gravitational wave.

In figure 17, R_{xy} is the radius of the xy plane, n is the whole number of small quantum distances $c \times t$, $2r$ is

Table 4. Formula obtained in modified general relativity, new expression of κ.

Parameters	Formula	Formula
Strains	$\varepsilon_{ii} = \frac{\delta L}{L}$	$\varepsilon_{ij} = \frac{\gamma}{2}$
Polarised wave $h_{\mu\nu}$	$A_+ \cos\left(\frac{\omega}{c}(ct - z)\right) \times \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{+1} & \mathbf{0} & \mathbf{0} \\ 0 & \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ 0 & 0 & \mathbf{0} & \mathbf{0} \end{bmatrix}$	$A_\times \cos\left(\frac{\omega}{c}(ct - z)\right) \times \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{0} & \mathbf{+1} & \mathbf{0} \\ 0 & \mathbf{+1} & \mathbf{0} & \mathbf{0} \\ 0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$
Associated stress tensor	$\sigma_{xy} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & -\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\sigma_{xy} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
Associated wave in the interferometer tube	Longitudinal compression/traction $c = \sqrt{\frac{E}{\rho}}$	Shear $c = \sqrt{\frac{\mu}{\rho}}$
Curvature = k energy density	$\frac{1}{L^2}(\varepsilon)^2 = 4\pi(1+\nu) \times \left(\frac{\pi f^2}{\rho}\right)\frac{1}{c^4} \times \frac{U}{V}$	$\frac{1}{L^2}(\gamma)^2 = 16\pi \frac{\pi f^2}{\rho}\frac{1}{c^4} \times \frac{U}{V}$
Einstein's gravitational field	$G^{\mu\nu} = -\frac{8\pi G}{c^4}T^{\mu\nu}$	$\partial^\lambda\partial_\lambda \overline{h}_{ij} = \Box\overline{h}_{ij} = -\frac{16\pi G}{c^4}T_{ij}$
Proposed Izabel's field equation	$G^{\mu\nu} = -(1+\nu)\rho\left(\frac{\omega}{E}\right)^2 T^{\mu\nu}$ $G^{\mu\nu} = -2\rho\left(\frac{\omega}{E}\right)^2 T^{\mu\nu}$	$G^{\mu\nu} = -2\rho\left(\frac{\omega}{\mu}\right)^2 T^{\mu\nu}$
G	$G = \pi\left(\frac{f^2}{E}\right)c^2$ $G = \pi f^2 \frac{1}{\rho} = \frac{\omega^2}{4\pi\rho}$	$G = \pi\left(\frac{f^2}{\mu}\right)c^2$ $G = \pi f^2 \frac{1}{\rho} = \frac{\omega^2}{4\pi\rho}$
Poisson's ratio	$\nu = 1$	$\nu = 1$
Young's modulus $E{=}Y$	$E = Y = \rho c^2$	$E = Y = 2\rho c^2(1+\nu)$ $E = Y = 4\rho c^2$
Speed c	$c = \frac{1}{f}\sqrt{\frac{GE}{\pi}}$	$c = \frac{1}{f}\sqrt{\frac{GE}{2\pi(1+\nu)}}$
Frequency f	$f = \frac{Gh}{2\pi V c^2}$	$f = \frac{Gh}{2\pi V c^2}$
Einstein's constant κ	$\kappa = -(1+\nu)\rho\left(\frac{\omega}{E}\right)^2$ $\kappa = -2\rho\left(\frac{\omega}{E}\right)^2$	$\kappa = -8(1+\nu)^2\rho\left(\frac{\omega}{E}\right)^2$ $\kappa = -2\rho\left(\frac{\omega}{\mu}\right)^2$

the thickness of the space sheet (r is the radius of the fibre of space = 1×10^{-35} m).

The passage of time is the succession of its instantaneous photos following z. We notice that

(1) The perturbation $h_{\mu\nu}$ of the metric depends on the variable , $ct - z$, where ct and $-z$ are at the same level but with opposite sign.
(2) In the $h_{\mu\nu}$ matrix, we see that all the time terms and z terms are correlated at zero (see (30a) and (30b)).

