

Relational Methodologies and Epistemology in Economics and Management Sciences

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Chapter 7

A Methodology to Measure the Hierarchical Degree of Formal Organizations

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ABSTRACT

Hierarchy is a fundamental phenomenon in management and organization science, a phenomenon which has marked the evolution of human societies over centuries. Among the many studies on this issue, the ones that adopt a formal approach of investigation are mainly based on social network analysis. Following this line, in this work we focus on organization distribution of formal direct authority in stylized, pure hierarchical archetypes. Past research, analyzing the share of asymmetric links in out-tree topologies, was not able to distinguish among different types of out-trees. Indeed since the out-trees can differ under substantial structural features, in order to measure the degree of hierarchy it is necessary to employ indicators of power concentration and distribution. Results show that the purest archetype of hierarchy is the star form, and not the typical org chart. Further, ceteris paribus, an organization with more hierarchical levels is less and not more hierarchical than an organization with fewer levels. Moreover, power tend to concentrate in lower levels, and especially into the penultimate one.

INTRODUCTION

The objective of this work is to develop a methodology and draw the main results of measuring the hierarchical degree of formal organizations, limitedly to direct relationships. More generally, authority (or power or status, according to the many sociological approaches) can be seen as an asymmetric decision. And a decision is a peculiar injunctive type of communication, which orients or strictly determines a receiver's behavior. In this perspective, formal organizations are essentially decision networks, and indeed this was exactly Simon's view of organizations (Simon, 1947; Simon & March, 1957). Now, after

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so long time from his approach, we know well that organizations are much more and else than that (for a broad view, see Clegg *et al.*, 2006). However, to a large extent, depicting organizations as decisions (and not only “simple” communications) networks still grasps a relevant aspect of what organizations do¹.

In its simplest and purest sense, a hierarchical relationship is an asymmetric relationship, characterized by the fact that one can be obeyed by another (or many other) one(s). As sociological literature suggests since the classic works of Weber and Simmel, the reasons for such obedience can be various: rituals, charisma, money, physical dominance, etc. Such issues, albeit very interesting, do not concern our analysis, for which it is enough to express the point in terms of asymmetric decision, regardless of the specific nature and source of power. In this sense, a digraph whose links are not-all-reciprocal decisions (which guide the actions of subordinates) is a hierarchy.

For in a formal hierarchy, someone decides what another (or many others) should do, the resulting structure can be viewed as a command (or asymmetric decisions) network, in particular an *out-tree* graph (see Krackhardt 1994; Krackhardt & Hanson, 1993). In fact, if we look at a stylized org chart, which Simon (1962) considers the archetype of hierarchy, and if we orient its connections from the vertex to the subordinates through intermediate managers, and if there are no collaborations (or joint/reciprocal decisions) between managers or subordinates, an org chart becomes a pure out-tree. Krackhardt (1994) refers to the hierarchical properties of out-tree graphs by means of four graph indicators - connectedness, hierarchy, efficiency and least upper boundedness - that measure an organization degree of hierarchy in terms of its ‘degree of out-treeness’. Because a pure out-tree would score 100% according to these four indicators, they are not able alone to grasp some salient features of hierarchies: they do not, strictly, discriminate between organizations with more or less power concentration. In fact organizations that exhibit the same degree of out-treeness could be saliently different in their configuration of *span of control* (SoC) and *number of ranks* (NoR), that is in two fundamental structural aspects. The former is an evident sign of the power of single individuals, because it indicates how many subordinates are managed. The latter indicates hierarchical levels, and it is an intuitively-grasped and popularly-used signal of an organization hierarchical degree. Hence, it appears difficult to accept a measure of organizational hierarchical degree (OHD) that overlooks both issues. Further, the degree of out-treeness does not catch how hierarchical power is distributed between hierarchical levels.

Therefore, a set of crucial questions, which cannot be answered with the degree of out-treeness, arise: how can we measure OHD besides the degree of out-treeness? That is, is there some other measure of power concentration else than the degree of out-treeness? In fact, this latter question is particularly interesting, because another way of looking at OHD is in terms of the extent to which decisions are concentrated in few hands or distributed evenly among most individuals, instead of looking at the share of asymmetric relationships, as the degree of out-treeness points out. As we will see below, these two perspectives on OHD differ remarkably. A further question concerns how can we compare the OHD of two or more organizations that are pure out-trees and have the same size, but differ in terms of SoC and/ or NoR. And finally, how OHD is distributed between people and ranks? In this chapter we provide an answer to these questions.

As we will see in the next sections, if OHD is meant as power concentration (PC) and it is measured by out-degree centralization indices, then such questions can be answered. Moving from the consideration that an organizational formal structure can be described by SoC and NoR, we are going to investigate on how a pure out-tree centralization, and thus, its PC depends on different configurations of these three variables. Furthermore, by calculating rank centrality, we will measure power distribution (PD) among them.

PREVIOUS RESEARCH

In this work we face the problem of OHD in terms of PC and PD moving from the traditional idea of power: one person decides what (and eventually how) another person can and/or must do something. We consider such a relation only in terms of legitimized power - authority - so that our analysis focuses only on formal organizations typically represented by bureaucratic structures (Weber, 1922). In this vein, authority can be defined as the power to make decisions which guide the actions of others (Simon, 1947).

The notions of hierarchy and power characterize the tradition of organization studies in the last century. Michels (1915) puts under the light the factual evidence that even democratic systems evolve toward centralistic structures. Mintzberg (1979) highlights that different types of hierarchical structures result from different types of environmental pressure. According to Simon (1962), in a hierarchical system it is possible to distinguish between interactions among subsystems and interactions within subsystem, moving from the consideration that those within a subsystem exhibits an higher frequency than those between subsystems. In this perspective a hierarchical system is characterized by the property of near-decomposability - which makes the system more evolvable, if compared with other types of organization - because of the possibility of treating in a quasi-additive manner the adaptiveness of its components.

In more recent decades, different theoretical perspectives on the notion of power and hierarchy are related to a number of prominent theoretical contributions flourished over years within social network analysis (for a review see Brass & Krackhardt, 2012; see also Carpenter *et al.*, 2012; Salancik, 1995). Power is a structural phenomenon to the extent organizational structure imposes ultimate constraints to individuals (Brass, 1984); and organizations can be seen as political arenas in which the exercise of power emerges when different interests and conflicts arise (Brass, 2002).

According to Burt (1992) power is related to structural holes, that is to say, the connections of an actor with other actors who are not themselves connected. The fundamental idea is that by connecting other actors who are not themselves connected, the actor in question possesses non redundant information. Within the organization, an actor may connect other actors to benefit of each, for example the boss connects subordinates with complementary skills and knowledge rather than just mediating their flow of information (Obstfeld, 2005).

Brass (1984) demonstrated that an actor with ties to dominant coalitions increases its own power. In particular Sparrowe & Liden (2005) demonstrated that ties with a central actor are source of power if the central actor in question share his own connections. With reference to power as centrality the idea is that actors occupying a central position in a network are associated with more power. This evidence has been confirmed with reference to different levels of analysis: interpersonal networks (Brass & Burkhardt, 1993; Krackhardt, 1990; Brass, 1984; Tushman & Romanelli, 1983), intergroup networks (Hinings *et al.*, 1974), interorganizational networks (Boje & Whetten, 1981; Galaskiewicz, 1979).

