Relational Methodologies and Epistemology in Economics and Management Sciences

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A volume in the Advances in Finance, Accounting, and Economics (AFAE) Book Series

Information Science REFERENCE
An Imprint of IGI Global

Published in the United States of America by
Information Science Reference (an imprint of IGI Global)
701 E. Chocolate Avenue
Hershey PA, USA 17033
Tel: 717-533-8845
Fax: 717-533-8661
E-mail: cust@igi-global.com
Web site: http://www.igi-global.com

Library of Congress Cataloging-in-Publication Data

Names: Biggiero, Lucio, 1955- editor.
Title: Relational methodologies and epistemology in economics and management sciences / Lucio Biggiero, Pier Paolo Angelini, Mario Basevi, Nunzia Carbonara, Antonio Matrogiorgio, Eliano Pessa, Enrico Sevi and Marco Valente, editors.
Description: Hershey, PA : Information Science Reference, [2016] | Includes bibliographical references and index.
Identifiers: LCCN 2015041972| ISBN 9781466697706 (hardcover) | ISBN 9781466697713 (ebook)
Subjects: LCSH: Social sciences--Methodology. | Science--Methodology. | Social epistemology. | Knowledge, Theory of.
Classification: LCC H61 .R45 2016 | DDC 300--dc23 LC record available at http://lccn.loc.gov/2015041972

This book is published in the IGI Global book series Advances in Finance, Accounting, and Economics (AFAE) (ISSN: 2327-5677; eISSN: 2327-5685)

British Cataloguing in Publication Data
A Cataloguing in Publication record for this book is available from the British Library.

Chapter 2
NK Simulation Modeling

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ABSTRACT

Launched and developed primarily by Kauffman from the end of sixties, NK simulation modelling candidates for capturing networks dynamics. Grounded in reference to biological networks, it has aroused a grate and durable interest in economics and management sciences too. This methodology is split into a version focused on studying proper Boolean networks dynamics, whose trajectories are substantially conditioned by Boolean functions, and a (much more frequented) version focused on systems co-evolutionary paths driven by the search for optimizing its fitness value. Besides the unquestionable value of Kauffman's work for the theoretical implications on evolutionary biology and the strong interest for economics and management sciences, in this chapter failures and limitations of both NK modelling versions are discussed. In particular, it is shown that as applications try to be more realistic, this modeling becomes hardly treatable from a computational point of view. On the other hand, it is underlined that, especially the fitness landscape version, NK simulation modelling is very useful to show general aspects of system's dynamics, and the impossibility to find general optima (excepted for very special and unrealistic cases). This result sounds a sharp criticism to general economic equilibrium, and it is perfectly consistent with Simon's contributions.

INTRODUCTION

A way to perform network dynamic analysis is through Boolean networks (BNs) methodology, which here is treated in one of its most important formulations, proposed by Kauffman in the sixties (1969a, 1969b) and seventies (1971a, 1971b), and then developed and refined in the eighties (1984, 1986, 1988) in two directions: one focused on the self-organizing dynamics followed by a single BN, and the other focused on a system's (possibly also a BN) potential adaptive evolution within a fitness landscape. BNs theory and modelling is rooted in various research streams developed in the 60s: the theory of dynamic systems (Brian, 1984; Klir, 1991; Weisbuch, 1991; Wuensche, 1994, 1998); automata studies and cellular automata theory (Gill, 1962; Hanson, 2009; Ilachinski, 2001; Shannon & McCarthy, 1953; Sutner, 2009; Trakhtenbrot & Barzin, 1973; Waldrop, 1992; Wolfram, 2002; Wuensche & Lesser, 1992), cyber-

DOI: 10.4018/978-1-4666-9770-6.ch002

netics (Ashby, 1956; Ashby & Walker, 1966[1]; Glushkov, 1966; Kauffman, 1984, 1993; Trappl, 1983, von Foerster, 1982, 2003), and complexity science (Arrow *et al.*, 1988; Arthur *et al.*, 1997; Casti, 1989, 2004; Khalil & Boulding, 1996; Mainzer, 1994; Mitleton-Kelly, 2003; Strogatz, 2001, 2003; Sulis & Trofimova, 2000)[2]. In fact, networks are discrete systems, and so they have much in common with the theory of dynamic systems and complexity science. Further, besides its roots in cybernetics and systems science, cellular automata can be seen as a BN sub-group (Wuensche, 1994). From the same scientific milieu developed in the Santa Fè Institute (Waldrop, 1992), an important stream has been founded and brought forth either in economics (Arrow *et al.*, 1988; Arthur *et al.*, 1997; Arthur, 2010; Blume & Durlauf, 2006; Lane *et al.*, 2009) or in management and organization sciences (McKelvey, 1999, 2004; Schneider & Somers, 2006; and the Special Issue of Organization Science, 1999)[3].

Following the exposition made by Kauffman in his two books "At home in the universe" and "The origins of order", the first part of this chapter is dedicated to Boolean network modelling, which will be labelled as NK-BN, and the second part to fitness-driven network evolution, which will be called NK-FL. In fact, for current literature does not distinguish the two models, and calls NK model also those based on FL, to avoid confusion this specification is made. Kauffman links the two models into his main book "The origins of order" (pp. 209-219), when FL is a Boolean network space. The separation of the two models is accompanied by two disciplinary orientations: authors in the field of biology, mathematics and computer science focus on improving mostly NK-BN, while those in the field of economics and management focused almost exclusively on NK-FL models. Consistently with the setting of this volume, both models will be applied in the second part (see chapter 9 and 10).

As it will be clear later on, the separation of the two models hides important issues, because the relevance of self-organizing properties of biological (and social) systems (networks) rests more into the RBN rather into the FL model, while vice versa for the role of selection. Thus, its separation addresses to the incomplete view of a self-organizing but not selective or a selective but not self-organizing world. On the contrary, it seems that the evolution of biological and especially social systems results from the conflicting forces of self-organization and selection (with its connected devices of mutation and retention).

PART I: THE NK MODEL AS "PURE" BOOLEAN NETWORK DYNAMICS

Built on the theory of random Boolean networks (Bollobas, 1985; Dorogovtsev & Mendes, 2003; Gershenson, 2002, 2004a; Serra & Villani, 2006; Somogyvari & Payrits, 2000), the idea of the standard version of NK modelling is that the dynamics of a *topologically invariant* system (or network) - that is a system that does not mutate neither its size nor its links distribution - is defined deterministically (or stochastically) by the interaction of its nodes according to (as well invariant) interaction rules. A brief description of this methodology, as it has been proposed by Kauffman and applied by him and some others, can be done as follows:

- Given a certain binary network topology, it is assumed that nodes can have a number of activation states, which are usually reduced to two: on or off (active or inactive);
- The (active) edges going to each node[4] can be combined according to Boolean functions, which can be called (dis)activation rules;
- As output of each combination, a node receiving edges will be active or inactive, thus, determining a distribution of active/inactive nodes at the whole network level;

- Given a certain set of (dis)activation rules[5], it is possible to know all the states in which such a network can be found until reaching a final stable state, called attractor, which can be a single state (fixed point) or a series of states that are consecutive and invariable (a cyclic attractor). What characterizes an attractor is that all successive steps in its dynamics will not change the distribution of active/inactive nodes;
- Each initial state triggers a specific dynamics (trajectory), which can meet (and hence, from that state/point on, it will coincide with) another trajectory;
- The set of all trajectories constitutes a network state space.

A BN modelling is defined by two essential information: network topology and the set of (dis)activation rules, which are nothing else than Boolean functions. For in the standard model it is assumed a random distribution of links, topology is reduced to two parameters: network size (*N*) and uniform in-degree centrality, which in this literature is usually indicated with *K*[6]. From previous definitions, it comes that a network possible states are 2^N and that the number of transitions from one state to another are 2^2^N. Moreover, if we indicate with *z* the number of a node's activation states[7], the number of its possible activation rules is 2^z^k. Therefore, adopting the standard assumption of just two states nodes, if K=2, then there are 16 possible activation rules, if K=3 then 256, if K=4 then 65,353, etc. It is clear, therefore, the importance of *K* in determining a dynamic network complexity.

In essence, given a topology and (a set of) activation rules governing each node's behaviour, for each initial state (characterized by a certain distribution of active and inactive nodes) at least one attractor always exists. Notice that when a network arrives in that state, it does not mean that the network stops its dynamics, but rather it means that its dynamics no longer causes any change in the distribution of the activation states, i.e. active nodes remain active, and inactive nodes remain inactive. Now, the point is that, even with a fixed topology, if one or more activation rules or the initial state do change, then the attractor can change.

Given the relevance of *K*, the main question that Kauffman faced with was the following: if we took *K* as an order parameter, can be discovered peculiar phenomena? The answer was positive: there are two phase transitions between three main "regimes", and attractor's length and number do change substantially. Indeed, *a fundamental property of BNs modelling is the importance, type and variations of a trajectory average length*, because – projecting this model to a real network behaviour - the longer it is, the more resources are spent to reach the stable state. Moreover, if it is assumed that during the trajectory damages can occur, the longer it is the riskier it is. The other property is attractor's number and the size of its basin associated to a given network[8].

Kauffman demonstrated[9] that, when increasing *K*, BNs go across three regimes: an *ordered regime* when $k<2$, where the network is not even fully connected; a *chaotic regime* when $K>2$, where the network is densely connected; and a *regime on the "edge of chaos"* when K~2, where the network is stable but not too much. Between each of these regimes there is a phase transition[10]. Let's depict briefly the main characteristics of each regime in terms of attractors' length, number and convergence, and network change of a node's initial state. In the ordered regime network density is very sparse, in a state that in social network analysis (Hanneman & Riddle, 2005; Jackson, 2010; Lewis, 2009; Newman, 2010; Prell, 2011; Wasserman & Faust, 1994) would be called of high efficiency, because there are few redundant connections: each node receives *in average* less than two in-degrees. In this regime trajectories average length varies in polynomial form with *N,* and the mean number of attractors varies as the square root of *N*. However, length (and perhaps also attractors number) is not normally distributed, but rather similarly to a power-law, with few long trajectories and most short ones. Further, a change of a node's state would produce very small (temporary) consequences on its future states and on the future states of the other nodes. More specifically, if the consequences of an alteration of nodes' states are called cascades (or avalanches), it has been demonstrated that in the ordered regime there is a power-law relationship between the frequency of cascades and the size of its consequences. Hence, very few alterations produce heavy effects, while most alterations have irrelevant effects. Moreover, consistently with the previous characteristic, two close initial states would trigger trajectories that most likely will converge to the same attractor[11].

