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PHD THESIS

Consistency and Naturalness Beyond the Standard Model

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Abstract

The fundamental features of the Standard Model of Particle Physics are revised. Despite a great success through various precision tests, it does not have all the answers. A lot of unsatisfactory and unexplained stuff ask for physics Beyond the Standard Model (BSM). I will address some of these fundamental problems, such as the mass hierarchy problem, the weak mixing pattern and the strong CP problem in a way I define *natural*. Related to this concept, I will discuss an important (and underrated) class of symmetries, called *emergent symmetries*, where we will see supersymmetry is one of the major examples in a lot of context.

Introduction

The Standard Model of Particle Physics represents our best knowledge about the strong and electroweak interactions. Although the various precision tests successfully agree with its predictions, it is known as well that it cannot be the final theory; indeed, a lot of mysteries, puzzles and unsatisfactory features are present therein. As canonical examples we could mention gravity, which is not included in the SM context, but also Dark Matter and Dark Energy. Outside from the astrophysical context we can mention the fermion mass hierarchy problem, the weak mixing angles pattern, the strong CP problem and the puzzle of replication of families; all these concepts will be developed in this work. In the literature these issues are usually addressed by introducing a lot of stuff like anomalous global symmetries or some heavy spectator fields; the aim of part of this work is to address these problems by relaxing almost all of these *ad hoc* assumptions, by relying on the more fundamental gauge structure of the theory.

Stability of Higgs mass against radiative corrections lead to the idea of Supersymmetry, which in its minimal realization introduces a fermionic (bosonic) partner for each known boson (fermion); among its implications, we cite the possibility of Grand Unification, that is, the idea that at some very high energy scale the three gauge couplings converge to the same value when we consider their running under the Renormalization Group Equations. We will stress that Supersymmetry and Grand Unification are deeply connected.

This work is organized as follows. After a brief review of the ideas behind and beyond the Standard Model in Chapter 1, I will discuss some interesting results about the coherence of the Grand Unification concept in Chapter 2, where also a detailed analysis of the thresholds effects is included, as well as an application to the fermion masses. Chapter 3 is dedicated to the idea of *emergent symmetry* arising from a Renormalization Group evolution; this concept could be used to have an alternative point of view about global internal symmetries (which are not really fundamental), but also Supersymmetry and gauge symmetries. In the last Chapter I address the so called *family problem*, as well as the Strong CP problem by the introduction of an extra abelian local symmetry, whose spontaneous breaking pattern at some high energy scale will provide generation dependent Yukawa couplings which could explain the observed fermion mass hierarchy and weak mixing angles.

Chapter 1

Behind and Beyond the Standard Model

In this chapter we briefly recall the basic ingredients needed to the construction of the Standard Model (SM) of particle physics [1, 2, 3], as well as its successes and its unsatisfactory features which demand for new physics.

1.1 Standard Model review

The role of symmetries in particle physics is crucial: they provide the classification of particles, dictate conservation laws in their interactions and are instrumental in solving dynamical problems. As far as we know, the only really fundamental symmetries are gauge symmetries, that are just a manifestation of a field definition redundancy in the theory; they must be anomaly free, and cannot be explicitly broken. On the contrary, global symmetries are not protected by any fundamental principle, so their origin is a mystery; indeed, they could arise as accidental symmetries (like lepton and baryon number or isospin symmetry in the Standard Model), but at some point they will be broken by e.g. non perturbative gravitational effects [4, 5, 6, 7, 8], due to the presence of Planck suppressed higher order operators which only respect gauge symmetries.
So, the basic principle which guides the construction of the all interactions (including gravity) is the local gauge invariance. The Standard Model, which is the theory of strong and electroweak interactions, is based on the local gauge group $SU(3) \times SU(2) \times U(1)$, where $SU(3)$ is the colour symmetry group acting on quarks and gluons, $SU(2)$ is the chiral gauge group of the weak interaction, while $U(1)$ is the so called hypercharge group.

All the fermion fields are Weyl spinors which fit in the following representations of the Standard Model group[1]

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim \left(3, 2, \frac{1}{6}\right) \qquad \ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \sim \left(1, 2, -\frac{1}{2}\right) \tag{1.1}$$

$$u_R \sim \left(3, 1, \frac{2}{3}\right) \qquad d_R \sim \left(3, 1, -\frac{1}{3}\right) \qquad e_R \sim (1, 1, -1) \tag{1.2}$$

In principle, together with these we can include also the right handed neutrino $\nu_R \sim (1, 1, 0)$, which is a gauge singlet; its relevance will be discussed later on. The above quantum numbers are the same for all the 3 known fermion families.

The Standard Model Lagrangian can be decomposed in the following sectors:

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yuk} \tag{1.3}$$

Here

$$\mathcal{L}_{gauge} = -\frac{1}{2}\,\mathrm{tr}\, G_{\mu\nu}\, G^{\mu\nu} - \frac{1}{2}\,\mathrm{tr}\, W_{\mu\nu}\, W^{\mu\nu} - \frac{1}{4}\, B_{\mu\nu}\, B^{\mu\nu} \tag{1.4}$$

is the kinetic term for the gauge bosons. $G_{\mu\nu} = G^a_{\mu\nu}\, T^a$ is the field strength for $SU(3)$, with generators $T^a = \lambda^a/2$, $a = 1, \ldots, 8$; $W_{\mu\nu} = W^a_{\mu\nu}\, T^a$ is the field strength for $SU(2)$, with generators $T^a = \sigma^a/2$, $a = 1, \ldots, 3$, while $B_{\mu\nu}$ is the abelian field strength. In general, if we call $A_\mu = A^a_\mu\, T^a$ the Lie-valued gauge field, the corresponding field strength is defined as

$$F_{\mu\nu} = \partial_\mu\, A_\nu - \partial_\nu\, A_\mu + i\, g\, [A_\mu, A_\nu] \tag{1.5}$$

where g is the gauge coupling constant. In this discussion we take the group generators in fundamental representation canonically normalized as

$$\mathrm{tr}\,(T^a\, T^b) = \frac{1}{2}\,\delta^{ab} \tag{1.6}$$

Next, we have the matter sector, including the kinetic term of the fermions, as well as their interactions with the gauge bosons

$$\mathcal{L}_{matter} = i\,\bar{q}^i_L\, \not{D}\, q^i_L + i\,\bar{u}^i_R\, \not{D}\, u^i_R + i\,\bar{d}^i_R\, \not{D}\, d^i_R + i\,\bar{\ell}^i_L\, \not{D}\, \ell^i_L + i\,\bar{e}^i_R\, \not{D}\, e^i_R \tag{1.7}$$

Here, the indices $i = 1, 2, 3$ are family indices, and D denotes the covariant derivative

$$D_\mu = \partial_\mu + i\, g_3\, G_\mu + i\, g_2\, W_\mu + i\, g'\, \frac{Y}{2}\, B_\mu \tag{1.8}$$

