

Article

Analysis of the Vibrations of Operators' Seats in Agricultural Machinery Using Dynamic Substructuring

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Abstract: The vibrations produced by an agricultural machine are transmitted to the seat of the tractor operator and must comply with the limitations imposed by international and national regulations. An agricultural machine is generally composed of a tractor that can be linked to different machines required to perform a large number of agricultural tasks. In this paper, substructuring techniques are proposed to investigate the dynamics of the agricultural machine and to evaluate the resulting vibration exposure to the tractor driver in different configurations of the machine. These techniques allow one to couple reduced-order models or experimental models of the component subsystems to obtain the response of the whole system. In the results, the vibration exposure of the tractor operator is evaluated for different configurations of the agricultural machine, by observing the frequency response function (inertance and transmissibility) and the transient response to a given excitation. In conclusion, these techniques allow one to investigate a large number of different configurations and a wide range of operating conditions with a light computational burden and without asking the manufacturers to share sensitive design details.

Keywords: dynamic substructuring; vibrations; vibration exposure; agricultural machinery; agricultural tractors



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1. Introduction

In everyday life, vibrations generated in the surrounding environment may be unpleasant for people and generate temporary discomfort [1–4]. On the other hand, in some working sectors, daily exposure to vibrations can irreversibly compromise, in the long term, the health of the operator [5–7]. To protect workers, regulations have been introduced that limit the level of the exposure to vibrations during daily work.

Agricultural machine operators, when driving vehicles and mobile work machines, are typically subjected to mechanical vibrations transmitted to the whole body in a seated position [8,9]. The ISO 2631-1:1997 standard [10] shows that the human body in a seated position is particularly sensitive to vibrations in the frequency band 0.4–100 Hz. In particular, for frequencies between 1 and 2 Hz, vibrations can produce effects of temporary discomfort, such as car sickness; for frequencies between 2 and 20 Hz they can give rise to lesions of the lumbar tract and trauma to the spine (this last effect has a critical frequency of between 3 and 10 Hz) [11]. The driving of tractors determines a postural overload due not only to a prolonged seated position but also to the frequent rotations of the lumbar tract for carrying out particular operations [12].

It is, therefore, necessary that the designers, since the initial phase of the project, focus on the dynamic behavior of the machine to limit the vibration exposure of the operator [13].

The activities carried out in agriculture are varied and of different nature and require the use of specialized machines to carry out individual operations. Typically, an agricultural

machine is composed of a driving machine (tractor) connected to one or more operating machines. In fact, the tractor is the most widespread mobile agricultural machine because, thanks to the coupling systems with which it is equipped, it can be connected to different operating machines, allowing several operations to be carried out. Depending on the activity and therefore on the machine used, the vibration exposure level of the tractor driver can vary considerably.

Focusing on the tractor, the main internal vibration sources are the engine, the gear changes, and sometimes hydraulic or pneumatic systems that generate high-frequency (over 10 Hz) vibrations. The main external source of vibrations is represented by the contact of the wheels with the ground, especially when the vehicle travels on compact surfaces at high speed [13]. As stated previously, these low-frequency (below 10 Hz) vibrations are particularly dangerous for the operators.

The main techniques adopted to reduce the vibration levels of the driver include the adoption of passive anti-vibration systems (suspension systems for the front axle, the cab, and the seat) designed to limit the transmission of vibrations to the operator [14].

A tractor is connected to an operating machine with a standard coupling system that allows the connection of different operating machines to the same tractor. Towed machinery is connected to the tractor by the towing hook. A detailed description of the necessary mechanical connection between the towed and towing vehicles is provided in the five parts of the standard ISO 6489 [15–19]; a detailed description of the mechanical connection between towed vehicles is provided in the three parts of standard ISO 5692 [20–22]; and the hitch rings and the attachment to the tractor draw bar are described in the standards ISO 21244 [23] and ISO 20019 [24].

Front and rear-mounted or semi-mounted machinery is connected to the tractor by the front or rear three-point linkages, respectively. A general description of the rear-mounted three-point linkage is provided by the standard ISO 730 [25]. Detailed descriptions of different kinds of hitch couplers are provided in the four parts of the standard ISO 11011 [26–29], and details about the clearance zones around the operating machines are provided in the standard ISO 2332 [30]. The front-mounted three-point linkage and its connection to the operating machine are defined in parts four and two of the standard ISO 8759 [31,32], respectively.

Finally, standard connections are defined for the power transmission from the tractor to the operating machine: in particular, the rear-mounted power take-off is detailed in the three parts of the standard ISO 500 [33–35], and the front-mounted power take-off is detailed in the parts one and three of the standard ISO 8759 [36,37].

As stated previously, the types of operating machine are numerous and different. However, to assess the vibrational impact on the operator, it is sufficient to identify and accurately describe the sources of excitation and the possible modes of transmission of the vibrations. To this end, the machines can be classified based on:

- The type of tractor attachment;
- The weight distribution;
- The presence of the power take-off and therefore of moving parts in the operating machine;
- The interaction with the soil or crops.

National regulations provide limits on the level of exposure to vibration, and if necessary, the reduction of this level by adopting appropriate actions. However, typical measurements of the vibration level (also according to particular Standards, e.g., ISO 2631-1:1997 [10]) are carried out on a given configuration of the agricultural machine that can not be representative of all the possible configurations.

In fact, the operator can be exposed to very different levels of vibration depending on whether or not there is an operating machine mounted on the tractor, on the type of operating machine, and on the working conditions.

Over time, manufacturers of the individual components of agricultural machines (tractors, linkages, operating machines) have developed and introduced several devices

to reduce vibration. However, it is necessary to evaluate the effects of these devices on the dynamic behavior of the whole agricultural machine. Moreover, it would be useful to identify the sources and the transmission paths that are most responsible for the vibration of the operator's seat to design optimal structural modifications [38] in order to remediate existing machines.

For this reason, a modeling technique based on substructuring is proposed in this paper. It effectively allows taking into account the complexity and variability of the entire agricultural machine. Furthermore, it allows one to highlight the most critical sources and transmission paths and provides useful information in the design and optimization of the individual components.

The paper is organized as follows. In Section 2 the dynamic substructuring method is introduced and typical models of the component substructures of an agricultural machine are presented; moreover, a frequency response function and displacement transmissibility are derived: these quantities are used in the sequel to compare different configurations of the agricultural machine. In Section 3 the effects of different vibration sources and different configurations of the agricultural machine are highlighted; moreover, the effectiveness of dampers at different locations is investigated. In order to deal with a realistic model, the response of the adopted tractor model is compared to the experimental response of a real tractor.

2. Models and Methods

As introduced in the previous section, agricultural machinery can assume different configurations and produce very different effects in terms of vibration exposure level to the operator. This means that a devoted analysis would be necessary for each configuration. Instead, it would be more effective to start from individual models of the components of the agricultural machine (tractor, linkage system, and operating machine) and to couple them through dynamic substructuring. This would allow analyzing different configurations of the agricultural machine (for example, different operating machines coupled to the same tractor) by varying only the model of one of the components.

The dynamics of the whole machine should be accurately defined within the frequency band and based on the DoFs of interest for the vibrational analysis (e.g., the operator's seat or the connections with other components). Hence, the model of each component should simply comply with the same requirements. This allows performing accurate vibration analysis of the agricultural machine by even adopting very simplified models.

2.1. Dynamic Substructuring

In the field of structural dynamics, dynamic substructuring methods have played a fundamental role because they allow studying the dynamic behavior of a complex system starting from the dynamic behavior of the component subsystems. The main advantages are the following [39]:

- The possibility of studying the dynamic behavior of systems that would be too complex to analyze with a single model. In particular, at the design stage, it allows analyzing the effects of local modifications in the complete system, by changing only the model of the subsystems involved in the modification.
- The analysis of the single subsystems allows more immediate recognition of the local dynamic behaviors and therefore very efficient global optimizations. At the same time, in the analysis, it is possible to neglect the local dynamic behaviors that do not have any significant effect on the dynamics of the whole system.
- The possibility of assembling subsystems whose dynamic behavior is described using numerical or analytical models, and subsystems described using models derived from experimental measurements.
- The possibility of combining subsystems modeled by different project groups even at different times and places.

Using the substructuring approach, once the numerical, analytical, or experimental models and the coupling conditions at the interface degrees of freedom (DoFs) between the subsystems are established, the dynamic behavior of the coupled system can be obtained. The coupling conditions impose the compatibility and the equilibrium at the interface between the given subsystems.

Therefore, the substructuring method, used in this paper, provides the following advantages in the dynamic analysis of agricultural machinery:

- It allows one to analyze the dynamics of the complete system (the agricultural machine) through the characterization of the single components (tractor, linkage system, and operating machine);
- It simplifies the analysis of the complete system when only one of the subsystems is modified (as in the case of several operating machines connected to the same tractor or of the same operating machine connected to different tractors);
- It allows one to focus the attention only on the elements that affect particular aspects of the dynamic behavior of the agricultural machine (e.g., the DoFs of the connector between the tractor and the operating machine to evaluate the vibration transmission from the machine to the tractor).