At the edge of chaos – that is, for *K~2* – trajectories average length varies strictly proportionally to the square root of *N*, and also attractors number is about the square root of *N*. Cascades and alterations are as well connected through a power-law function, even though its skewness is not so sharp than into the deep ordered regime, meaning that proportionally some more alterations can produce devastating consequences. Close initial states trigger trajectories that have no preference to converge or diverge. Finally, in the chaotic regime – that is, for *K>2* –attractors average length varies exponentially with the square root of *N*, and thus, for a just small size like 200 nodes, it would be an astronomic number. Recalling a Kauffman's example, if we take into account that the age of the universe is believed to be of the order of 10^{23} seconds and a network of 200 nodes would have 2^{200} states, which is a number roughly equivalent to 10^{60}, a computer that calculates a state for each microsecond-age would take much longer than the age of the universe to compute even a single trajectory[12]. While trajectories length increases exponentially, attractors number grows linearly with *N*, and when *K=N* reduces to *N/e* (where *e* means the natural logarithm). Moreover, besides the set of alterations connected to cascades occurs in a power-law shape (as in the two other regimes), there is a "spike" of alterations that would produce devastating avalanches. Further, trajectories starting from two close initial states would rapidly and unpredictably diverge, likely reaching different attractors.

In sum, moving from the ordered to the chaotic "regime", while increasing *k* as order parameter, trajectories average length increases dramatically, node's state alterations produce more devastating consequences, and trajectories rapidly diverge preventing to infer any future development of a given initial state as well – a fortiori - of its similar states. This is due to the fact that with increasing network complexity the number of cyclic attractor's increases constantly, and therefore many attractors that at first were independent fixed points, now are part of the same cyclic attractor. This means that the basin of attraction of each attractor increases greatly, and thus, it provides a kind of order even in the midst of chaos. The problem is that trajectories length becomes enormous, and that with even small changes in topology or activation rules, trajectory becomes very unpredictable.

Kauffman was a biologist, and he developed his NK model finalized to and consistent with the construction of a theoretical framework for evolutionary biology. After the early 25 years of research and computational experiments on his model and empirical research on biological networks it was clear that many of them, like cellular networks, gene networks, and protein networks are very large and – most problematic respect to NK modelling - have *K* much larger than 2, but at the same time it resulted that they are not chaotic. For instance, it has been recently discovered (Zimmer, 2011) that a Boolean dynamic network *par excellence*, like the brain neuronal network, has about K=1000, because 100 trillion connections are distributed between 100 billion neurons. So, there should be some more order parameter governing the behaviour of these networks, able to keep them into an ordered regime even with high levels of connectivity. Not surprisingly, there can be many physical or biochemical factors playing this role, but what interested Kauffman was the mathematical expression of such factors. Two parameters were found, one with the help of other colleagues – namely Derrida and Pomeau – and another introduced by Kauffman himself. Both refer to differences between Boolean functions, and in particular they deal with the fact that some basic Boolean functions[13] reduce (compress) the variety (complexity) of potential outputs. The former one is the so-called *P* parameter, which refers to the fraction of 0 and 1 characterizing each Boolean function, which varies between 0.5 and 1. In fact, there are functions very unbalanced on 0 or 1, and therefore they can reduce the variety of output combinations with other nodes because they operate as variety reducers: in front of many possible states of inputs they produce most likely the same – 0 or 1 – output. Thus, they act as complexity compressors hindering a network to grow towards the chaotic regime, and it has been demonstrated (Kauffman, 1993, 2000) that, in a plane with *P* parameter in the vertical axis and *K* in the horizontal axis, the phase transition to the chaotic regime moves from level of K=2 to K=10 while increasing *P* from 0.5 to 0.9.

The other parameter is represented by canalizing functions (CF), which are defined as the ones in which – in the simplest case - the state of one of the two inputs is enough to guarantee the output. In other words, one input constraints (canalizes) the output, leaving irrelevant the state of the other inputs. As for the other order parameter *P*, even in this case it has been computationally demonstrated that, while growing *K*, an appropriate “dosage” of *CF* can “delay” the phase transition to chaos[14]. Unfortunately, the *CF* fraction sharply reduces with k increases. In particular, it reduces shifting between K=2 and K=4, with 87.5% and 4% respectively of *CF*. After K=4, that fraction reduces asymptotically with *K* increment. However, in some experiment this property explained real networks high connectivity, because even a small fraction of *CF* revealed to be enough to prevent chaos. Moreover, some *CF* has also a high *P* parameter, and anyway the combination of the two parameters can keep a highly *K* connected network into the ordered regime.

So far, it has been sketched the standard version of NK model. Further studies during last 15 years are developing and extending it in several directions. The main effort is to adapt NK model to be more corresponding to real networks, and thus, to “relax” many assumptions of the standard version. A direction was to study distributions that are very different from random or regular ones, and in particular the scale-free type (Aldana, 2003; Serra *et al.*, 2004), because they seem to be very diffused in real natural and social networks (Barabasi, 2003; Caldarelli, 2007)[15]. Another development removes the assumption of simultaneous update of all nodes’ states, replaced by the adoption of asynchronous dynamics. Indeed, many (perhaps most) real networks work in this way, because time intervenes in different ways to each single node. Several studies are underway (Correale *et al.*, 2006; Denga *et al.*, 2007; Di Paolo, 2001; Gershenson, 2002, 2003, 2004a, 2004b; Gershenson *et al.*, 2003; Luqua *et al.*, 2005; Oosawa & Savageau, 2002; Solè *et al.*, 2000) in order to understand what changes respect to the standard model: how do modify the three regimes? How *K* and *N* combine with the other order parameters? Etc.

Kauffman himself (1993) was aware that other order parameters could likely spring out in future studies. Klüver & Schmidt (2007) made an interesting advancement drawing attention to network topology: it is true, they argue, that what primarily matters is in-degrees distribution, because Boolean functions are associated to them, but they claim that, especially if random or regular distributions are not employed, out-degrees distribution is not irrelevant. They analyze a network composed by six nodes, with uniform k=2, and 2-state nodes, from which 10^6 possible digraphs could be built[16], each one assuming 2^6 possible initial states. The 16 Boolean functions generate 16^6 possible combinations associated to each initial state. It produces an enormous number of combinations to be explored, even assuming a constant K[17]. And to each of its 2^6 possible initial states had to be applied the 16^6 possible combinations of the 16 Boolean functions. Indeed, 10^6 possible digraphs could be reduced focusing only the 1499 non-isomorphic digraphs, and 2 out of the 16 Boolean functions – the contradiction and the tautology, that is, 0000 and 1111, respectively - can be eliminated because they have no practical meanings in the real world. However, even with this focus only on non-isomorphic digraphs and 14 functions, it remains an enormous number of experiments to be done. Following the work done by Klüver & Schmidt (2007), "out of the remaining set of 14 Boolean functions 22 subsets of three functions each were selected; from any of these subsets 3^6 combinations yielding six functions were formed" (pg. 31). In sum, $1.6*10^9$ digraphs should be analyzed, corresponding to $1499*2^6*22*3^6$. They define an index that can capture many topological features and it is based on the sequence of nodes' out-degree[18]. In other words, the logic is the following: with a given (constant[19]) in-degree, a number of different digraphs can be built varying out-degrees combinations. For an out-degree sequence contains some information about network topology, they propone to use it jointly with *P* and *CF* as the three order parameters that, given *N* and *K*, allow understanding BN dynamics.

All in all, and considering that their experiments are done with N=6 and uniform K=2, their main results can be summed up as follows: i) only 1% of all configurations generates long trajectories; ii) the presence of "terminal" nodes[20] strongly reduces the frequency of long trajectories[21]; iii) *long trajectories – which can be assumed as an indicator of complexity – prevail when out-degrees are evenly distributed rather than in extreme configurations*; iv) none of the three parameters – *N*, *K*, and topology[22] – is enough to determine complex dynamics; v) among the three parameters, topology is the most important to determine complex dynamics; vi); there is a "principle of minor inhomogeneity" of activation rules and *CF*, meaning that complex dynamics requires a configuration of quasi-homogeneity "poisoned" with few diversity; vii) in any case, in order to generate complex dynamics, the three parameters should range in a certain area of its respective state space, but this is a necessary and not a sufficient condition. Besides these results, they confirm other already established results, like that only activation rules whose *P* parameter is lowest or low *CF* can trigger long trajectories

In-degrees distribution investigated by Gershenson and others, and out-degrees distribution investigated by Gershenson and Klüver & Schmidt are interesting and relevant parameters contributing to understand BN dynamics. They are consistent in underlying the crucial role played by network topology for network dynamics[23], and they give also various general results, whose synthesis can be that when topologies are not regular or random, but rather very irregular, then trajectories shorten dramatically, and thus, also large dense networks tend to remain in ordered regimes or at the edge of chaos. This would give sense to the fact that, at the very end, many real large and interconnected systems are not chaotic, even if its density would suggest chaotic behaviors. Especially if a significant portion of its nodes are governed by activation rules characterized by high *P* and *CF* parameters. In other words, when these parameters values are diffused in a very unbalanced links distribution either in input or in output, then complex behaviors is limited to a small fraction of combinations - be they concerning particular topologies, initial states or activation rules (or some combination of them). This result is also very consistent with the empirical fact - discovered during last 15 years of studies in network analysis – that such complex (in the sense of large and relatively dense) networks are very nonlinear, and often shaped in a scale-free structure. If confirmed by future researches, this conclusion strengthens one another: nonlinear topologies of this kind would make networks more resilient not only in a static, but also in a dynamic dimension.

Applications in Economics and Management

Kauffman applied and developed this methodology firstly into life sciences[24]: evolutionism (1993, 1996), molecular biology (1969a and 1969b, 1993) and, in collaboration with others, to medicine (Huang & Kauffman, 2009; Huang *et al.*, 2009). The main applications have so far been heavily concentrated in the study of gene regulatory or other biological networks[25] (Correale *et al.*, 2006; Das *et al.*, 2010; Krawitz & Shmulevich, 2009). Outside biology, the fascination of organization and management sciences for NK models concerned almost exclusively NK-FL.

Carroll & Burton (2000) and Klüver & Schmidt (2007) apply NK-BN modelling to the same topic of organization science: the complexity and efficiency of centralized vs. decentralized workgroup structures[26]. In particular, revisiting an old experiment in communication networks research (Bavelas, 1948, 1950; Leavitt, 1951; Shaw, 1954), Carroll & Burton looked for confirmations and extensions of already established concepts concerning the relative (dis)advantages of the two structures. Their main aim is to develop a better understanding of the relationship between group structure and individual's interdependence, and its effects on group performance under various task complexities. Note that here the level of interdependence is meant as depending only on the number of communicational connections between activities-agents. Carroll and Burton find consistency between the results of their model and NK-BN models, even though their model and experiments have little to do with them, because nodes have just one (active) state – and therefore there are no activation rules. Moreover, K is assumed to be averaged and not constant, and undirected. Further, model and experiments were built to measure group performance not in terms of attractor's length and number, but rather in terms of efficiency (execution time) and effectiveness (task outcome quality). Something that – as Biggiero & Sevi (2009, and in Chapter 13 of this volume) - has much to do with the relationships between task complexity and workgroup structure rather than with NK modelling.

