Application of DOE to estimate the variability of SEA solution

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Abstract

Statistical Energy Analysis (SEA) is the most acknowledged method to predict the averaged sound and vibration levels in mechanical systems in the high frequency range. A limit of this analysis is that of providing only the mean value of the variables of interest. The mean value provided by SEA equations is the mean of the responses of a set of similar systems, averaged on frequency bands. Two systems are considered similar if their physical parameters are slightly different. No information on the standard deviation is obtained by SEA as it would be expected by a true statistical approach. In this paper, the variability of SEA parameters (coupling loss factors, internal loss factor and injected powers) to uncertainties in the physical properties of the considered mechanical system (Young modulus, material density, geometry, ...) is investigated using a Design of Experiment (DoE) approach. This is done in order to take into account the idea of similar systems. Subsequently, the variability of SEA solution to the uncertainties on SEA parameters found at the previous step is investigated by using again a DoE approach.

1 Introduction

The meaning of "statistical" in Statistical Energy Analysis implies that the studied systems belong to a random population of similar systems [1]. Two systems are considered similar if their physical parameters are slightly different. SEA considers a structure as the union of several subsystems. Each of them is a modal group, i.e. a set of similar modes. For example, let us consider two welded plates: six modal groups can be identified, one set of flexural modes and two sets of in plane modes for each plate.

SEA estimates the mean value of the energy stored in the modal groups constituting the studied system. The mean value provided by SEA equations is in principle the average response of a set of similar systems. However, SEA equations are represented by a linear system of equations for each frequency, or better for each frequency band. The solution of each linear system gives the energy of each subsystem in a given frequency band. No average operation is explicitly performed, but all the statistics lies at an invisible level for the user. In general, this is not a problem because many simple relationships used by physicists and engineers are the result of more complicated mathematical procedures. Unfortunately, in this case this simple model holds only under many strong hypotheses, listed in Section 2.

The linear system results from some mathematical manipulations, that include also averages on frequency bands, on the classical equations of motion of multi degrees of freedom systems, and the observance of the strong hypotheses mentioned before. The coefficients of the linear system, named coupling loss factor (CLF) and internal loss factor (ILF), are the result of these average processes and account for the parameters of the native physical system. Therefore, SEA gives the energy of each modal group belonging to the studied system. This energy is the most representative sample of a statistical population of similar systems and on a frequency band. No information is given about the dispersion of the data around the result. In order to provide a true statistic solution it is necessary, at least, to know the variance of the result. A correct procedure should calculate the variance of the energy by starting from the equation of motion following a similar procedure like that performed for the mean calculation. Lyon [1, 2] estimates the variance by introducing a particular probability distribution of natural frequencies. Therefore this procedure neglects a direct dependence of the modal parameters on the randomness of the physical properties. However, he concludes that ”There is a considerable area of interesting research work that needs to be done in analysing variance of interacting systems.”

Radcliffe and Huang [3] study the problem by introducing a stochastic perturbation in SEA equations, by using a first order approximation of them in terms of this random perturbation and by calculating the variance of the new linear equations. They state that the lack of information of SEA solution may be filled by calculating the solution variance due to the random perturbation of SEA parameters (coupling loss factor, injected power, ...). Langley and Cotoni [4] follow the Lyon's approach and study the problem by considering the Gaussian orthogonal ensemble, detailed in Weaver [5]. Some other authors [6, 7] assume that uncertainties lie on the system parameters and they achieve results not exhibiting the same tendency of the Lyon's prediction. Bussow et al. [8] propose an analytical approach for the investigation of the problem of uncertain system parameters. The analysis is performed by partial derivatives of the energies. It shows the effect of uncertainties of a given parameter.

In this work the variability of CLFs and SEA energies is investigated by considering uncertainties on some physical parameters of a structure. To be more precise, in a first step, the variability of CLFs is studied by considering some material and geometric property as uncertain. Subsequently, the variability of energy is calculated by considering uncertainties on ILFs and CLFs. In this second type of analysis, the nominal values of CLFs and their range of variability are those resulting from the previous analysis. The effect of uncertainties is modelled by using a DoE approach [9, 10]. A set of numerical experiments is considered by combining the values of the variable parameters. DoE provides a regression model of CLFs and energies and the coefficients of this model show the influence of the uncertain parameters on the outcome of the experiments.

2 SEA equations

To solve a vibro-acoustic problem by SEA it is necessary to subdivide the system into subsystems representing groups of similar modes. A group of modes is considered as an energy storage. The energy flowing into each subsystem from external sources is balanced by the dissipated power and the power transferred to other modal groups.

Under some particular hypotheses, it is possible to assume that the transmitted power between two subsystems is proportional to the difference of the energy stored in each subsystem. A list of these hypotheses is presented below:

- all the modes of a subsystem must be similar (i.e. they must have almost the same energy, damping, coupling with the other subsystems and they must be almost excited by the same input power),
- the coupling between the subsystems must be conservative,
- the eigenfrequencies must be uniformly probable in the frequency range,
- the force exciting the subsystems must be random and not-correlated,
- the interactions between the subsystems must be weak.