In adddition, as the z-axis is confused with the time axis, we therefore have thin sheets of xy plane space, of thickness $2r$ (2×10^{-35} m) which follow each other in time along the z-axis.Therefore, we propose to relate rate of time lapse to the thickness of the fibre that are said to make up space (following z in the case of gravitational wave). As the gravitational wave passes from one plane of fibres to another (see figure 17), the different time lapses can be counted by these successive passages from one spatial diameter of the fibre to another (quantum of time). Time and space therefore appear to be somewhat quantified:

$$t_q = \frac{2r}{c}. \tag{150}$$

Time has a minimum duration corresponding to the time necessary to transmit information at the speed c from one fibre plane to another (multisandwich sheets). Based on (134) and (17), taking into account the definition V (148e) function of r, we obtain the quantified time lapse t_q:

$$t_q = \frac{2r}{c} = \left[\frac{18}{\pi^3}\frac{G(1+\nu)}{Ec^8}\right]^{1/6} h^{1/3}. \quad (151a)$$

With the new definition of G:

$$t_q = \left[\frac{9}{\pi^2}\frac{f^2}{\rho\mu c^8}\right]^{1/6} h^{1/3} \approx \left[\frac{f^2 h^2}{\rho\mu c^8}\right]^{1/6}. \quad (151b)$$

With the values of table 3:

$t_q = \quad 1.0451 \times 10^{-43}$ s

Value compatible with the Planck time:

$t_p \approx 1.0000 \times 10^{-43}$ s.

If we define $E_t = hf$ the time motor (energy), we get

$$t_q \approx \left[\frac{1}{\rho\mu c^8}\right]^{\frac{1}{6}} E_t^{\frac{1}{3}}. \quad (152)$$

We confirm that if the frequency f is zero, the spatial material disappears and time too disappears. Gravitation can bend time because it bends the fibres of the space fabric which therefore expands. The time passes step by step through each of its fibre, one after the other to transmit information, and so it expands like these fibres. This elasticity of time is a characteristic of its elastic behaviour defined by the strain ε_{00}.

11. Conclusions

In the light of the above, it seems logical to propose a new mechanical and physics expression of the Einstein's constant κ based on:

(1) The elastic behaviour of space and time.
(2) The postulate that space is a substance, an elastic material characterised by its characteristic frequency f, its elastic properties ($E = Y$, ν) and its density ρ (or energy via $E = mc^2$).
(3) The parallelism between the elasticity theory and general relativity (see §5.1.2 and [10]).
(4) The perfect correlation between the two gravitational waves polarisations ($h_{\mu\nu(A_+)}$, $h_{\mu\nu(A_\times)}$) with the two compression/traction and pure shear tensors of the space medium as a function of the facet considered in elasticity (case of a space medium cylinder twisted by rotation of two massive objects that merge) [10–13] and the link between the graviton of spin 2 with the image of the Mohr's circle in terms of rotation (see [10]).
(5) The intrinsic quantum characteristics of this elastic space medium with a ground state different from 0 (ρ/vacuum energy and circular frequency ω).
(6) The quantified space microstructure with a fibre dimension of the Planck scale, and quantified lapse time.
(7) The new definition of G (macroscopic manifestation of the said frequency f of space medium (see 58)).

We obtain a new set of formulas defined in table 4 as a function of the tensors (polarisations) considered:
Numerically we obtain the following values:

(1) The fibre length constituting the substance texture of the space fabric is 1.566×10^{-35} m compatible with the string dimension defined in the string theory.
(2) The Young's modulus $E = Y = 1.00 \times 10^{113}$ Pa (from the longitudinal wave) and 4.00×10^{113} Pa (from the shear wave) compatible with the results of ref. [11] (4.4×10^{113} Pa, see §3.4, formula 3.13). This value is compatible with the extreme stiffness of space.
(3) The density $\rho = 1.11 \times 10^{96}$ kg/m^3 is also compatible with the results of ref. [11] (1.30×10^{96} kg/m^3, see §3.4, formula 3.14).

As a result:

(1) The gravitational constant G does not seem to be a universal stable constant as it would depend on the characteristics of the material constituting the space elastic fabric (ρ, f) calculated from the vacuum properties.
(2) G could thus have varied in time as the density or the vacuum natural frequency.
(3) The speed limit of light could find an explanation via the limited ratio ρ/E characterising the penetration degree of light inside the elastic space medium (analogy of water that becomes a concrete wall at high speed).