Klüver & Schmidt (2007) perform a true NK-BN study, and in their research design the topological parameter of out-degree sequence was handled in order to contrast three kinds of structures: an equally distributed, a centralized, and an intermediate. The first one corresponds to what in social network or organization studies is indicated as one of the prototypical forms: the circle[27]. Indeed, it is not the most equally distributed structure, which indeed is the team[28] (or clique, in the language of social network analysis). The centralized structure is designed as one in which two nodes influences all the others asymmetrically and there are three reciprocal connections. They interpret them as prototypes of, respectively, democracy and hierarchy, and they find that, if measured in terms of trajectory length, the former is more likely to have a complex dynamics. In other words, order parameters typical of the democratic topology have values that, even if with very low probability, can allow complex behaviors. Conversely, to the hierarchical structure are associated order parameters values out of the range that admit complex behaviors.

Biggiero & Basevi (2012) have applied this methodology to the Lazio Region (Italy) aerospace industrial cluster[29]. The actors constituting this network are profit and not-for-profit organizations, like institutions of local government, trade associations, universities, etc. These actors can exchange different things, like goods and services, information, knowledge, etc. In the application in question the network of knowledge has been studied, and in particular the managerial one, consisting of what agents learn from each other during their collaboration for innovation: e.g. the development of new products, the participation in research consortia, etc. In the concrete application of the NK model, the node activation rule was linked to the capabilities of the agents that insist on that node. The resulting value is then further "weighted" by the number of incoming agents. In addition, the activation is measured by means of thresholds and not as an on/off condition, and in this way one was able to take into account the continuous variations in the knowledge that a node receives from other agents. According to their absorptive capacity (Cohen & Levinthal, 1992)[30], agents retain a certain specific amount of knowledge. The role of absorptive capacity (and hence the responsiveness to the received knowledge) has been explored in the intermediate value, modifying it slightly: > 5 or ≥ 5.

The sensitivity analysis of this threshold is measured in terms of how many attractors are generated, how many nodes are active in each of them, and consequently how much knowledge is exchanged and by whom. What emerges is that the minimum difference between a strictly higher constraint - including also the intermediate value - determines quite significant changes in the results: the number of the attractors rises strongly from 131 to 193, while the number of active nodes drops considerably. Without going over the other results and possible simulations, it is helpful to understand what they may suggest, for example, in terms of industrial intervention policies on the cluster evolution. Given the sensitivity of the dynamics related to the parameters (and possibly even to the topology), a local government institution may obtain useful information from it to orient the incentives towards the collaboration for co-operation, since this depends consistently on how much knowledge is exchanged and on the agents producing it. It may happen that slight changes in the topology of the network, namely in the relations of cooperation between companies, lead to significant changes in the attractors. In fact, the results discussed here suggest that subtle changes in their activation threshold can produce significant effects.

Even Easton and colleagues (2008) take into consideration an industrial cluster, and in particular its hierarchical subcontracting networks of small-medium sized enterprises. Fairly innovative, in their approach nodes are not made of individual firms but of their exchange relations: in the simplest case there are four actors and three corresponding relations. Vendor A sells its product to the distributor B, which in turn sells it to customers C and D. So the BC and BD exchanges cannot take place without the previous AB exchange, and without having reached a certain agreement to produce. The analysis of Easton *et al.* shall be held within the standard of Kauffman's examples to keep the average value of the centrality at 2, and therefore it has the disadvantage of not really looking to the specific case of the industrial cluster of reference, but on the other side it has the advantage of being quite general to give directions in all the hierarchic networks of size 4 and with an upsilon shape, which represents the prototype of the multi-level hierarchy.

Failures and Limitations

Let's now get a synthetic view of what does mean a deep understanding of the dynamics of a simple network reminding Klüver & Schmidt's (2007) work: even a simple network composed by six nodes, with uniform K=2, and 2-state nodes and generates $1.6*10^9$ digraphs. This makes clear that the use of this modeling for studying real networks *in an extensive way* is terribly heavy in terms of time and computational resources. Further, if it were taken into account also asynchronous interactions and nodes' multiple states, the analysis would become more and more complex. That is why most researches employing this methodology limit to extract random samples of networks built with random or regular topologies with two-state nodes and synchronous update. Indeed, an empirically driven research could limit the exploration only to the analysis of the set of (digraphs and activation rules) combinations that seem more realistic or more interesting.

Further, let's remark that – at least the standard model – assumes that during a trajectory there is *topological and regulative invariance*, which means that size, links distribution, and rules of activation do not change. Indeed one or all these three parameters are usually varied to run sensitivity analysis and network dynamics resilience respect to small perturbations, even though K is made varying only in terms of constant value, and not in term of network topology. When some more incisive change is made in order to simulate what would happen under very different circumstances, all of them vary at the same time, but never during a trajectory. Perhaps in the biological networks specifically related to the human body that limit may be not very compelling, as it is into the field of social sciences, because the distribution of connections and the way to interact with each node can change, and they generally change, indeed, relatively easy and often. Imagine considering the network of trade relations in the computer industry in Italy: hundreds of companies are created and die every year, the distribution of their relationship changes, and, by virtue of changes in prices, technology, labour market, learning processes, etc., the activation rules change with high frequency. The non-invariance of topology is perhaps the main reason that makes NK-BNs modelling difficult to apply.

In principle, nothing prevents to deal with this major problem, but unfortunately, when size or topology or activation rules change during a trajectory, it is as if a completely new network starts a new dynamics from a possibly new initial state[31]. This fact has two consequences. The first one is that the computational problem remains unsolved, because just shifted to the "new" networks. The second one is that, if the "new" network (and its following "new" networks) has an appreciable complexity, there is a serious risk that they will never reach any attractor.

Respect to the methodology of social network analysis, standard NK-BN modelling requires some more information: it can be applied only to digraphs, and activation rules. Provided that a precious understanding of network dynamics can be obtained either with longitudinal network analysis and exponential random graph models (see Chapter 1 of this book), NK-BN modelling has, in addition to operational difficulties and to computational load, some other limitations. The first one is that dynamics concerns only node's activation state, assuming invariant all other parameters. Secondly, nodes have the only attribute of its states, and no mutual influence between incoming or outgoing links with a node's state is hypothesized. A third one is the (above discussed) requirement of topological and regulatory (activation rules) invariance. A fourth limitation consists in the fact that, in order to study a BN dynamics, its topology should be built on a single *type* of connection between nodes, while nodes can exchange more than one type of things. In fact, socio-economic networks can have a multitude of types of connections: economic, political, informational, psychological, etc.[32] Now, as each type of connection likely discloses a specific topology among the same set of nodes, and because the dynamics of (types of) connections influence each other and influence - and are influenced – by nodes states, the study of the dynamics of a single type of connection in such a multiplex gives only a very partial, and potentially distorting, view of its behaviour. Moreover, there is the problem how to compare and interpret the results of the different maps, i.e. of the states' space (and especially of the attractors), generated by the various topologies.

A further limitation is represented by the condition of memoryless nodes, and not only to keep nodes with just two (or few) states. Letting memory to intervene into a node's behaviour would mean equalizing nodes to *non-trivial machines* (von Foerster, 1982, 2003), which is a fundamental trait of the complexity of living systems and their components. Indeed, in a way that is much less incisive and fundamental respect to that of giving nodes' autonomy and internal states, there is a research stream of NK-like models that renew the standard version also in the direction to introduce some learning mechanism (Kauffman, 1986; Patarnello & Carnevali, 1986). This research stream is close to what is typical of neural network modelling[33] (see Eliano Pessa with Chapter 4). In fact, the logic is that to "train" a BN to reach certain attractors – that is, certain points in the state space – by searching and feedback algorithms that modify network topology or activation rules (Benedettini *et al.*, 2013).

Another weakness of this methodology is that it requires fully *connected networks*, while real networks can be more or less fragmented, and there is not always a major component on which you can focus the analysis, hoping to capture the essential behaviour of the entire network. Likely, this requirement may be less stringent in biology, where networks are likely to possess this property naturally, but it is quite constraining in socio-economic systems. Finally, one cannot forget to mention the fact that this methodology, especially into the fields of social sciences, has received yet few strong and accurate virtual and much less (empirical) experimental confirmation, which means that few specific predictions - that were subsequently confirmed experimentally - have been made with it so far.

PART II: THE NK MODEL AS FITNESS LANDSCAPE

From NK-BN to NK-FL

As we have seen, given a network of size *N*, there are many parameters that determine its dynamics: initial states number, Boolean functions number[34], in-degrees distribution (which Kauffman and most followers reduce to constant or average *K*[35]), Derrida's *P* parameter concerning the degree of unbalance between 0s and 1s of one or many Boolean functions, Kauffman's *CF* concerning the degree of "forcing" functions among the Boolean functions, out-degrees distribution (that Klüver & Schmidt underlined), many other topological parameters that could be salient for determining a specific network dynamics, and finally synchronous or the many (exponential) ways to introduce asynchronicity. Therefore, we are dealing with at least seven parameters (likely ten or a dozen), most of which scales squarely or exponentially with *N*. To have an idea of how many combinations they can generate, let's come back to Klüver & Schmidt's work (2007), which for a standard BN (with only 2-states nodes) of N=6 and K=2 identified about 10^9 combinations among order parameters. Even though 10^9 is a number enormously inferior to the true number, because they overlooked asynchronicity and other topological parameters, it is anyway sufficient to give an idea of the construction of a FL.

Each combination triggers a specific dynamics which, with more or less long and unpredictable trajectories, leads to a given attractor[36]. Let's now consider the set of all possible attractors and call it attractor's space. The idea is that, according to the specific ontological characteristics of that BN, there are better or worse attractors, let say preferable or dangerous stable states. Further, let suppose that it is possible to measure such fitness value from the best to the worst. We would get a set of stable states, each with its fitness value: this is the FL corresponding to a given BN. FL's points are not independent one another, because they are produced by variations of order parameters. In other words, if average *P*, ceteris paribus, changes from 0.71 to 0.72, then another combination would occur, and thus, another trajectory would be triggered, and - possibly, not necessarily - another attractor reached. Hence, another point in attractor's space could be reached, and it could have a different fitness value, higher or lower respect to the previous one. If a BN represented a cognitive system, and it had the capability to "perceive" such fitness value, because i.e. the external environment sent it a feedback in terms of resource consumption or achieved goals[37], and if the fitness value associated to the current attractor were not satisfying, then the BN could make another change in one or more of its parameters values. In other words, the BN could move within its FL.