[1]The $U(1)$ quantum number is defined by the relation $Q = I_3 + Y/2$, where Q is the electric charge and I_3 is the third component of the weak isospin.

where g_3, G_μ and g_2, W_μ are the gauge coupling constants and gauge fields of $SU(3)$ and $SU(2)$, respectively, while Y is the eigenvalue of the hypercharge. Of course, this operator has to be understood in the corresponding representation when acting on the fields; e.g. $D_\mu \ell_L$ will have no contribution from G_μ, being a colour singlet, and so on.
The Higgs sector

$$\mathcal{L}_{Higgs} = |D_\mu H|^2 - V(H) \tag{1.9}$$

is responsible for the electroweak symmetry breaking; indeed, once the Higgs field $H \sim \left(1, 2, \frac{1}{2}\right)$ acquires a vacuum expectation value (VEV) $\langle H \rangle = v/\sqrt{2} = 174$ GeV, we get the symmetry breaking pattern

$$SU(2)_L \times U(1)_Y \to U(1)_{EM} \tag{1.10}$$

leaving as the only unbroken generator the one of electromagnetic interaction; as a result, the gauge boson of $U(1)_{EM}$ remains massless; we identify it as the photon, which will be a linear combination of the hypercharge gauge boson and the neutral one of $SU(2)$:

$$A_\mu = \cos\theta_W\, B_\mu + \sin\theta_W\, W^3_\mu \tag{1.11}$$

while the orthogonal linear combination

$$Z_\mu = -\sin\theta_W\, B_\mu + \cos\theta_W\, W^3_\mu \tag{1.12}$$

together with the other 2 charged $SU(2)$ gauge bosons $W^\pm$ acquires a mass due to the Higgs mechanism

$$M_W^2 = g_2^2\, v^2 \qquad\qquad M_Z^2 = \frac{g_2^2\, v^2}{\cos^2\theta_W} \tag{1.13}$$

where the rotation angle θ_W is known as the Weinberg angle, defined through the ratio of the coupling constants of the electroweak group as

$$\tan\theta_W = \frac{g'}{g_2} \qquad\qquad (\text{exp. } \sin^2\theta_W \simeq 0.23117 \text{ at EW scale}) \tag{1.14}$$

The Higgs potential can be written as

$$V(H) = -\frac{\mu^2}{2}\, H^\dagger H + \frac{\lambda}{4}\,(H^\dagger H)^2 \tag{1.15}$$

by minimizing this potential we get an expression for the Higgs vev

$$\langle H \rangle = \sqrt{\frac{2\,\mu^2}{\lambda}} \tag{1.16}$$

If one parametrizes the Higgs doublet as

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{1.17}$$

we get a physical Higgs mass $m_h = \mu = \sqrt{\lambda/2}\, v$.

Finally, the Yukawa sector, which provides the fermion mass terms after the EW symmetry breaking, is given by

$$\mathcal{L}_{Yuk} = Y_u^{ij}\, \bar{u}_R^i\, q_L^j\, H + Y_d^{ij}\, \bar{d}_R^i\, q_L^j\, \tilde{H} + Y_e^{ij}\, \bar{e}_R^i\, \ell_L^j\, \tilde{H} + \text{h.c.} \tag{1.18}$$

Here $\tilde{H}$ represents the charge conjugated Higgs field, defined by $\tilde{H} = i\, \sigma_2\, H^*$; a remarkable fact in the Standard Model is indeed that we only need one Higgs field in order to write mass terms for both up-type and down-type fermions. We will have not anymore this freedom when we construct a supersymmetric model, where the charge conjugated field $\tilde{H}$ is forbidden since the superpotential has to be holomorphic.

So far, we used both left handed and right handed fields notation; in the following, instead of writing right handed fermion fields, we will use their left handed complex conjugates antifields, e.g. $u_L^c = C\, \overline{u}_R^T$, so that we will have only left handed Weyl spinors, see Appendix A for details. This chiral notation is extremely convenient in order to extend the discussion to a supersymmetric or a GUT context.

In this sense, the Yukawa sector can be written in a more simple way as

$$\mathcal{L}_{Yuk} = Y_u^{ij}\, u_i^c\, q_j\, H + Y_d^{ij}\, d_i^c\, q_j\, \tilde{H} + Y_e^{ij}\, e_i^c\, \ell_j\, \tilde{H} + \text{h.c.} \tag{1.19}$$

After the EW symmetry breaking, these Yukawa couplings originate the fermion mass matrices $M_f = Y_f\, v/\sqrt{2}$, which can be diagonalized by a bi-unitary transformation

$$M_f^{\text{diag}} = V_f^c\, M_f\, V_f \tag{1.20}$$

The V_f^c matrices rotating the RH states are not physically relevant in the SM, while the left handed rotations $V_{u,d}$ give rise to the mixing in the quark (and lepton) charged currents coupled to the $W^\pm$ bosons; in the case of quarks, this is determined by the Cabibbo-Kobayashi-Maskawa (CKM) unitary matrix

$$V_{\text{CKM}} = V_u^\dagger\, V_d = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1.21}$$

which in the standard parametrization takes the form

$$V_{\rm CKM} = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\,\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\,\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\,\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\,\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\,\delta} & c_{23}\, c_{13} \end{pmatrix} \tag{1.22}$$

with $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. Then, the mixing pattern is described by only 4 parameters, the 3 mixing angles and the weak CP phase; actually, as a measure of CP violation, it is sometimes useful to consider the Jarlskog invariant, which in the standard parametrization reads

$$J = s_{12}\, s_{23}\, s_{13}\, c_{12}\, c_{23}\, c_{13}^2\, \sin\delta \tag{1.23}$$

Finally, to be honest we should mention another operator in the SM Lagrangian, which is the so called QCD θ-term

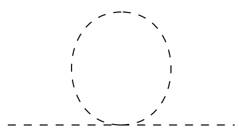
$$\mathcal{L}_\theta = \theta\, \frac{g_3^2}{32\,\pi^2}\, \varepsilon^{\alpha\beta\mu\nu}\, G^a_{\alpha\beta}\, G^a_{\mu\nu} \tag{1.24}$$

which, although it is a total derivative, it has important physical implications we will discuss in the following. One can also show that analogous topological terms for $SU(2)$ and $U(1)$ are instead completely unphysical.

1.2 Further arguments: Supersymmetry and GUT

In the modern point of view, a given theory (e.g. the Standard Model) is always the effective theory of a more complete underlying theory, which adequately describes physics at a energy scale higher than a threshold M. This threshold is physical in the sense that the complete physical spectrum includes particles with a mass of order M. For example, in the case of the seesaw mechanism for neutrino masses, there is a neutrino field of mass M. That scale acts as an UV cutoff on loop momenta.