Thus, the SEA equations of M coupled subsystems can be written as follows:

$$P_{i,inj} \;=\; \omega\,\eta_i\,E_i \;+\; \omega \sum_{j=1,j\neq i}^{M} (\eta_{ij}\,E_i \;-\; \eta_{ji}\,E_j) \tag{1}$$

where i and j are indexes of the subsystems, η_i and η_{ij} are the internal loss factors (ILF) and the coupling loss factors (CLF), respectively, $P_{i,inj}$ is the power injected into the subsystem i, E is the energy in a given subsystem and ω is the central frequency of the considered band. Equations (1) represents the energy balance of the subsystems. The power dissipated in the subsystem i is:

$$P_{i,d} \;=\; \omega\,\eta_i\,E_i \tag{2}$$

The power transmitted from subsystem i to the subsystem j is:

$$P_{ij} \;=\; \omega\;(\eta_{ij}\,E_i \;-\; \eta_{ji}\,E_j) \tag{3}$$

The solution of the linear system (1) provides the energy stored in each subsystem. The set of equations (1) can be rewritten in a more convenient way as follows:

$$\{P\} \;=\; \omega\;[\eta]\;\{E\} \tag{4}$$

where the coefficients of matrix $[\eta]$ are combinations of ILFs and CLFs and the solution is obtained by inverting the matrix $[\eta]$.

CLFs and injected powers depend on the geometry and the material of the subsystems. In fact, CLFs are complicated functions of the wave transmission coefficients and the injected powers of the mobilities of the excited subsystems.

3 Design of Experiments

An experiment is a test or a series of tests in which the values of the variables that affect an output response are appropriately modified to identify the reasons for changes in the response. Therefore, the objective of an experiment may be to find which variables are most significant in determining the response f, and to find how to set the significant variables so that either the response f is near the desired value or the variability in the response f is small. This definition does not prevent from performing numerical experiments whenever this may be convenient for a better understanding of the numerical problem under investigation.

3.1 Factorial design and Central Composite Design

Many experiments involve the study of the effects of two or more variables or factors. In general, factorial designs investigate all possible combinations of the levels of the factors and are very efficient for this task.

A regression model representation of a factorial experiment with two factors A and B at two levels could be written as:

$$f = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_{12} x_1 x_2 + \varepsilon \tag{5}$$

where the α's are parameters whose values are to be determined, the variables x_1 and x_2 are defined on a coded scale from -1 to $+1$ (the low and high levels of A and B) and ε is an error term.

Specifically, if p factors at two levels are considered, a complete series of experiments requires 2^p observations and is called a *two-level* 2^p *full factorial design*. Usually, each series of experiments should be replicated several times using the same value of the factors to average out the effects of noise. Of course, this is unnecessary if experiments are numerical.

A potential concern in the use of two-level factorial design is the assumption of linearity in the factor effects. Actually, the interaction term in Eq. (5) introduces a bilinear effect, but each section of $f(x_1, x_2)$ with a plane with either $x_1 = \text{const}$ or $x_2 = \text{const}$ would be a straight line. To account for possible non linear effects, a logical extension is to consider quadratic terms, as in the following expression:

$$f = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_{12} x_1 x_2 + \alpha_{11} x_1^2 + \alpha_{22} x_2^2 + \varepsilon \tag{6}$$

Of course, a three level (low level -1, intermediate level 0, high level $+1$) factorial design, involving 3^p observations, is a possible choice if quadratic terms are important. However, a more efficient alternative is the Central Composite Design (CCD) that starts from the 2^p design augmented with:

- the *center point*, a single observation with all factors at intermediate level;
- *axial runs*: each factor is considered at two levels (the low level -1 and the high level $+1$) while the remaining factors are at the intermediate level, for a total of $2p$ observations.

Overall, a central composite design for p factors requires $n = 2^p + 2p + 1$ observations instead of 3^p observations required by the three level factorial design, with advantages for $p \geq 3$.

3.2 Response surface model for Central Composite Design

For p control factors, the experimental response can be expressed as:

$$\begin{aligned} f = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_i + \sum_{i=1}^{p} \sum_{j=1}^{i-1} \alpha_{ji} x_j x_i + \ldots + \\ + \sum_{i=1}^{p} \sum_{j=1}^{i-1} \cdots \sum_{n=1}^{m-1} \alpha_{nm \cdots ji}\, x_n x_m \cdots x_j x_i + \sum_{i=1}^{p} \alpha_{ii} x_i^2 + \varepsilon \end{aligned} \tag{7}$$

where a regression model representation of a 2^p full factorial experiment (involving 2^p terms), augmented with p quadratic terms, is used. Overall, the expression contains $2^p + p$ parameters α. Each parameter provides an estimate of the effect of a single factor (linear or quadratic) or of a combination of factors.

Note that Eq. (7) is linear in the parameters α, and it can be rewritten as:

$$f = \begin{bmatrix} 1 & x_1 & \cdots & x_p^2 \end{bmatrix} \begin{Bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{pp} \end{Bmatrix} + \varepsilon \tag{8}$$

having arranged the parameters in a vector $\{\alpha\}$. A different equation can be written for each observation by varying the factors $(x_1, \ldots, x_p)$.

In a Central Composite Design, a set of equations can be written by choosing the factors in a standard order [10]. By arranging the experimental responses in a vector $\{f\}$, a linear relationship between $\{f\}$ and $\{\alpha\}$ can be expressed in matrix notation as:

$$\{f\} = [X]\{\alpha\} + \{\varepsilon\} \tag{9}$$

where $[X]$ is a $(2^p + 2p + 1) \times (2^p + p)$ matrix.

The least square estimate of $\{\alpha\}$ is given by:

$$\{\hat{\alpha}\} = ([X]^T [X])^{-1} [X]^T \{f\} \tag{10}$$

where $([X]^T [X])^{-1} [X]^T = [X]^+$ is the pseudo-inverse of $[X]$.