Since decades, centrality measures have represented fundamental tools to analyze the overall level of centrality in a structure providing a specific articulation of the factual level of prominence of its members (see Frantz *et al.*, 2009; Borgatti *et al.*, 2006; Borgatti, 2005; Borgatti & Everett, 2005; Costenbader & Valente, 2003; Friedkin, 1991; Bolland, 1988; Bonacich, 1987; Freeman *et al.*, 1980; Freeman, 1979, 1977; Nieminen, 1974; Sabidussi, 1966; Bavelas, 1948). Such kind of studies looked at hierarchical phenomena focusing mainly of informal social relations among actor (Freeman, 1997; Monge & Contractor, 2003), as it was the dominant view until now. Moreover, most of them were considering mostly the combination of direct and indirect power, and not only the former one, as in our work. Others look at org charts, like Likert (1961), among whom some employed network analysis (Watts, 2003), mostly

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focusing on formal and informal coordination mechanisms, or on the efficiency of communications, and hence on the advantages that would be obtained by inserting horizontal (intra-rank) relationships. This would shorten the average distances within hierarchical (and thus, vertical) decision chains (Watts, 2003).

Further, some of them are based on positional analysis rather than on link asymmetry or centrality. Doreian (1988) assumes the ostensive hierarchical form of an out-tree of 15 people with SoC=2, and the consequent 3 ranks (vertical differentiation) and 2 departments (horizontal differentiation) required by that structure. His goal is to test two algorithms (CONCOR and STRUCTURE) for calculating structural equivalence and another one (REGE) for regular equivalence, so to check whether they are able to correctly discover vertical and horizontal equivalent positions. In this approach the hierarchical relationships between single (or group of) nodes would emerge from the hierarchical clustering generated by each algorithm. There is no measure of OHD at whole network level, nor a measure of PD among ranks or groups of nodes – at most, their vertical or horizontal groupings.

However, the notion of formal power cannot be too much separated by the notion of hierarchy as represented in organizational charts: a multilevel communication structure in which each agent reports to his or her immediate supervisor, who in turn reports to the top-level manager, who makes the ultimate organizational decision (Lin, 1994). The power of an actor can be considered in terms of his status within the organization: when considering also indirect relationships, status can be seen as the number of its immediate subordinates, plus twice the status of their subordinates, etc., so that power identifies a correspondence of the actor's status and his position within the organizational structure (Harary, 1959a, 1959b).

Krackhardt (1994) provides an effective definition of “what” is a hierarchy by means of an ideal-typical construct: an out-tree graph, which is a directed-graph where “every point, with the exception of the one point at the “top” of the outtree, has exactly one arrow pointing to it, although the points may have several arrows emanating from them. If these arrows represented authority relationship, then we might interpret this statement as nothing that each point has one and only one “boss” but each point may have any number of subordinates” (Krackhardt, 1994, p. 93; see also Friedkin, 1991, 1986; French, 1956).

So, a hierarchical relationship is an asymmetric relationship, characterized by the fact that one can be obeyed by another (or many other) one(s), but can obey only to one boss. In other words, contrarily to Taylor's theory of scientific management and according to Fayol and the schools of classical organization design, a person can have only one boss and one or more subordinates. Simon (1962) refers to such structures as the *archetypical formal hierarchy*, which embeds the fundamental principles of classic organizations, as unity of command, chain of command, and scalar principle.

Following a misunderstanding common during the eighties and nineties, Ishida & Ohta (2001) compare hierarchy and network, and contrast them with market coordination mechanisms. The misunderstanding concerns the fact that the “network” form cannot be juxtaposed to hierarchy, because both forms are networks, which differ in terms of topology. Our paper clarifies also this point by allowing to measure an OHD of any form. In this (methodologically and formally correct) perspective, what differs is, among other aspect, its OHD: maximal for out-trees, and in particular for star forms, and minimal for clique-like forms (called also team-structures or all-channel structures). The main mistake was induced by a misinterpretation of Williamson's theory made by Powell (1990) and Thorelli (1986), because the “network” implicated by transaction cost economics was that of inter-firm networks and not that of single organization network-form. That misunderstanding propagated and was amplified by some other fortunate papers (van Alstyne, 1997; Jones *et al.*, 1997; Miles & Snow, 1992). A third mistake made by Ishida & Ohta (2001) and that was (and it is still) common concerns the fact that centralized decisions

are intended as opposed to reciprocal decisions: that is, if decisions are reciprocal, then they are not centralized. Consequently, hierarchies would be characterized by centralized decisions while networks by reciprocal decisions. However, as we show in this paper, this view is not correct, because it is based on a confusion between two network properties: the degree of reciprocity and the form of link distribution. In fact, the effects of links reciprocity on OHD is grasped by the degree of out-treeness, while link distribution by nodes respective positions. And this latter is independent on the former: that is, the same link distribution can be accompanied by different degrees of link reciprocity. At the extreme, a network with all reciprocated links – or an undirected network – can have a star structure, and as such it can be maximally hierarchical.

A rather different research stream comes from organizational economics, not only meant as the field of transaction cost economics, but also – and even more pertinently respect to the issue investigated here – as that of the economics of design, and its precursors in the theory of teams (Alchian & Demsetz, 1972; Marschak & Radner, 1972). In fact, though transaction cost economics² deals with organizational structures and the sources of authority to guarantee coordination and avoid opportunism, it does not go so deeply into the issues of hierarchy and its measurement. Conversely, the economic theory of mechanisms design (Hurwicz, 1973; Hurwicz & Reiter, 2006) investigated more closely various aspects of hierarchy and organizational structures, focusing on the efficiency of decision making processes (Li, 1999; Sah & Stiglitz, 1986; Van Zandt, 1995), and the optimal hierarchy, in terms of SoC, NoR, and size, depending on managerial wage, cost of decision delay, production costs, etc. (Calvo & Wellisz, 1978; Hurwicz, 1973; Williamson, 1967 among the earlier, and then Bolton & Dewatripont, 1994; Qian, 1994; Marschak & Reichelstein, 1998; Radner, 1993, 1998). Many contributions took a model elaborated by Kere & Levhari (1979, 1983, 1989) as point of reference for these analyses.

Within this literature, Marschak and Radner were prominent scholars, and Radner (1992) in particular, in an interesting and meaningful contribution on a leading economic journal, recognizes that economics came late to analyze firms structures³, and “the economic significance of hierarchy in the organization and management of large firms”. Focusing on the reasons why it is useful a division of labor, that he calls decentralization, he comes to define a hierarchy as a *ranked tree*. Assuming Simon’s perspective of looking at organizations as information processing systems – that is as cognitive systems in a strictly computational sense (Biggiero, 2009) – whose components (people) are single processors, Radner underlines that “hierarchical structures, which are usually thought of as the epitome of the centralization of authority, are also remarkably effective in decentralizing the activities of information processing” (pg. 1393). This sentence, and other parts of Radner’s thinking (1997), suggests that he was aware that an organization’s vertical differentiation by introducing hierarchical levels implies a progressive reduction (decentralization) of power, and not its concentration (centralization). A thesis that is so important as it has been overlooked, especially in popular thinking, but that has not been demonstrated through precise measures, a goal that is prior in this work.

