Mechanics Research Communications 125 (2022) 103990

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Experimental analysis on metamaterials boundary layers by means of a pantographic structure under large deformations

Marco Laudato [a], Luca Manzari [a], Peter Göransson [a], Ivan Giorgio [b], Bilen Emek Abali [c,*]

[a] Department of Engineering Mechanics, KTH Royal Institute of Technology, Stockholm, Sweden
[b] Department of Civil, Construction-Architectural and Environmental Engineering (DICEAA), International Research Center on Mathematics and Mechanics of Complex Systems (M&MoCS), University of L'Aquila, L'Aquila, Italy
[c] Division of Applied Mechanics, Department of Materials Science and Engineering, Uppsala University, Uppsala, Sweden

ARTICLE INFO

Keywords:
Metamaterial dynamics
Pantographic sheets
Second gradient materials
Digital image correlation analysis

ABSTRACT

In this work, the dynamical behavior of a pantographic structure undergoing large deformations is analyzed. A harmonic force is applied on the structure, while its motion in time is recorded by means of a high-speed camera system. The resulting displacement field is obtained via the digital image correlation (DIC) method for the whole domain. A parametric study with respect to the amplitude of the imposed force exhibits the appearance of non-linear effects in the system response as well as the presence of a so-called boundary layer region. Such geometric nonlinearities are of paramount interest for studying the accuracy of a potential computational model. Boundary layers are known to be effected by the microstructure, but their roles in the system response are yet to be comprehended. The experimental results obtained in this work may be used as a benchmark case in modeling metamaterials.

1. Introduction

New technological possibilities owing to fast-prototyping techniques allow to purposely develop structures whose characteristic dimensions span multiple length scales. This possibility entails developing new mathematical and numerical models in order to efficiently capture the mechanical response of these so-called metamaterials [1]. A paradigmatic example in this flourishing sector is represented by the so-called pantographic structure [2–4]. This "multiscale" mechanical system is built by a microstructure made of (at least) two parallel arrays of mutually orthogonal fibers [5,6]. These arrays are connected via cylindrical elements called pivots (see Fig. 1). The macroscopic behavior of this system has promising technological properties [7], which are tailored as a remarkable feature concerning the constitutive modeling [8] because of a non-local interaction effected by the microstructure [9]. A so-called generalized elasticity theory [10–13] is needed to model metamaterials accurately [14–16]. Most of the studies in the field concern static or quasi-static boundary conditions [17–21], whereas the dynamical behavior has been studied experimentally and numerically only recently [22–24].

This work aims at an experimental analysis of the dynamics of pantographic structures: in particular, the analysis of the dynamical behavior of the system undergoing large deformation—up to ≈ 10% of the longest geometric dimension in the system. Harmonic oscillations are imposed, causing non-linear effects to be studied once the deformation is large enough. We investigate the boundary layer of this metamaterial as the amplitude of an imposed sinusoidal force signal varies. We propose the acquired results to be used as a benchmark for constitutive models in high-order gradient continuum theories.

2. Experimental analysis

The physical specimen under investigation is a rectangular pantographic metamaterial sheet made of polyamide EOS PA2200 printed by a Formiga P100 3D printer. Its geometric dimensions are 235 mm × 78 mm with a thickness of 6.3 mm composing its weight of 24.37 g. The distance between two pivots is 7.00 mm with a pivot diameter of 1.8 mm.

The aim of the experimental measurement is to record the structure displacement vector field in time. The pantographic sheet under analysis has one of its short sides clamped. A controlled force is applied on the other short side by a Brüel & Kjær type 4809 shaker. The excitation amplitude is controlled by a Brüel & Kjær type 2718 power amplifier. The imposed force signal is sinusoidal with a frequency of 10 Hz. This frequency has been chosen because it is well below the first eigenfrequency of the system, which occurs at 60.8 Hz [25]. By

* Corresponding author.
E-mail address: bilenemek@abali.org (B.E. Abali).

https://doi.org/10.1016/j.mechrescom.2022.103990
Received 14 March 2022; Received in revised form 9 July 2022; Accepted 12 September 2022
Available online 16 September 2022