This vision of the medium of space is a new vision of a relativistic Ether [20,21] without any correlation with the luminescent Ether which does not exist. This medium can be made up of infinitesimal beams of small sizes (quantum) in perpetual oscillation forming a three-dimensional frame of stiffness density rather than strings without a bending stiffness of 1×10^{-35} m. This three-dimensional fabric can be characterised by an elastic material itself, characterised by the Young's modulus E, Poisson's ratio ν and density ρ. In this case, we characterise the space by a sort of elastic substance, at the fundamental state (minimum vacuum energy E_v) based on the quantum field theory.

In addition, we have shown that we could consider a new type of wave, space torsion waves, which can generate longitudinal and shear waves. It would therefore be extremely interesting to use current interferometers to measure possible shear strains, that is to say, use the potential lateral movements of the laser in the interferometers to determine the angular shear strain γ and thus the shear modulus μ of space. This will also confirm whether, in addition to transverse shear wave, we should also consider longitudinal waves in the gravitational waves.

We have shown in tables 2 and 3 that it is possible to recalculate the Einstein's constant κ based on the theory of elasticity and volume wave theory. A research conducted by Ringermacher and Mead [19] seems to show that the Universe can sound like a crystal. The analysis of these 'special frequencies' is also an open door to explore and to obtain information about the space fabric structure.

In this study, we focussed on the elastic approach of space in weak fields. The presence of black hole being confirmed, it seems logical that the gravitation in strong field approaches the plasticity of the space material, in which case, it would be interesting to study what becomes of the Einstein's constant in strong fields using the plastic theory of the strength of the materials [12,13].

In this paper we found an expression of the Einstein's constant κ based on the spatial part only of the gravitational field tensors. For this, we used the two-dimensional stress tensor and the Poisson's ratio ν. We recalled that Einstein established its coupling constant κ by using the temporal part of its gravitational field tensor in weak fields (Poisson's formula and Newton's gravitational field equation). It naturally comes from the idea of extending the stress tensor obtained from the elastic theory at four dimensions and to deduce informations about the potential elastic characteristics of time. Such approaches are adopted in [11] and in §10 we showed that it is possible to relate time lapse to the thickness of the space fibres with an elastic time.

Dark energy and the associated cosmological constant Λ are characterised by an accelerated expansion of space. Based on the new definition of $G = \pi f^2/\rho$, we might consider that G varied over time as a function of density ρ. This variation of density can be explained by the intrinsic nature of the space material related to the dark energy (correlation with the expansion coefficient of the elastic material).

Dark matter was proposed to explain why the stars on the periphery of a galaxy were spinning at high speed without being ejected from galaxies (normally speeds decrease when moving from the centre of the disc to its periphery (153)). Considering the elastic deformation of a disc with a hole reamed (analogy of a galaxy with a massive black hole in the centre), made of elastic material (elastic fabric of the space) rotating at high speed, we see by an elastic calculation [57] that the shape taken by it redistributes the material and changes the apparent density ρ of the disc along the radius (thinning near the centre, thickening in the periphery). With the new definition of G as a function of this density ρ (153), it becomes possible to consider a variation of G along the radius of the galaxy (rather to increase the mass M and to look for dark matter) and thus to recalculate different velocities of the stars with a variable value of G. Indeed,we see that the spinning disc gets thinner and stretches. Therefore, the density ρ decreases which, taking into account the new definition of G, implies that $1/\rho$ increases. This can compensate the decrease of $1/R$ when R approaches the periphery of the disc and can thus make it possible to maintain an almost constant speed of rotation of stars in the disc (see also the MOND's approach). In addition, the tensions inside the space fabric disc in rotation allow to maintain the stars in their place, ensuring a global motion of the rotating disc, and a constant speed of the stars in the centre and in the periphery of the disc.

$$v = \sqrt{\frac{M}{R} \times G} = \sqrt{\frac{M}{R} \times \frac{\pi f^2}{\rho}}, \tag{153}$$

where M is the rotating mass and v is the speed of rotation of the stars around the centre of the galaxy.

In addition, if the vacuum is full of fluctuating substance as this article seems to prove via the quantum field theory of the vacuum energy at the fundamental state, it implies that in Young's experience of the double slit experiment, this substance has to be taken into account on the interpretation of the duality wave/particle. Indeed, the fluctuations of the vacuum present everywhere in the medium during the experiment disrupts the trajectory of the particles between the moment it is drawn and the moment it strikes the screen having passed through the two slits.