A remarkable research stream of management and organization sciences (for a general view see Carley & Prietula, 1994; Prietula *et al.*, 1998) developed most of the same subjects of the economic theory of organization just mentioned, with some salient differences: i) less constraints deriving from the requisites of neoclassical economics on organizational goals and on agent’s behaviors⁴; ii) less use of analytic algebra (with its continuous-time and continuous-event methods) and more use of network analysis⁵; iii) more attention for effects due to task complexity and environmental uncertainty, and for socio-cognitive and psycho-social aspects, like bounded rationality, cognitive maps, trust, etc. Within this wide research area there are studies concerning either issues very close to the ones treated by econom-

ics of design and organizational economics, or issues more connected with, let say, primitive or more fundamental aspects of hierarchy analysis⁶, like the ones investigated by Krackhardt, and with whom is related the present paper.

Respect to many contributions we have briefly reviewed of current literature, our work posits in a different way. First, we are not looking for any optimal hierarchy, as the economic (and partly also organizational-management) approaches, nor for individuating (extracting) hierarchical levels or power groups from a given (abstract or empirical) network. Further, we do not look at power in generic terms, nor in terms of relative (dis)advantages of different coordination mechanisms, and neither as (indirect) influence or authority. Conversely, we remind again that this work is that here we deal only with direct power, that is, a power that derives from direct relationships, and that can be measured through various forms of single or collective direct centrality or centralization (see the methodological section). Of course, there is also indirect authority, which is exerted and channeled through intermediate actors alongside a command chain. This form of power has a lot of implications, among which these two seem particularly noteworthy: i) it can overcome direct power, especially in large organizations, where command chains are quite long (even more than 5 hierarchical levels); ii) it can be distributed in a rather different way than that of direct power. Consequently, a complete evaluation of HD had to comprise both direct and indirect forms of power. Despite its relevance, we decided here not to treat indirect power because its measuring introduces many methodological and conceptual problems, which would make this chapter too long and complex.

Indeed, a complete analysis of OHD in formal organizations would require to investigate all the following dimensions: i) overall concentration of direct and indirect power; ii) rank distribution of direct and indirect power; iii) dichotomous and valued links; iv) constant and varying span of control; v) application to pure vs. hybrid out-trees; vi) intra-rank links; vii) not only asymmetric links. Respect to these seven dimensions (and its combinations), here we face with only direct power in terms of: a) overall concentration; b) rank distribution; c) dichotomous and valued links; d) constant span of control; e) pure out-trees. Therefore, results and conclusions cannot be extended to the other cases, but their usefulness is twofold: on one side many interesting results reached here have a good scope of generality, and on the other side they would be anyway required to allow effective interpretations of future studies comprising either the missing dimension of indirect power or the many other combinations of direct power analysis – varying span of control and hybrid out-trees – that here have not been developed.

RESEARCH QUESTIONS

By anchoring the archetype of hierarchy to a pure *out-tree*, and hence by implicitly arguing that a hierarchy is a form of network and not the opposite of network (as some authors argued during the eighties and nineties), Krackhardt (1994) shows that the following four characteristics, defining and quantifying the degree of out-treeness, define and quantify OHD too:

- **Connectedness (Index):** The ratio of the number of pairs in the graph that are reachable relative to the number of ordered pairs;
- **Hierarchy (Index):** The proportion of pairs that, in the derived reachability digraph, has reciprocated ties relative to the number of pairs where there is a tie (note that the maximum hierarchy corresponds to the absence of reciprocated ties);

- **Efficiency (Index):** The degree to which each component in a network contains the minimum relations possible to keep it connected;
- **Least Upper Boundedness (Index):** The degree to which pairs of agents have a common boss (except pairs formed between the ultimate boss and others).

Krackhardt's indicators represent fundamental theoretical dimensions that allow us to compare the degree of hierarchy of a graph when compared with a random, real or generated one. For each indicator varying between 0 and 1, a pure out-tree would score 1, and this value represents the benchmark respect to which any real organization's OHD should be measured. In particular, his measure of hierarchy is a crucial step forward respect to applying the same measure to the source graph, because it would limit the measure of OHD only to direct links. Conversely, applying it to the derived reachability digraph allows to capture both direct and indirect power relationships.

At a closer look, we see that, for a "standard" organization analyzed in terms of decision network, connectedness can be taken for granted, because a disconnected node would indicate an organization member who does not obey to and does not command anybody. It's hard to believe that somebody can be part of an organization but at the same time is completely disconnected in terms of decisions. Hierarchy refers to the existence of reciprocal decisions in the reachability digraph (which implies also the original digraph). This indicator is the most strictly related to the concept of hierarchical power as decisional asymmetry: it could be called asymmetry-based hierarchy, which distinguishes from the position-based hierarchy. Efficiency depends on how many decisions are made beyond the minimum necessary: clearly, any (horizontal) intra-rank decision, that is decisions made between colleagues lying at the same hierarchical level, and any reciprocal decision is redundant respect to that minimum⁷. These three indicators do not tell very much about the topology of typical org charts, because they are compatible with a number of different forms else than them. For instance, a chain of n nodes oriented towards the same direction or converging on (or beginning from) a central node, etc., all can satisfy a maximum score of connectedness, hierarchy and efficiency. The proper out-tree form is expressed by the characteristic of least upper boundedness, because it embodies the elementary superior-subordinate structure beyond a single dyad: a boss managing two (or more) subordinates. Hence, if we took for granted maximum connectedness and efficiency, and if we assumed asymmetric dyads, then it would become clear that the network form depends mostly on the latter property.

Nevertheless, this analysis is not able to *fully* articulate the extent to which an organization - characterized by specific feature in its span of control and number of ranks - is characterized by a higher or lower level of hierarchy. In fact, the limit of Krackhardt's approach is that it is unable to make distinctions between perfect out-trees that, given a certain size, score a different SoC and/or NoR. In other words, pure out-trees differing in one or more of these dimensions would score the same maximum OHD. This outcome prompts out a certain surprise, because it is, intuitively, rather difficult to believe that, in reality, an organization of 2 levels, shaped as a perfect out-tree is as much hierarchical as an organization (supposedly as well shaped as a perfect out-tree) of 5 levels. Indeed, the archetype of hierarchy shown in Figure 1b is a pure out-tree of a very special type, generated by Equation (1), supposing three kinds of extreme regularity at the same time: constant intra- and inter-rank SoC, and an organizational unit assigned to each manager (intermediate node). The two first conditions implies the property of self-similarity, and thus, that the out-tree assumes the form of a fractal⁸, and that consequently it shows, for large sizes, the typical scale-free distribution of direct centrality (Biggiero & Angelini, 2015; Cardarelli, 2007). The third condition implies that all operatives are place into the last (highest) rank. We chose to

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employ this form because it expresses results in a very clear and interpretable way, but it is important to bear in mind that this fractal structure is only a special way to build out-trees. There are innumerable forms of out-trees that, for each pair of the variables involved in Equation (1), would be perfect as well, scoring the maximum degree of out-treeness, and hence reminding to the same limit of that composed indicator.