A BN's cognitive capacity is intermediate between zero, in which case only random mutations can take place, and infinite, in which case, being a sort of god, it could calculate the astronomic number of all combinations and select that (or those) that leads to the optimal fitness value, that is to the best attractor. Notice that the dosage of cognitive capacity assigned to the BN significantly matters respect to its possibilities to find a fit-for-survival or satisfying stable state. However, it should be remarked that all basic results hold also if the BN had not cognitive capacity at all, and changes occurred only randomly. We can call BN as agents, be them cognitive or non-cognitive natural systems or human agents or organizations (Biggiero, 2009). Notice that, if BNs were assigned infinite cognitive capacity, this hypothesis would correspond exactly to that of perfectly rationality assumed by neoclassical economic theory, especially in its standard versions of general economic equilibrium. In fact, in such models agents know or can calculate optimal solutions.

So we have an agent, whose fitness is determined by a set of parameters, whose combinations determine the form of the agent's FL. Consequently, imaging that the fitness value of each point were represented by the height of the symbolic landscape, agent's ways to move within the FL to improve its fitness depend on FL "morphology". As we will see right below, many concepts – like trajectories, basins of attraction, etc. - used for NK-BN are valid and necessary also in NK-FL. In fact, the more complex is agent's structure, the longer and unpredictable will be the trajectories to reach a better position, that is to climb higher hills in FL, because an agent's structure is made – as said right before – by order parameters with its variations. However, in the language of NK-FL modeling, such order parameters are called agent's elements (or components), and among them, in line with Kauffman's (1993) original presentations of both NK-BN and NK-FL models, *K* is given a special role. Jointly with *N*, the crucial parameter is still *K*. Kauffman demonstrates that the higher *K*, the more "rugged" becomes FL, and consequently the more likely an agent is entrapped into a local little maximum, because most reasonable – that is, limited and near – changes that the agent can do will not lead to other and better hills. Further, even with "long jumps" – that is, radical changes of its parameters – the agent will be entrapped in another little hill instead of a remarkable improvement. If the change were realistically meant as a resource consuming action - and consequently, radical changes were rare (if any) - then it comes that if an agent is a complex BN, then its evolution will be likely confined into small fitness states. Further, Kauffman (1993) demonstrates that, for N=100 and depending on *K* values, there will be an ordered regime until K=2, a chaotic regime beyond 5, and a phase transition between these two values. In the ordered regime there are few (and relatively high) hills, and thus, FL is smooth, while in the chaotic regime FL is rugged.

A final remark is that in the ordered regime BNs not only have very few attractors, which - in the simplified Kauffman approach that overlooks the variety of topologies and synchronizations - are the square root of *N*[38], but further they can be reached with short trajectories, and therefore in a short evolutionary time. Even more interesting is now the fact that all possible attractors are correlated into a smooth FL, and further many of them will get high fitness values. Hence, it is relatively easy to do an evolutionary walk exploring large areas of the landscape, and to find relatively high peaks. Conversely, in the chaotic regime, not only trajectories will be long and FL rugged, but peaks will have a smaller fitness and its evolutionary exploration will be very long as well, because each next fitness gain requires doubling the search space. Consequently, a BN trying to evolve in such a landscape will take a lot of time just to end in a local small optimum. This is what Kauffman called the "complexity catastrophe". In this context, the larger *N* and the higher *K*, even radical changes under many parameters, which in principle had to improve the probability to get a better fitness outcome at the price of a deep transformation, will not help much more than a simple random choice. As for BN dynamics, depending to *N* and other order parameters, there should be “somewhere” a threshold (a phase transition) between the ordered and the chaotic regime, that is between the smooth and the rugged landscape. “Somewhere” refers to the fact that, once the oversimplifications of Kauffman’s original approach have been removed, it is not yet clear neither if *K* is the main order parameter nor how phase transition occurs depending on several possible combinations of the various order parameters. Likely, there will be a “space of phase transitions”, which has to be still studied. From an empirical point of view, the question becomes that of getting realistic ranges of variation of each parameter, and focus on that portion of space.

NK-FL Models

Before starting the discussion of NK-FL modelling and its applications, failures and limitations, it is necessary to warn that, breaking the conceptual and methodological relationship with NK-BN, some fundamental meaning changes[39]. The most important are the following:

- *N* represents the number of elements (components) constituting an agent’s characteristics;
- *K* represents the average number of links between them[40];
- Agent’s fitness value is given by the mean of its fitness elements.

As for the NK-BN model, here Kauffman proposes as well *K* as the order parameter to determine the FL form and the phase transition between smooth and rugged landscapes. However, as discussed below, here too *N* and *K* are not the only two order parameters, because there are at least two more ones[41]: system’s topology, that is links distribution and direction, and the number of other interacting systems. As we will see, agent’s “explorative capacities” – and more generally, cognitive capacities – could matter as well. By now, let’s notice that leaving the view of FL as BN attractor’s space simplifies its construction and interpretation. In fact, if they do not derive from BN dynamics, it is just enough to conceive an FL as the space of all the forms that a system can assume with its associated fitness values. These, in turn, just depend on the type and density of connections of its (binary) elements, which can be active or inactive. However, though the almost unique approach concretely applied, this way to build an FL freely and randomly seem quite disputable, especially because if a given agent’s state were not a stable state, then it had to follow its inner dynamics reaching a stable state, which would locate the agent in another point of the FL. In other words, if points were not stable states, then they would be inherently unstable, regardless of the fitness value and the seeking for a better fitness. Put differently, the search for increasing a fitness value refers to the relationship between a system and its environment, while the system’s stability depends on whether the system is in a stable or unstable state respect to its own inner dynamics.

While in the NK-BN modelling the most important properties were attractors number and size of its basin and trajectories length, here they are the following others: i) number of fitness peaks in states space, ii) average lengths of walks via fitter neighbours to reach fitness optima, iii) total number of states encountered until an optimum is reached, iv) ratio of accepted to tried mutations on a walk, v) number of alternative optima to which one system can climb, vi) number of systems which can climb to the same optimum, vii) rate at which the fraction of fitter neighbours dwindles to zero along walks to fitness peaks, viii) similarity of local optima (adapted from Kauffman, 1993: 45). Despite the three potentially involved parameters, working on random uniform distributions of fitness values, through numerical experiments[42] Kauffman demonstrates that most likely such properties are influenced only by *N* and *K*[43], leaving aside agent’s inner topological characteristics else than *K*.

Even in this different framework, analogies with NK-BN are strict and straight: given *N* system's elements, as well as BN states[44] there are 2^N possible points that an FL can have. If systems states are placed into FL following a logic of binary adjacency, that is by differing for just one bit (0 or 1), then each system is a one-characteristic neighbour of other *N-1* states. An element's fitness contribution depends on its inner value and on the value of the elements that influence it: the number of possible combinations is $2^{(k+1)}$.

The more interconnected are its elements, the more rugged the landscape. Moreover, for a system structure is nothing else than a network of elements, each form they can assume is nothing else than a specific instantiation of its topology. That is, an agent of this kind is a BN, and he moves among its possible states within FL, regardless whether some of these states are attractors or not. Agent's evolutionary walk is not more determined by activation rules, but simply by the purpose to reach a better fitness point. In short, this new conceptual and methodological framework is very much simplified respect of NK-BN, because a system's dynamics is not more determined by topology and activation rules. Therefore, many order parameters disappear in the FL standard model, which overlooks the relevance of out-degree distribution.

Now, suppose that these values are distributed randomly to each system's element and that the binary identification of each state determines its adjacency to other states by 1-bit difference. The resulting landscape will be made by points whose proximity is calculated in terms of string similarity[45], but whose fitness values are relatively disentangled from it. If we assume that a system can change the state of one or more of its elements, it could walk in FL to achieve a higher fitness. Now, the key point is the following: the more rugged the landscape, the more likely a system will be trapped in a local maximum rather than reach the global or even a relatively high maximum.

Regardless whether random or specific, if *K* is very small, then the landscape is smooth, and states with similar values are adjacent, so that one-neighbour change would modify fitness by only 1/N. In this landscape there is only one global optimum, that is one best state, which can be reached by any other states in a maximum number of N/2 steps. Notice that, given the definition of "system", which is a set of interacting elements, K=0 implies that elements do not interact, and thus, it follows that such an "entity" ceases to be properly a system. It loses the connotation of being a system.

Kauffman shows that, ceteris paribus, the higher *k* and the higher the roughness of the fitness. At the lowest bound of $K=0$, that is when each node fitness value is independent from the others, there is only one maximum value, which obviously corresponds to the global maximum, and hence natural selection does not encounter much difficulty in "showing the way" for the system to "climb" to the optimal state. On the contrary, if $K=(N-1)$, which corresponds to its maximum possible value, the landscape is totally rugged, in which case there would still be one global optimum, but it would be lower than its analogous in a smooth landscape. At the extreme, if all elements were depending each other, there would be $2^N/(N+1)$ intermediate "peaks", which therefore augment exponentially with *N*. In fact, each state "has *N* neighbours and the probability that the group made by itself and by his neighbours is the strongest [i.e. the most suitable, in sense of having the highest fitness score] is equal to $1/(N+1)$" (Kauffman, 1995: 234). If *N* is equal to 100, there would about 10^{28} intermediate peaks. This also means that a system that attempted to improve its fitness would go through very short paths (an average length of *lnN*), and probably it would always reach relatively low peaks.

Besides the exponential growth of peaks and its capacity to trap agents with few steps, a maximally rugged landscape has the following other remarkable properties: i) the expected fraction of fitter one-mutant variants dwindles by ½ at each improvement step[46], ii) system's variants tried to reach an optimum is proportional to the number of one-mutant neighbours, iii) the ratio of accepted to tried variants scales in lnN/N for the case 2-states elements, iv) a system can climb to only a small fraction of the local optima, v) only a small fraction of systems can climb to any given optimum, vii) as N increases, the fitness of local optima falls toward the mean fitness of the whole landscape. Kauffman underlines the relevance of this latter property, which is a complexity catastrophe common to all evolutionary complex systems. It displaces the role of natural selection, because if FL is rugged, then a systems population will be mostly entrapped in such small peaks.

So far we have supposed adaptive walks based on minimum changes through one-bit variants. What happens if 2- or M-bit (with M<N) variants occur? Well, a property of FL is that correlation distance exponentially lowers as (Hamming) distance increases. Therefore, doing long jumps is a truly hazardous choice, because the expected fitness gain is uncertain and uncorrelated to the initial one. Further, a long jump strategy is very costly in terms of exploration efforts, because the number of variants to be tried to find with a further long jump another fitter variant doubles. Moreover, fitness gains through long jumps follow a law of diminishing returns with *N*.

Complex FL models combine two forces that determine a system's adaptive walk: one is due to inner parameters, and in particular by *N* and *K*, while the other is due to possible connections with other (different) systems. While in simple models there is just one agent or a population of independent (but equal) agents, in more complex models there is also another agent – and in principle, many others - who co-evolves into the same FL through inter-agent connections *C*: these are called NKC models. In fact, in real world a system does not evolve in isolation, but rather in competitive or cooperative relationships with other systems of the same or different species. In modern evolutionary industrial economics and in management and organization sciences this view is fully shared and evident: organizations are not islands, but rather they are interconnected in a web of many relationships of many different types. For instance, the literature on inter-organizational networks and on local production systems offers a rich and wide body of knowledge in this field[47].