It seems therefore that in order to make progress in physics today, to renounce the Newton's gravitation concept, as proposed by the Eisntein's general relativity, is not sufficient. As this gravitation force is only an illusion, it seems logical that the universal and constant character of this constant G created with this force also have to be given up. We show in this article that if Newton and his gravitational force had not existed, Einstein could have found his coupling constant κ without going through the temporal component of his theory (μ, $\nu = 0$), but going through the mechanical and spatial parts of its theory (μ, $\nu = 1, 2, 3$) via the elasticity theory, the quantum field theory and the string theory data. If this had been the case, we would not have had the idea of constraining, to a universal constant, the parameters $\pi f^2/\rho$ related to the substance constituting the space that without knowing it Newton had called G in his equation. We also show that by proceeding in this way, and by calculating the parameter G as a function of the vacuum data via quantum field theory, we find elements on the infinitesimal substance constituting the space: the dimension of its granulometry 1×10^{-35} is compatible with string theory; a high Young modulus is necessary considering the very small curvatures generated at the space by extremely massive bodies; $Y = E = 1 \times 10^{113}$; a Poisson's ratio equal to 1. The 'metal' constituting the space is 5×10^{101} times harder than steel at the speed of light (and rather fluid at very low speed)! To advance further in the elastic approach of space, it seems necessary to conduct measurements of Young's modulus (see Casimir test described in this article) and of shear modulus (measurement of the lateral motion of the laser in the large interferometer). This will also make it possible to settle the paradox of the energies of the vacuum resulting from the cosmological constant Λ on the one hand and the quantum field theory on the other hand by quantifying once and for all the energy of the vacuum that have to be considered in the calculation. To summarise, let G vary, and probably many of today's physics problems will in turn disappear. So we shall conclude by quoting one of the first great scientist Alhazen Ibn al-Haytham (965–1040) who said that:

"The search for the truth is difficult, the road that leads to it is full of pitfalls, to find the truth, it is advisable to leave aside its opinions and not to trust the writings of the ancients. You must question them and submit each of their assertions to your critical mind. Trust only logic and experimentation, never the affirmation of one or the other, because every human being is subject to all sorts of imperfections; in our quest for truth, we must also question our own theories, each of our research to avoid succumbing to prejudices and intellectual laziness. Do this and the truth will be revealed to you. "

Acknowledgements

The author would like to thank Christian Liegeois, Doctor in Physics, for the useful comments and discussions, Daniel Spagni for the English and the two *Pramana* reviewers for their very effective and thoughtful reading that allowed the author to further develop and improve this article. The author would also like to thank the late R Gregoire, the great mechanician, who through his teaching based on the research of "how it works" guided the author.

Appendix A. Demonstration of the equivalence between the stress tensor $\sigma_{\mu\nu}$ and the stress energy tensor $T_{\mu\nu}$ [49]

In the theory of elasticity, resulting from the continuum mechanics, the relation between the stress tensor σ_{ij}, (with $T_i = \sigma_{ij} n_j$ where $\vec{T}$ is a stress vector attached to the facet of normal vector $\vec{n}$), and the applied force Q_i on a surface S_j can be written as follows:

$$Q_i = \sigma_{ij} S_j. \tag{A1}$$

In the field of variational approach, the stress tensor can be written as follows:

$$\sigma_{ij} = \frac{\Delta Q_i}{\Delta S_j} \quad \text{with } \Delta S_j \to 0, \tag{A2a}$$

where ΔS_i is an area.

So, with m as the mass, ρ as the density of mass energy, V as the volume and a_i as acceleration, we have:

$$\sigma_{ij} = \frac{\Delta Q_i}{\Delta S_j} = \frac{\Delta(m \times a_i)}{\Delta S_j} = \frac{\Delta(\rho \times V \times a_i)}{\Delta S_j}. \tag{A2b}$$

Assuming that the variation of the force is due only to the variation of volume V as a function of time t we obtain with, $a_i = \frac{v_i}{\Delta t}$,

$$\sigma_{ij} = \frac{\Delta(\rho \times V \times a_i)}{\Delta S_j} = \rho \frac{1}{\Delta S_j}\left(\frac{\Delta V}{\Delta t}\right) v_i. \tag{A3a}$$

Thus, we get with $V = \Delta x_i \times \Delta x_j \times \Delta x_k$

$$\sigma_{ij} = \rho \frac{1}{\Delta S_j}\left(\frac{\Delta x_i \times \Delta x_j \times \Delta x_k}{\Delta t}\right) v_i. \tag{A3b}$$