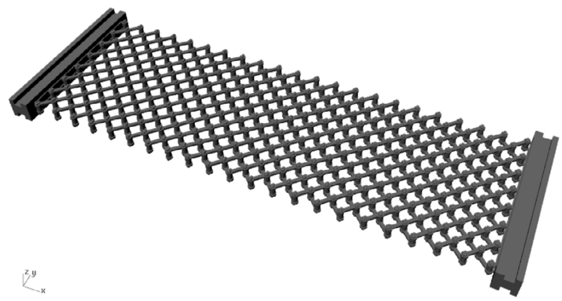

Fig. 1. CAD modeling of a pantographic metamaterial characterized by two layers of fibers connected by cylindrical elements called pivots.

Fig. 2. The setup of the experimental measurement. From left to right: the two cameras, four spotlights, the sample clamped to the floor and connected to the shaker, and the signal amplifier.

Table 1
Values of the amplitude of the imposed oscillation. The frequency of the oscillation is 10 Hz.

Amplitude	A_1	A_2	A_3	A_4
Value (mm)	7.3	13.5	16.6	18

means of soft springs, the shaker is suspended from the ceiling and its height relative to the ground is adjusted to impose a pre-stretch to avoid buckling effects during the experiment. A consistent alignment of the sample and the determination of the pre-stretch is ensured by means of laser distance meter and a laser level. The measurement of the displacement vector field in time is obtained by leveraging digital image correlation (DIC) techniques on the stereo frames captured by two Phantom v1612 high-speed cameras equipped with 200 mm lenses and located at 1.5 m from the specimen (see Fig. 2). In this setting, the theoretical resolving power is in the order of 10^{-5} m with a DIC interrogation window of 19×19 pixels.

The boundary conditions for the various experimental routines are compiled in Table 1 in terms of the maximum amplitudes of the displacement obtained as the result of the imposed force signal.

For each amplitude, the following measurement routine is performed:

- Four 250 W spotlights are switched on. A camera exposure time of 250 μs is chosen for blur-free images.
- A sinusoidal signal at a given amplitude is sent to the shaker via a power amplifier. The sample starts to move.
- After 2 s the system is assumed to have reached the steady state. The cameras start recording with a sampling rate of 1024 Hz. The length of the record ensures that at least 4 complete periods of oscillations are captured. The data is copied from the RAM memory of the cameras to a persistent storage. Then the lights are switched off.

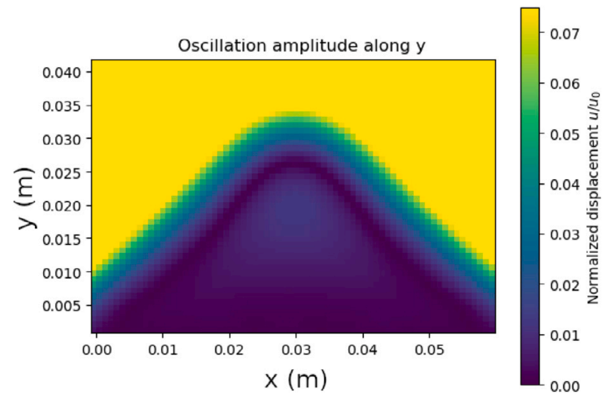

Fig. 3. Focus on the clamping area of the specimen (simulated data). Color represents the displacement along the y direction normalized on the amplitude of the imposed displacement. The boundaries of the triangular region influenced by the effects of the clamping are called boundary layers.

3. Results

In this section, the experimental data obtained via the aforementioned procedure are analyzed. In particular, we discuss the effects of an increasing force amplitude on the dynamical response of the system. We also focus on the observed non-linear response and anticipate a connection to the aforementioned non-locality caused by the microstructure. Moreover, we investigate the mechanical deformation near the clamping region in dynamical regime. The region of the system, which is influenced by the effects of the clamping, is depicted in Fig. 3. A "triangular" region characterized by almost negligible displacement amplitude is distinguishable, one of its three sides coincident with the clamped end of the specimen. The boundaries between the region, where the displacement is almost negligible, and the rest of the specimen are called *boundary layers*. It is of interest to analyze the displacement of the pantographic fibers whose orientation is orthogonal (in-plane) to at least one boundary layer. In previous experimental studies of the static behavior of the system [26–29], the equilibrium configuration of such fibers has been considered one of the clear signs of deviation from the standard theory of elasticity in pantographic structures [30–36]. Here we present an analogous experimental observation under dynamical loading.