The presence of fundamental scalar fields leads to the well known problem of quadratic divergences as soon as one introduces a finite cutoff Λ in the theory. Indeed a diagram of this type



generically gives a contribution

$$\delta m^2 = \lambda \int^{\Lambda} \frac{d^4k}{(2\,\pi)^4}\,\frac{1}{k^2} \sim \frac{\lambda}{16\,\pi^2} \int^{\Lambda} d^2k \sim \frac{\lambda\,\Lambda^2}{16\,\pi^2} \tag{1.25}$$

to the scalar mass squared (here λ denotes the dimensionless quartic self coupling constant). Namely, if we denote with m_0 the bare mass, then at one loop level we obtain the mass squared

$$m^2 = m_0^2 + \alpha\,\lambda\,\frac{\Lambda^2}{16\,\pi^2} \tag{1.26}$$

with α an order 1 constant. Plugging typical numbers, e.g. $m \sim 100$ GeV (EW scale) and $\Lambda \sim M_{pl} \sim 10^{19}$ GeV, we see that m_0^2/Λ^2 has to be adjusted to more than 30 orders of magnitude, which is a very unsatisfactory fine tuning.

This happens because the scalar masses are not protected by any symmetry; more precisely, in the case of a fermion mass loop, the correction will ever be proportional to the mass itself, so the proportionality coefficient is dimensionless and it behaves as $\log\Lambda$. This is due to the fact that in the limit $m \to 0$ the theory exhibits an enhanced symmetry, which is the chiral symmetry. In this sense, those parameters are technically natural, following the 't Hooft criterion.

The observation that, on the contrary of the fermion case, setting to zero the scalar mass does not enhance the symmetry of the theory suggests the idea of relating a scalar field to a fermion field by a new symmetry. Supersymmetry [9, 10, 11, 12, 13] is born in this sense, solving the hierarchy problem [14, 15] by connecting representations of the Poincaré group of different spin; the contributions of fermions to quadratic divergencies cancel that of bosons, see Appendix B.

From a more technical point of view, supersymmetry transformations are just translations in a generalized space, known as superspace, where we add anticommuting Grassmann variables to the standard spacetime coordinates. In this context, standard fields are promoted to superfields, which describe general supermultiplets.

The most general supersymmetric and gauge invariant action involving a chiral superfield Φ and a gauge vector superfield V can be written as

$$\mathcal{L} = \int d^4\theta\,\Phi^\dagger\,e^{2\,g\,V}\,\Phi + \int d^2\theta\;(W(\Phi) + \text{h.c.}) + \frac{1}{4}\int d^2\theta\;(\mathcal{W}^\alpha\,\mathcal{W}_\alpha + \text{h.c.}) \tag{1.27}$$

where g is the gauge coupling constant, W is the superpotential which is holomorphic in the chiral superfields and $\mathcal{W}_\alpha$ is the supersymmetric field strength that, in the most general case of a non abelian gauge theory, is given by

$$\mathcal{W}_\alpha = -\frac{1}{8\,g}\,(\overline{\mathcal{D}\mathcal{D}})\,\left(e^{-2\,g\,V}\,\mathcal{D}_\alpha\,e^{2\,g\,V}\right) \tag{1.28}$$

where $\mathcal{D}_\alpha$ is the supercovariant derivative.

It is important to mention that, in the case of an abelian gauge theory, there is an additional term in the supersymmetric Lagrangian above, which is the Fayet-Iliopoulos $D-$term

$$\mathcal{L}_{FI} = \xi \int d^4\theta\, V \tag{1.29}$$

In the case of the supersymmetric extension of the SM, so called Minimal Supersymmetric Standard Model (MSSM), we promote the fields we wrote at the beginning with the corresponding superfields containing the standard fermions as well as their superpartners:

$$Q = \begin{pmatrix} U \\ D \end{pmatrix} \sim \left(3, 2, \frac{1}{6}\right) \qquad L = \begin{pmatrix} N \\ E \end{pmatrix} \sim \left(1, 2, -\frac{1}{2}\right) \tag{1.30}$$

$$U^c \sim \left(\overline{3}, 1, -\frac{2}{3}\right) \qquad D^c \sim \left(\overline{3}, 1, \frac{1}{3}\right) \qquad E^c \sim (1, 1, 1) \tag{1.31}$$

Moreover, due to the holomorphicity of the superpotential, we need to introduce 2 independent Higgs doublets

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \sim \left(1, 2, \frac{1}{2}\right) \tag{1.32}$$

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \sim \left(1, 2, -\frac{1}{2}\right) \tag{1.33}$$

The superpotential with these notation has the simple form

$$W = Y_u^{ij}\,U_i^c\,Q_j\,H_u + Y_d^{ij}\,D_i^c\,Q_j\,H_d + Y_e^{ij}\,E_i^c\,L_j\,H_d + \mu\,H_u\,H_d + W_{\not{B}\not{L}} \tag{1.34}$$

where μ is the only dimensionful supersymmetric parameter, and $W_{\not{B}\not{L}}$ contains the following gauge invariant terms

$$W_{\not{B}\not{L}} = \lambda_1\,E^c\,L\,L + \lambda_2\,D^c\,L\,Q + \lambda_3\,U^c\,D^c\,D^c + \mu'\,L\,H_u \tag{1.35}$$

which are dangerous since each of them breaks both baryon number and lepton number; these terms are not present in the SM Lagrangian since there baryon and lepton number are accidental symmetries. This part of the superpotential can be forbidden by introducing an extra symmetry, known as $R-$symmetry or $R-$parity, defined as

$$R = (-1)^{3\,(B-L)+2\,S} = \begin{cases} +1: \text{ all observed particles} \\ -1: \text{ superpartners} \end{cases} \tag{1.36}$$

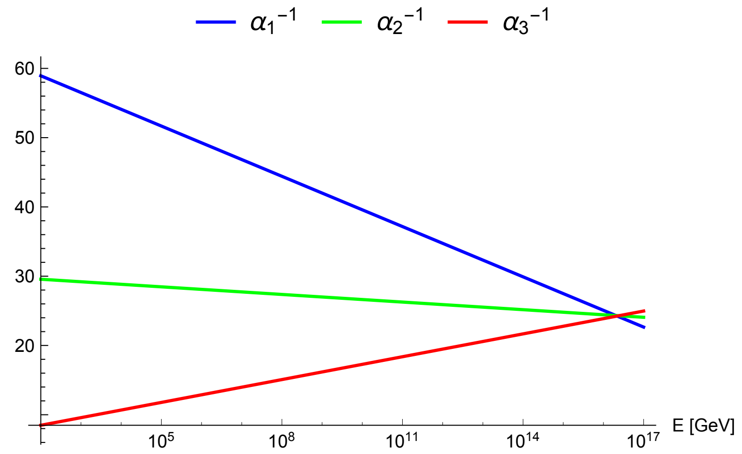


Figure 1.1: Running for the inverse gauge coupling constants in the MSSM scenario.