The main point to be underlined when different systems co-evolve into the same FL is that FL itself deforms due to the influence exerted by other system(s). In fact, depending on *C* and *K*, even adjacent states could score very different fitness values. In principle, especially if *C* and *K* are high, evolutionary paths could become very unstable and unpredictable, and life itself would become (again) improbable, as it was objected by creationists before the results obtained by NK modelling discussed so far. However, in simplified 2-systems co-evolutionary approach conceived for biological networks, Kauffman shows that, within a certain portion of the combined parameters space of *N*, *K* and *C*, evolutionary paths seem to converge towards ordered areas. In other words, the reciprocal influences exerted on each other does not prevent to find, sooner or later, a situation in which each system reaches a relative local optimum, and hence a mutually stable state, analogue to a Nash equilibrium[48]. Kauffman (1993) considers a first set of simulations concerning 100 co-evolving pairs of $N=24$ systems interconnected through 8 relationships ($C=8$), each with three different degrees of *K*: 16, 8, and 2. For all *K* degrees a large fraction of the co-evolving pairs reach a Nash equilibrium, and hence stop evolving, but – remarkably – the time needed to reach that point lowers as *K* increases, with a sharp major gain when $K>C$. Conversely, if *K* and *C* are kept fixed and *N* made increasing, it happens the opposite: the waiting time to hit Nash equilibria increases. The explanations are that, in the former case, higher *K* makes FL more rugged, and thus, easier to be entrapped in a local optimum, while in the latter case, increasing size produces the opposite effect of reducing local optima[49].

In an enlarged computational experiment with eight species (systems) mutually influencing through 13 elements evolving over 2500 generations, Kauffman (1993: 245) shows that "the mean fitness of the whole set of species increases, rapidly at first, then more slowly. Increasingly long intervals with no change occur, reflecting the fact that, as fitness increases, the waiting time to fitter variants increases for each partner… By about 1600 generations, a Nash equilibrium has been found such that each species is locally fitter than all one-mutant variants, granted that the other species do not change".

Depending on the ontology of the phenomenon on which it is applied, the logic and language of NK-FL should change: if the object of research is a biological system like genome or cells, then its behaviour can be thought of as blind or unintentional, and thus, it should be supposed that chance produce variations. Therefore, a number (a population) of systems can be supposed to be immersed into the same FL, and each system – be it a system of genes in a genome or a system of cell components in a cell – walks according to its specific *blind variations*. As Kauffman underlines - and puts at the ground of his contribution for a reformulation of evolutionary theory – natural selection operates in conjunction with BN and FL self-organizing properties. In fact, if FL is smooth, then most systems will achieve good and near fitness values, and natural selection can retain its form. Conversely, if FL is rugged, then most systems will be trapped into small hills with low fitness values. Hence, natural selection cannot contrast and counterbalance self-organization, but just choose among common forms[50]. As Kauffman says, in these environments evolution proceeds not because of, but (rather) despite of natural selection.

If the object of research modelled with NK-FL is not a biological system but rather an intentional and highly cognitive system like human systems, then system's states variations can be thought of as – at least partially – deliberated. The model language, in fact, does changes accordingly: an agent (a system) tries to improve his fitness by trial and error, and largely through learning processes. Therefore, as we will see here below, extension and characteristics of agent's cognitive capabilities are the main traits that distinguish various approaches and applications to economics and management sciences. Moreover, cognitive capabilities make plausible – and indeed, likely – changes that are not limited to one-bit variation, but to wide range variations: at the extreme, a radical mutation of all *N* elements. Further, a system's path dependency, which in the standard model is of Markovian chains, that is entirely based on the system's previous state, can assume a much more complex form. In fact, a cognitive system can remember and elaborate much more than the last or few previous states. A cognitive agent (Carley, 1986, 1989; Carley & Newell, 1994), like individuals or organizations, could have a deep knowledge of its past, and hence it could significantly modify its walk respect to what it should be by a simple blind one-bit variation. Finally, highly cognitive agents have strategies and even if in NK-FL the objective to be maximized is just fitness value, if an agent understands that its landscape co-evolves with it and with other competing or collaborating agents, the agent's "strategic actions space" increases dramatically. At the very end, it would open to so much possible strategies to make simply impossible the discovery of an optimal strategy.

When reframed in this way, it can be seen that, even keeping invariant the basic structure of NK-FL modelling, the number of order parameters is, for a given N-element system, much more than the number of system's states[51], *K* and *C*. There are at least: i) a system's inner topology, which is a specification of *K* intra-system connections; ii) the topology of inter-systems connection, which is a specification of *C*; iii) systems cognitive capabilities; iv) systems strategies.

Applications in Economics and Management

An agent is supposed to be compound by a number *N* of components. In the two fields that are more extensively explored in this paper, that is technological interdependence patterns and workgroups, a system's elements correspond respectively to activities (or product components) and group members. Agents are supposed to be able to compare their own fitness value with that of nearest neighbours and to adopt its combination (cover its position) if fitness is higher. In other versions of the model, it is supposed that agents can compare its fitness value with that of far agents, through long jumps within FL. How far it is possible to "see" and how detailed is the comparison depends on single variants of FL-like models. Usually, this is related to cognitive capabilities assigned to agents. A number of different combinations can be supposed: for instance, it could be hypothesized that if the rate of growth of fitness functions is under a certain threshold, then the agent can look farer.

Apart from biological applications, which firstly started with Kauffman's early works, most applications in economics and in management and organization sciences overlooked the interpretation of FLs attractors' space and therefore its relationships with BN dynamics. In other words, conceptual, methodological, and epistemological connections between NK-BN and NK-FL get lost. Indeed, one can use the NK-FL methodology in itself, breaking or neglecting the problem of how a certain FL has been built. Actually, even from an historical point of view, FL early formulations preceded of about 30 years BN early formulations.

Differently from NK-BN, NK-FL model has been frequently applied in economics and management and organization sciences, and therefore here only few contributions are commented[52]. Dealing with organizational issues, a work by Westhoff *et al.* (1996), applies both sides of NK modelling, and anticipates those of Levinthal (1997), Levinthal & Warglien (1999), and a Special Issue of Organization Science in 1999, which "squeezed" the whole field of complexity theory and self-organization only on Kauffman's model (Anderson, 1999; Anderson *et al.*, 1999). This siding likely contributed significantly to address to NK-FL modelling a large part of the following research on complexity in organization and management sciences (Chang & Harrington, 1997, 2003, 2004; Fleming, 2001; Fleming & Sorensen, 2001; Gavetti & Levinthal, 2000; Lacks, 2004; Levitan *et al.*, 2003; Rivkin, 2000, 2001, Rivkin & Siggelkow, 2002, 2003; Siggelkow & Levinthal, 2003; Siggelkow & Rivkin, 2005; Yuan & McKelvey, 2004, among the many).

Levinthal (1997) applies the FL model to the capacity of survivability, which is measured by the corresponding fitness score of an organization within a population of different organizational forms. Diversity is measured here by *K*, which represents the degree of interdependence of the organizational units and *N*, representing their size. Levinthal & Warglien (1999) retake this approach and examine how organizational structures, different in terms of degree of interdependence between their units and size, get different fitness scores in terms of economic performance, and how they have to climb the highest peaks in order to achieve the best locations. Gavetti & Levinthal (2000) study the value of organizations experience and expectations seeking to represent the competitive environment within a cognitive space of possible representations, which would correspond to FL. Rivkin (2000) shows that complex competitive strategies become very difficult to imitate, because small errors can lead the imitators to regions of very low fitness (possibly trapped in local peaks) of the "strategic space". This model is particularly interesting because it suggests that the role of learning, and then of the Lamarkian typical evolution of social phenomena, would cover a much lower importance than it is often supposed.

Yuan & McKelvey (2004) contrast situated and cognitive learning theories, and argue that the former is to be preferred because they view organizations as complex adaptive systems rather than mere information processors. This judgment is shared by others (Biggiero, 2009, 2012; Amin & Cohendet, 2004), who further link this aspect with that of the ability to knowledge creation and transfer, and in particular to produce and treat tacit knowledge. Yuan & McKelvey use a "humanized" version of Kauffman's model to study the situated nature of learning. Indeed, McKelvey (1999) was one of the first who transferred Kauffman's concept of "complexity catastrophe" to organization science. The "humanization" of Kauffman's model means here that uniform *K* is replaced by its differentiation.

In this version of NK-FL models, agents are groups and its components are group members. Agent's fitness function measures group learning, so that group level dominates individual learning. The structure of the fitness function guarantees that a new state, that is a new composition of the group, is accepted only if its whole learning is superior. It could happen, in fact, that individual learning of a single group member increases but leaves the whole learning (value) unchanged or even diminished. Yuan & McKelvey formulate seven hypotheses, out of which H1 and H2 are hypotheses on the amount of learning, while H3, H4 and H5 concern the rate of learning, and H7 and H8 the effects of task interdependence[53]. They consider a group (agent) compounded of 48 members (components).

They found that the first two hypotheses are confirmed, and thus, that over time and across many interactions, (ceteris paribus) the amount of group learning is a nonlinear non-monotonic (inverted U) function of *K,* and that (ceteris paribus) the amount of group learning is influenced by group size. The rationale is that when groups are too small then members have no adequate opportunities to learn from each other, presumably because they become soon homogeneous. Conversely, due to bounded rationality, when groups are too large or too interconnected, then the cognitive and non-cognitive costs of holding many communications (connections) can inhibit their learning capacities. A confirmation of the third hypothesis is obtained as well, because the rate of learning is a non-monotonic (U-shaped) function of *N*. The rationale is that, as group size increases, group learning "get trapped on lower suboptimal peaks that are easier to reach quickly" (pg. 74). However, the degree of interconnectedness in terms of average direct centrality, does not produce the same curvilinear effect of size, and consequently the fourth hypothesis is only partially confirmed, because *K* effect on the rate of learning is a positive function. While the former three findings are consistent with those obtained by Carroll and Burton (2000), the latter is not.

In the adjacency-based setting level and learning rate are lower than that in the random walk, but values are statistically not significant (pp. 89-90). Finally, as concerning the effect of members' differentiation in terms of degree centrality, that is the presence of central and peripheral members, the amount of learning in differentiated groups is impaired. Though Yuan & McKelvey do not give an explanation for this effect and neither a rationale for the corresponding hypothesis, this effect (albeit weak in statistical terms) is indeed rather interesting, because it confirms that topology matters. Though not directly comparable, to a certain extent, this result is consistent with that obtained by Biggiero & Basevi in Chapter 12 of this volume: unbalanced topologies favour knowledge growth and learning. In synthesis, Kauffman's "complexity catastrophe" effect seems confirmed: as communication interactivity becomes denser, and rate of learning speeds up, there are diminishing returns to improving group learning. However, density in communication interactivity is not independent of group size. Once we adjust for this effect standardizing *K* by *N-1*, we find that the curvilinear effect disappears, but the catastrophe effect continues as a function of two linear variables. *Rate* of group learning is a positive linear function of communication interactivity, but *amount* of learning becomes a negative linear function of interactivity density" (pg. 93).