3.1. Non-linearity

By increasing the amplitude of the imposed force signal, we obtain a significant change in the mechanical behavior of the system. The displacement vector field for each point of the mesh is decomposed along the two orthogonal directions along the short and the long side of the sheet, referred to as x and y directions, respectively.

Let us consider a generic non-nodal point of the pantographic sheet. We analyze the behavior at such a point by spanning different amplitudes of the loading force. From Figs. 4 and 5, the mechanical response appears to be linear for small amplitudes, deviating from this linear behavior as the applied amplitude increases. The distortions appear gradually, without a clear threshold effect: this phenomenon is an indication that the non-linearity of the system is caused by geometric nonlinearities. Analogous behavior is observed for all the non-nodal points of the domain.

It is not possible to resolve the nature of these deviations from linearity, which may be due to the multiscale nature of the pantographic sheet. Such deviations may be modeled using higher order inertia terms [37,38]. Hyperelasticity may also be employed, given the expected behavior of the material used to produce the experimental sample when undergoing large deformations [39,40]. Further numerical and experimental investigations are required in order to determine an accurate constitutive equation modeling this behavior.

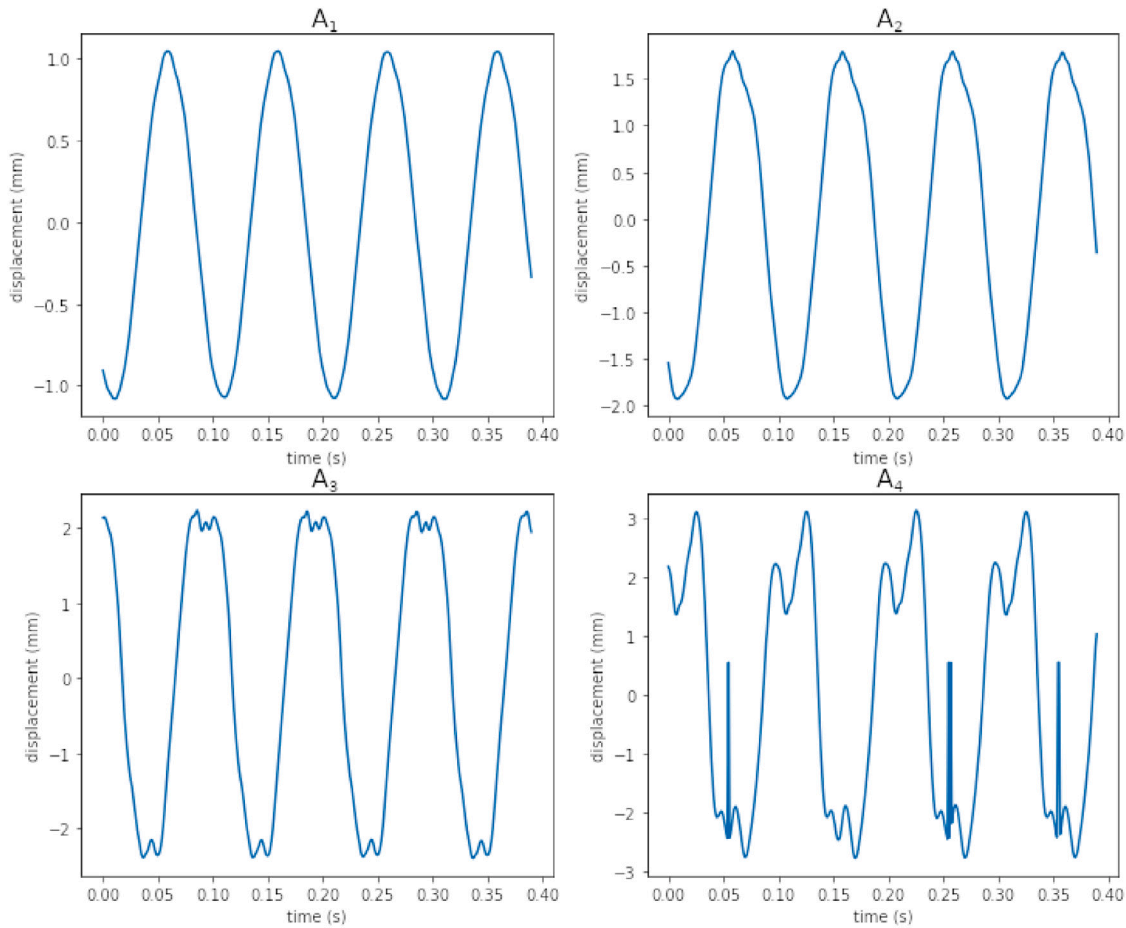

Fig. 4. Displacement in time along the x direction of a generic non-nodal point of the pantographic sheet. The input signal is a sinusoidal force with an oscillation frequency of 10 Hz. By increasing the amplitude of the force (A_3 and A_4 in figure), we observe that non-linear response becomes significant.

Table 2
Experimental data for a benchmark analysis: At the maximum of four different amplitudes, A_1, A_2, A_3, A_4 given in Table 1, numerical values of the displacement in mm, Δx along the x-direction and Δy along the y-direction of the representative points as shown in Fig. 6 located at x^P, y^P on a coordinate system with its origin at the bottom left.