The fitted regression model is:

$$\{\hat{f}\} = [X] \{\hat{\alpha}\} \tag{11}$$

The difference between the actual observation f and the corresponding fitted value $\hat{f}$ is the residual $e = f - \hat{f}$. The residual accounts both for the modelling error ε and for the fitting error due to the least square estimation. A vector of residuals can be defined as:

$$\{e\} = \{f\} - \{\hat{f}\} \tag{12}$$

3.3 Error sum of squares

If the squared deviations of each response f_i from its average value $\bar{f} = \sum_{i=1}^{n} f_i/n$ are considered, their sum is the so-called total sum of squares SS_T that can be computed as:

$$\mathrm{SS}_T = \sum_{i=1}^{n} (f_i - \bar{f})^2 = \sum f_i^2 - n\bar{f}^2 = \{f\}^T\{f\} - \{\bar{f}\}^T\{\bar{f}\} \tag{13}$$

Furthermore, the total sum of squares can be partitioned into a sum of squares due to the model (or regression) and a sum of squares due to residual (or error):

$$\mathrm{SS}_T = \mathrm{SS}_R + \mathrm{SS}_E \tag{14}$$

The sum of squares of the residuals can be computed as:

$$\mathrm{SS}_E = \{f\}^T\{f\} - \{\hat{\alpha}\}^T [X]^T \{f\} \tag{15}$$

A low value of the ratio $\mathrm{SS}_E/\mathrm{SS}_T$ between the error sum of squares and the total sum of squares indicates that the chosen regression variables provide a good fit.

4 Estimate of CLFs and energy variability

SEA gives only the mean value of the energy of a set of similar systems. This is not a complete statistical information, because at least the dispersion of the data around the mean is lacking. The correct variance of the solution could be obtained by working directly on the equation of motion as it was done to provide the SEA equations. Here a study on the variability of SEA coefficient and results is performed. Therefore, the goal of this research is not a way to achieve the variance matching the classical SEA solution, but to understand how much the CLFs are dependent on uncertainties on the physical parameters and how much the energies (SEA solution) depend on uncertainties on CLFs and ILFs.

SEA equations are deterministic, and CLFs are deterministic functions of the physical parameters as well. The solution of this deterministic set of equations, the energies of the modal groups, depends on the ILFs, the CLFs and the injected powers. In order to study the variability of SEA coefficients and solution, many techniques can be followed (Monte Carlo, algebraic derivatives, etc.), but DoE seems to be the method giving as many information as possible with the less computational burden.

Let us consider a given mechanical system made of M subsystems. CLFs depend on the material properties and the geometric parameters of the coupled subsystems. Therefore, a given η_{ij} depends, for instance, on the Young modulus of the systems i and j, Y_i and Y_j, and on the thickness of these subsystems, t_i and t_j. By defining a range of variability of these parameters, a DoE procedure can be developed in order to obtain the coefficients of a regression model and therefore to quantify the influence of the considered physical parameters on the CLFs.

The energy of each subsystem is calculated by solving equation (4) with the obvious implication that energies depend on the CLFs and the ILFs of the considered system. Similarly to the procedure followed before, a range of variability of CLFs and ILFs is defined, and a DoE procedure is developed to obtain a regression model that accounts for the dependence of the energy on the variability of SEA coefficients.

5 Numerical examples

The studied structure is a system of three Aluminum plates with the same thickness of 3 mm and different sizes: plate 1 (600 mm $\times$ 400 mm), plate 2 (300 mm $\times$ 400 mm) and plate 3 (400 mm $\times$ 400 mm). These plates are welded along the 400 mm side (Figure 1). Two problems are considered.

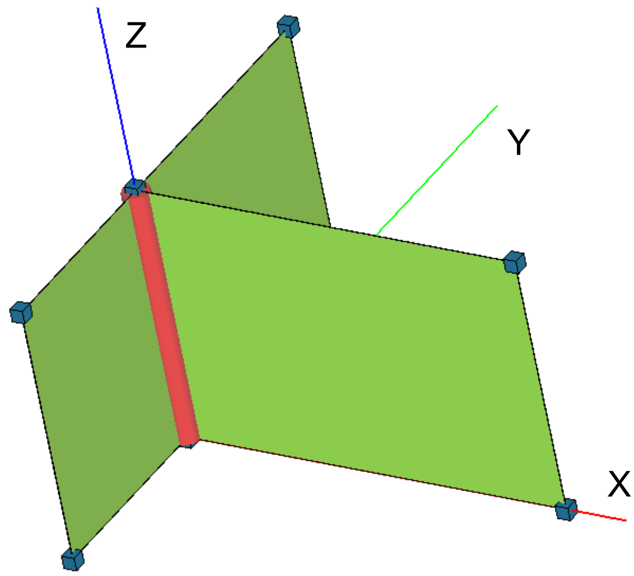


Figure 1: Three plates system

The first one takes into account the variability of the physical parameters, i.e. Young modulus and thickness of each plate (6 parameters), to calculate the CLFs by DOE procedure: $2^6 + 2 \cdot 6 + 1 = 77$ experiments for each of 14 third octave bands from 100 Hz to 2000 Hz.

The second problem concerns the computation of the energy of each subsystem, represented by the flexural modes of each plate, when the variability of CLFs and ILFs (9 parameters) is considered. In this second case the DoE procedure requires 521 numerical experiments for each of the third octave bands indicated before.

5.1 First problem

Uncertainties on the Young modulus and the thickness are considered by varying the parameters in a range of $\pm 10\%$ around the nominal values: 7×10^{10} Pa and 3 mm, respectively.

The CLFs are calculated by one of the most diffused and tested commercial software, AutoSEA. In figure 2 the DOE results, in terms of regression coefficients α, of the 6 CLFs at 500 Hz are shown. Each tick of the η_{ij} axis represents a given coupling loss factor at a frequency, each tick of the regression coefficient index axis represents each α_{ij}. Only the linear, quadratic and bilinear terms of the regression model are shown. The fit is very good as witnessed by the low values of the ratio $\mathrm{SS}_E/\mathrm{SS}_T$ shown in Table 1.