We can replace S_j by its value:

$$\Delta S_j = \Delta x_i \times \Delta x_k. \tag{A4}$$

So the new expression of the stress tensor is

$$\sigma_{ij} = \rho \frac{v_i}{\Delta t}\left(\frac{\Delta x_i \times \Delta x_j \times \Delta x_k}{\Delta x_i \times \Delta x_k}\right). \tag{A5}$$

After simplification we obtain

$$\sigma_{ij} = \rho v_i \left(\frac{\Delta x_j}{\Delta t}\right). \tag{A6}$$

By definition of speed v_j, we have

$$v_j = \left(\frac{\Delta x_j}{\Delta t}\right). \tag{A7}$$

We finally obtain the expression of the stress tensor at low speed as a function of energy density ρ and based on the multiplication of the velocities v_i and v_j:

$$\sigma_{ij} = \rho v_i v_j. \tag{A8}$$

The stress energy tensor results from the product of the energy density and the multiplication of the four-velocities (four dimensions of the space–time) resulting from the general relativity:

$$T_{\mu\nu} = \rho u_\mu u_\nu \tag{A9}$$

and for a perfect fluid with P as pressure:

$$T_{\mu\nu} = P g_{\mu\nu} - (\rho + P) u_\mu u_\nu. \tag{A10}$$

For the four-velocities:

$$u_\mu \left| \begin{array}{l} \gamma \times c \\ \gamma \times v_x \\ \gamma \times v_y \\ \gamma \times v_z \end{array} \right. . \tag{A11}$$

The Lorentz factor γ results from the Lorentz transformation which implies that the speed of light remains constant in all the referentials.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{A12}$$

Comparing (A9) and (A8), we can see the similarity between the four-dimensional general relativity stress energy tensor and the three-dimensional elastic stress tensor.