It is important to say that also in the MSSM the Yukawa matrices remain totally arbitrary; moreover, the presence of two independent Higgses with vevs $\langle H_u \rangle = v_u = v \sin\beta$ and $\langle H_d \rangle = v_d = v \cos\beta$ ($v = 256$ GeV) introduces another free parameter $\tan\beta = v_u/v_d$.
In this framework, also the idea of a grand unification of the fundamental forces (except for gravity) is suggested [16, 17, 18]. Indeed, in the MSSM the one loop renormalization group equations for the gauge coupling constants read

$$\frac{dg_i}{dt} = \frac{B_i^{(1)}}{16\,\pi^2}\,g_i^3 \qquad\qquad i = 1,2,3 \tag{1.37}$$

where $t = \ln\mu$ with μ renormalization scale, and $B_i^{(1)} = (\frac{33}{5}, 1, -3)$ are the beta functions for the gauge coupling constants of $U(1)$, $SU(2)$ and $SU(3)$ respectively. In order to solve these equations, we use the fact that, at $\mu = M_Z$, the fine structure constants[2] is measured as $\alpha_{em}^{-1}(M_Z) = 127.943 \pm 0.027$; this value is related to those for the EW gauge group as

$$\alpha_1(M_Z) = \frac{5}{3}\frac{\alpha_{em}(M_Z)}{\cos^2\theta_W(M_Z)}, \qquad\qquad \alpha_2(M_Z) = \frac{\alpha_{em}(M_Z)}{\sin^2\theta_W(M_Z)} \tag{1.38}$$

where $\sin^2\theta_W(M_Z) = 0.23117 \pm 0.00016$ and the 5/3 factor in the expression of α_1 comes from the canonical normalization of the hypercharge generator. With these initial conditions, we can see that the 2 coupling constants met each other at a scale $M_G \simeq 2 \cdot 10^{16}$ GeV; moreover, putting inside also the running of α_3, using the experimental value $\alpha_3(M_Z) = 0.1185 \pm 0.0020$ as initial condition, we see that all the 3 constants met at the

[2]For each gauge coupling constant g, the corresponding 'fine structure constant' is defined by $\alpha = g^2/4\pi$.

same energy scale, within the experimental errors, see Fig. 1.1. A detailed discussion can be found in the next chapter.

1.2.1 $SU(5)$ unification

At scales of order M_G, the Standard Model gauge group $SU(3) \times SU(2) \times U(1)$ can be consistently embedded into $SU(5)$ [16], which at larger scales could be extended to larger groups. Electroweak precision tests are in very good agreement with the predictions of supersymmetric $SU(5)$ model, while they exclude the non supersymmetric one[3] [19, 20]. In the context of $SU(5)$, known quarks and leptons fit into the antifundamental and symmetric representations as

$$(d^c + \ell)_i \sim \overline{5}_i, \qquad (u^c + q + e^c)_i \sim 10_i \tag{1.39}$$

more precisely

$$\overline{5}_i^\alpha = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_i, \qquad 10_{\alpha\beta,\, i} = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}_i \tag{1.40}$$

where $\alpha, \beta = 1, \ldots, 5$ are $SU(5)$ indices, while $i = 1, 2, 3$ is a family index. Together with these there are of course the 2 Higgses $H \sim 5$ and $\overline{H} \sim \overline{5}$, which contain the SM Higgs doublet and an heavy color triplet

$$5_H = (T_1, T_2, T_3, H^+, H^0)^t \tag{1.41}$$

With these notations, in the minimal fashion of the model the superpotential terms responsible for the fermion masses are

$$W = Y_u^{ij}\, 10_i\, 10_j\, H + Y_d^{ij}\, \overline{5}_i\, 10_j\, \overline{H} + \frac{Y_\nu^{ij}}{M_{pl}}\, (\overline{5}_i\, H)\, (\overline{5}_j\, H) \tag{1.42}$$

where we include also a neutrino mass term. At GUT scale these Yukawa coupling reduces to the MSSM ones with $Y_d^{ij} = Y_e^{ji}$, and therefore $y_{d,s,b} = y_{e,\mu,\tau}$. Although the $b - \tau$ unification $y_b = y_\tau$ is a definite success of the model [21], on the other hand the predictions

[3]Notice that also a MSSM without GUT does not work, since unification at string scale implies a too small value of the Weinberg angle at EW scale.

$y_s = y_\mu$ and $y_d = y_e$ are completely wrong, since they would imply $m_s/m_d = m_\mu/m_e \sim 200$. So, it is natural to consider that the Yukawa couplings in the superpotential are just functions of the adjoint superfield of $SU(5)$, that is, as a series expansion

$$Y_{ij} = Y_{ij}(\Sigma) = Y_{ij}^{(0)} + Y_{ij}^{(1)} \frac{\Sigma}{M_{pl}} + \cdots \tag{1.43}$$

This means that we can assume that the operator $Y_{ij}\, \overline{5}_i\, 10_j\, \overline{H}$ contains the higher order operator $Y_{ij}^{(1)} \frac{\Sigma}{M_{pl}}\, \overline{5}_i\, 10_j\, \overline{H}$ and so on. Since in general $\Sigma \cdot H$ contains $24 \times 5 = \overline{5} + \overline{45}$, it can distinguish the corresponding entries in Y_e and Y_d, giving rise to deviations from the wrong prediction of minimal $SU(5)$.

Moreover, there is still no explanation neither for the mass hierarchy, nor for the CKM mixing pattern, since the Yukawa matrices are still arbitrary. For this reason we are motivated to go beyond the minimal $SU(5)$ unification in order to implement new ideas that could shed some more light on the origin of fermion masses and mixing.

The idea of Grand Unification solves interesting issues which are present in the Standard Model, such as the electric charge quantization, which would remain unexplained otherwise, and of course it addresses the gauge hierarchy problem. However, this is strictly connected to the so called Doublet-Triplet splitting problem: indeed, as we wrote above, the two Higgs doublets H_u and H_d, when embedded in the GUT multiplets, are unavoidably accompanied by the colour triplet partners T and $\overline{T}$, which would mediate a very fast proton decay process unless their masses are of the order M_G. The Higgs sector in the minimal SUSY $SU(5)$ model consists of two Higgses in fundamental 5 and antifundamental $\overline{5}$ representations

$$H = (T + H_u) \sim 5, \qquad \overline{H} = (\overline{T} + H_d) \sim \overline{5} \tag{1.44}$$

and a chiral superfield $\Sigma \sim 24$ in adjoint representation. The most general superpotential involving all these fields have the form

$$W = \frac{M}{2}\, \Sigma^2 + \frac{h}{3}\, \Sigma^3 + M_H\, \overline{H}\, H + f\, \overline{H}\, \Sigma\, H \tag{1.45}$$

The supersymmetric ground state $\langle \Sigma \rangle = (M/h)\,\mathrm{diag}(2, 2, 2, -3, -3)$, $\langle H \rangle = \langle \overline{H} \rangle = 0$ provides the symmetry breaking pattern $SU(5) \to SU(3) \times SU(2) \times U(1)$. In this case, the masses of the T and $H_{u,d}$ superfields are respectively

$$M_3 = M_H + \frac{2\,f}{h}\, M \qquad \text{and} \qquad \mu = M_H - \frac{3\,f}{h}\, M \tag{1.46}$$

Therefore, the light doublet $\mu \sim M_Z$ versus the heavy triplet $M_3 \sim M_G$ requires $M_H\, h \simeq 3\, f\, M$ with an accuracy of order 10^{-14}; although supersymmetry makes this constraint stable against radiative corrections, this is nothing but a precise fine tuning of the parameters in the superpotential.