Following Krackhardt's idea of representing a pure hierarchy with an out-tree, we can model its features using SoC and NoR. A third variable - size (S) - is dependent on the others and represents the total number of individuals. Such variables identify the fundamental features which, embedded in any organization charts, are able to articulate authority relations. They can be expressed through the following formula:

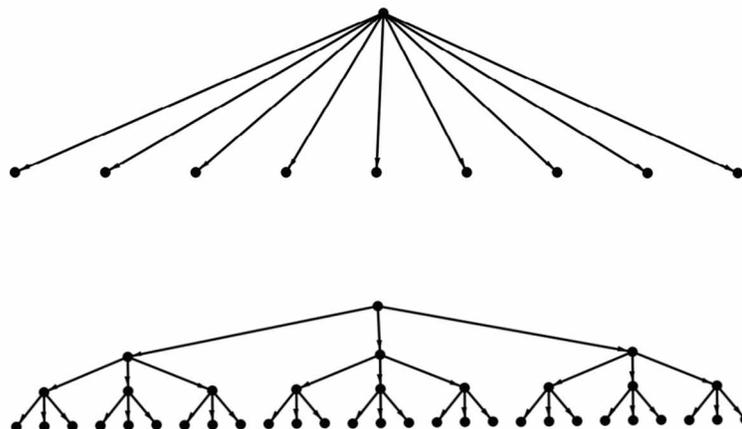
$$S = \sum_{i=0}^{NoR} SoC^i \quad (1)$$

Note that it's much more realistic to suppose that SoC increases as climbing to the next rank, because, "descending" down the hierarchical levels, activities become less complex, and thus, more subordinates can be managed by a single person⁹. In our analysis we assume for simplicity that SoC is constant for each hierarchical level, postponing to the future agenda the investigation on different articulations of *span of control* for different ranks.

Different configurations of the three variables generate different hierarchical structures¹⁰. The ideal-typical hierarchical structure is the *star form* in which one individual (the boss) controls directly all the individuals of the organization (see the upper image in Figure 1). Because this form is by definition characterized by 1 hierarchical level, then NoR = 1 and SoC = S - NoR. The star form (the upper image of Figure 1) is characterized by a configuration: NoR = 1, SoC = 9, and the resulting size (S) is 10.

Note that the *star form* is a pure *out-tree*, and thus, it scores 100% in terms of all the four indicators of out-treeness. If we replicate on different layers the *star form* we obtain Simon's archetype of hierarchy, in which one head decides what his subordinates should do in a typical pyramidal fashion (see the lower image of Figure 1).

Figure 1. A star form and an example of hierarchical structure



Even in this case, being a pure out-tree, the degree of out-treeness is 100%. Hence, they are not sufficient to grasp a precise analysis of OHD, lacking measures able to capture differences between organizations scoring the same degree of “out-treeness”¹¹. Of course, innumerable cases could be made of organizations scoring the same degree of out-treeness but having significantly different forms and size. In order to capture this aspect, it is possible to consider the measure of individual’s power par excellence: *out-degree centrality* (Out-Dc) (see Landherr *et al.*, 2010; Freeman, 1979). Out-Dc is an actor level measure that expresses the number of nodes toward which a node is linked (note that the measure can be standardized considering the number of nodes of the network). Referred to our theoretical framework, Out-Dc catches “how much” an individual directs his formal power toward subordinates. For example, the head’s Out-Dc in the star form of Figure 1 is 9: embedding the number of subordinates involved in the authority relation, the index expresses how much power a person possess and equals SoC. Clearly, while Out-Dc is an actor-level measure, we need here to employ a network-level measure, that is Out-Dc *centralization* (Out-Dc CE) which will be discussed in next section.

It should be underlined that, once we leave the measurement of OHD in terms of out-treeness and replace it with that of (direct) PC, we shift the concept of OHD from the share of asymmetric links to its overall distribution. These two aspects could be rather disjoint, because the share of asymmetric links is (relatively) independent on network topology. This phenomenon is particularly evident when looking at pure out-trees, as we point in this paper, because in the comparison of pure out-trees the degree of out-treeness becomes obviously useless. On the other hand, if were not analyzed and compared pure out-trees, then all the OHD measures – share of asymmetric links and Out-Dc distribution and centralization – would be required. Note that the hierarchical index contributing to Krackhardt’s degree of out-treeness takes into account not only direct power, but also indirect power, because the count of asymmetric links is not made on the original digraph, but rather on its derived reachability digraph. At first sight, using that index as a point of reference for our work, which is limited only to direct power – that is, direct relationships - could seem an incongruence. However, it is not, because if we would apply the same measure of hierarchical degree to the original digraph we would obtain a congruent index, and because each pure out-tree is made by only asymmetric links, it would score as well 100% of hierarchy. Hence, respect to the present analysis and our experiments, nothing would change. It would if, for instance, some intra-rank link were added, and so if the out-tree would be “imperfect”.

Our general hypothesis is that an OHD, when measured in terms of PC through Out-Dc CE, decreases with NoR increase. In the common sense an organization is considered hierarchical if it exhibits a high *number of ranks*, that is, an organization with, say, 6 ranks is considered more hierarchical than an organization with, say, 3 ranks. In this view the more an organization is pyramidal, the more it is perceived as hierarchical. Here, we move from a different and, to some extent, opposite consideration, which can be understood by means of a mental experiment. Let us consider an organization with, say, 6 hierarchical levels, and let us hypothesize that its PC is x . Now let us imagine to transform this organization in a flatter organization keeping equal its size. The consequence is that its OHD (x) will be distributed among a lower *number of ranks*, and this implies that each rank will be characterized by a higher level of power. In other words, keeping equal PC, a decrease of the *number of ranks* implies that each rank is characterized by more power. Hence, SoC will absorb the decrease of NoR. For example, if we transform an organization with 40 nodes and 2 ranks (the lower image in Figure 1) in a 1-rank star organization, we will obtain that SoC will increase from 3, to 39.

The ostensible paradox - that an organization with higher NoR is, *ceteris paribus*, less hierarchical than one with a few levels - is a matter at stake, and it represents the first main objective of this work.

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Objective 1: Analyzing the dependence of OHD - measured in terms of PC through centralization - on different configurations of NoR and SoC.

That OHD changes for different configurations addresses to the further question about how power is distributed among ranks. In fact, the common sense that power is concentrated on the higher ranks had to be tested (especially when limiting the analysis only to direct power). If we consider each rank as a group we can measure its centrality, considering the group as an actor, characterized by an *out-degree* (and an in-degree). Everett & Borgatti (1999) developed a *group centrality* index in order to avoid the methodological problem that the traditional centrality measures apply only to individual actors. In particular, *group degree centrality* is the number of non-group nodes that are connected to group members, normalized by dividing by the number of non-group actors. In this work, in order to analyze PD among the different organizational ranks we can calculate *group degree centrality*¹² referred to the *out-degree* (called here R-Out-Dc) in order to know if power is distributed more/less on higher ranks with respect to lower ones. This is the second main objective of this work:

Objective 2: Analyzing PD - measured in terms of rank centrality - among the different ranks of the organization.