References

[1] A Einstein, *Phys. Math.* **1**, 844 (1915)
[2] A Einstein, *Phys. Math*. **1**, 688 (1916)
[3] A Einstein, *Querido. Verlag*. **6**, 220 (1934)
[4] W Dyson, A S Eddington and C Davidson, *Philos. Trans. Royal. Soc. London A* **220**, 291 (1920)
[5] A Einstein, *Sitz. Preus. Akad. Wissenschaften*. **1**, 831 (1915)
[6] K T McDonald, *J. Henry. Lab. Princ. Univ*. **NJ08544**, 1 (2018)
[7] R Chaubey and A K Shukla, *Pramana – J. Phys*. **88**, 65 (2017)
[8] T Damour, *Conf. Mardi. Esp. Des. Scien*. **1**, 1 (2016)
[9] T Damour, *Conf. Assos. Amis. Lab, Arago*. **1**, 1 (2017)
[10] R Feynman, F Morinigo and W Wagner, T and F Group Lecture 3, Chapter 3.4, figures 3.3 and 3.4, *Lectures on gravitation* (CRC Press, 1962–1963) Vol. 1, p. 53
[11] T G Tenev and M F Horstemeyer, *Int. J. Mod. Phys. D* **27**, 1850083 (2018)
[12] K Thorne, conference, UCI, School, Reines, Lecture, Exploring, the, Universe, with, Gravitational, Waves., **1**, 1 (2018)
[13] K Thorne, *Geometrodynamics Conference on the Space–Time* (Charles University, Prague, 2019) Vol. 1, p. 1
[14] E Hubble, *Astronmy* **15**, 168 (1929)
[15] C W F Everitt, *Phys. Rev. Lett*. **106**, 221101 (2011)
[16] Collective LIGO Scientific Collaboration and Virgo Collaboration, *Phys. Rev. Lett.* **116**, 061102 (2016)
[17] Collective LIGO Scientific Collaboration and Virgo Collaboration, *Phys. Rev. Lett*. **119**, 161101 (2017)
[18] A Einstein, *Sitz. König. Preuß. Akad. Wiss*. **1**, 154 (1918)
[19] H I Ringermacher and L R Mead, *R. Astron*. **149**, 137 (2015)
[20] A Einstein, *Einstein Arch. Univ. Héb. J*. **16**, 453 (1916)
[21] A Einstein, *Univ. Leyde. Gauthier. Villars. Cie*. **1**, 1 (1921)
[22] S Timoshenko and J N Goodier, *Theory of elasticity* (McGraw Hill, New York, 1951) Vol. 1, p. 146
[23] S Timoshenko, *Strength of material* (Van Nostrand, 1930) Vol. 1, p. 91
[24] A D Sakharov, *Sov. Phys*. **12**, 1040 (1968)
[25] J L A Synge, *Math. Zeitschrift*. **72**, 82 (1959)
[26] C B Rayner, *Proc. R. Soc. A. Math. Phys. Eng*. **272**, 44 (1963)
[27] K Kondo, *Int. J. Eng. Sci*. **2**, 219 (1964)
[28] C Truesdell, *The elements of continuum mechanics* (Springer-Verlag, New York, 1966) Vol. 1, p. 1
[29] R Grot and A Eringen, *Int. J. Eng. Sci*. **2**, 1 (1966)
[30] J E Marsden and T J Hughes, *Mathematical foundations of elasticity* (Prentice-Hall, Englewoods Cliffs, NJ, 1983) Vol. 1, p. 1
[31] U H Gerlach and J F Scott, *Phys. Rev*. **34**, 3638 (1986)
[32] G A Maugin, *An. Inst. Henri. Poincare* **4**, 275 (1971)
[33] J Kijowski, *Leonardo Centre Phys. Th. CNRS Luminy*, arXiv:hep-th/9411212, **CPT-94**, 3102 (1994)
[34] A Tartaglia, *Front. Fund. Phys*. **1**, 147 (1994)
[35] S Antoci and L Mihich, *Nuovo Cimento* **1**, 8 (1999)
[36] M Karlovini and L Samuelsson, *Quantum. Grav.* **20**, 3613 (2002)
[37] T Padmanabhan, *Int. J. Mod. Phys*. **13**, 2293 (2004)
[38] M Rangamani, *Quantum Grav.* **26**, 224003 (2009)
[39] M Beau, *Fondation. Louis de Broglie* **40**, 1 (2015)
[40] C G Böhmer, N Tamanini and M Wright, *Int. J. Mod. Phys*. **27**, 1850007 (2018)
[41] W Hehl and C Kiefer, *Gen. Rel. Gravit*. **8**, 50 (2017)
[42] A Tartaglia and N Radicella, arXiv:0903.4096 [gr-qc] (2019)
[43] M Beau, arXiv:1805.03020v2 (2018)
[44] T Tenev and M F Horstemeyer, *Adv. Phys. Sci*. **2**, 1 (2018)
[45] T Tenev and M F Horstemeyer, *Int. J. Mod. Phys. D* **28**, 1 (2019)
[46] T Tenev and M F Horstemeyer, *Int. J. Mod. Phys. D* **28**, 9 (2019)
[47] C W Misner, K S Thorne and J A Wheele, *Gravitation* (W H Freeman and Company, S Francisco, 1973) Vol. 1, p. 941

[48] G Mihai, *The bases of the unifying theory of physics* (University of Craiova, Faculty of Electrotechnics, Romania, 2012) Vol. 1, p. 12
[49] Collectif, Cours. De. Physique. *Gratuit.,* **1**, 1 (2019)
[50] R Taillet, e-Lecture Introduction à la relativité générale (Pod cast, Universite de Savoie, 2013) Vol. 24, p. 1
[51] E Ventsel and T Krauthammer, *Thin plates and shells theory and applications* (Dekker, New York, 2001) Vol. 1, p. 347
[52] S Timoshenko and S Woinowsky Krieger, *Theory of plate and shell* (McGraw Hill, 1961) Vol. 1, p. 432
[53] J Pascal, *Vibration et acoustique 2* (ENSIM, 2A, 2008–2009) Vol. 1, p. 14
[54] G Lhermet and G Vessiere, *Module F312 Elasticité linéaire* (problème et corrigés semestre 3, 2008-2009) Vol. 1, p. 3
[55] H Casimir, *R. Netherlands Academy Arts Sciences* **1**, 793 (1948)
[56] A Michelson and W Morley, *Am. J. Sci.* **1**, 1 (1887)
[57] Collectif, mms2.ENSMP.fr.mmc.Paris.annales.corrige.dim.disc.helicop. **1**, 1 (2006)
[58] R Owen, J Brink,Y Chen, J Kaplan, G Lovelace, K Matthew, D Nichols, M Scheel, F Zhang, A Zimmerman and K S Thorne, *Phys. Rev. Lett.* **106**, 151101 (2011)