1.2.2 Further ideas: $SO(10)$ and $SU(6)$

Next to $SU(5)$, we can consider $SO(10)$ as grand unification group [22]; indeed, it is the smallest group in which all the fermions in one family fit in the same representation, which is the spinorial 16. In addition to the quarks and leptons of the SM, it also includes the RH neutrino ν^c, which is a singlet of $SU(5)$. This implies that now all the Yukawa matrices could be in principle related by the $SO(10)$ Clebsch factors, reducing the number of fundamental parameters in the fermion sector.

The $SO(10)$ symmetry can break down to the SM via two interesting channels: $SO(10) \to SU(5)$ and $SO(10) \to SU(4) \times SU(2) \times SU(2)'$. In order to break this symmetry down to the SM we need a set of Higgses in representations 45, 54 and $16 + \overline{16}$. In terms of the $SU(5)$ subgroup, their contents are

$$45 = 1 + 24 + 10 + \overline{10}, \qquad 54 = 24 + 15 + \overline{15}, \qquad 16 = 1 + \overline{5} + 10 \tag{1.47}$$

Moreover, the Higgs doublets $H_{u,d}$ fit in the fundamental representation H of $SO(10)$, that in terms of $SU(5)$ is

$$10_H = 5(T, H_u) + \overline{5}(\overline{T}, H_d) \tag{1.48}$$

while the three fermion families are arranged in chiral superfields 16_i, $i = 1, 2, 3$ as

$$16_i = \overline{5}(d^c, \ell)_i + 10(u^c, q, e^c)_i + 1(\nu^c)_i \tag{1.49}$$

As a minimal extension of $SU(5)$, we have the $SU(6)$ model [23, 24, 25, 26, 27, 28, 29]: the Higgs sector in this case consists in the supermultiplet $\Sigma \sim 35$ in adjoint representation and $H + \overline{H}$ in $6 + \overline{6}$ representation, in analogy to 24 and $5 + \overline{5}$ of $SU(5)$. Differently from the other GUT models, where the Higgs sector consists in two different sets (one for the GUT symmetry breaking and another for the EW symmetry breaking), in $SU(6)$ we have no preferred superfield for the EW transition; indeed, 35 and $6 + \overline{6}$ constitute a minimal Higgs content needed for the local symmetry breaking pattern $SU(6) \to SU(3) \times SU(2) \times U(1)$.

1.3 Standard Model: the Good, the Bad and the Ugly

Although the Standard Model works pretty well for the available precision tests (up to TeV scale, except for some yet unconfirmed anomalies). Among its remarkable properties, we can mention that the origin of mass is related to a unique dimensional order parameter, which is the Higgs vev. In addition, we have a natural flavour conservation, since at tree level the Z boson interacts diagonally with all the fermion fields.
However, it is clear that it cannot be the end of the story, since there are a lot of unanswered questions therein. In the following, we briefly recall the most challenging puzzles of modern particle physics, together with the most common approaches which try to explain them.

- **Family Problem**
  The replication of families is one of the main puzzle in particle physics; indeed, the 3 fermion families are in identical representation of the Standard Model gauge group. In this context, precision tests also exclude the existence of a fourth sequential chiral family, but on the other hand extra vector-like families could exist, even at TeV scale; the issue in this case would be that, being vector-like, their masses will be not anymore linked to the Higgs vev, but they are arbitrary parameters of the theory. Related to this, there is the problem of the fermion mass hierarchy and mixing angles pattern: indeed, the mass eigenstates obey $m_t : m_c : m_u = 1 : \epsilon^2 : \epsilon^4$ and $m_b : m_s : m_d = 1 : \epsilon : \epsilon^2$, with $\epsilon \sim 1/20$; moreover $V_{us} \sim \sqrt{\epsilon}$, $V_{cb} \sim \epsilon$, $V_{ub} \sim \epsilon^2$. The Standard Model does not contain any theoretical input that could explain this; we can say it is technically natural in the sense that it can tolerate any pattern of the Yukawa matrices, but their structures remain arbitrary. In this sense the origin of the fermion mass hierarchy and the weak mixing pattern remains a mystery.
  A common approach to this puzzle is to introduce a family symmetry [30, 31, 32, 33, 34, 35, 36, 37, 38], or horizontal symmetry, which is realized at some high scale and then spontaneously broken by the vev of some flavon field(s). Then, depending on the specific model, the breaking pattern of this symmetry will be reflected in the fermion mass hierarchy and mixing. A quite complete picture can be found in the non abelian $SU(3)_H$ gauge symmetry between the three families; this has to have a chiral character, with LH and RH fermions transforming in the fundamental and

antifundamental representation [39, 40], respectively, so that they cannot acquire a mass without the breaking of $SU(3)_H$ [41, 42]. In chiral notation this means that

$$q_i,\, u_i^c,\, d_i^c,\, \ell_i,\, e_i^c \sim 3 \tag{1.50}$$

with $i = 1, 2, 3$ family index. This arrangement is compatible with a GUT extension of the SM. For example, in an $SU(5)$ context, we have the following representations of $SU(5) \times SU(3)_H$

$$(d^c,\, \ell)_i \sim (\overline{5}, 3), \qquad (u^c,\, q,\, e^c)_i \sim (10, 3) \tag{1.51}$$

while in the context of $SO(10)$ all these fermions, along with the RH neutrinos $\nu_L^c = C\,\overline{\nu_R}^T$ can be packed into the unique multiplet in the spinor representation of $SO(10)$, $\Psi_i = (d^c,\, \ell,\, u^c,\, q,\, e^c,\, \nu^c)_i \sim (16, 3)$.

As far as the fermion bilinears $u_i^c\, q_j$, $d_i^c\, q_j$ and $e_i^c\, \ell_j$ transform in representations $3 \times 3 = 6 + \overline{3}$, the fermion masses can only be induced via the higher order operators

$$\frac{\chi^{ij}}{M}\, u_i^c\, q_j\, H + \frac{\chi^{ij}}{M}\, d_i^c\, q_j\, \tilde{H} + \frac{\chi^{ij}}{M}\, e_i^c\, \ell_j\, \tilde{H} + \text{h.c.} \tag{1.52}$$