2.2. Models

The agricultural machine considered in this work was an agricultural tractor that can be joined to one or more operating machines. Our interest is limited to evaluating the vertical vibrations of the operator’s seat. In this subsection, the models of the component subsystems are described: the tractor, the three-point linkage, and the operating machine.

2.2.1. Lumped Parameter Models of the Agricultural Tractor

In order to investigate the vertical vibration of the operator’s seat, it is possible to model different tractors using lumped parameter models (see Figure 1) in the longitudinal plane, namely:

- A 3-DoFs tractor with seat suspension (see Figure 1a);
- A 4-DoFs tractor with seat and front wheel suspension systems (see Figure 1b);
- A 6-DoFs tractor with seat, front wheel, and cabin suspension systems (see Figure 1c).

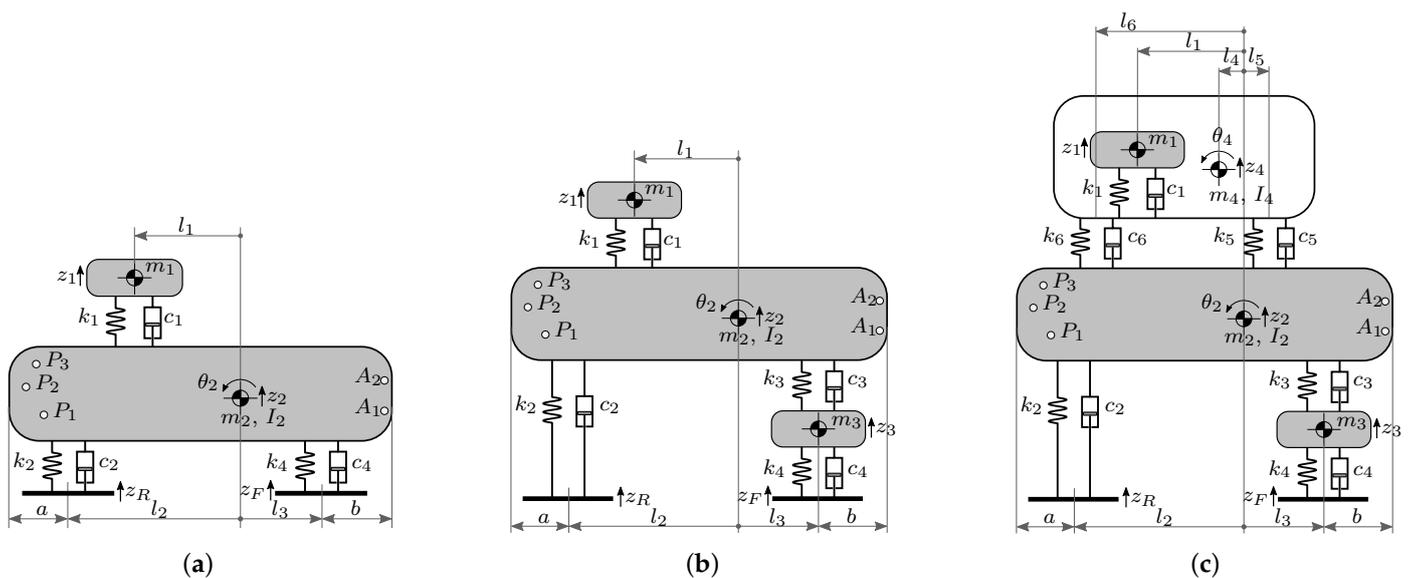


Figure 1. Lumped parameter models of different agricultural tractors: (a) a tractor with seat suspension; (b) a tractor with seat and front wheel suspension systems; (c) a tractor with seat, front wheel and cabin suspension systems.

The tractor modeled in Figure 1c accounted for two lumped masses and two rigid bodies. The lumped mass m_1 represents the mass of the operator, including that of the seat, and mass m_3 represented the mass of the front axle. Conversely, the tractor frame and the cabin were modeled as rigid bodies characterized by masses m_2 and m_4 , and moments of inertia I_2 and I_4 , respectively.

The seat suspension was composed by a viscoelastic element with stiffness k_1 and damping coefficient c_1 . The cabin was suspended to the tractor frame by two viscoelastic elements characterized by stiffnesses k_5 and k_6 , and damping coefficients c_5 and c_6 , respectively. The rear axle was rigidly connected to the tractor's frame. The elasticity and the damping of the rear wheels were modeled by a stiffness constant k_2 and a damping coefficient c_2 . The tractor frame was suspended to the front axle by a viscoelastic element composed of a spring k_3 and a damper c_3 . The elasticity and the damping of the front wheels were modeled by a stiffness constant k_4 and a damping coefficient c_4 .

In Figure 1c the DoFs of the model are also shown. For the seat and the front axle, the DoFs z_1 and z_3 represent the vertical displacements of the two masses. The DoFs z_2 and z_4 represent the vertical displacements of the CoGs of the tractor frame and the cabin, and the DoFs θ_2 and θ_4 represent the respective rotations. Moreover, the DoFs z_F and z_R represent the vertical displacements of the contact points between an irregular soil and the front and rear wheels. Finally, the points A_i and P_i denote, respectively, the connection points between the tractor and the front and rear three-point linkages [25,31,32].

The models in Figure 1a,b can be easily obtained by the model in Figure 1c by considering the absence of the cabin suspension and the axle suspension.

2.2.2. Reduced-Order Model of the Rear-Mounted Three-Point Linkage

The rear-mounted or semi-mounted machines are connected to the tractor through the rear three-point linkage. Figure 2a shows a typical rear three-point linkage and highlights the main components as defined in the international standard ISO 730:2009 [25]: ① lower links; ② upper link; ③ lift rods; ④ implement; ⑤ stabilizers. The implement is part of the operating machine, whilst all the other bodies are connected to the tractor.

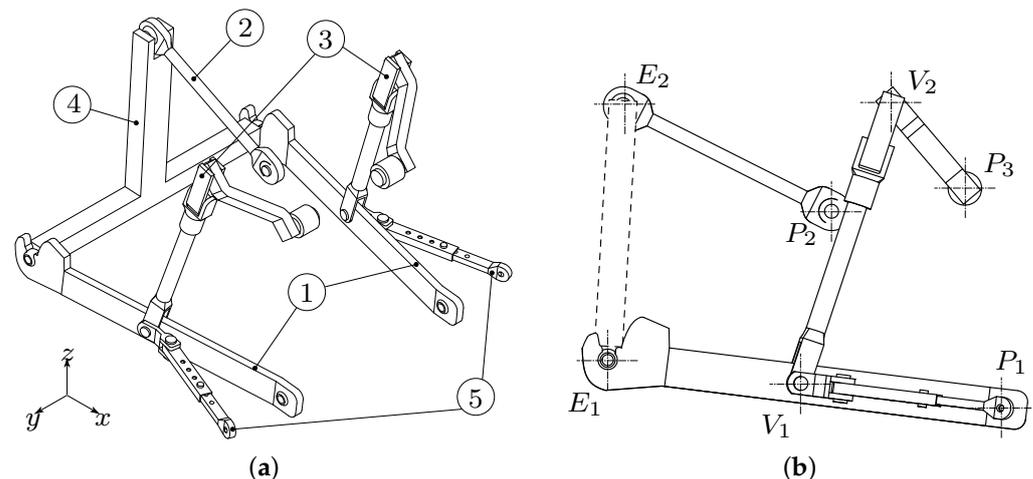


Figure 2. Rear-mounted three-point linkage: (a) assembly; (b) boundary nodes of the reduced-order model.

In Figure 2b the boundary nodes of the different components are highlighted: P_i and E_i indicate the boundary nodes connecting the three-point linkage to the tractor and the operating machine, respectively; V_i are the boundary nodes connecting the components of the linkage each other.

To obtain the model of the rear three-point linkage, each component was modeled using commercial FE software; subsequently, a modal reduction was performed, using the Craig Bampton approach and retaining only the physical connecting DoFs with other

components. Finally, the model of the whole rear-mounted three-point linkage was built by coupling the reduced-order models of the different components. Note that the linkage can assume different geometric configurations and the coupling conditions must be defined accordingly.

2.2.3. Reduced Order Model of the Front-Mounted Three-Point Linkage

The front-mounted or semi-mounted machines are connected to the tractor through the front three-point linkage. Figure 3a shows a typical front three-point linkage and highlights the main components as defined in the international standard ISO 8759-4:2018 [32]: ① lower links; ② upper link; ③ lift cylinders; ④ implement.