Figure 3 shows the regression coefficients α of the CLF $\eta 12$ for all the third octave bands: figure 3a shows the actual α, figure 3b shows the same values normalised with respect to the mean values α_0 and denoted as $\tilde{\alpha}$. It is clear that the α values decrease as the frequency increases, and that the behaviour of regression coefficients related to a given CLF is similar at all frequencies.

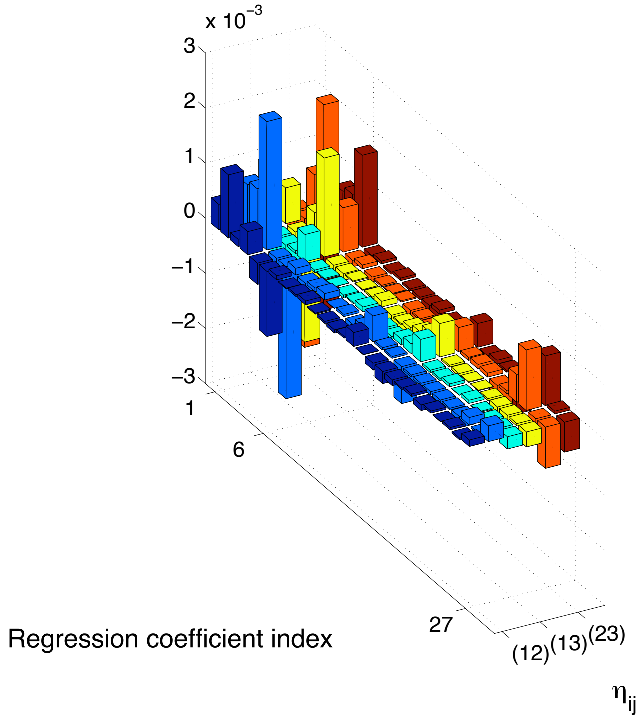
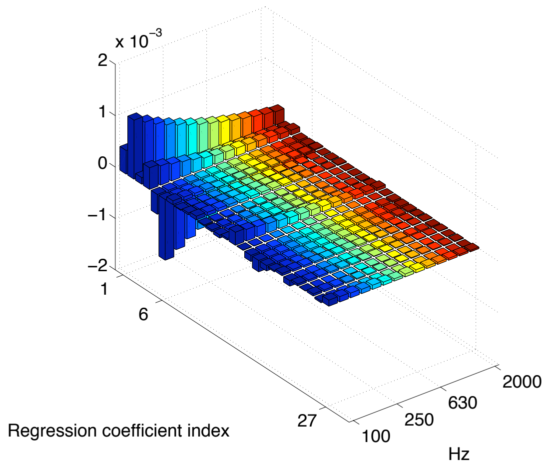


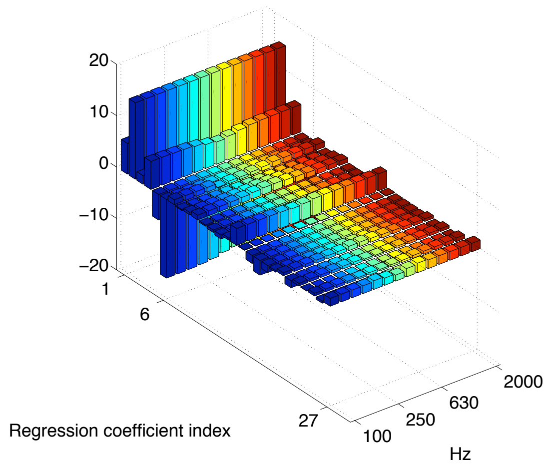
Figure 2: Regression coefficients α for all the CLFs at 500 Hz

Table 1: $\mathrm{SS}_E/\mathrm{SS}_T$ of the CLFs regression model

Frequenze	η_{12}	η_{21}	η_{13}	η_{31}	η_{23}	η_{32}
100	0.0006	0.0020	0.0029	0.0026	0.0208	0.0181
125	0.0006	0.0020	0.0028	0.0026	0.0096	0.0073
160	0.0006	0.0020	0.0028	0.0026	0.0084	0.0063
200	0.0006	0.0020	0.0027	0.0025	0.0097	0.0075
250	0.0007	0.0021	0.0026	0.0024	0.0114	0.0092
315	0.0004	0.0017	0.0024	0.0022	0.0109	0.0086
400	0.0003	0.0014	0.0023	0.0020	0.0070	0.0069
500	0.0004	0.0016	0.0027	0.0025	0.0162	0.0161
630	0.0006	0.0019	0.0030	0.0028	0.0122	0.0123
800	0.0007	0.0021	0.0030	0.0028	0.0085	0.0071
1000	0.0007	0.0021	0.0030	0.0028	0.0075	0.0062
1250	0.0007	0.0021	0.0031	0.0029	0.0071	0.0058
1600	0.0008	0.0022	0.0031	0.0029	0.0067	0.0055
2000	0.0008	0.0022	0.0031	0.0029	0.0062	0.0050



(a)



(b)

Figure 3: Regression coefficients α for η_{12} from 100 Hz to 2000 Hz: (a) actual and (b) percent normalised values

A more accurate analysis can be performed by looking at Table 2 where the values of $\tilde{\alpha}$ normalised by the mean values at 500 Hz are shown. The results show that the α of the linear terms are always the most relevant, and they assume a positive value if they correspond to a parameter with the same index of the considered CLF, e.g. α_1, the coefficient of Y_1, is positive for η_{12} and negative for η_{23}. This law is denied for the coefficients of the quadratic and bilinear terms. It can be noticed that also some of these coefficients are relevant, as for instance those related to Y_2^2 for all η_{ij} and those related to Y_1^2 and Y_3^2 for η_{23} and η_{32}.