involving some horizontal scalars χ in symmetric $\chi^{\{ij\}} \sim \overline{6}$ or antisymmetric $\chi^{[ij]} = \varepsilon^{ijk}\,\chi_k \sim 3$ representations of $SU(3)_H$, and M is some effective scale. The effective operators we wrote are invariant under $SU(3)_H$ by construction, but they actually have a larger symmetry group $U(3)_H$; indeed, they feature an accidental global chiral $U(1)$ symmetry. Therefore, the three fermion families become massive only if $U(3)_H$ is fully broken. In a natural realization, in the first step $U(3)_H$ is broken down to $U(2)_H$ and the third family becomes massive, while mixing angles are all zero; then, $U(2)_H$ breaks to $U(1)_H$, the second family acquires mass and the CKM angle θ_{23} could be non zero; at this stage, the first family remain massless and unmixed with the heavier generations. In the last step, $U(1)_H$ is also broken, so the first family becomes massive and their mixings arise. In this way, the interfamily mass hierarchy can be related to the hierarchy of flavon vevs inducing the horizontal symmetry breaking $U(3)_H \to U(2)_H \to U(1)_H \to$ nothing. As a simplest set of flavons we can choose two triplets and one antisextet with the following vevs

$$\langle \chi_3^{\{ij\}} \rangle = \text{diag}(0, 0, V_3), \qquad \langle \chi_{2i} \rangle = \begin{pmatrix} V_2 \\ 0 \\ 0 \end{pmatrix}, \qquad \langle \chi_{1i} \rangle = \begin{pmatrix} 0 \\ 0 \\ V_1 \end{pmatrix} \tag{1.53}$$

so that the total matrix of flavon vevs has the form

$$\langle \chi^{ij} \rangle = \langle \chi_3^{\{ij\}} + \chi_2^{[ij]} + \chi_1^{[ij]} \rangle = \begin{pmatrix} 0 & V_1 & 0 \\ -V_1 & 0 & V_2 \\ 0 & -V_2 & V_3 \end{pmatrix} \tag{1.54}$$

The hierarchies between the different Yukawa entries, corresponding to the interfamily mass hierarchies, can be related to a hierarchy $V_3 \gg V_2 \gg V_1$ in the horizontal symmetry breaking chain $U(3)_H \to U(2)_H \to U(1)_H \to$ nothing. Moreover, it is important to note that the chiral global $U(1)_H$ symmetry can be associated with the Peccei-Quinn symmetry provided that it is also respected by the Lagrangian of the flavon fields. After this breaking, the theory reduces to the SM with one standard Higgs doublet H, and so, in our construction the flavor will be naturally conserved in neutral currents.

- **B & L violating Operators**

  In the Standard Model Lagrangian we wrote previously, lepton number and baryon number are realized as accidental global symmetries of the renormalizable interactions; however, it is known that they have to be broken in order to have a good baryogenesis mechanism (Sakharov conditions). Moreover, at this level there is no possibility to have a mass term for the neutrino; this is because we typically do not include a right handed neutrino in the Lagrangian, being a gauge singlet, and no Majorana mass terms are allowed.

  If we include a left handed antineutrino field, then it can have a mass term

$$\mathcal{L}_\nu = \frac{1}{2} M_{ij} \, \nu_i^c \, \nu_j^c \tag{1.55}$$

  which breaks lepton number; together with the standard Dirac mass term, this generates the so called seesaw mechanism: greater is the mass scale M, lighter the LH neutrino will be.

  Without the introduction of the RH neutrino, by the way, it is possible to get lepton number violating masses by writing higher order operators in the so called Standard Model Effective Field Theory. The Weinberg operator [43]

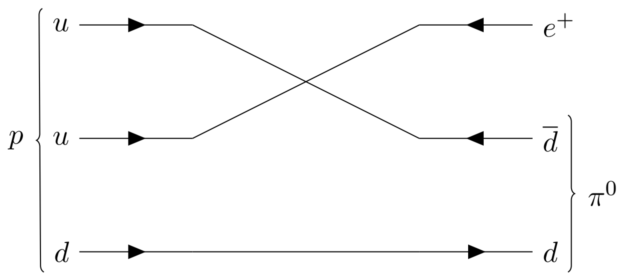
$$\mathcal{L}_5 = \frac{b_{ij}}{M} \, \ell_i \, \ell_j \, H \, H \tag{1.56}$$

with $|b_{ij}| \sim \mathcal{O}(1)$ provides a Majorana mass term for the LH neutrino after the electroweak symmetry breaking, $m_\nu \sim v^2/M$; in the seesaw picture, this operator can be induced by exchange of heavy RH neutrinos [44, 45, 46]. The mass scale M can be estimated by phenomenology: indeed, observations related to neutrino flavour oscillations could be used to get $M \sim 10^{15}$ GeV, which is of the order of the Grand Unification scale. Notice that, in the abscence of such external information, the natural mass scale would be the Planck scale.

In the same way, other operators will break the accidental global symmetries of the renormalizable Lagrangian, such as the dimension six operator

$$\mathcal{L}_6 \propto \frac{1}{M^2}\, q\, q\, q\, \ell \tag{1.57}$$

which breaks baryon number, and contributes to the proton decay process $p \to \pi^0\, e^+$



Proton decay is one of the key points when we discuss a Grand Unified Theory; limits on the proton lifetime can be used to estimate the suppression scale in the dimension six operator, which results to be of the order of the GUT scale, $M \sim 10^{15}$ GeV.

- **Strong CP problem**

  Another unclear fact present in the Standard Model is the so called strong CP problem, or strong CP puzzle, which in the standard discussion is presented as a naturalness problem related to the following term in the QCD Lagrangian [47, 48]

$$\mathcal{L}_\theta = \theta\, \frac{g_3^2}{32\, \pi^2}\, \varepsilon^{\alpha\beta\mu\nu}\, G^a_{\alpha\beta}\, G^a_{\mu\nu} \tag{1.58}$$

  which is just a topological term, but it could have physical consequences due to the non triviality of the QCD vacuum structure; indeed, QCD has a continuum of vacua, labelled by the CP violating vacuum angle θ, which belong to different superselection sectors. More precisely, the physical measurable quantity is

$$\overline{\theta} = \theta + \arg\det M_q \tag{1.59}$$

where M_q is the quark mass matrix. In the quantum theory, $\overline{\theta}$ induces an electric dipole moment for the neutron which, according to the current experimental bounds, provides the upper limit

$$|\overline{\theta}| < 10^{-10} \tag{1.60}$$

Thus, the puzzle relies in the fact that observations indicate that we live in a sector with an extremely small (or zero) $\overline{\theta}$.

The axion solution eliminates this vacuum structure by making $\overline{\theta}$ dynamical, relaxing it to a CP invariant ground state. In the original model of Peccei and Quinn [49, 50, 51, 52], the axion field $a(x)$ arises as a pseudo Goldstone boson of a spontaneously broken anomalous $U(1)$ global symmetry [53, 54, 55, 56, 57, 58, 33, 59]; then, by a chiral rotation it is possible to move this field into the $\overline{\theta}$ parameter, so that the ground state energy will be proportional to

$$E \propto \cos\left(\overline{\theta} - \frac{a(x)}{f_a}\right) \tag{1.61}$$

where f_a is the axion decay constant. Now if $\langle a \rangle / f_a = \overline{\theta}$ the ground state has no effective $\overline{\theta}$ dependence; moreover, a vanishing axion vev dynamically relaxes $\overline{\theta}$ to zero.