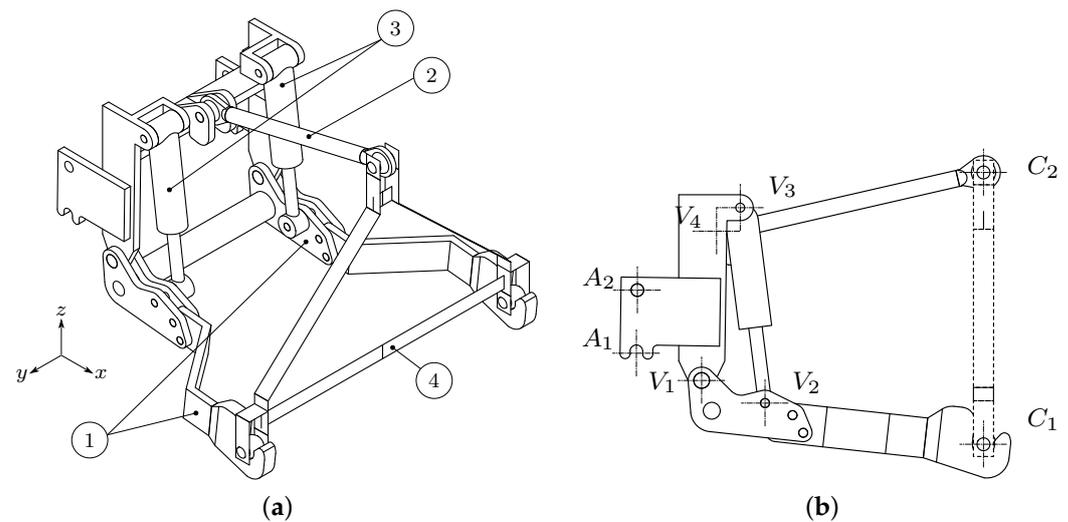


Figure 3. Front-mounted three-point linkage: (a) Assembly; (b) Boundary nodes of the reduced-order model.

In Figure 3b the boundary nodes of the different components are highlighted: in this case, A_i and C_i indicate the boundary nodes connecting the three-point linkage to the tractor and the operating machine, respectively; V_i are the boundary nodes connecting the components of the linkage each other. The model of the front three-point linkage is obtained following the same procedure detailed in the previous subsection for the rear three-point linkage.

2.2.4. Mounted and Semi-Mounted Machines

To evaluate the effect of operating machines on the vertical vibration of the operator's seat, a rigid body model was used to represent the behavior of mounted and semi-mounted machines in the longitudinal plane. Figure 4a,b shows the adopted models of rear and front machines, where O is the CoG of the rigid body. Note that the viscoelastic component (with stiffness k_o and damping coefficient c_o) was present only in the semi-mounted machine; in this case, a further DoF z_s represents the vertical displacements of the contact points between an irregular soil and the wheels.

According to Figures 2b and 3b, nodes E_i in Figure 4a and C_i in Figure 4b are the nodes connected to the rear and front three-point linkages, respectively. All the possible standard couplers [26–29] can be represented with a slight modification of the proposed model.

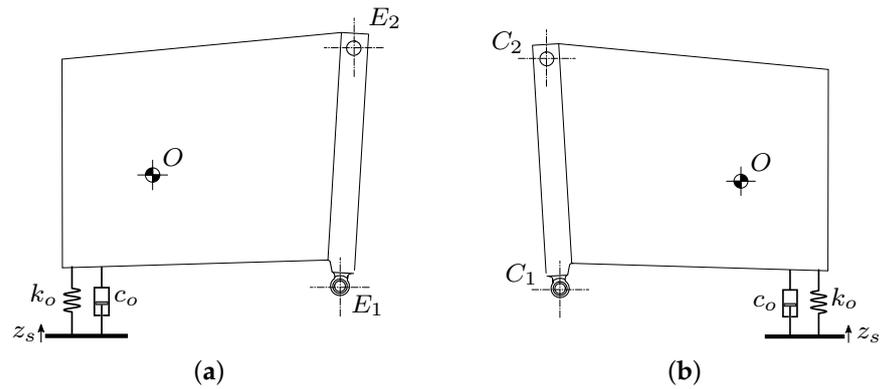


Figure 4. Lumped parameter models of mounted or semi-mounted machines: (a) rear machine; (b) front machine.

2.3. Substructure Coupling

Once the models of the component subsystems of the agricultural machine are defined, they can be assembled using the substructure coupling technique. For a mechanical system consisting of n linear subsystems, the equation of motion of each subsystem r may be expressed as:

$$\mathbf{M}^{(r)}\ddot{\mathbf{u}}(t)^{(r)} + \mathbf{C}^{(r)}\dot{\mathbf{u}}(t)^{(r)} + \mathbf{K}^{(r)}\mathbf{u}(t)^{(r)} = \mathbf{f}(t)^{(r)} + \mathbf{g}(t)^{(r)} \tag{1}$$

where:

$\mathbf{M}^{(r)}, \mathbf{C}^{(r)}, \mathbf{K}^{(r)}$ are the matrices accounting for the mass, damping, and stiffness;

$\mathbf{u}(t)^{(r)}$ is the displacement vector;

$\mathbf{f}(t)^{(r)}$ is the external force vector;

$\mathbf{g}(t)^{(r)}$ is the internal constraint force vector, accounting for the connecting forces with other subsystems.

The equations of motion of n subsystems can be gathered in a block diagonal format as:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) + \mathbf{g}(t), \tag{2}$$

with:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}^{(1)} & & \\ & \ddots & \\ & & \mathbf{M}^{(n)} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}^{(1)} & & \\ & \ddots & \\ & & \mathbf{C}^{(n)} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \mathbf{K}^{(1)} & & \\ & \ddots & \\ & & \mathbf{K}^{(n)} \end{bmatrix} \tag{3}$$

$$\mathbf{u}(t) = \begin{Bmatrix} \mathbf{u}(t)^{(1)} \\ \vdots \\ \mathbf{u}(t)^{(n)} \end{Bmatrix}, \quad \mathbf{f}(t) = \begin{Bmatrix} \mathbf{f}(t)^{(1)} \\ \vdots \\ \mathbf{f}(t)^{(n)} \end{Bmatrix}, \quad \mathbf{g}(t) = \begin{Bmatrix} \mathbf{g}(t)^{(1)} \\ \vdots \\ \mathbf{g}(t)^{(n)} \end{Bmatrix} \tag{4}$$

The compatibility condition is generally expressed as:

$$\mathbf{B}\mathbf{u}(t) = \mathbf{0} \tag{5}$$

where each row of \mathbf{B} states that the displacement of a pair of matching DoFs, i.e., DoF l belonging to subsystem r and DoF m belonging to subsystem s , must be the same ($u_l^{(r)} - u_m^{(s)} = 0$).

The equilibrium condition for internal constraint forces is generally expressed as:

$$\mathbf{L}^T\mathbf{g}(t) = \mathbf{0} \tag{6}$$

where each row of matrix \mathbf{L}^T states that: for any pair of matching DoFs, the sum of internal constraint forces must be zero (i.e., $g_l^{(r)} + g_m^{(s)} = 0$); and for non connecting DoFs, the corresponding internal constraint forces must be zero. Note that the number of rows of the matrix \mathbf{L}^T is equal to the sum of the number of non connecting DoFs and the number of pairs of matching DoFs.

Gathering Equations (2), (5) and (6), one obtains the three-field formulation that describes the coupling among n subsystems:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) + \mathbf{g}(t) \\ \mathbf{B}\mathbf{u}(t) = \mathbf{0} \\ \mathbf{L}^T\mathbf{g}(t) = \mathbf{0} \end{cases} \quad (7)$$

To solve the coupling problem, here a unique set of DoFs \mathbf{q} is considered, composed by the unique set of interface DoFs and by non-interface DoFs. By stating that

$$\mathbf{u}(t) = \mathbf{L}\mathbf{q}(t), \quad (8)$$

the interface equilibrium is automatically satisfied and the interface forces are eliminated from the first line of Equation (7). In fact, since the full set of DoFs \mathbf{u} is obtained from the unique set \mathbf{q} , the compatibility condition becomes:

$$\mathbf{B}\mathbf{u}(t) = \mathbf{B}\mathbf{L}\mathbf{q}(t) = \mathbf{0} \quad (9)$$

and holds for all values of \mathbf{q} . Therefore, the compatibility condition, i.e., the second line in Equation (7), is always satisfied and the three-field formulation reduces to:

$$\begin{cases} \mathbf{M}\mathbf{L}\ddot{\mathbf{q}}(t) + \mathbf{C}\mathbf{L}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{L}\mathbf{q}(t) = \mathbf{f}(t) + \mathbf{g}(t) \\ \mathbf{L}^T\mathbf{g}(t) = \mathbf{0} \end{cases} \quad (10)$$