METHODOLOGY

An overall network-level measure of Out-Dc is the *Freeman's index of centralization* (Out-Dc CE) in the form that has been introduced by Freeman (hereafter Out-Dc CE(F)), which indicates how much in a network the formal power is concentrated in one or few nodes (see Wasserman & Faust, 1994; Freeman, 1979). The range of Out-Dc CE(F) is between 0 and 1: it equals 0 when all actors have the same centrality value and it equals 1 when one actor dominates the others.

The formula of Freeman's centralization index is:

$$\text{Out-Dc CE(F)} = \frac{\sum_{i=1}^S (x^* - x_i)}{(S-1)(S-2)} \quad (2)$$

where x_i is the centrality of the actor i , and x^* is the largest value of the index, occurring among the S actors.

The index computes the sum of differences between the largest degree and the other observed values (numerator) and compares it with the theoretical maximum possible sum of differences (denominator), which occurs for the *star form*. Put in other words, in the numerator the index computes how much the actors' degrees differ from the most central actor (the one with highest degree), and, in the denominator, the index weights such measure using the theoretical maximum possible, corresponding to the case of a *star form*. The theoretical maximum possible is a theoretical quantity and is not calculated using an observable graph but taking into account all possible theoretical networks (with a fixed size). Notice that the theoretical maximum possible is calculated leaving aside any consideration about network density.

Freeman's index of centralization is subject to some limits discussed in Snijders (1981). In particular, Freeman's index, using as a benchmark the single most central actor and comparing it with the others, is

not able to take into account that there can be more central actors. For example we can have a network with more-than-one very central actors and we could be interested in measuring how much all the other nodes differ from them, and not just from the most central one (as in Freeman's index). Snijders (1981) develops an *index of heterogeneity* - called J^2 - which takes into account the differences in actors centrality among all actors: such index compares the degree of each node with the average degree by calculating the variance of observed degrees. Like Freeman's index, *Snijders' index of heterogeneity* is still a measure of dispersion, but can take into account more extensively the network topology. The original Snijders' index weights the variance of degrees (numerator) with the theoretical maximum variance of a network (denominator), using as a parameter the network density. In other words, (and differently from Freeman's index), the index of heterogeneity allows calculating the relative theoretical maximum, because it gives the option of specifying the network density.

Here in order to compare Snijders' index (in which the density is a parameter for calculating the theoretical maximum) with Freeman's index (in which the density is not a parameter for calculating the theoretical maximum) we will consider the maximum possible variance, without restriction on the network density.

The Snijders' index ranges between 0 (minimum heterogeneity among degrees) and 1 (maximum heterogeneity among degrees). The index - that we name Out-Dc CE(S), so to make it strictly similar to the other one - is expressed by the following formula:

$$\text{Out-Dc CE}(S) = \frac{V}{V_{\max}} = \frac{S^{-1} \sum_{i=1}^S x_i^2 - \bar{x}^2}{(1 - S^{-2})h^2 / 4} \quad (3)$$

The numerator is

$$V = S^{-1} \sum_{i=1}^S (x_i - \bar{x})^2 = S^{-1} \sum_{i=1}^S x_i^2 - \bar{x}^2 \quad (4)$$

and expresses the variance of the *out-degree*, representing the dispersion of centrality of each node x_i from the mean degree \bar{x}

The denominator is

$$V_{\max} = (1 - S^{-2})h^2/4 \text{ (if } S \text{ is odd, that is our case)} \quad (5)$$

or

$$V_{\max} = h^2/4 \text{ (if } S \text{ is even)} \quad (6)$$

where $h = S - 1$. The denominator expresses the maximum possible theoretical variance of degrees: it occurs when half actors are characterized by the highest degree and the other half by the lower degree (without restrictions on the network density). This case (half and half) corresponds to the situation in which, in a network, half of the nodes are connected to all the nodes, and the other half presents no con-

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nections, that is to say, an organization in which half of actors are the heads and control all the others, and half are the subordinates.

That half of nodes are heads (each one controlling all the other nodes) implies that each head is the central node of a star form, hence the theoretical maximum is reached in a graph composed of as many star forms as the half of the nodes. This property is fundamental because it reveals that also in Snijders' index the star form represents the benchmark-topology used to measure the dispersion of degrees among nodes.

The role played by the star form as a benchmark-topology is strictly related to the role of network density. Indeed, when calculating the denominator of Snijders' index, if we do not impose any restriction on network density (our case), we reach the maximum theoretical possible value when half of the nodes possess all connections and the other half possess no connections. On the contrary if we impose restrictions on network density we obtain relative theoretical maxima. For example if we impose the density of an out-tree graph (characterized by definition by low redundancy) we reach its relative theoretical maximum when one node possesses all the connections and the others possess no connections, that is to say, we reach the maximum in the case of a single *star form*. Notice that the possibility of specifying the network density, as a parameter of theoretical maximum, is not just an ancillary option. Being the density a linear function of the average degree, a restriction on the density implies a restriction on the average degree, that is to say, calculating the maximum variance given a specific average degree.

In this work we opted for calculating the Snijders' index without any restriction on density (and on the average degree) in order to make a comparison with Freeman's index for which density is not a parameter. In other words we opted for the theoretical maximum possible (as for Freeman's index) and not for a relative theoretical maximum.

Now, with reference to Objective 1, we can analyze the two centralization indices corresponding to organizations with different configurations of SoC and NoR.

In a stylized manner:

$$Y = f(X_1, X_2)$$

where Y is Out-Dc CE; X_1 is SoC; X_2 is NoR; and f is a function which expresses the *out-tree* structure.

In our experimental design, the independent variables can assume these values:

SoC = 2, 4, 8, 16.

The values indicate the number of people that can be managed by a single boss. According to management and organization science (Baligh, 2006; Biggiero & Sevi, 2009; Burton *et al.*, 2006; Burton & Obel, 1998; Colombo & Delmastro, 1999; Delmastro, 2002; Lin, 1994; Mintzberg, 1979), 4 and 8 represent typical numbers for small groups, while 16 is a medium size group. Though in some organizations there are units in which, under highly standardized and routinized contexts, a boss manages more than 40 subordinates, such cases are not considered in our analysis, because the four values we have analyzed are enough to allow useful generalizations.

NoR = 1, 2, 3, 4.

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Though some very large organization is characterized by 5-7 hierarchical levels, we consider a maximum of 4 ranks to keep our analysis as simple as possible but enough accurate for generalizations. Moreover, Colombo & Delmastro (1999) and Delmastro (2002) show that these are the most diffused structures.

Notice that, as in Equation (1), S is dependent on NoR and SoC. For our experimental design, we assume a uniform SoC intra- and inter-ranks, and thus, we deal not only with pure out-trees, but more specifically with that sub-class of fractal out-trees (Li *et al.*, 2008). In fact, excepted for the lowest rank, each node reproduces the same micro-pattern (the elementary configuration), which is represented by SoC¹³. Indeed, this facilitates analysis and interpretations, especially for future statistical modeling of this analysis, because fractality generates regularities in the ways that our power indicators (the dependent variables) change by varying SoC and NoR (the independent variables). However, if out-trees were not fractal, every result would hold as well.

The dependent variable – OHD in terms of PC - will be calculated by imposing the values indicated above to the two independent variables, and – by computing Out-Dc CE(F) and Out-Dc CE(S) - for each configuration, as shown in Table 1 in the next section (for an overview on approaches and methodologies see Newman, 2003; Wasserman & Faust, 1994; Wellman & Berkowitz, 1988; Burt, 1980).