Figure 4 shows the regression coefficients α corresponding to Table 2.

The analysis of this first problem shows that CLFs sensitivity is of the same order of magnitude as the variation of subsystem geometry and stiffness (it is worth to recall that the range of variability of physical parameters is $\pm 10\%$). The values of α decrease with the frequency, whilst the values of $\tilde{\alpha}$ are almost constant.

5.2 Second problem

The second problem considers the variability of the SEA solution, the energy of each subsystem, when uncertainties are assumed on the CLFs and ILFs. A correspondence between the uncertainty of the physical parameters and the variability of the CLFs is preserved, because the nominal value of the CLFs corresponds to the nominal value of the physical parameters. The considered uncertainty is the maximum variability around the nominal value obtained in the first problem. The ILFs are varied of $\pm 10\%$ around 0.01.

In figure 5 the DoE results, in terms of regression coefficients α, of the energies of the three subsystems at 500 Hz are shown. Each tick of the energy axis represents an energy, E_1, E_2 and E_3, each tick of the regression coefficient index axis represents a regression coefficient α_{ij}. Again, only the linear, quadratic and bilinear terms of the regression model are drawn. Also in this case, the fit is very good because low values of $\mathrm{SS}_E/\mathrm{SS}_T$ (not shown) are found.

Figure 6 shows the regression coefficients α for the energy E_1 at all the third octave bands: figure 6a shows the actual α, figure 6b shows $\tilde{\alpha}$. As for the CLFs, also for the energies the values of α decrease as the frequency increases, and the behaviour of regression coefficients related to energy is similar at all frequencies.

Table 2: Regression coefficients $\tilde{\alpha}$ [%] for CLFs at 500 Hz (in bold if $|\tilde{\alpha}| > 2\%$)

α index	parameter	η_{12}	η_{21}	η_{13}	η_{31}	η_{23}	η_{32}
0		100.0000	100.0000	100.0000	100.0000	100.0000	100.0000
1	Y_1	**6.0458**	1.3582	**5.9395**	1.2396	**-7.7516**	**-7.7230**
2	t_1	**14.8799**	**5.7142**	**14.7061**	**5.5880**	**-10.2437**	**-10.2206**
3	Y_2	1.5227	**6.0373**	**-4.1137**	**-4.4234**	**5.3347**	1.0855
4	t_2	**4.5900**	**14.2064**	**-15.4263**	**-15.0905**	**13.1460**	**4.6747**
5	Y_3	**-4.9002**	**-4.6761**	0.4910	**5.4656**	1.7973	**5.9572**
6	t_3	**-14.4683**	**-14.6663**	**5.5480**	**14.6306**	**4.2137**	**12.6850**
7	Y_1Y_1	0.6039	0.4784	-0.7083	-1.2452	**-4.8205**	**-5.3124**
8	Y_1t_1	-0.2718	-1.1129	-0.1622	-0.9905	-1.9665	-1.9960
9	Y_1Y_2	0.7074	0.6865	-0.2062	-0.0368	**3.3105**	**3.8851**
10	Y_1t_2	1.3315	1.2202	-1.1195	-0.3712	-0.8170	-0.0119
11	Y_1Y_3	-0.4273	-0.1888	0.4862	0.5345	**3.2975**	**2.7229**
12	Y_1t_3	-0.8984	-0.2193	1.5526	1.3722	0.5757	-0.2294
13	t_1t_1	-0.7521	-0.9624	**-2.0642**	**-2.6859**	**2.8957**	**2.4037**
14	t_1Y_2	0.4314	0.8751	-0.9163	-0.1884	0.9394	1.4579
15	t_1t_2	**3.3166**	**3.1512**	-0.6431	0.4816	-0.9242	0.0997
16	t_1Y_3	0.0716	0.2490	1.4194	1.3126	0.9825	0.4640
17	t_1t_3	-1.6310	0.0441	**2.3287**	**2.7137**	0.5751	-0.4488
18	Y_2Y_2	**-2.4620**	**-4.0359**	**4.7030**	**4.6621**	**-7.5489**	**-5.6042**
19	Y_2t_2	-1.6015	-0.5349	-0.3469	-0.0438	0.2051	-0.5818
20	Y_2Y_3	-0.3544	-0.3246	-0.5336	-0.4548	1.8675	1.8913
21	Y_2t_3	0.8183	-0.1415	0.4376	-0.2942	0.3915	0.3286
22	t_2t_2	-1.1589	-0.3484	0.3005	0.2595	**4.7892**	**3.9014**
23	t_2Y_3	0.2584	-0.4244	0.9976	-0.0113	0.3524	0.3677
24	t_2t_3	-0.1257	-1.2987	-0.3049	-1.4289	1.8258	1.8496
25	Y_3Y_3	0.3547	0.7252	-1.3414	-0.8943	**-13.3875**	**-13.3727**
26	Y_3t_3	-0.1676	0.0864	-1.7807	-0.6651	-0.6056	0.2289
27	t_3t_3	1.6126	1.9831	**-2.4711**	**-2.0721**	**4.3934**	**4.2972**