By pre-multiplying the dynamic equilibrium equation by \mathbf{L}^T and considering that the interface equilibrium is automatically satisfied ($\mathbf{L}^T\mathbf{g}(t) = \mathbf{0}$), the assembled system becomes:

$$\mathbf{L}^T\mathbf{M}\mathbf{L}\ddot{\mathbf{q}}(t) + \mathbf{L}^T\mathbf{C}\mathbf{L}\dot{\mathbf{q}}(t) + \mathbf{L}^T\mathbf{K}\mathbf{L}\mathbf{q}(t) = \mathbf{L}^T\mathbf{f}(t) \quad (11)$$

i.e.,

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}}(t) + \tilde{\mathbf{C}}\dot{\mathbf{q}}(t) + \tilde{\mathbf{K}}\mathbf{q}(t) = \tilde{\mathbf{f}}(t). \quad (12)$$

To obtain the receptance matrix $\mathbf{H}(j\omega)$ of the assembled system, Equation (12) must be expressed in the frequency domain considering $\tilde{\mathbf{f}}(t) = \tilde{\mathbf{F}}(\omega) e^{j\omega t}$ and $\mathbf{q}(t) = \mathbf{Q}(\omega) e^{j\omega t}$:

$$\left[-\omega^2\tilde{\mathbf{M}} + j\omega\tilde{\mathbf{C}} + \tilde{\mathbf{K}} \right] \mathbf{Q}(\omega) = \tilde{\mathbf{F}}(\omega) \quad (13)$$

Therefore,

$$\mathbf{H}(j\omega) = \left[-\omega^2\tilde{\mathbf{M}} + j\omega\tilde{\mathbf{C}} + \tilde{\mathbf{K}} \right]^{-1}. \quad (14)$$

2.4. Modal Reduction through the Craig–Bampton Method

In some situations, it could be convenient to reduce the number of degrees of freedom involved in the dynamic model of a system, for example, in the case of the finite element models of the linkages introduced in Sections 2.2.2 and 2.2.3, where only the physical DoFs that connect one component with the other components are retained.

For this purpose, the Craig–Bampton approach [40] can be used. Considering a given subsystem (r), the full set of DoFs $\mathbf{u}^{(r)}$ can be partitioned as:

$$\mathbf{u}^{(r)} = \begin{Bmatrix} \mathbf{u}_b \\ \mathbf{u}_i \end{Bmatrix} \quad (15)$$

where subscripts b and i indicate boundary and interior DoFs, respectively, and the superscript (r) is dropped. Note that the r th substructure is connected to the other substructures through the boundary DoFs. The displacements $\mathbf{u}^{(r)}$ can be approximated considering the static deformation modes Φ_c and the fixed interface modes Φ as follows:

$$\mathbf{u}^{(r)} = \begin{Bmatrix} \mathbf{u}_b \\ \mathbf{u}_i \end{Bmatrix} \simeq [\Phi_c \quad \Phi] \begin{Bmatrix} \mathbf{u}_b \\ \mathbf{q}_m \end{Bmatrix} \tag{16}$$

where \mathbf{q}_m represents the modal coordinates associated with the fixed interface modes.

Static deformation modes are expressed as:

$$\Phi_c = \begin{bmatrix} \mathbf{I}_b \\ \Phi_{c,i} \end{bmatrix} \tag{17}$$

where each column b of the sub-matrix $\Phi_{c,i}$ represents the static deformation of the interior DoFs when a unit displacement is imposed on the boundary DoF b .

Fixed interface modes are expressed as:

$$\Phi = \begin{bmatrix} \mathbf{0}_b \\ \Phi_i \end{bmatrix}. \tag{18}$$

Generally, a truncated set of m fixed interface modes, with $m \ll i$, is used to approximate the dynamics in a limited frequency band.

Therefore,

$$\mathbf{u}^{(r)} \simeq \begin{bmatrix} \mathbf{I}_b & \mathbf{0}_b \\ \Phi_{c,i} & \Phi_i \end{bmatrix} \begin{Bmatrix} \mathbf{u}_b \\ \mathbf{q}_m \end{Bmatrix} = \Gamma^{(r)} \mathbf{q}^{(r)} \tag{19}$$

where $\Gamma^{(r)}$ indicates the transformation matrix associated with the reduced set of generalized coordinates $\mathbf{q}^{(r)}$.

By using the coordinate transformation of Equation (19) in Equation (1) and premultiplying by $\Gamma^{(r)T}$, one obtains:

$$\hat{\mathbf{M}}^{(r)} \ddot{\mathbf{q}}(t)^{(r)} + \hat{\mathbf{C}}^{(r)} \dot{\mathbf{q}}(t)^{(r)} + \hat{\mathbf{K}}^{(r)} \mathbf{q}(t)^{(r)} = \hat{\mathbf{f}}(t)^{(r)} + \hat{\mathbf{g}}(t)^{(r)} \tag{20}$$

which approximates the equation of motion in the reduced set of generalized coordinates $\mathbf{q}^{(r)}$.

2.5. Transmissibility Ratios of Multi-DoF Systems

In the dynamic analysis of a complex system, it is often important to evaluate whether the vibrations transmitted from one point to another are attenuated or amplified. To that purpose, transmissibility, defined as the dimensionless ratio between forces or displacements at different DoFs, gives useful insights into the dynamic behavior of a system. For a multi-DoF system, its definition is not trivial. Here the displacement transmissibility ratio is used:

$$T_{ij}^d = \frac{x_i}{x_j}; \tag{21}$$

in particular, the transmissibility from a given wheel (axle) j to an internal DoF i of the system is evaluated considering the displacement only on that wheel (DoF j), with the other wheels being grounded and no other force acting on the internal DoFs.

3. Results

The models described in the previous section are here used to analyze the dynamics of the agricultural machine. The objective is to highlight the effects of different sources on the level of vibration of the operator’s seat. Moreover, to highlight the effects of operating machines on the vibrations transmitted to the operator’s seat, different configurations of the agricultural machine are considered. Finally, the effects of dampers at different

locations are investigated to evaluate their effectiveness in the reduction of vibration levels of the operator's seat. To obtain reliable results, it is desirable that the numerical model of the tractor be representative of a real-life tractor, especially when significant simplifications are introduced into the models of other component subsystems. In the following, the experimental response of the real tractor is compared to the response of the lumped parameter model.

3.1. Setup of the Tractor Model

A medium-size tractor, employed in the experimental tests in [41], was modeled by the lumped parameter model shown in Figure 1c. The parameters of the lumped model were initially estimated and subsequently adjusted to provide dynamic behavior close to that of the tractor. The final parameter values of the tractor model are listed and described in Table 1.

Table 1. Numerical parameters of the 6-DoFs tractor.

Description	Symbol	Value	Unit
Mass of the seat with operator	m_1	150	kg
Mass of the tractor frame	m_2	3350	kg
Mass of the front axle	m_3	700	kg
Mass of the cabin	m_4	350	kg
Moment of inertia of the tractor frame	I_2	3959	kg m ²
Moment of inertia of the cabin	I_4	77	kg m ²
Stiffness of the seat suspension	k_1	9.8	kN/m
Stiffness of the rear wheels	k_2	712.5	kN/m
Stiffness of the front axle suspension	k_3	85.6	kN/m
Stiffness of the front wheels	k_4	546.2	kN/m
Stiffness of the front cabin suspension	k_5	145.8	kN/m
Stiffness of the rear cabin suspension	k_6	54.3	kN/m
Damping coefficient of the seat suspension	c_1	6065	N s/m
Damping coefficient of the rear wheels	c_2	28.5	N s/m
Damping coefficient of the front axle suspension	c_3	12.4	N s/m
Damping coefficient of the front wheels	c_4	21.8	N s/m
Damping coefficient of the front cabin suspension	c_5	5703	N s/m
Damping coefficient of the rear cabin suspension	c_6	3880	N s/m
Stiffness proportional viscous damping coefficient	β	5×10^{-5}	s ⁻¹
Horizontal position of the seat suspension with respect to the CoG of the tractor frame	l_1	-0.50	m
Horizontal position of the rear axle with respect to the CoG of the tractor frame	l_2	-0.83	m
Horizontal position of the front axle with respect to the CoG of the tractor frame	l_3	1.54	m
Horizontal position of the CoG of the cabin with respect to the CoG of the tractor frame	l_4	-0.07	m
Horizontal position of the front cabin suspension with respect to the CoG of the tractor frame	l_5	-0.31	m
Horizontal position of the rear cabin suspension with respect to the CoG of the tractor frame	l_6	-0.83	m
Radius of the rear wheels	R_r	0.83	m
Radius of the front wheels	R_f	0.62	m

By forcing with a given soil excitation, the experimental response of the medium-sized tractor and the response of the numerical model are compared. Numerical and experimental vertical accelerations of the operator's seat are compared in terms of their power spectral density (PSD). Figure 5 shows the results obtained by the authors using the excitation provided by the rougher track defined in the standard [42] when the forward speed of the tractor was $v = 6.3$ km/h.

The results show that the measured and simulated responses are similar and that the numerical model of the tractor can be considered representative of the actual machine for the purpose of this paper.