In order to further articulate the Objective 1 we also investigate on the role of weighted connections. We move from the consideration that higher ranks should make more important decisions if compared with lowest ones. In this perspective we will analyze Out-Dc CE(F) for weighted relations, assigning value 1 to the first rank, and dividing by 2 the weight for other ranks: weight = 0.5 for the second level, 0.25 for the third, and 0.125 for the fourth. We calculate only Out Dc CE(F), because there is no corresponding valued-links algorithm for the Snijders' index.

With reference to Objective 2 we will calculate R-Out-Dc. Therefore, our analysis is characterized by the construction of 16 out-trees, each one described by NoR and SoC. For each out-tree we calculate the Out-Dc CE(F), the Out-Dc CE(S), and within each out-tree the R-Out-Dc for each rank (as in Table 1 in the next section).

Table 1. Results of Out-Dc CE(F) and Out-Dc CE(S)

Values of Out-Dc CE (given SoC and NoR)		NoR			
		1	2	3	4
SoC	2	Out-Dc CE(F): 1.000 Out-Dc CE(S): 1.000	Out-Dc CE(F): 0.222 Out-Dc CE(S): 0.111	Out-Dc CE(F): 0.082 Out-Dc CE(S): 0.020	Out-Dc CE(F): 0.036 Out-Dc CE(S): 0.004
	4	Out-Dc CE(F): 1.000 Out-Dc CE(S): 0.667	Out-Dc CE(F): 0.160 Out-Dc CE(S): 0.029	Out-Dc CE(F): 0.036 Out-Dc CE(S): 0.002	Out-Dc CE(F): 0.009 Out-Dc CE(S): 0.000
	8	Out-Dc CE(F): 1.000 Out-Dc CE(S): 0.400	Out-Dc CE(F): 0.099 Out-Dc CE(S): 0.005	Out-Dc CE(F): 0.012 Out-Dc CE(S): 0.000	Out-Dc CE(F): 0.001 Out-Dc CE(S): 0.000
	16	Out-Dc CE(F): 1.000 Out-Dc CE(S): 0.222	Out-Dc CE(F): 0.055 Out-Dc CE(S): 0.001	Out-Dc CE(F): 0.003 Out-Dc CE(S): 0.000	Out-Dc CE(F): 0.000 Out-Dc CE(S): 0.000

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Table 2. Results of R-Out-Dc for each rank

Values of R-Out-Dc (given SoC and NoR)		NoR = 1			
		Rank 1			
SoC	2	0			
	4	0			
	8	0			
	16	0			
Values of R-Out-Dc (given SoC and NoR)		NoR = 2			
		Rank 1	Rank 2		
SoC	2	0.667	0.000		
	4	0.800	0.000		
	8	0.889	0.000		
	16	0.941	0.000		
Values of R-Out-Dc (given SoC and NoR)		NoR = 3			
		Rank 1	Rank 2	Rank 3	
SoC	2	0.286	0.571	0.000	
	4	0.190	0.762	0.000	
	8	0.110	0.877	0.000	
	16	0.059	0.938	0.000	
Values of R-Out-Dc (given SoC and NoR)		NoR = 4			
		Rank 1	Rank 2	Rank 3	Rank 4
SoC	2	0.133	0.267	0.533	0.000
	4	0.047	0.188	0.753	0.000
	8	0.014	0.109	0.875	0.000
	16	0.004	0.059	0.938	0.000

RESULTS

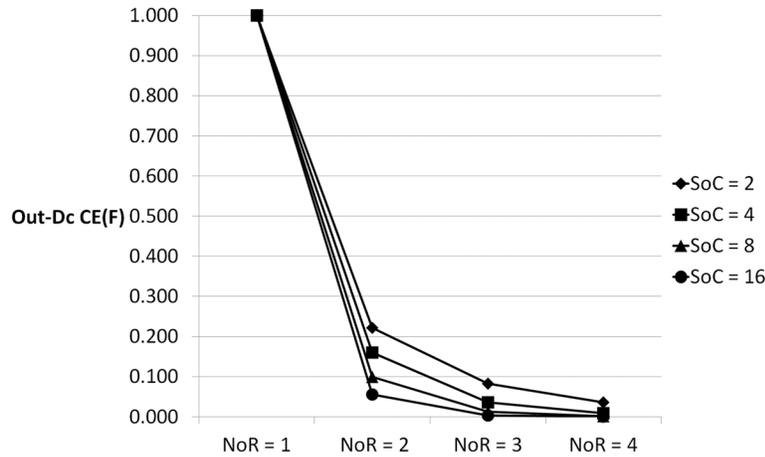
With reference to Objective 1, Out-Dc CE(F) and Out-Dc CE(S) for each one of the 16 graphs are shown in Table 1.

With reference to Objective 2 of analysis, R-Out-Dc (normalized by the number of connections) for each rank, for organizations characterized by increasing NoR and SoC are in Table 2.

DISCUSSION

The analysis of Out-Dc CE(F) - see Table 1 and Figure 2 - shows that for the star forms (NoR = 1) the value of the index is 1, which is the same value of the degree of out-treeness of the upper image in Figure 1. Therefore, the *star form* – and not the typical org chart like the lower image of Figure 1 - can be seen as the prototype of hierarchical structure, because it score the maximum OHD value either for the degree of out-treeness or for PC (in terms of both centralization indexes). Moving from the star

Figure 2. Out-Dc CE(F) (given NoR and SoC)

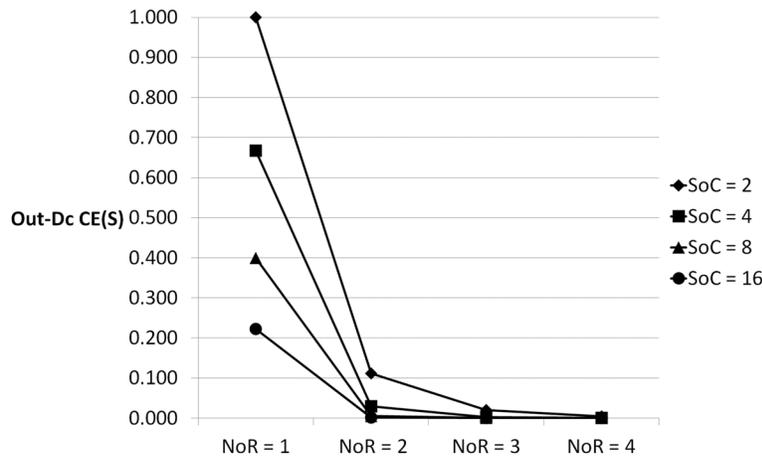


form, with NoR increase, centralization drastically decreases (about 80%). This is a fundamental and somehow surprisingly result, which shows that, ceteris paribus, an organization with more ranks is less hierarchical, that is to say, *a flat organization is, in terms of only direct power, more hierarchical than a pyramidal organization.*

The magnitude of OHD decrease is monotonic but nonlinear (Figure 2), and it depends on SoC: from 1 to 0.222 to 0.082 to 0.036 for SoC = 2, and it is 1 to 0.16 to 0.036 to 0.009 for SoC = 4, and so on. SoC affects the slope of Out-Dc CE(F) decrease to the extent an organization with a lower SoC is more hierarchical with respect to an organization with a larger SoC. In fact, for a SoC = 16 the index tends rapidly to zero for each NoR > 1.