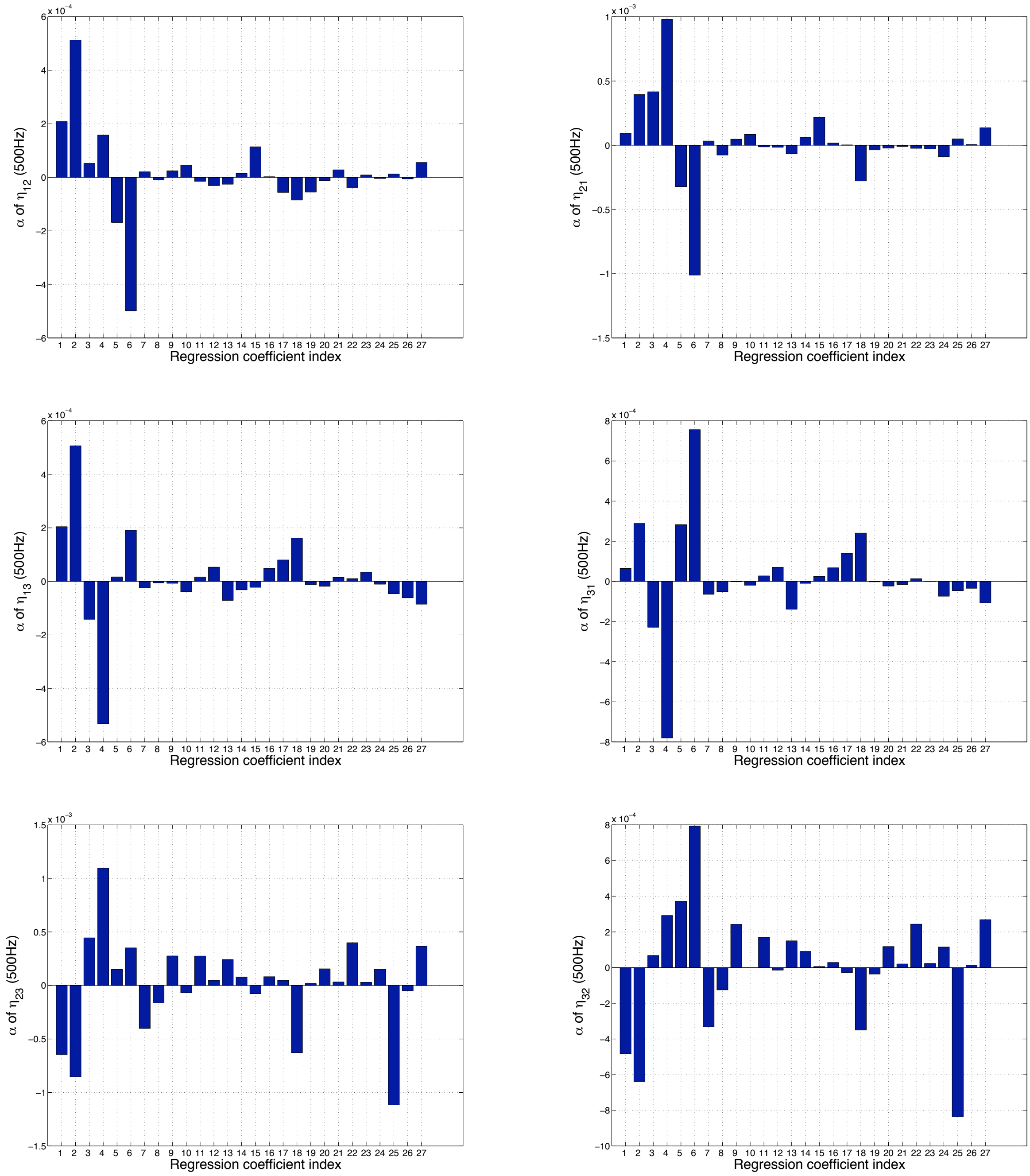


Figure 4: Regression coefficients α for CLFs at 500 Hz

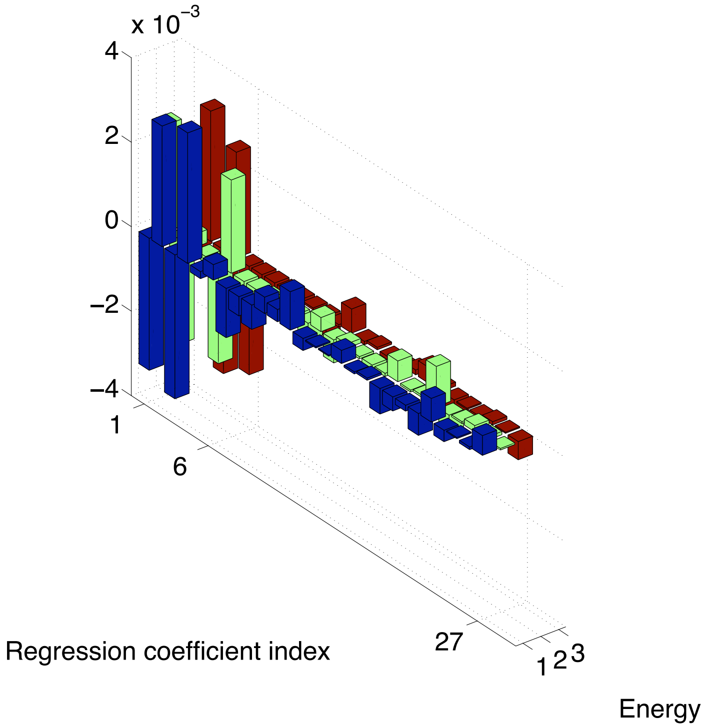
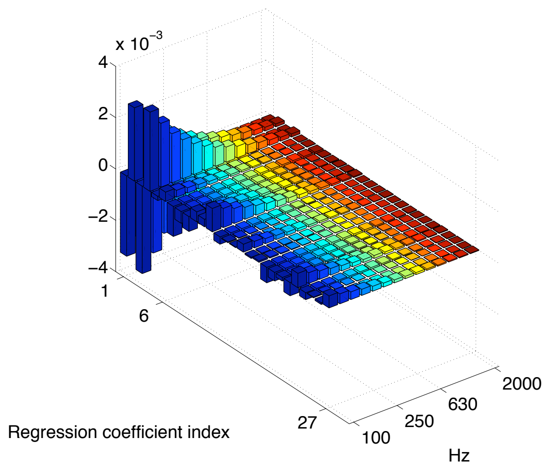
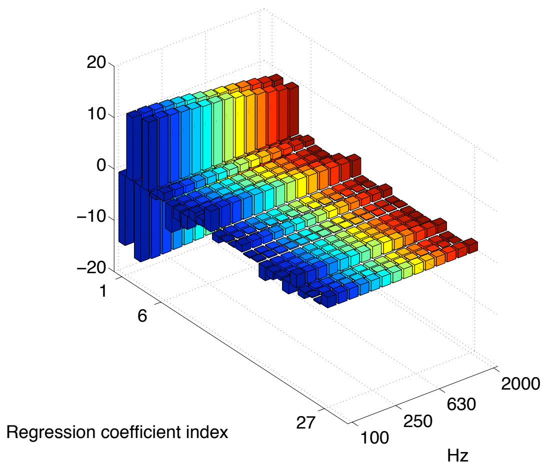