One of the most delicate aspects of the PQ mechanism is the fact that it relies on a global $U(1)$ symmetry, which has to be preserved to a great degree of accuracy in order for the axion vev to be relaxed to zero, a precision compatible with the non observation of the neutron electric dipole moment; this issue is known as the PQ quality problem.

As we said before, non perturbative gravitational effects will break global symmetries at some point; we can consider the effective operator

$$\lambda\, e^{-i\,\delta}\, \frac{\phi^n}{M_{pl}^{n-4}} + \text{h.c.} \tag{1.62}$$

with λ real and δ is the phase coupling. In this parametrization we can take

$$\phi = \frac{f_a}{\sqrt{2}}\, e^{i\,a/f_a} \tag{1.63}$$

The effect of this PQ breaking operator is to move the minimum of the axion potential away from the CP conserving minimum of the QCD induced potential

$V(\theta) = -m_a^2 \, f_a^2 \, \cos\theta$ and shift it to [47]

$$\langle\theta\rangle = \frac{n^2 \, m_*^2 \, \sin\delta}{m_a^2 + n^2 \, m_*^2 \, \cos\delta} \tag{1.64}$$

where we defined

$$m_*^2 = \frac{\lambda}{2} \, f_a^2 \left(\frac{f_a}{\sqrt{2} \, M_{pl}} \right)^{n-4} \tag{1.65}$$

From that analysis we get that, in order to be compatible with the observational bounds on neutron electric dipole moment, the minimal PQ breaking operator has to have $n \gtrsim 12$. The issue at this point will be to understand how to forbid lower order operators without ad hoc assumptions.

Chapter 2

Consistency of Grand Unification

We review the standard calculation of the gauge coupling running under renormalization group at both one and two loop level. We study the consistency of Grand Unification by looking at the behaviour of the strong coupling constant α_3 near the Grand Unification scale. Although at one loop level the three couplings meet each other with a very high precision, things are definitely worse at two loop level; the behaviour of the Yukawa coupling for the top quark y_t plays a crucial role in the discussion. An analysis of the behaviour of y_t as a function of its value at GUT scale provides interesting information about the possible values of this coupling at electroweak (EW) scale, where we show it is precisely constrained, as well as lower bound limits on the $\tan\beta$ parameter. Moreover, we try to improve the discussion on gauge coupling unification by taking into account the so called threshold effects; we find that the three gauge couplings meet each other with a very good precision if the supersymmetric thresholds are not all equal: following an argument based on the gaugino mass parameters renormalization we explain why and what the relation between these scales can be.

2.1 Gauge Coupling Renormalization

Let us consider a gauge group $SU(3) \times SU(2) \times U(1)$, with corresponding gauge couplings g_i, $i = 1, 2, 3$. Then the renormalization group equations (RGE) for each of them can be written as

$$\frac{dg_i}{dt} = \frac{1}{16\,\pi^2}\,\beta_{g_i} = \frac{1}{16\,\pi^2}\,\left[\beta_{g_i}^{(1)} + \frac{1}{16\,\pi^2}\,\beta_{g_i}^{(2)}\right] \tag{2.1}$$

where $t = \ln \mu$, with μ renormalization scale. Here we have included the one and two loop contributions to the beta function.

2.1.1 One Loop Results

At one loop level these beta functions take the form

$$\beta_{g_i}^{(1)} = B_i^{(1)}\, g_i^3 \tag{2.2}$$

where the coefficient $B^{(1)}$ is given by the well known result

$$B^{(1)} = -\frac{11}{3}\, C_2(G) + \frac{2}{3}\, T(R)|_f + \frac{1}{3}\, T(R)|_s \tag{2.3}$$

Here $C_2(G)$ is the quadratic Casimir of the group G, which equals N for $SU(N)$ and vanishes for an abelian group, while $T(R)$ is defined by the normalization of the generators in the representation R:

$$\mathrm{Tr}(T^a\, T^b) = T(R)\, \delta^{ab} \tag{2.4}$$

In the fundamental representation, $T(R) = 1/2$, while for the adjoint representation $T(R) = C_2(G)$. The sum over all the fermions and scalars in (2.3) is understood.
In the non supersymmetric case, one can see that (2.3) takes the form

$$B_i^{(1)} = \left(\frac{4}{3}\, N_f + \frac{1}{10}\, N_H,\ \frac{4}{3}\, N_f + \frac{1}{6}\, N_H - \frac{22}{3},\ \frac{4}{3}\, N_f - 11 \right) \tag{2.5}$$

where N_f is the number of families, and N_H is the number of Higgs doublets. For $N_f = 3$ and $N_H = 1$ this provides [60]

$$B_{i,\,SM}^{(1)} = \left(\frac{41}{10}, -\frac{19}{6}, -7 \right) \tag{2.6}$$

while in the Minimal Supersymmetric Standard Model (MSSM) equation (2.3) becomes

$$B_i^{(1)} = \left(2N_f + \frac{3}{5}\, N_{ud},\ 2N_f + N_{ud} - 6,\ 2N_f - 9 \right) \tag{2.7}$$

where N_f is the number of generations and N_{ud} is the number of pairs of Higgs doublets; plugging $N_f = 3$ and $N_{ud} = 1$ we get [60]

$$B_{i,\,MSSM}^{(1)} = \left(\frac{33}{5}, 1, -3 \right) \tag{2.8}$$

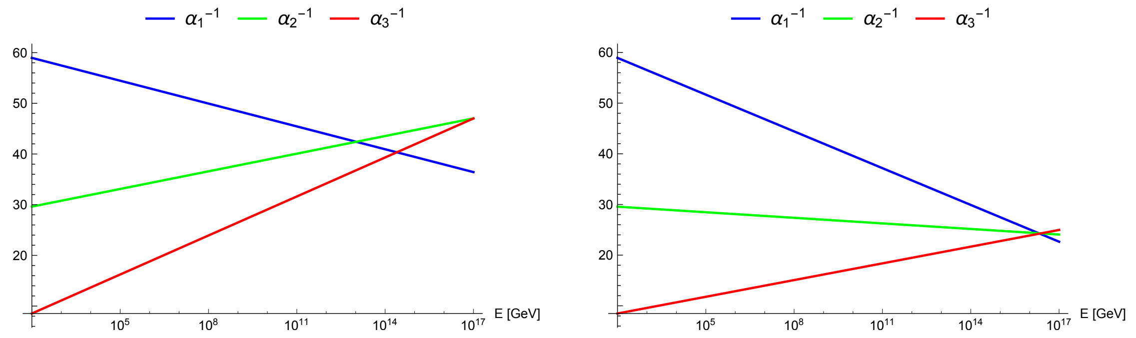


Figure 2.1: One Loop running for inverse gauge couplings. Left: Standard Model. Right: Minimal Supersymmetric Standard Model.