Kauffman himself has contributed to some applications of NK model to economics with a first paper in 1988, which has been followed by a number of other papers (Auerswald *et al.*, 2000; Kauffman *et al.*, 2000; Kauffman & Macready, 1995; Frenken, 2006; Frenken *et al.*, 1999, 2007; Levitan *et al.*, 2003; Marengo & Valente, 2010). In a paper with Lobo and Macready (Kauffman *et al.*, 2000), preceded by another introductive work (Kauffman & Macready, 1995), FL is constituted by a *technological landscape*, representing the space of the technological possibilities in which companies are moving to reach the best available technology. With the constraint of a research cost, the technological distance between two alternatives, namely two points in the technological landscape space, is measured. Simulations show that the best strategies depend on the company's initial position: if we start from a low position, then substantial changes should be undertaken to move very far, while if the starting point is already good, then it is more advantageous to remain in same space area. This means that the morphology (one might even say the topography) of the space has closely related characteristics: the "hills" are close one another, and so are the "valleys". Unfortunately, the authors do not explain how the companies are able to understand what are the minimum, the maximum and the average scores (and possibly the variance), so that one can then place his relative distance. They may be "simply" pushed or placed, against their will, by the competitive selection, but then their strategies for finding the best technology would be blind, and thus the decision to make small or large technological changes, i.e. small or large "jumps" in the space of the technological possibilities, would be connected to it only through past experience. In this model, the competitive selection of the companies and the natural species act basically in the same way.

Building on previous contributions, and in particular on that of Auerswald and colleagues (2000), Frenken (2006) introduces and explores the problem of components interdependence and its constraints on the vertical disintegration of production. The central theme becomes the production of modular architectures, which allow the outsourcing of the stages of production without binding the production too much and losing too many production skills needed to perform subsequent innovations. Through an application of Altenberg's (1995, 1997) FL model prepared for genetic biology, Frenken proposes to introduce elements that function as standards that mediate the interdependencies between other elements. Analogously with Auerswald model, the landscape is a "design space", where fitness measures the suitability of different combinations of components (modules) of a product[54]. A product is described on the basis of the N components (that in genetics would correspond to the genotype) and of F functions that it may develop (that in genetics would correspond to the phenotype). Each element contributes to a function (performance) of the product, alone or together with other N elements, and through K "epistatic" connections it is linked to a number of other elements. Following the track marked by Altenberg, this model is a generalization of Kauffman's original model, in which each N element coincides with an F performance.

In another work (Levitan *et al.*, 2003) Kauffman contributes to study organizational performance and extends the model to take into account, in each choice, also interactions between organizations, which are divided into large and small. Here the space to be explored regards the choice of organizational efficient size. Moreover, this work investigates the co-evolution of different organisms (organizations) interrelated within the same space (Kauffman, 1995, Kauffman & Johnsen, 1991). Fitness corresponds here to organizational performance, and it is demonstrated that an optimal organizational size depends on the extent of its connections with the others and on the amount of random attempts accumulated in the past. It happens that in the short term (and thus, with few cumulated attempts) a large size is more efficient, while in the long run the advantage moves to the small sizes with a low number of connections (interdependencies) with other organizations. Also in the long run, these small organizations would benefit from any increase in the size of its external connections or of their own size.

Failures and Limitations

Kauffman's most important argument is that, given the genome size of the humans and other species, if its complexity (measured in terms of K) were very high, natural selection would not be able to guide towards an "improving" evolutionary path of greater fitness. The corresponding state space - to be randomly explored by blind natural selection - would be too large to give significant results even over millions of years. Therefore, Kauffman says, the "real" K must be quite low and there must also be mechanisms - which we discussed previously concerning NK-BN - to reduce complexity[55]. If this is so, then natural selection actually becomes a good "algorithm of evolution", because it really would be helped by the "self-organization laws" of complex systems (Biggiero, 2001b; Casti, 1994; Kauffman, 1993, 1995, 2000; McKelvey, 1999, 2004, Strogatz, 2001), which are partly those described by NK-BN dynamics discussed above.

The main result of Kauffman's contributions is the demonstration that life is far to be unlikely. On the contrary, due to self-organizing properties of biological systems, and indeed of any kind of system, even when the number of connections among entities is not very large, these entities start producing ordered structures. Moreover, when such a degree of connection is not too high, emerging structures become sufficiently stable to be able to build superior structures, ordered at a higher level of aggregation. To take place, such a process does not need long time and neither complex rules or basic entities: simple Boolean functions and binary-state entities are enough.

There are two important implications. The first one is that the fundamental engine of evolution is not only selection, as evolutionary Darwinism argued so far, but also self-organization. Indeed, von Foerster (1962) and Ashby (1962) demonstrated it earlier, and showed that self-organizing properties do not characterize only biological systems, but basically any kind of complex system. The second one is that, in order to explain the origin of life and ordered structures, it is not necessary to suppose a god's intervention. Actually, even after Simon's (1962) argument of complex systems decomposability and progressive aggregation from elementary components in a nested hierarchy of sub-systems, creationists were arguing that, due to the hyper-astronomic number of possible combinations of elementary organic molecules, it was definitely not plausible supposing only the work of selection of best alternatives. A complete scanning of all alternatives could have employed several times the age of universe. Simon's argument was not enough, and thus, *Kauffman's demonstration of spontaneous and intrinsic self-organizing processes completed the scientific explanation of the origins of life*.

As for NK-BN, the standard model (and its near variants) should be distinguished from what, in principles, could be done with more advanced versions. In fact, the results obtained from an NK-FL model depend essentially on the type of FL used and even more on the search algorithms assigned agents to move into that space. These algorithms describe essentially agents' cognitive capabilities, which in the standard model are almost absent, in the sense that agents are myopic – they see only neighbours - and inertial – they change only one item at a time[56]. Of course, these traits were inherited by the early applications to biological entities, for which was not reasonable to not give high cognitive capacities. Conversely, they appear unrealistic limitations when agents represent individuals, or (even more) organizations. When agents are very small and the states of each element are just two, like in the standard model, myopia and inertia are still reasonable, but when an agent is large, they seem less plausible. Such defects have been acknowledged by researchers, and in fact recent models progressively increase agents' cognitive capacities and learning attitudes. Another problem lies in the oversimplification of FL and its environment, in the sense that during FL exploration, agents do not change their structural and cognitive characteristics.

When considering more complex versions of the model, which includes at least another (type of) agent[57], and especially if the model is built for empirical applications, the complexity of intra- and inter-agent connections – that is, *K* and *C* - should be taken into account, because it affects the degree of computational tractability. In fact, the simple mean of single elements' values employed to calculate global fitness of each point could be totally unrealistic, because any sort of super-additive function could hold between some element, either within the same agent or between two agents. Of course, this problem makes the empirical application of NK-FL modelling much more complicated, especially if *N*, *K* and *C* are not small. On the other hand, for social organizations are complex systems and are not islands, but rather strictly intra- and inter-connected[58], and therefore they are complex in the double sides of internal structure and functioning and of internal-external relationship[59], *K* and *C* are necessary large, and indeed in many organizations also *N* is large. They are multi-product and multi-business, and hence *N*, meant at least[60] as production elements of a space of possible production recipes - à la Auswald *et al.* (2000) and à la Frenken (2006) – is very large as well. Further, for many different agents are connected, it is clear that, if one aimed at employ this methodology to face with real contexts, terribly hard computational (and analytical) problems would be raised.

A further problem is that FL deforms not only because of multi-systems co-evolution, but also because of the co-evolution between a system and its environment. In other words, if a system reached a high fitness point, it could happen that its environmental niche changes: for instance, a species success could exploit niche resources and induce other transformations. In turn, they can modify fitness values of many other states of the FL, thus deforming it: states that previously had a low value now can have a high value. This situation is particularly evident for economic systems, where a single system could be so large to heavily influence its niche[61]. It this is true, then a system's walk could be very unpredictable and never ending.

Besides these specific problems, there is one very broad that, impacting very similarly to biology and economics, deserves a discussion. It is the question of the heuristic appropriateness – that is, of its correspondence with reality - of the concept (and scalar measure) of fitness function. Fitness had to be considered as composed by a number of aspects: financial, productive, cognitive, personnel-based, etc. Now, the relationships between some of them are governed by some (more or less known and clear) function, and so they can be expressed by a vector, and maybe also reduced to a scalar by applying some algorithm. However, besides the fact that such relationships could be nonlinear, and so hardly treatable to build the state (system) space, the connections between some other aspects cannot be expressed through algorithms for the simple reason that there are no connections at all. Put in other words, some aspects could be independent one another. In these cases, how could be express this independency with a function? It cannot be, and in fact when we build a fitness function we are replacing independency with dependency, for the simple reason that a function – whatever it can be – establishes a relationship, that is a dependency. Therefore, building a fitness function means manipulating a strong qualitative aspect of reality – independence – to replace and reduce it with its opposite – dependence.

Put differently, we can observe that the elements of a fitness function could – and usually are – connected (in different ways) by different types of connections. If we look – accordingly with Kauffman and all the NK-FL following literature – at a system's average fitness value as composed by a network of *N* elements, each with its own fitness value and *K* connections among these elements, they can be connected by rather different types of links. In other words, that system is a multiplex. But then, in a multiplex each type of relationship can generate its own specific topology, which concerns also its own *K*. Therefore, we had to calculate the mean fitness value according to each specific system's topology (each specific dimension of the multiplex). This calculation would make NK-FL mulch more complex and computationally heavy than what it already is. Moreover, one could speculate that each *type* of relationship generates its own FL, and that a certain type of relationship can influence another one, so that an FL is deformed by another FL even without putting in play inter-systems connections – that is, even in the simplest model configuration, which overlooks C[62].

There is a further aspect strictly related, but nevertheless distinct from the last two: the problem of multiple criteria. In fact, not only a multiplex is a multi-dimensional entity whose fitness value cannot be expressed by a fitness function, but the whole FL is an even more complex multidimensional entity, whose points cannot be expressed by a single fitness function. Hence, *natural selection operates as a "multicriteria evaluator", whose algorithm of selection is unknown*. Indeed, a complex system is more or less fit according to a number of different aspects (criteria): the more complex it is, the many reciprocally independent aspects it has. A system could be unfit according to one aspect and well fit according to another, and if these aspects are independent – that is, if a trade-off does not hold – optimization algorithms cannot be employed (see Biggiero & Laise, 2003, 2007). Further, for an environment co-evolves with its constituting systems, what is fit today could not be tomorrow, and vice versa[63]. In sum, *the representation-reduction of such complexity with a fitness function and a scalar measure is largely misleading*[64].