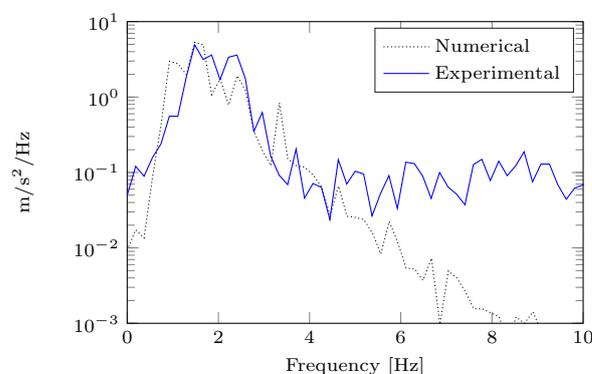


Figure 5. Power spectral density of the numerical and experimental acceleration at the operator's seat of the tractor with an operative speed $v = 6.3$ km/h on the rougher track.

In the following, models of simpler tractors are also considered: the 3-DoFs tractor in Figure 1a and the 4-DoFs tractor in Figure 1b. The 4-DoFs model was obtained by assuming that the cabin was rigidly connected to the tractor frame; instead, the 3-DoFs model was obtained by assuming that both the cabin and the front axle were rigidly connected to the tractor frame.

3.2. Effects of Different Vibration Sources on the Operator's Seat

This subsection aims to identify the vibration sources that most influence the vibration exposure of the operator.

For this reason, the analyses were carried out in the frequency band 0.2–20 Hz, which is relevant for vibrations transmitted to the whole body, as shown by the weighting curves provided by [10].

The main sources of vibrations to be considered are:

- Tractor wheel–soil contact;
- The motion of internal tractor components;
- Wheel–soil contact of the tool;
- The motion of internal operating machine components;
- Tool–soil (or crop) interaction.

To evaluate the effects of these sources on the operator, the following quantities are used:

- The displacement transmissibility ratio (see Equation (21)), concerning the vibrations generated by the soil irregularities and transmitted by the wheels;
- The frequency response function (acceleration/force), concerning the forces acting on the agricultural machine (motion of internal components or tool–soil interaction).

Once the most critical sources and vibration paths are identified, it is possible to outline the most effective actions to mitigate the effects on the operator.

3.2.1. Vibrations Generated by Tractor Wheel–Soil Contact

The irregularity of the soil and of the tires generates a vertical motion of the axle and consequently vibrates the operator's seat, the level of which depends on the inertial and elastic properties of the tractor including the presence of suspension systems.

Figure 6 shows the transmissibility from the rear and front wheels of the tractor to the operator's seat. Results refer to tractors with seat suspension only (3-DoFs tractor in Figure 1a); suspension systems for the seat and front axle (4-DoFs tractor in Figure 1b); and

suspension system for the seat, axle, and cabin (6-DoFs tractor in Figure 1c) introduced in Section 2.2.1.

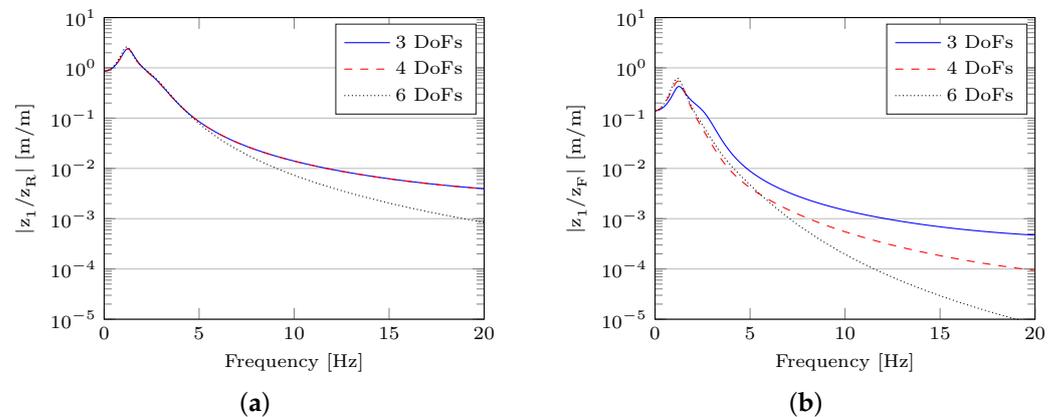


Figure 6. Transmissibility between the tractor wheels and the operator’s seat for different tractors: (a) rear wheel z_1/z_R ; (b) front wheel z_1/z_F .

The transmissibility z_1/z_R in Figure 6a shows that the vibrations due to the rear wheel are amplified at the operator’s seat at frequencies below 2 Hz, and they are attenuated in the remaining range of frequencies. The transmissibility z_1/z_F shows, instead, that the vibrations due to the front wheel are attenuated in the whole range of frequencies.

Figure 6b shows the influence of the front axle suspension on the transmissibility between the front wheels and the seat. Note that the suspension on the front axle does not influence the transmissibility z_1/z_R between the rear wheels and the seat, since it does not affect this vibration transmission path. In the cases under examination, the suspension of the cabin does not affect the transmissibility below 6 Hz, whereas it is effective for higher frequencies. It is in fact generally used to improve the acoustic comfort of the operator at frequencies above 20 Hz. In general, suspension systems provide good isolation at higher frequencies, but they amplify vibrations at low frequencies. The effect of the axle suspension is clearly shown in the transmissibility z_1/z_F .

Figure 7 shows the transmissibility ratios—with or without the damper c_3 —of the front wheels z_F to the vertical displacement of the seat z_1 , the vertical displacement of the frame z_2 , and the rotation of the frame θ_2 , respectively. Note that all the other parameters were set to the values in Table 1.

Figure 7a highlights that the damping of the front axle suspension c_3 significantly reduces the vibrations transmitted to the operator’s seat (z_1/z_F). Figure 7b,c shows that the reduction of the vibrations transmitted to the operator is due to the attenuation of the vibrations transmitted to the frame.

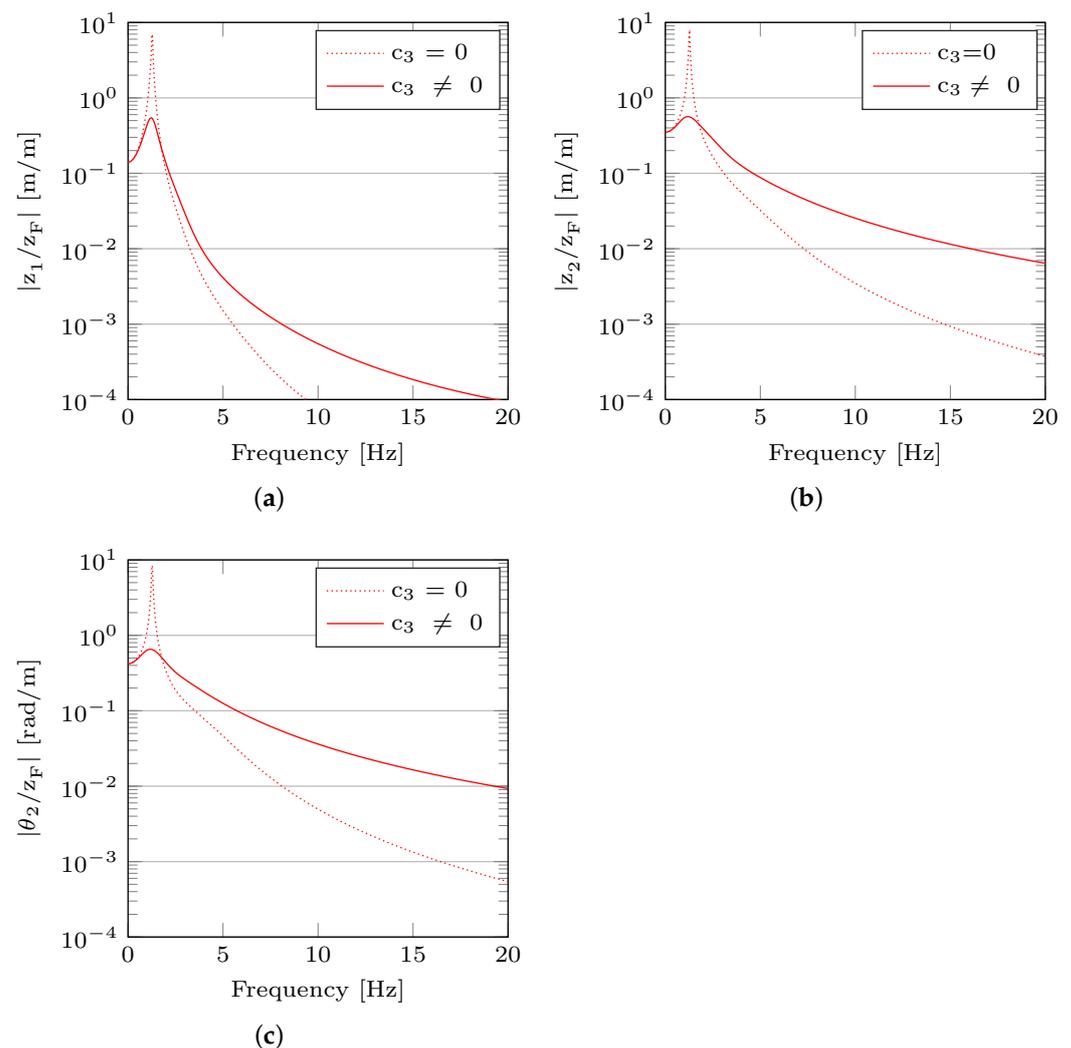


Figure 7. Transmissibility between the vertical displacement of the tractor front wheels and: (a) the vertical displacement of the operator's seat z_1/z_F ; (b) the vertical displacement of the tractor frame z_2/z_F ; (c) the rotation of the tractor frame θ_2/z_F .