Figure 5: Regression coefficients α for all the energies at 500 Hz



(a)



(b)

Figure 6: Regression coefficients α of E_1 from 100 Hz to 2000 Hz: (a) actual and (b) percent normalised values

A more accurate analysis can be performed by looking at Table 3 where the values of $\tilde{\alpha}$, normalised by the mean values at 500 Hz are shown. The results show that the α of the linear terms are always the most relevant. It can be noticed that also some of the coefficients of bilinear and quadratic terms are relevant.

Figure 7 shows the regression coefficients α corresponding to Table 3.

Table 3: Regression coefficients $\tilde{\alpha}$ [%] for energies at 500 Hz (in bold if $|\tilde{\alpha}| > 2\%$)

α index	loss factor	E_1	E_2	E_3	α index	loss factor	E_1	E_2	E_3
0		100.0000	100.0000	100.0000					
1	η_{12}	**-13.3132**	**22.7597**	**6.0767**	28	$\eta_{13}\eta_{31}$	0.9341	-0.2725	-1.4881
2	η_{21}	**10.7494**	**-19.1484**	**-4.3496**	29	$\eta_{13}\eta_{23}$	0.0598	-0.3882	0.1989
3	η_{13}	**-14.2923**	**4.1651**	**21.9607**	30	$\eta_{13}\eta_{32}$	0.0696	**4.6187**	**-3.6648**
4	η_{31}	**12.0929**	**-2.3519**	**-19.5190**	31	$\eta_{13}\eta_{1}$	1.4657	0.4170	-0.3872
5	η_{23}	-0.3603	**-10.9916**	**8.9980**	32	$\eta_{13}\eta_{2}$	0.1506	-0.2024	-0.4274
6	η_{32}	1.4181	**21.8591**	**-19.1614**	33	$\eta_{13}\eta_{3}$	-0.1882	-0.6549	-1.3673
7	η_{1}	**-5.2885**	**-4.6119**	**-4.7820**	34	$\eta_{31}\eta_{31}$	**-2.1123**	0.5721	**3.3107**
8	η_{2}	-1.6701	**-3.0211**	**-2.2906**	35	$\eta_{31}\eta_{23}$	1.0533	-0.0461	-1.8296
9	η_{3}	**-2.2319**	**-2.9293**	**-4.2322**	36	$\eta_{31}\eta_{32}$	**-2.3422**	**-3.4602**	**6.7872**
10	$\eta_{12}\eta_{12}$	1.6106	**-2.8807**	-0.6375	37	$\eta_{31}\eta_{1}$	-1.1824	-0.4323	0.2860
11	$\eta_{12}\eta_{21}$	1.1518	**-2.0233**	-0.5239	38	$\eta_{31}\eta_{2}$	-0.2148	0.1368	0.4529
12	$\eta_{12}\eta_{13}$	**3.7498**	**-3.7181**	**-3.7259**	39	$\eta_{31}\eta_{3}$	-0.0233	0.5800	1.5503
13	$\eta_{12}\eta_{31}$	-0.8392	**2.6425**	-0.5687	40	$\eta_{23}\eta_{23}$	0.0762	1.0716	-0.9328
14	$\eta_{12}\eta_{23}$	-0.0548	**-2.6084**	**2.0767**	41	$\eta_{23}\eta_{32}$	0.0102	-0.5466	0.3776
15	$\eta_{12}\eta_{32}$	-0.0697	1.4955	-1.0093	42	$\eta_{23}\eta_{1}$	0.0346	0.5138	-0.3890
16	$\eta_{12}\eta_{1}$	1.3715	-0.5122	0.3152	43	$\eta_{23}\eta_{2}$	0.1852	0.6640	0.0061
17	$\eta_{12}\eta_{2}$	-0.1904	-1.0485	-0.6012	44	$\eta_{23}\eta_{3}$	-0.1774	0.0536	-0.6288
18	$\eta_{12}\eta_{3}$	0.1461	-0.6680	-0.3554	45	$\eta_{32}\eta_{32}$	-0.1961	**-3.7222**	**3.1920**
19	$\eta_{21}\eta_{21}$	**-2.1369**	**3.7730**	0.9208	46	$\eta_{32}\eta_{1}$	-0.1365	-1.0453	0.7886
20	$\eta_{21}\eta_{13}$	-1.0337	-0.4964	**2.1657**	47	$\eta_{32}\eta_{2}$	-0.3794	-1.1603	-0.1152
21	$\eta_{21}\eta_{31}$	-0.8376	0.6923	0.9739	48	$\eta_{32}\eta_{3}$	0.3399	-0.2842	1.5342
22	$\eta_{21}\eta_{23}$	-1.0410	**4.0307**	-1.2303	49	$\eta_{1}\eta_{1}$	0.2642	0.2076	0.2783
23	$\eta_{21}\eta_{32}$	**2.2980**	**-5.1375**	-0.1422	50	$\eta_{1}\eta_{2}$	0.1681	0.2243	0.1796
24	$\eta_{21}\eta_{1}$	-1.0581	0.3646	-0.3039	51	$\eta_{1}\eta_{3}$	0.2225	0.2309	0.3049
25	$\eta_{21}\eta_{2}$	0.0316	1.1286	0.5436	52	$\eta_{2}\eta_{2}$	0.0628	0.0285	0.0883
26	$\eta_{21}\eta_{3}$	-0.2090	0.6798	0.2849	53	$\eta_{2}\eta_{3}$	0.0934	0.1633	0.1649
27	$\eta_{13}\eta_{13}$	1.8922	-0.5884	-2.8770	54	$\eta_{3}\eta_{3}$	0.0972	0.0598	0.1492