CONCLUSION

NK modelling evolved considerably from its very beginning in both its forms: BN and FL. Kauffman has great merits on many respects, especially into the field of biology and evolutionism[65]. The first (and perhaps the most important) one is that he offered a persuasive explanation of evolution not based only on natural selection, but also on self-organization as a spontaneous and blind generator of stability. A typical attack of creationism to evolutionism was that the rarity of viable and effective forms is so incredibly high that natural selection could not efficiently work. Without a "transcendental operator" life would be an almost impossible event from a combinatorial point of view. He demonstrated that, especially at the edge of chaos, biological networks can find spontaneously stable states, which then can be "submitted" to natural selection[66]. This way the *selection task becomes definitely realistic and affordable, and the "transcendental operator" is replaced by the "immanent operator" of self-organization*.

The edge of chaos represents an ideal evolutionary condition, because with too much stability there would be few combinations to choose from, whereas in chaos no selected combination would have enough time to remain stable, and therefore to be able to produce its biological effects. Too much order "freezes" a network into stable states preventing or hindering its adaptation capabilities. At the opposite extreme, too much disorder prevents stability, and thus, deprives (social or natural) life of the time and resources required to reproduction and learning. In this perspective, and this is actually the most interesting result of Kauffman studies on evolutionism, and the emergence of life as a likely event. In contrast, at the edge of chaos a network tends to stabilize itself and to be quite robust, even in the presence of large dimensions, but without "blocking" itself and not producing enough variety to evolve afterwards. Natural selection would act essentially on K, so as to exclude the networks with an "improper" K. This result is an extraordinarily strong proof of the falsity of the creationist view, which tries to leverage on the combinatory improbability of life.

This theoretical perspective can be confirmed as valid for networks and social phenomena as well, as Kauffman suggests (2000, Kauffman *et al.*, 2000; Levitan *et al.*, 2003). In this context, the role of the natural selection would take place by such mechanisms as an "invisible hand" associated to a self-organizing (Feltz, 2006; Foster & Metcalf, 2001; Geisendorf, 2009; Guastello, 1998; von Foerster & Zopf, 1962; Jantsch, 1979; Witt, 1997; Yovits & Cameron, 1960) and decentralized processes of social systems (Chang & Harrington, 2000, 2004), or by design interventions at the level of individual organizations or of territorial areas, e.g. an institution that governs an industrial cluster (Biggiero & Basevi, 2012; Easton *et al.*, 2008). If these theories were sufficiently corroborated by the simulation models and by the empirical research, then they could be translated in terms of policy objectives too.

Kauffman's results have had a remarkable impact not only in evolutionary theory, but also in economics and management sciences, as shown before. Indeed, implications for economic theory are very important for supporting criticisms to mainstream neoclassical approaches. They are a further knock at standard economic theory, because NK-FL models demonstrate the impossibility of globally optimal choices, without which general or partial economic equilibrium crashes down. The idea that economic agents - be them individuals or organizations or organizational units - evolve within a FL that most likely has not a single peak destroys de facto the possibility of optimizing behaviours, and consequently of optimal choices, optimal, prices, etc. In short, *NK-FL modelling destroys the fundamental ground on which the whole neoclassical edifice is built: the effectiveness of optimization algorithm*[67].

As we have seen, both BN and FK models have been improved, extended and enriched a lot during the five decades from beginning. Respect to the initial theoretical framework, the former has been transformed perhaps more substantially than the latter. For instance, once topology is considered more attentively and added to *P* and *CF* parameters, it comes that the *K* parameter, which in the original models was the crucial factor determining and discriminating between complex or simple networks, becomes less fundamental: *K* effects can be "neutralized" or substantially modified. Consequently, in a modern view, phase transitions between order and chaos can be many and at different values of *K*. *Phase transitions become much more network-specific*. Therefore, NK-BN loses generality, but gains realism.

According to published papers in economics and management sciences – and indeed in all social sciences – NK-FL has been considered much more appealing and fruitful than the NK-BN modelling. However, this unbalanced preference does not mean that NK-BN modelling does not provide interesting theoretical and empirical advantages. One of the most interesting is what Klüver calls the "conjecture of inequality" (2000: 103), which brings a lot of remarkable implications for economics and management sciences, and more generally for social sciences. It says that the more equal are nodes in terms of five parameters – activation rules, *P*, *CF*, *K*, and out-degree distribution (the latter two being topological parameters) – the more complex is, in probabilistic terms, its network dynamics. Moreover, there are good reasons to believe that this effect is cumulative, in the sense that higher equality of one or more parameters gives higher complexity. This conjecture, which can find strong support in modern social network analysis (see the previous chapter), could trigger and shape some future developments of social sciences. At the extreme of the apotheosis of hierarchy (and thus, of inequality) as it is represented by an out-tree (see Biggiero & Mastrogiorgio's Chapter 7 of this volume), the corresponding BN would be so simple that it would be almost unable to evolve at all. In fact, it is made by *K* slightly inferior of 1[68], it is composed by many terminal nodes (the operators at the lowest hierarchical level), and rather than inter-dependence relationships there are just dependence relationships, because all direct and indirect links are asymmetric, and there are no cycles. If we consider that most organizations are strictly and strongly hierarchical – and some of them very much, like armies and most large public administrations or large companies of standardized consumer goods – it comes that, *from a social evolutionary theory point of view, humanity is equipping itself with evolutionary poor systems*.

The conjecture of inequality could be then reformulated as the conjecture of "complex and evolutionary power of democracy", conjecture that is supported by two other and well consolidated research streams: those on small-group management and social psychology. With partially coinciding arguments they show that teams – the pure opposite of hierarchy – consume much more informative and time (and often also space) resources than hierarchies, and that these overloads can be compensated only under specific contexts: namely, when tasks are complex and size is large, as demonstrated by Biggiero & Sevi (2009, and in Chapter 13 of this volume). This result is due mostly because of the number of links, which is the old and well-known argument addressed already in the thirties by Graicunas (1937). This result depends also on the argument of potential variety of rationality, which means complexity of rationality, if variety is a way to express and measure complexity[69] (Ashby, 1962; von Foerster & Zopf, 1962). The rationale is that the higher is group variety the more difficult is the accomplishment of a common view in decision making. If collective reasoning is interpreted as a BN trajectory of a communication network whose links are opinions, then these research streams are perfectly consistent with the conjecture of "complex and evolutionary power of democracy". Let's recall von Foerster's motto "act always to improve variety".

In conclusion, from an epistemological and theoretical point of view, both parts of NK modelling are very interesting and increased our understanding of complex phenomena not only in natural sciences, but also in economics and management sciences. In particular, this methodology is very helpful to understand evolutionary and behavioural aspects of socio-economic networks. From a methodological point of view there is, instead, some doubt about the real explanatory power of single facts. In fact, while in principle it is possible to overcome the strong limitations and oversimplifying assumptions of the modelling standard version, in practice the heavy computational power required by this methodology to cope with enriched versions raises the question whether other methodologies could be more appropriate.

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ENDNOTES

1 In most Ashby's works - and especially in this article with Walker - there are all seeds and basic ideas of the theory of cellular automata, self-organization, and autopoiesis (Maturana & Varela, 1980).

2 An interesting and rather extensive presentation of how these research streams cross with NK modeling can be found in Klüver (2000). This work makes connections also with the field of genetic algorithms and neural networks, and thus, it deals with a methodology that is discussed in this volume (see Chapters 3 and 11). A further reason of interest for Klüver's book is that, differently from most works in these fields of formal modeling, its orientation is toward applications and theorization in social sciences instead of natural sciences. I will come back on this point in the conclusions of this Chapter and in the Conclusions Chapter of the whole volume.

3 More references are given in this chapter, and in Chapter 1 of this volume.

4 In-degrees, in the language of social network analysis (see Chapter 5, 6, 7, 8).

5 In standard formulations, such rules are random for all nodes. See later on in this chapter as concerning new versions of this methodology.

6 As it has been demonstrated in later studies (Klüver, 2000; Klüver & Schmidt, 2007), the exclusive focus on in-degrees is a limitation of the standard NK model, because trajectories depend, of course, also on out-degrees distribution. In other words, network topology matters on both sides. Indeed, when it is made the "heroic" assumption that in-edges distribution is uniform, it is enough to know in-degrees average connectivity (k), as the standard model does. Besides these formal aspects, it can be added that, at least in social real phenomena, it is not hard to imagine that a node (dis)activation depends on out-degrees distribution too. Consider, for example, a trade network and, therefore, how important it is for the behavior of a company (node), if it sells or not its products, that is, the measure of its out-degree. Since the inputs' activation depends on out-degree distribution too, different out-degree distributions will determine different dynamics.

7 This is the number of a node's possible states, which can be also more than two, even though most applications assumes just two states: active/inactive. It is excluded – and never considered neither theoretically – the possibility that a node has internal states. The presence of an internal state function would imply that, depending on the actual internal state, the same input pattern – the set of in-edges with its specific Boolean functions – could determine a different node's state. This is the core concept of complexity in von Foerster's approach (1982, 2003), who calls systems with internal states "non-trivial machines". As the founder of second-order cybernetics, he shows that, due to computational complexity, a non-trivial machine behavior cannot be calculated even with very small inputs, outputs and internal states (Biggiero, 2001). Moreover, he argues that the whole universe and almost any social and natural system is a non-trivial machine. Therefore, almost all system is affected by computational complexity. This clarification is intended to underlie that

activation states do not refer to internal states, because the standard NK model assumes *a single internal state.*

8 Notice that, because it is assumed an uniform in-degrees (K) connectivity, in the standard version of this methodology a network is entirely defined by two parameters only: N (network size) and K (connectivity). Therefore, the label NK model. Most Kauffman's (and his followers') experiments statistically studied number and length of all attractors associated to all possible activation rules. However, if one removes the assumption concerning K, then in order to study network dynamics with its attractor's length and number, it becomes necessary to specify network topology, that is the specific link distribution. Later on, we will see that indeed also out-degrees matter, and that when networks are large and dense (with high K) there are also two more parameters that – besides network topology and activation rules - play an important role: P, and (hierarchical) canalizing (Boolean) functions (CF).

9 Kauffman's most important book concerning his theoretical framework and basic concepts in analytical terms is "The origins of order" (1993), while "At home in the universe" (1996) and "Investigations" (2000), besides a popular exposition of his researches, offer an exciting extension and development of implications in many fields of natural and social sciences. His other technical contributes are distributed over a number of papers, out of which here only those strictly necessary are referred to.

10 Indeed, there is an earlier phase transition when $0.5<k<0.75$, because in that area, even though not all nodes are connected, a giant component percolates – that is, agglomerates - from a highly fragmented network.

11 For a network state can be expressed by a string of 0 ad 1, the "distance" of two states can be measured through the Hamming distance, which counts the differences of each element in each place of the two strings.