With reference to Out-Dc CE(S) (see Table 1 and Figure 3) the trend is similar to Out-Dc CE(F): an increasing NoR corresponds to a decreasing OHD. In particular, the decrease is larger for lower SoC, that is to say, organizations with lower SoC are more hierarchical than organizations with larger SoC. When measured through Out-Dc CE(S), OHD is strictly affected by SoC which, contrarily to Out-Dc CE(F), progressively and remarkably reduces OHD for any SoC value >2.

Figure 3. Out-Dc CE(S) (given NoR and SoC)



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It is fundamental to notice that for $NoR = 1$ – the star form – the Out-Dc CE(S) does not reach the maximum value = 1 because the reasons discussed in the section of research questions. In fact, we opted for calculating the theoretical maximum possible and not the relative maximum. And the theoretical maximum possible is reached not for a single star-form graph but for a graph composed of as many star forms as the half of the nodes (that is, a graph in which half of the nodes are connected to all the nodes, and the other half present no connections).

Both indices reveal, in different ways, the same result: an organization with a higher *number of ranks* is, in terms of only direct power, less hierarchical than one with few ones; and this evidence is magnified considering the role played by SoC. In fact, the higher the power of single managers as expressed by their SoC (which corresponds to their Out-Dc), the less concentrated is overall power.

In order to explore in a deeper manner the first objective - as we discussed in the methodological section - we also investigated on the role of weighted connections in the graphs, moving from the intuitive consideration that in higher ranks make more important decisions than in lower ranks. So, following this assumption, we assigned value 1 to the first rank and we divided by 2 the weight for other ranks: weight = 0.5 for the second level, 0.25 for the third, and 0.125 for the fourth. We calculated only Out Dc CE(F), because there is no corresponding valued algorithm for the Snijders' multi-centered approach to centralization. Results show that the impact of weighted relations on OHD overall analysis is marginal (so we omitted to indicate in a table): values were very similar to the ones of no weighted graphs (Table 1) and there is no difference in the monotonic configuration of patterns. For a relative weight of double size between each rank pair seems already significant and almost nothing changes, it is likely that to counterbalance the dichotomous OHD it would be necessary to increase links weight differences between ranks to extreme values. However, they would be hardly legitimate from an empirical point of view. Therefore, we argue that this evidence presents a deeper and more general implication about the nature of hierarchy in terms of direct power: it is intrinsically specified in terms of its topology seen as a pyramidal out-tree structure. It is the topology itself to embody the power relations and, in particular, to determine its concentration degree, and its distribution between ranks.

With reference to the Objective 2 (see Table 2), starting from the star form we observe that power is not present only in the first rank, because it is fully possessed by the unique actor. If we add a rank (see

Figure 4. R-Out-Dc (for 2-ranks organization)

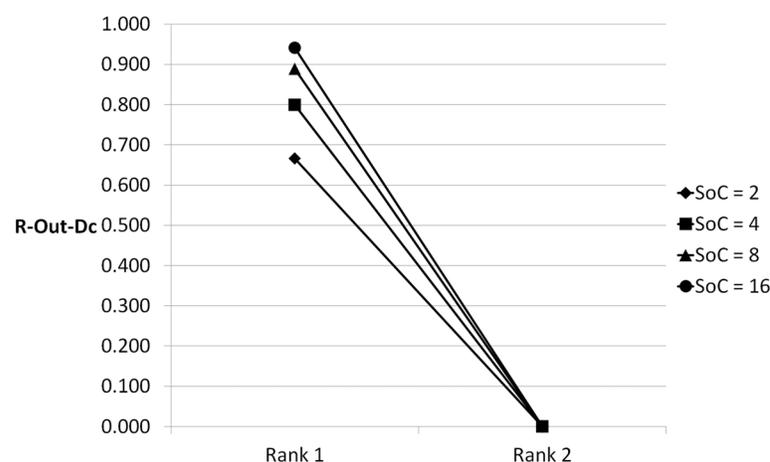


Table 2 and Figure 4), we note that power is mainly concentrated in the first rank, respect to the second (and last) rank (rank 2). And this result is magnified for larger SoC, that is to say, within the rank 1 there is more power if the SoC is larger.

If we further increase NoR to 3 ranks, we can observe an interesting nonlinearity in the dynamics of power: as shown in Table 2 and Figure 5 power tends to be concentrated within the intermediated rank (Rank 2) in which R-Out-Dc reaches its maximum. It is interesting to note that in this intermediate level (Rank 2) the effect of the SoC control is inverse with respect to the effect in Rank 1. If in Rank 1 centrality presents higher values in correspondence of lower SoC, in Rank 2 centrality reaches its maximum for SoC = 16.

If we build a configuration with 4 ranks, we are able to generalize our results (see Table 2 and Figure 6). The interesting evidence is that power tends to be concentrated in Rank 3, that is to say, observing other cases with NoR = 1, 2, 3, we are able to state that power tend to be concentrated in the penultimate rank of the organization. In fact if the last rank presents an absence of power (being R-Out-Dc = 0) the other ranks, except the penultimate, present an increasing level of power, that is to say, *power tends to be concentrated not in the top of the organization but in the lower ranks, and reaches its maximum in the penultimate rank*. Notice that for this penultimate rank the effect of SoC is inverse: if in the other ranks a larger SoC corresponds to larger centrality, in the penultimate level the maximum of power is characterized by the maximum SoC (16).

Unfortunately, despite the plethora of different approaches and objectives hinted in the section on previous researches and the many social network studies on hierarchical structures, our results cannot be directly confronted with any other, if not confirming what Freeman (1977, 1979) argued in his early works on centrality measures, that the star form has the most hierarchical degree. Other studies do not focus on direct hierarchical power and its concentration and distribution, while instead they treat aspects that here have not been dealt with, like indirect hierarchical power, informal power, positions equivalence within hierarchies, optimal hierarchical structure according to organizational decision making process or communication cost, efficiency, etc.

Figure 5. R-Out-Dc (for 3-ranks organization)