3.2.2. Vibrations Generated by Moving Parts on the Tractor

As shown in Section 1, the engine and the drive-train represent sources of high-frequency vibrations. Although they are generally outside of the frequency band of interest for the whole body vibrations, they can be sources of direct noise. In the case of tractors, they can produce vibrations of the cabin panels, transforming the passenger compartment into a soundbox. The introduction of the cabin suspension, although mainly designed to attenuate high-frequency vibrations that can generate noise, has a beneficial effect even in the frequency band of interest for the whole-body vibrations.

Figure 8 shows the inertance, with and without cabin suspension dampers (c_5 and c_6), of the operator's seat z_1 (Figure 8a) and of the CoG of cabin z_4 (Figure 8b) for an excitation on the CoG of the tractor frame z_2 . Note that forces generated by tractor parts can be turned into forces and moments acting on the CoG of the tractor frame.

It can be noted that the suspension dampers of the cabin attenuate the peaks of the frequency response even within the band 0–20 Hz of interest for the estimation of the operator's exposure to vibrations.

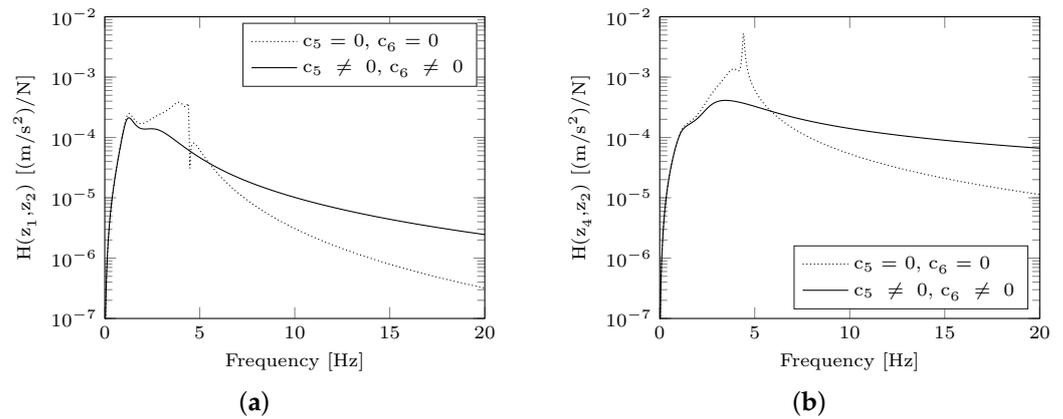


Figure 8. Inertance of a 6-DoFs tractor for a force acting along the vertical direction on the CoG of the tractor frame z_2 . (a) The response measured on the operator’s seat z_1 ; (b) the response measured on the CoG of the cabin z_4 .

3.2.3. Vibrations Generated by Operating Machine Wheel–Soil Contact

The interaction of the wheels of the operating machine with the soil produces vertical motion in the tires as well. The vibrations can be transmitted to the operator’s seat through the connections between the operating machine and the tractor.

The operating machines concerned with this vibration source are semi-mounted and trailed machines. Figure 9 shows the transmissibility between the wheels of front or rear semi-mounted operating machines z_s and the driver’s seat of the tractor z_1 . The results show that the same operating machine coupled with different tractors transmits different vibrations to the operator. In particular, the considered front semi-mounted machine is less burdensome than the rear one. Results in Figure 9a highlight the effectiveness of the cabin suspension in the reduction of the vibrations transmitted from the wheel of the rear semi-mounted machine to the operator at higher frequencies. On the other hand, results in Figure 9b show a detrimental effect of the front axle suspension for the vibrations below 5 Hz generated by the wheels of front operating machines, although this variation concerns low levels of the transmissibility ratio. Different results were obtained for the transmissibility between the tractor wheels and the operator’s seat (see Figure 6). This suggests that a tractor designed to minimize the transmission of vibrations due to the interaction of the tractor wheels with the soil may not be equally effective in attenuating the vibrations due to the interaction of the wheels of the operating machine with the ground.

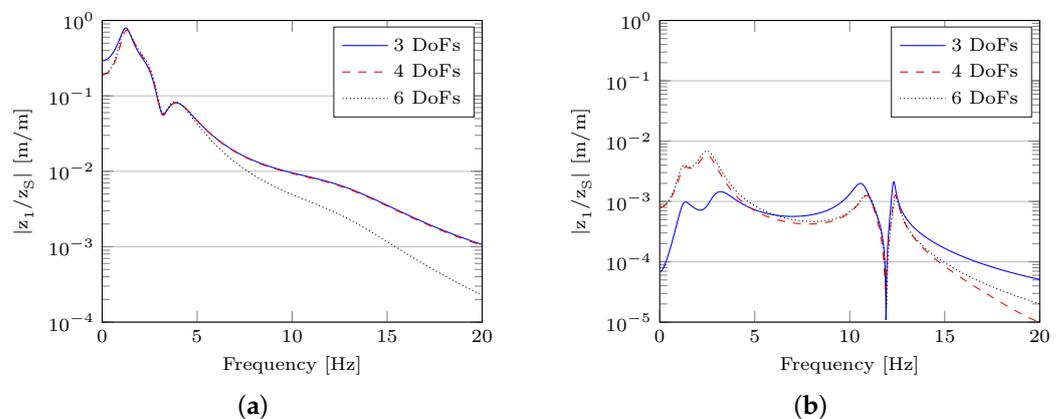


Figure 9. Transmissibility between the operating machine wheel z_s and the operator’s seat on the tractor z_1 : (a) for a rear semi-mounted machine; (b) for a front semi-mounted machine.

3.2.4. Vibrations Generated by the Operating Machine

The operating machine connected to the tractor can represent a source of vibrations that are transmitted through the linkage system to the agricultural tractor and therefore to the operator. These vibrations can be generated by the moving parts of the machine and by the interactions of the tools with the ground or crops. Furthermore, the presence of the operating machine itself can significantly alter the dynamic behavior of the whole agricultural machine. Figure 10 shows the inertance between the center of gravity of the operating machine and the operator's seat. In each plot, the same operating machine (rear-mounted in Figure 10a or front-mounted in Figure 10b) is coupled with the three different tractors shown in Figure 1. Even in this case, a burdensome effect of the rear-mounted machine can be observed. Moreover, the results in Figure 10 lead to the same conclusion made in Section 3.2.3 about the effects of the cabin suspension and the front axle suspension on the vibrations transmitted from the operating machines to the operator's seat.

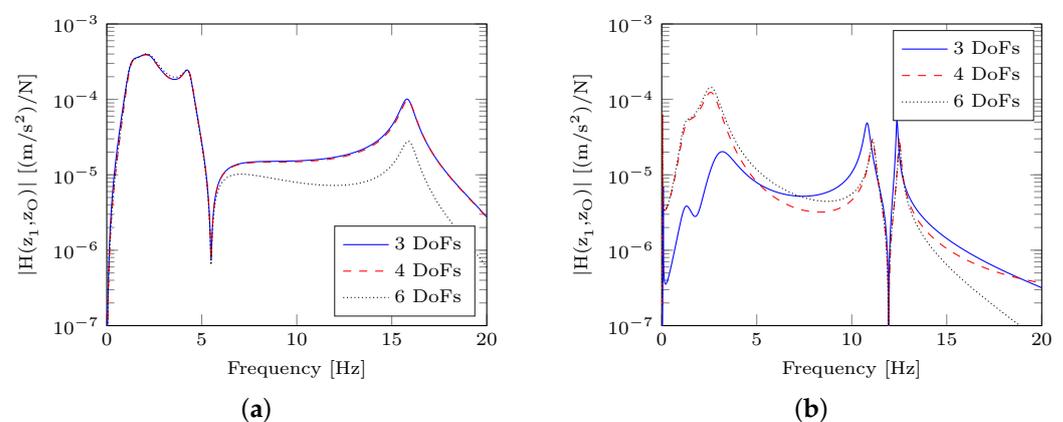


Figure 10. Inertance of different tractors for a force acting on the CoG of the operating machine z_O and the response measured on the operating seat z_1 : (a) for a rear-mounted machine; (b) for a front-mounted machine.