Point	x^P	y^P	A_1		A_2		A_3		A_4	
			Δx	Δy	Δx	Δy	Δx	Δy	Δx	Δy
1	44.092	55.465	0.121	0.082	0.211	0.153	2.856	2.045	3.532	2.599
2	48.653	50.173	0.084	0.061	0.208	0.126	2.074	1.684	2.942	2.046
3	53.214	44.852	0.052	0.038	0.103	0.101	1.363	0.936	1.638	1.408
4	57.775	39.530	0.019	0.023	0.032	0.065	0.743	0.412	0.670	0.877
5	62.336	34.209	0.006	0.019	0.004	0.035	0.279	0.109	0.118	0.377
6	66.898	28.888	−0.001	0.010	0.006	0.012	0.108	−0.031	−0.007	0.058
7	71.459	23.946	−0.003	−0.011	0.006	0.003	0.074	−0.010	−0.028	−0.030
8	76.020	18.625	0.001	−0.001	0.009	0.015	0.022	0.024	0.007	−0.003
9	80.581	13.304	0.003	−0.005	0.007	0.011	0.005	0.008	0.008	0.002

3.2. Boundary layers

We now analyze the displacement of one fiber orthogonally crossing the boundary layer, i.e. the fiber highlighted in Fig. 6. The deformed configuration at the instant of time corresponding to the maximum stretch of the oscillation is plotted for each excitation amplitude in Fig. 7.

For a quantitative analysis, in Table 2, we present the values of the displacement components measured on pivots – red dots in Fig. 6 – for the four different amplitudes at the maximum of each imposed excitation amplitude.

A peculiarity of this result is the fact that the deformed configuration resembles that of a cantilever beam, "clamped" when within the region enclosed by the boundary layer (i.e. values of x from 62 to 82 mm) in Fig. 7. This microstructure-dependent displacement gradient, which cannot be realized using the standard theory of elasticity, effectively makes this measurement a benchmark case to be used against second-gradient theories.

4. Conclusions

Generalized mechanics is used to model multiscale materials or metamaterials: pantographic structures present an ideal design for using such tools. In this work, the dynamical behavior of such a structure undergoing large deformations is analyzed experimentally. In particular, a sinusoidal force at a fixed frequency and a range of amplitudes is imposed to one of the short sides of the specimen, while the displacement vector field in time of the whole domain is captured