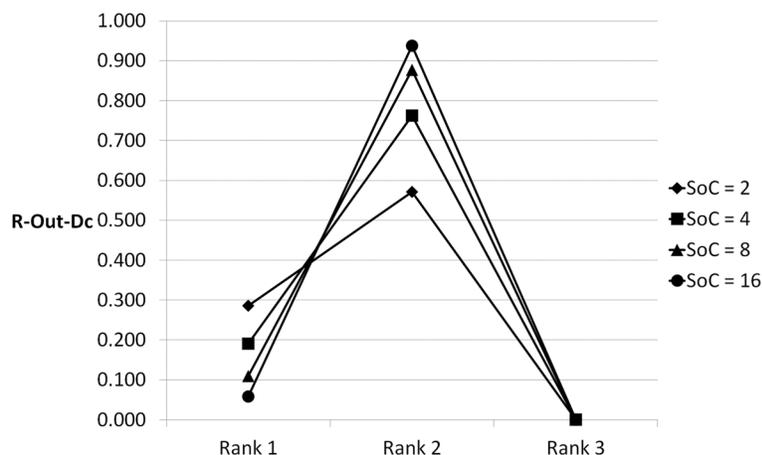
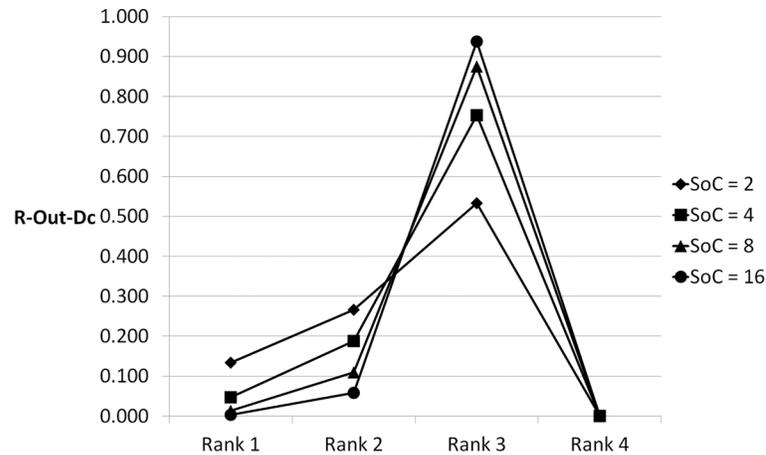


Figure 6. R-Out-Dc (for 4-ranks organization)



CONCLUSION

If and how direct-power hierarchical degree of a formal organization depends on the configuration of number of ranks and span of control is the point we analyzed in this work. Besides only direct power, we limited the analysis to pure out-tree structures (thus, excluding decisions made between individuals placed at the same rank), and to the case of uniform intra- and inter-rank individual power (span of control).

The first fundamental result is that a hierarchical degree measured in terms of out-degree centralization is inversely related to the number of ranks, that is, a flat organization is more hierarchical than a pyramidal organization. The second result is that the span of control plays the role of amplifying the impact of the pyramidal slope (indicated by the number of ranks): in particular, a smaller span of control corresponds to higher power centralization. The third result is that power tends to be concentrated in the lower ranks of the organization and reaches its maximum in the penultimate rank. The fourth result is that at the penultimate rank power is higher in correspondence of larger span of control, contrary to other ranks in which it is lower in correspondence of larger span of control. Finally, we highlight that, even taking into account that decisions do not have the same relevance in each rank but rather descend from top to lower ranks, the picture of direct power concentration and distribution does not change. Indeed, it is the out-tree topology itself to embody the power relations and, in particular, to determine its concentration degree, and its distribution between ranks. It is worth to highlight that if the hierarchical structure represents the archetypal form of hierarchy, the star form – and not the typical org chart - can be considered its ancestor, because it is characterized by the maximal hierarchical degree in terms of all the measures of (direct) power concentration and existence (the degree of out-treeness).

However, the claim of the star form as the purest hierarchical benchmark, which is actually implicit in Freeman's centralization index, becomes progressively less appropriate as an organization's archetypal form mutates from a perfect out-tree towards ramified, larger, and self-similar structures, as the ones that we have analyzed here. Its inappropriateness as archetype heavily increases to the extent that intra-rank links or reciprocal links are added or replaced to old ones, that is to say, the organization design becomes more modular and based on bilateral decisions. In fact, to the extent that an organization is more

decentralized and multi-centered, Freeman's centralization index becomes less appropriate to measure power concentration, and it should be replaced by Snijders' centralization index.

The major limitation of this work is that it does consider only direct power, which supposedly constitutes only a progressively smaller fraction of power as organization size increases by adding ranks. Indeed, the appropriate measurement of indirect power represents a task not so simple as could appear at first sight, because many different aspects should be taken into account, each one opening different options for choosing the appropriate indicator. The second important limitation is that of considering only pure out-trees, and thus, to exclude any symmetric or intra-rank (within or between organizational units) relationships. As the previous one, also this analysis would be rather challenging, because it opens a number of conceptual and methodological problems. Two, among the many, are noteworthy here. The first one is that would imply to "combine" the four indicators of out-treeness degree with the ones of power concentration. The second one is that, since they could move in opposite directions, it would become necessary to face with multicriteria problems. A third important limitation is that we did not employ a statistical model nor a formalization of the relationships between variables that we obtained through the numerical experiments.

Nevertheless, our work clarifies some issues about hierarchy measurement that, though they have crossed the whole history of sociological and organization studies, were remaining unclear. Moreover, limitedly to the understanding of direct power in pure out-trees, the action exerted by the main structural variables of organizations have been explored and quantified through numerical experiments. Although without a mathematical or statistical model, we know now how such structural variables interact to determine the degree of hierarchical power.

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ENDNOTES

¹ A different issue is whether such decisions reflect ex-ante rationality – as economics and large part of management sciences induce to think - or, instead, ex-post rationality – as the garbage can model (Cohen *et al.*, 1972; Bendor *et al.*, 2001; Fioretti & Lomi, 2010) and another remarkable part of organization science and sociology of organization proposes to think. The premises and results of this is paper are somehow neutral respect to the two perspectives, because they deal to the degree and distribution of direct power, regardless to the aims for which it is used.

² See Williamson (1975, 1981, 1985, 1994, 1996) and the followers.

³ For an explanation and general theoretical framework where placing such a delay, see Biggiero's contribution in the first chapter of this book.

⁴ For a deeper discussion of this point see Biggiero's contribution in the first chapter of this book.

⁵ Krackhardt's fundamental contribution of 1994 was published exactly into Carley & Prietula's readings.

⁶ This is a perspective taken by cybernetic studies, that here we did not review.

⁷ A network has maximum efficiency if it has the minimum number of links, which is $n-1$. Interestingly, if we consider a chain with all links oriented in the same direction, it would be sufficient to insert just one link connecting the two extremes to obtain a cycle. Consequently, this minimal redundancy would set to zero the hierarchy index, because through cyclic topology, in the reachability digraph all relationships would become reciprocal.

⁸ According to Li *et al.* (2008) “even if the network size is large enough to meet the demand of large-scale networks, the scale-free property can emerge only when a hierarchical tree lies in two extreme situations: 1) the exact span of control exists at all levels of an organization; 2) the node out-degree (i.e. span of control) distribution obeys a power-law distribution”. Therefore, these are not realistic conditions for organizational structures.

⁹ On the relationships between connection modes (interdependence), task complexity, coordination mechanisms, and group size see Biggiero & Sevi in this volume (Chapter 13).

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¹⁰ Note that the word “configuration” is used here with a general meaning with respect to organizational literature (Meyer *et al.*, 1993), because our configurations do not pretend to closely correspond to specific families of real organizations.

¹¹ The other limit of Krackhardt’s index of hierarchy is that, by definition, being based on counting asymmetric links, it cannot be applied to undirected graphs. However, the other three indexes composing the out-treeness degree can be applied to undirected networks too.

¹² In this work we normalized by the number of actual connections of the graph.

¹³ In the jargon of hierarchy studies, when SoC is constant only intra-rank, then the structure is called a *balance* hierarchy, while when it is constant also inter-rank, then it is called a uniform hierarchy. So, a uniform hierarchy is a fractal structure.