3.3. Influences of Operating Machines on the Global Dynamics

In this subsection, different configurations of the agricultural machine are considered in order to highlight the influence of the operating machine on the global dynamics of the agricultural machine. In fact, the dynamic behavior of the agricultural tractor, and therefore the level of vibrations transmitted to the seat, can change when an operating machine is connected to the tractor. For example, by connecting an operating machine, the mass distribution of the entire agricultural machine varies considerably.

3.3.1. The Effects of the Operating Machine on the Transmissibility between the Tractor Wheels and the Operator's Seat

Here, the influences of different operating machines on the transmissibility between the wheels and the operator's seat of the same tractor are analyzed. Figure 11 shows that the presence of a rear-mounted or semi-mounted operating machine on the 6-DoFs tractor modifies the transmissibility of the tractor. In particular, the mounted machines introduce a more relevant modification on the transmissibility than the semi-mounted ones. In fact, the attachment of an operating machine introduces additional vibration modes to the complete system due to the compliance of the linkage system. Since the mounted machine is a mass suspended on the linkage system, its effect is more relevant than that of a semi-mounted machine whose mass is partially supported by a dedicated suspension.

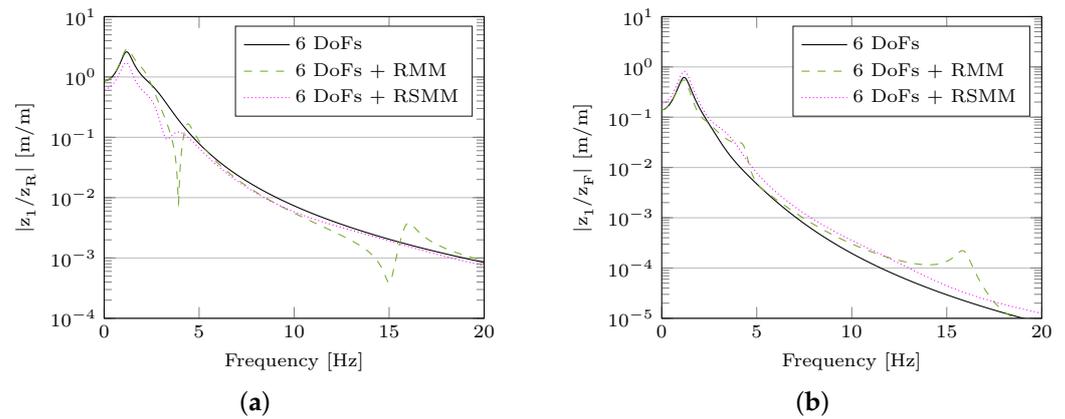


Figure 11. Transmissibility between the wheels of the 6-DoFs tractor and the operator’s seat z_1 with a rear-mounted (RMM) or a rear semi-mounted (RSMM) operating machine: (a) rear wheels z_1/z_R ; (b) front wheels z_1/z_F .

Figure 12 shows that the presence of a selected front-mounted or a semi-mounted operating machine on the same 6-DoFs tractor slightly modifies the transmissibility of rear wheels of the tractor, but a more relevant variation can be noticed for the front wheel transmissibility. The general observation made for the rear operating machine about the modification of the system dynamics can be extended to this case too. However, the effects of the front semi-mounted machine suspension are less relevant than the rear semi-mounted machine suspension: one of the reasons could be the proximity of the front axle suspension.

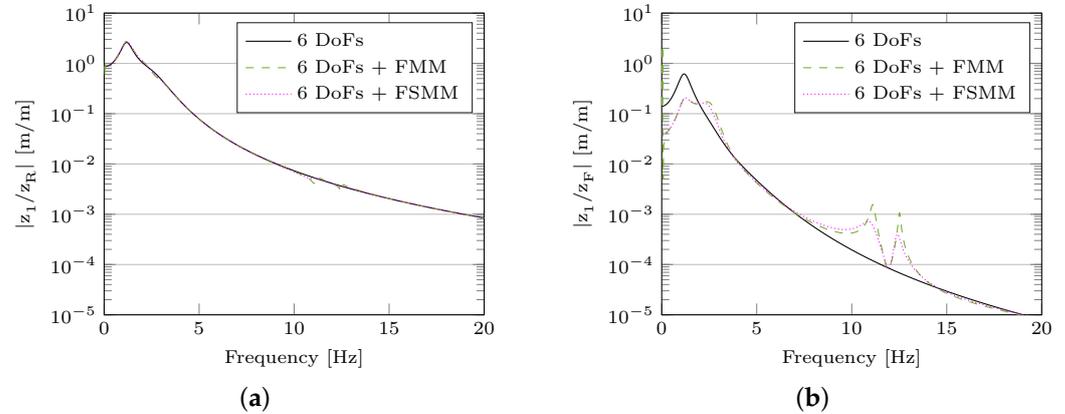


Figure 12. Transmissibility between the wheels of the 6-DoFs tractor and the operator’s seat z_1 with a front-mounted (FMM) or a front semi-mounted (FSMM) operating machine: (a) rear wheels z_1/z_R ; (b) front wheels z_1/z_F .

3.3.2. The Combined Effect of Rear and Front-Mounted Machines

Front and rear-mounted machines can be connected at the same time to the same tractor; the combined effect on the transmissibility between the wheels of the tractor and the operator’s seat should be verified. Even the presence of a ballast applied to the front three-point linkage can be represented with this approach. Figure 13 shows the transmissibility between the tractor wheels and the operator’s seat. The simultaneous presence of rear and front-mounted machines produces an effect that can be traced back to a combination of the effects already shown separately in Figures 11 and 12.

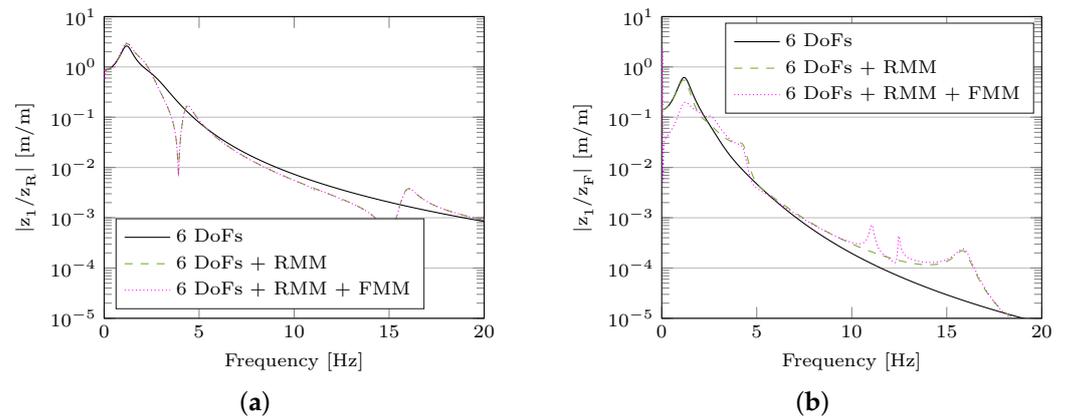


Figure 13. Transmissibility between the wheels of the 6-DoFs tractor and the operator’s seat z_1 with rear-mounted (RMM) operating machine and front-mounted (FMM) operating machine: (a) rear wheels z_1/z_R ; (b) front wheels z_1/z_F .

3.4. Estimation of the Vibration Exposure Level in the Operator’s Seat

In this section, the vibration exposure of the operator is evaluated using the weighted r.m.s. acceleration in the vertical direction (a_{wz}), computed using the definitions and the weighting curves provided in [10]. Several transient simulations have been performed considering the 6 DoFs tractor, with and without a rear-mounted machine (RMM), wherein the tractor was driven over the rougher and the smoother track proposed in [42] with different velocities. The operating machine considered in the simulations was characterized by a mass m_O of 700 kg and a moment of inertia I_O of 235 kg m².

Results in Figure 14 show that, as expected, the a_{wz} was higher for the rougher track than for the smoother track. Moreover, the presence of the rear-mounted machine had a more relevant effect on the rougher track than on the smoother track. In particular, the selected rear-mounted machine worsened the weighted r.m.s. acceleration when the operating speed was slower than 10 km/h, and gave rise to lower values of the a_{wz} for faster operating speed. Note that the standard [42] requires determining the weighted vibrations along the three axes, for operating speeds equal to 4, 5, and 7 km/h. Therefore, a higher velocity on the rougher track was considered for illustrative purposes only.