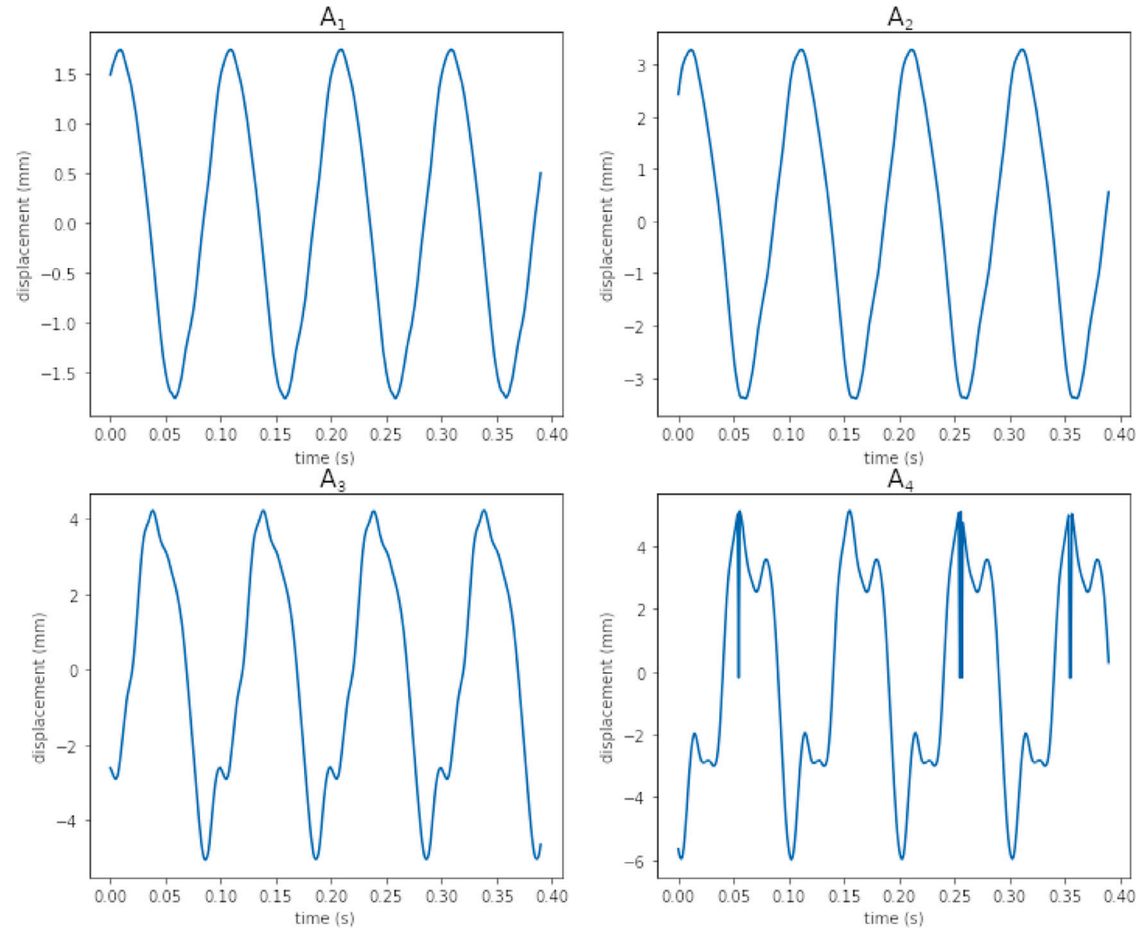

Fig. 5. Displacement in time along the y direction with the conditions as in Fig. 4.

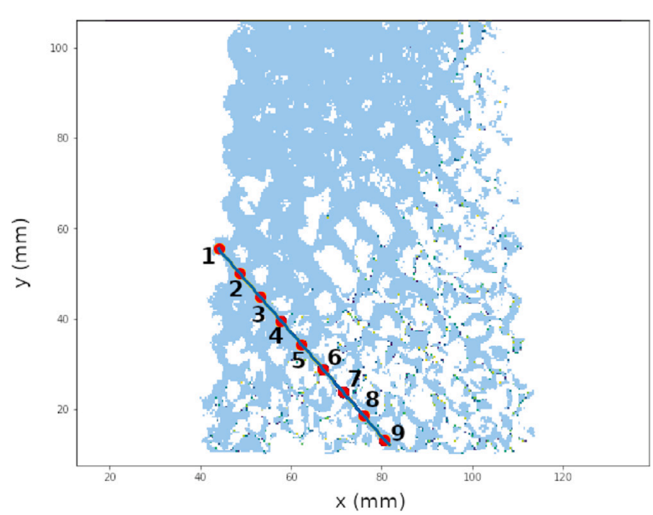

Fig. 6. The fiber whose displacements are analyzed, highlighted on top of the reference configuration.

by means of a high-speed camera system. The displacement is computed using DIC.