12 Its average length is in fact of 10^{30}.

13 Let's remind that they are 16, deriving from the minimum combinatory framework, where the binary behavior (state) of one target node depends on the interaction of 2 nodes with binary states. All other more complicated combinations with multiple interacting nodes or with multiple states are built as combinations of the 16 basic functions.

14 After this idea some other contribution followed (Kauffman *et al.*, 2004; Nikolajewa *et al.*, 2006; Rauh & Ay, 2014).

15 See also chapters 1, 5 and 12 of this book.

16 This holds if considering K as a constant and not as an average value, in which case the number of possible digraphs would increase.

17 Further, if the assumption of regular connections were removed, then – still with the same K=2 – a further huge number of possible networks would be generated.

18 Their v parameter is a sort of measure of out-degree concentration: the lower it is, the more balanced are out-degrees.

19 It should be warn that in this literature it is not always clear whether K represents average or uniform (constant) in-degree, while the two things have very different implications.

20 That is, nodes without any out-degree.

21 In small networks it is enough the presence of just one terminal node to prevent the existence of long trajectories.

22 Indeed, as discussed deeply in next note, topology is not a parameter, because it is a complex phenomenon that can be described only through a set of parameters.

23 Most likely, not only in- and out-degrees distribution matters for determining network dynamics, but rather most main indicators of network topology, as social network analysis suggests. Indeed, also Kauffman's *K* parameter is nothing else than an indicator of network connectivity, especially if intended in the broader meaning of average degree centrality and not as a uniform degree centrality. Other crucial relationships could be average path length, clusterization, small-worldliness, reciprocity, cyclicity, and assortativity (see Chapter 1 and basic literature, as Borgatti *et al.*, 2013; Hanneman & Riddle, 2005; Lewis, 2009; Newman, 2010; Prell, 2011; Scott, 1992; Scott & Carrington, 2011; Wasserman & Faust, 1994). (To read definitions see Glossary, and applications in chapters 5, 6, 7, and 8.) Indeed, some of them are correlated, but the understanding of which and how are correlated is still in progress (Valente, 2008). In any case, even when there were correlation it would be useful to know the relations between the different topological static properties and the corresponding BN dynamics.

24 This does not prevent that his approach has been – sometimes sharply – criticized by other biologists, i.e. by Gould (2002). Though this is not within the scope of this volume, some of these criticisms, to the extent they are pertinent also for socio-economic networks, will be reminded in the concluding section of this chapter.

25 To know more about these applications, see Biggiero (2011).

26 Though publishing much later, Klüver & Schmidt do not refer to Carroll & Burton's work, likely because they belong to a (substantially) different research tradition: theoretical sociology vs. management and organization science. Chang & Harrington (2006) deal with the same topic and use NK modelling as well, but in the FL version (see below in this chapter). Even Frenken (2006) and Levinthal & Warglien (1999) use NK modelling to analyse the effects of task complexity on group performance, but their approach differs, because the *K* connections among activities represent technological and not communicational links, and they employ the FL version (therefore, it is briefly commented below in this chapter). Further, workgroup structure and performance and task complexity is the same topic faced with by Biggiero & Sevi in Chapter 13, but dealt with agent-based simulation modeling.

27 Of course, it can be seen as a "closed chain".

28 It should be underlined that in the NK-BN model maximum *K* is *N*, because this parameter expresses only in-degrees. Conversely, when also out-degrees are taken into account, then maximum connectivity would be N^2. In fact, in network analysis, maximum connectivity is given by this algorithm, and corresponds just to team (or clique) connectivity. Therefore, all Kauffman's suggestions and results had to be reformulated and calculated in this way.

29 To know more on this cluster and its knowledge network, see Sammarra & Biggiero (2008) and Biggiero & Sammarra (2010).

30 To know more on this concept and the literature background with special reference to the role that it plays within industrial clusters, see Biggiero & Basevi (Chapter 12 of this volume).

31 This becomes inevitable if size is modified. An interesting case of this type has been made by Samal & Jain (2008) for a metabolic network.

32 And each of them may be classified in several sub-types.

[33] Kauffman (1993) himself discusses differences and similarities between NK and ANN (Artificial Neural Network) modelling.

[34] Let's remind that they are 2^z^k with z=2 only if nodes can have only two states. Otherwise they grow exponentially with z.

[35] Indeed, as we have seen in the previous part of this contribution, it matters not only average or constant *K*, but also its distribution.

[36] As we know, the more complex is the network, the longer its trajectories and the lesser its attractors. Consequently, many combinations can lead to the same attractor.

[37] Notice that it is not necessary to suppose that such perceptions and changes should be intentional or aware. It is sufficient some sort of feedback mechanism and a blind (random) choice of parameters variations. In fact, as argued by early cyberneticists (Ashby, 1956, 1960, 1962; Heims, 1982, 1991; von Foerster, 1982), once feedback mechanisms are in play, even non-biological natural systems can be acknowledged to have cognitive capacities. Of course, the higher a BN cognitive capacities the better its chance to improve fast and efficiently its fitness. However, this intuitive argument is not so trivial or granted, because, as we will see below, under some circumstances a deliberated and intelligent strategy is not necessarily better than a random one.

[38] Most likely, though the number of possible combinations – as Klüver & Schmidt show - grows exponentially after removing the simplifications, the number of attractors remains in the order of its squared root.

[39] Unfortunately, Kauffman has "nested" FL derivation from BN in just two (pp. 211-212) out of the 700 pages of his "The origins of order", and he has never (or at least not in the main contributions, as far as I know) came back again on this issue. Hence it's clear that most scholars did not pay too much attention on this fundamental conceptual connection, and took FL as a separated and independent tool. Indeed, this is not wrong, even (but not only) because the original idea and formulation was elaborated by Sewall Wright about 25 years before Kauffman. So, most researchers in economics and management and organization sciences identify tout court NK only with NK-FL methodology.

[40] Referring to biology, Kauffman calls them epistatic connections, which are those that connect genes within a genome. Consistently, genome is the agent trying to improve its fitness; and components are genes, which can assume the form of two alleles.

[41] When considering – as usually done by Kauffman and most scholars – random distributions, there is also a fifth parameter to be taken into account: the underlying distribution from which fitness values are assigned to each element, which can be (among the most important types) uniform, normal, U-shaped, exponential or scale-free.

[42] In these numerical simulations adaptive walks initiate from a random initial state and then proceed in a randomly chosen fitter one-mutant variant to the next fitter state.

[43] As for NK-BN there are many works studying FL with variations in one or more of the other parameters (Iguchi *et al.*, 2005).

[44] Correspondingly, if possible states are not binary, then the base should be increased accordingly, while the exponent remains the same.

[45] String similarity is measured through Hamming distance, which counts the number of bits that, being in the same position, have a different value in two strings.

[46] Conversely, it dwindles by 1 when K=0.

47 For some more cues on this issue, see Biggiero's Chapter 1 and Biggiero & Basevi's Chapter 12 of this volume.

48 Kauffman refers to Nash equilibrium as a situation in which no agent has convenience to move in an adjacent position. That is, each agent is in a local optimum.

49 Letting the two systems varying its inter-connections, that is with an uneven number of reciprocal connections, Kauffman obtains a number of other interesting results showing that Nash equilibria are reached rather soon (about 250 generations) within significantly large areas in the combined parameters space.

50 Let's remember that, analogously with the unpredictably different state of sequential states of BNs chaotic trajectories, two adjacent states in rugged landscapes are as well very different one another.

51 It is self-evident that an individual's or an organization's reduction to only two states is absolutely unrealistic.

52 A new way to modelling NK-FL, called pNK, is provided by Marco Valente in Chapter 10 of this volume. It allows eliminating some restrictions and difficulties of the original model.

53 Indeed it is not true task interdependence, but rather (in H7) a difference between adjacency and random walk, and (in H8) members' differentiation in terms of centrality index. The analogy (in H7) with Thompson's (1967) distinction between sequential and pooled task interdependence does not hold. On this point see also Biggiero & Sevi (2009, and Chapter 13 in this volume).

54 They would correspond to the genes of genome.

55 We find here the arguments uttered for BNs, namely, that K should be around 2, but not more than 5, and the role played by P and CN parameters as "reducers of complexity". We remind that, as discussed above, there are also other complexity reducers, like out-degrees topology, and in particular, its power-lawness shape, and the number of pendants (nodes with only in-edges).

56 However (and paradoxically), they can calculate instantaneously and without cost their own fitness value, and value gains from possible adjacent positions.

57 Indeed, another complication would be letting multiple independent agents evolve in the same FL, while taking into account also carrying capacity of each peak, and thus, resource consumption due to agents. The model would become even more complex if all or part of these agents were interconnected.

58 A medium sized organization has hundreds suppliers, and a large one thousands.

59 See Chapter 1 in this volume.

60 The expression "at least" refers to the fact that an organization's elements are much more than only production characteristics.

61 Indeed, this question addresses also to the interesting and still under-explored issue of the ontological differences between socio-economic and biological systems. Among them, phenotype heterogeneity and time scale seem particularly important to suggest strong caution to transfer results obtained in one field into the other.

62 Not to say that relationships can influence nodes attributes – that is, in FL language, a system's elements – and vice versa. See Biggiero's Chapter 1 discussing ERGM network modeling. A consideration of these eventualities would make NK-FL modeling extremely more complex and computationally heavy.

63 With a different language and arguments this is partly claimed by Gould criticizing NK-FL modeling, and grounding the explanation of the phenomenon of exaptation (Cattani, 2006; Gould & Vrba, 1982).

[64] For a view on correct multicriteria evaluation methods see Bouyssou *et al.* (2000) and Bouyssou (2001). For applications into the field of economics and management science see Biggiero & Laise (2003a, 2003b, 2007), and for a critique to fitness functions in evolutionary processes see Biggiero & Laise (2015).

[65] In Kauffman's works there are many more theoretical suggestions and contributions - like the emphasis on universe's non-ergodic nature or the view of economic production as co-evolving and co-constructing activity – that here cannot be discussed.

[66] It should be reminded that the other answer to this creationist criticism is that supplied by Simon (1962), among others, and it is perfectly compatible with that of Kauffman. It deals with the issue of systems decomposability and modularity, according to which, respectively, evolution creates small components at different levels of aggregations, and some of them are, fully or partially, substitutes. Other references to this issue can be found, in this volume, in Chapter 13.

[67] Noteworthy, even for this great contribution Kauffman and Simon go down the same road, because Simon (1957, 1976, 1978, 1979,1982) dedicated almost his whole life (Augier & March, 2004) to demolish neoclassical economic theory, especially in the form of utility function theory and general economic equilibrium theory.

[68] In an out-tree there are *n-1* links (see Krackhardt, 1994, and Chapter 7 in this volume).

[69] Biggiero (2001) and Casti (1994) argue that some of the main measures of complexity – i.e. algorithmic, informative, and variety-based measures - are substitutes one another.