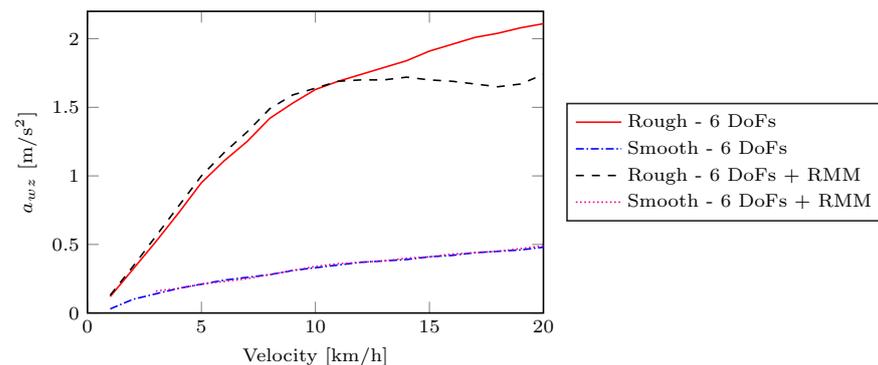


Figure 14. The effect of the forward speed on the root mean square value of the vertical acceleration a_{wz} of the seat.

3.5. The Effect of Tractor Dampers on the Vibrations Transmitted to the Operator’s Seat

In this section, the results of several analyses are compared in order to highlight the effect of each tractor damper on the vibrations that are transmitted to the operator’s seat. In the 6 DoFs tractor model, suspension dampers were those indicated in Figure 1c as c_1 , c_3 , c_5 , and c_6 , along with c_2 and c_4 , which represent the tire damping. Figure 15a,b shows the effect of each damping value listed in Table 1 on the transmissibility between the wheels

and the operator’s seat. Note that the undamped system gives rise to very high levels of transmissibility (dotted black line); therefore, the presence of the dampers is essential to preserve the health of the operator. A single damper affects each modal damping in a different way and thus the transmissibility in relation to the relative mode frequency. For example, in Figure 15b the effect of the damper of the seat suspension (c_1) is clearly visible: it reduces the transmissibility of the undamped system at the resonance frequencies of modes one (at 1.19 Hz), three (at 2.77 Hz), and four (at 4.09 Hz); but the transmissibility remains the same of the undamped system at the resonance frequencies of modes two (at 1.28 Hz), five (at 4.42 Hz), and six (at 4.82 Hz). Moreover, the transmissibility plots show that the combined effects of the six dampers (red line) attenuate the vibrations of the operator’s seat due to the vertical displacement of the front wheels (red line in Figure 15b). The vertical displacement of rear wheels is amplified at the operator’s seat for frequencies below 2 Hz and is attenuated for higher frequencies (red line in Figure 15a).

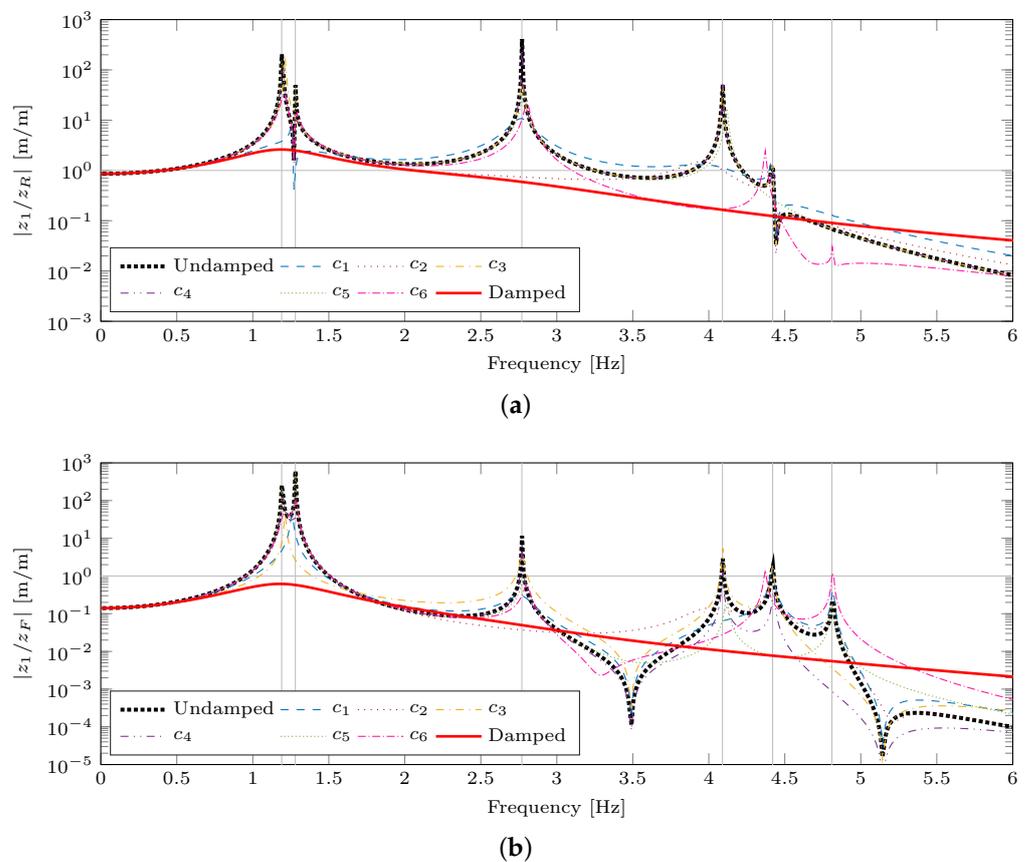


Figure 15. The effects of tractor dampers on the transmissibility between the tractor wheels and the operator’s seat z_1 : (a) rear wheels z_1/z_R ; (b) front wheels z_1/z_F .

To better understand the effect of a given damper on the vibration isolation of the operator’s seat, Table 2 compares the modal damping ratios ζ_i of the 6-DoFs tractor modes obtained accounting for the effects of each damper c_i . For each column, the values in bold indicate the modes with the most relevant variations of the modal damping ratio ζ_i compared to ζ_0 , accounting only for the stiffness proportional damping coefficient β . The modal damping ratio ζ accounts for the combined effect of the six dampers of the 6 DoFs tractor. The values in Table 2 are in agreement with the effects of each damper c_i on the transmissibility in Figure 15.

Table 2. The effect of suspension damping on the modal damping ratio of the tractor. Highlighted in bold the modal damping ratios of modes that are more affected by each damper.

Mode	f [Hz]	ζ_0 [%]	ζ_1 [%]	ζ_2 [%]	ζ_3 [%]	ζ_4 [%]	ζ_5 [%]	ζ_6 [%]	ζ [%]
1	1.19	0.02	20.51	0.25	0.20	0.47	0.16	1.53	23.98
2	1.28	0.02	0.52	0.03	52.46	1.86	0.02	0.67	50.82
3	2.77	0.04	2.59	32.89	0.31	0.06	0.50	0.63	36.36
4	4.09	0.06	6.06	3.32	0.07	0.07	0.15	82.63	89.78
5	4.42	0.07	0.17	0.22	36.26	0.10	52.41	0.16	54.08
6	4.82	0.08	0.08	0.08	0.07	51.43	0.10	0.08	91.26
a_{wz}	[m/s ²]	10.87	5.89	6.30	8.10	9.41	8.62	6.42	1.33

The weighted r.m.s. value a_{wz} of the vertical acceleration of the operator's seat is shown in the last row of Table 2. It was evaluated for the 6-DoFs tractor with an operative speed of $v = 7.0$ km/h on the rougher track defined in [42].

The r.m.s results highlight that dampers c_1 , c_2 , and c_6 are the most effective on the attenuation of the vibration transmitted to the operator. In fact, the damping ratios ζ_i of modes one, three, and four, are significantly affected by the presence of dampers c_1 , c_2 , and c_6 , respectively.

4. Conclusions

In this paper, substructuring techniques were employed to investigate the vibrating of the operator's seat of an agricultural machine. The proposed approach allows getting the dynamic model of the whole agricultural machine by coupling together models (even of different types) of its components: the tractor, the three-point linkage, and the machine. Several models of tractors were considered and coupled, using the substructuring technique, with front or rear, mounted or semi-mounted devices. The results highlighted the effectiveness of the proposed approach when investigating a large number of different configurations and a wide range of operating conditions, given the reduced modeling burden. In fact, mathematical operations needed for the computation of frequency response functions and of transmissibilities have a complexity order of $O(n^3)$, where n is the number of degrees of freedom. Therefore, by splitting the complete system into m smaller subsystems, the computational burden is reduced approximately by m^2 . The subsequent step of analysis requires the coupling of the subsystem models and it can be performed using reduced-order models, including only coupling and boundary DoFs, instead of full models, thereby limiting the computational cost. Furthermore, some subsystems can be characterized experimentally when the numerical model is not very accurate, and modifications introduced to a single subsystem do not require reanalysis of the remaining subsystems. Finally, the use of reduced-order models that are usually expressed in the modal or in the frequency domain allows one to carry out the dynamic analysis of a complex agricultural machine without sharing sensitive design details of component subsystems.

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