This parametric study shows a gradual change in the system response from linear to nonlinear, suggesting a geometric nonlinearity. However, given that the material response at the microscale is possibly hyperelastic, the overall contribution of material nonlinearity and geometric nonlinearity is difficult to quantify. The higher the imposed amplitude values – corresponding to the values of A_3 and A_4 in Table 1 – the higher the deviation from linearity in the system response. Additional spectral components are observed in the displacement plots. The obtained experimental results provide insights in modeling choices that may be used for simulating this complex multiscale behavior. In particular, the role of the microinertia might become relevant when studying the system under a dynamical load. The values presented in Table 2 may be used to validate such different modeling approaches.

A study of the boundary layer of the system has been presented as well. We have analyzed the displacement along a fiber of the microstructure. The behavior of such fiber crossing the microstructure-dependent boundary layer is of interest when compared to numerical simulations of different constitutive models under the same boundary and loading conditions.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

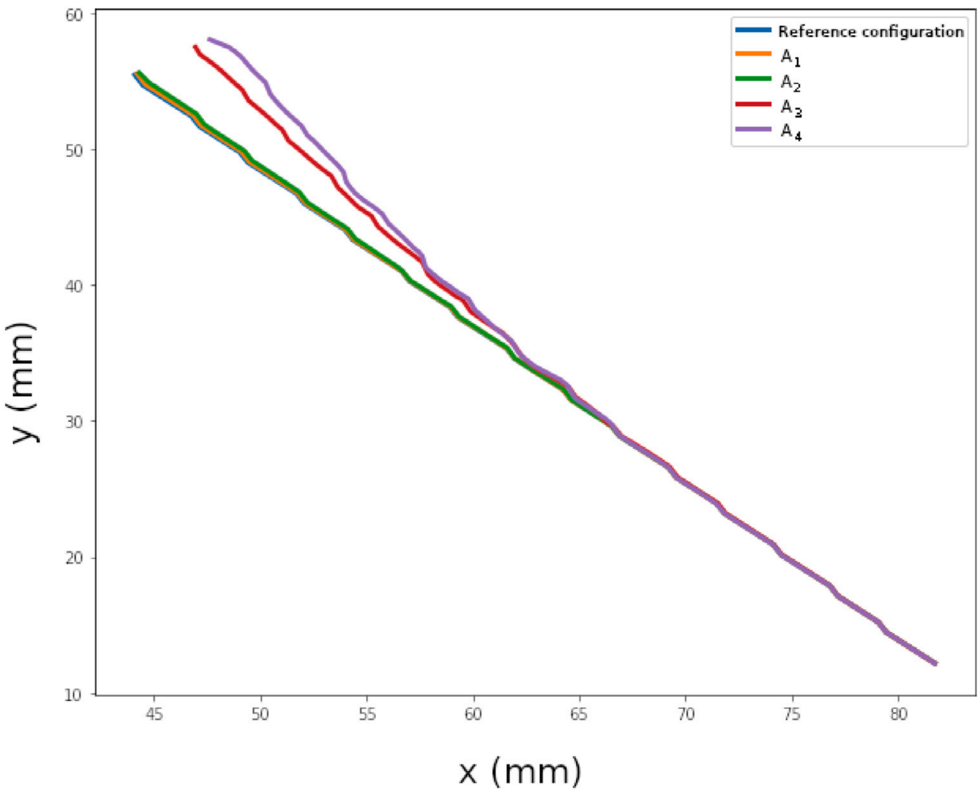

Fig. 7. Displacement along the chosen line highlighted in Fig. 6 for the four amplitudes of Table 1.

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