6 Comments and conclusions

A procedure to evaluate the sensitivity of the CLFs to physical parameters and of the SEA solution, i.e. the energy of each subsystem, to CLFs and ILFs is considered. The procedure is based on Design of Experiment technique. The coefficients of a regression model are calculated and they quantify the influence of single parameters and combinations of them on each quantity.

The results show a dependence of CLFs on the Young modulus and on the thickness of the coupled subsystems. To be more precise, the linear coefficients are of the same order of magnitude as the variation of geometry and stiffness. Similarly, the dependence of the energies on the CLFs and ILFS is of the same order of magnitude as the variations of coupling and internal loss factors.

DoE proves to be a suitable tool to perform this kind of study. In fact, it gives a lot of accurate information on the investigated dependencies and it allows to understand the influence of many parameters with a low computational burden.

Next activities will consider the study of more complicated systems to investigate the dependency of the SEA solution on the parameters of not directly connected subsystems.

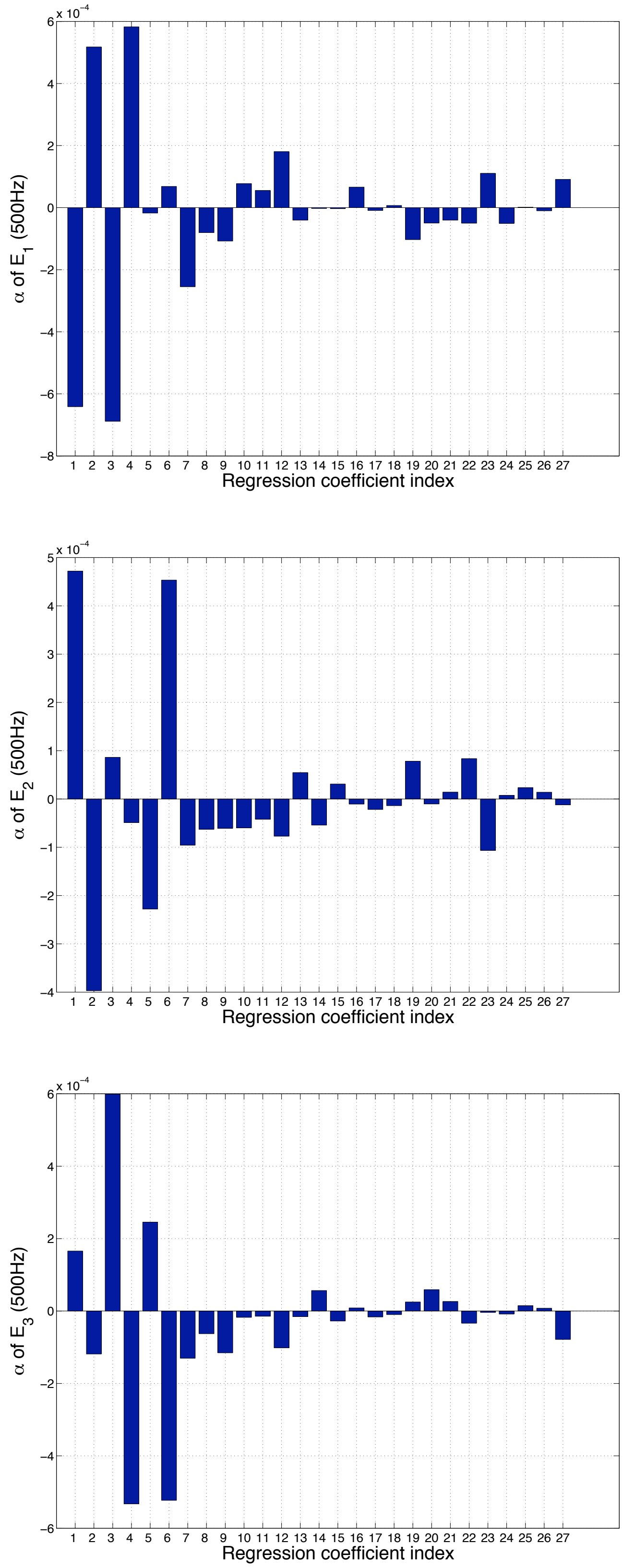


Figure 7: Regression coefficients α for energies at 500 Hz

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