In Fig. 2.1 we show the one loop running for the inverse coupling constants defined as

$$\alpha_i = \frac{g_i^2}{4\,\pi} \tag{2.9}$$

for both SM and MSSM. We set the renormalization scale at the Z mass, $M_Z = 92\,\text{GeV}$, where we know the measured values of the electromagnetic fine structure constant and the Weinberg angle [60]:

$$\alpha_{em}^{-1}(M_Z) = 127.943 \pm 0.027\,, \qquad \sin^2\theta_W(M_Z) = 0.23117 \pm 0.00016 \tag{2.10}$$

Using these we can set the initial values for α_1 and α_2, as well as the measured value for α_3 [61]

$$\alpha_1(M_Z) = \frac{5}{3}\frac{\alpha_{em}(M_Z)}{\cos^2\theta_W(M_Z)}\,, \qquad \alpha_2(M_Z) = \frac{\alpha_{em}(M_Z)}{\sin^2\theta_W(M_Z)}\,, \qquad \alpha_3(M_Z) = 0.1185 \pm 0.0020 \tag{2.11}$$

which corresponds to

$$\alpha_1^{-1}(M_Z) = 59.0199 \pm 0.0002\,, \qquad \alpha_2^{-1}(M_Z) = 29.58 \pm 0.03 \tag{2.12}$$

In the supersymmetric case we have room for grand unification; the energy scale at which the two above couplings meet each other is our estimate for the Grand Unification scale

$$M_{GUT} = (2.046 \pm 0.060) \cdot 10^{16}\,\text{GeV} \tag{2.13}$$

where

$$\alpha_{1,2}^{-1}\,(M_{GUT}) = \alpha_G^{-1} = 24.319 \pm 0.031 \tag{2.14}$$

Moreover, it follows from the unification condition that, at EW scale, [62]

$$\frac{\alpha_i^{-1}(M_Z) - \alpha_j^{-1}(M_Z)}{\alpha_j^{-1}(M_Z) - \alpha_k^{-1}(M_Z)} = \frac{B_i^{(1)} - B_j^{(1)}}{B_j^{(1)} - B_k^{(1)}}\,, \qquad i,j,k = 1,2,3 \tag{2.15}$$

We can study the quality of unification by looking at the value of α_3 at GUT scale by taking into account also the uncertainty in (2.11); we get

$$\alpha_3^{-1}(M_{GUT}) = 24.21 \pm 0.14 \tag{2.16}$$

in quite good agreement with (2.14).

2.1.2 Two Loop Results

Let us now see how much the next order corrections affect our result in the supersymmetric case. We write again the one loop beta functions for the gauge couplings, as weel as the two loop ones (see [63] for details)

$$\begin{aligned} \beta_{g_i}^{(1)} &= B_i^{(1)}\, g_i^3 \\ \beta_{g_i}^{(2)} &= g_i^3 \left[\sum_{j=1}^{3} B_{ij}^{(2)}\, g_j^2 - \sum_{x=u,d,e} C_i^x\, \mathrm{Tr}(Y_x^\dagger\, Y_x) \right] \end{aligned} \tag{2.17}$$

where

$$B_i^{(1)} = \left(\frac{33}{5}, 1, -3\right)\,, \qquad B_{ij}^{(2)} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix}\,, \qquad C_i^{u,d,e} = \begin{pmatrix} \frac{26}{5} & \frac{14}{5} & \frac{18}{5} \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix} \tag{2.18}$$

Here Y_u, Y_d and Y_e are generic 3×3 complex Yukawa matrices, whose eigenvalues are the couplings (y_u, y_c, y_t), (y_d, y_s, y_b) and (y_e, y_μ, y_τ), respectively, and the index i in the last matrix labels its rows.

We immediately see that, as expected, at two loop level the gauge coupling running depends also on the Yukawa contributions; the one and two loop beta functions for these couplings, written in matrix form, are given by [63]

$$\beta^{(1)}_{Y_u} = Y_u \left[\, 3\,\mathrm{Tr}(Y_u\,Y_u^\dagger) + 3\,Y_u^\dagger\,Y_u + Y_d^\dagger\,Y_d - \frac{16}{3}\,g_3^2 - 3\,g_2^2 - \frac{13}{15}\,g_1^2\right]$$

$$\begin{aligned}\beta^{(2)}_{Y_u} = Y_u \Big[& -3\,\mathrm{Tr}(3\,Y_u\,Y_u^\dagger\,Y_u\,Y_u^\dagger + Y_u\,Y_d^\dagger\,Y_d\,Y_u^\dagger) - Y_d^\dagger\,Y_d\,\mathrm{Tr}(3\,Y_d\,Y_d^\dagger + Y_e\,Y_e^\dagger) - 9\,Y_u^\dagger\,Y_u\,\mathrm{Tr}(Y_u^\dagger\,Y_u) \\ & - 4\,Y_u^\dagger\,Y_u\,Y_u^\dagger\,Y_u - 2\,Y_d^\dagger\,Y_d\,Y_d^\dagger\,Y_d - 2\,Y_d^\dagger\,Y_d\,Y_u^\dagger\,Y_u + \left(16\,g_3^2 + \frac{4}{5}\,g_1^2\right)\,\mathrm{Tr}(Y_u^\dagger\,Y_u) + 8\,g_3^2\,g_2^2 \\ & + \left(6\,g_2^2 + \frac{2}{5}\,g_1^2\right)\,Y_u^\dagger\,Y_u + \frac{2}{5}\,g_1^2\,Y_d^\dagger\,Y_d - \frac{16}{9}\,g_3^4 + \frac{136}{45}\,g_3^2\,g_1^2 + \frac{15}{2}\,g_2^4 + g_2^2\,g_1^2 + \frac{2743}{450}\,g_1^4\Big]\end{aligned} \tag{2.19}$$

$$\beta^{(1)}_{Y_d} = Y_d \left[\, \mathrm{Tr}(3\,Y_d\,Y_d^\dagger + Y_e\,Y_e^\dagger) + 3\,Y_d^\dagger\,Y_d + Y_u^\dagger\,Y_u - \frac{16}{3}\,g_3^2 - 3\,g_2^2 - \frac{7}{15}\,g_1^2\right]$$

$$\begin{aligned}\beta^{(2)}_{Y_d} = Y_d \Big[& -3\,\mathrm{Tr}(3\,Y_d\,Y_d^\dagger\,Y_d\,Y_d^\dagger + Y_u\,Y_d^\dagger\,Y_d\,Y_u^\dagger + Y_e\,Y_e^\dagger\,Y_e\,Y_e^\dagger) - 3\,Y_d^\dagger\,Y_d\,\mathrm{Tr}(3\,Y_d^\dagger\,Y_d + Y_e^\dagger\,Y_e) \\ & - 3\,Y_u^\dagger\,Y_u\,\mathrm{Tr}(Y_u^\dagger\,Y_u) - 4\,Y_d^\dagger\,Y_d\,Y_d^\dagger\,Y_d - 2\,Y_u^\dagger\,Y_u\,Y_u^\dagger\,Y_u - 2\,Y_u^\dagger\,Y_u\,Y_d^\dagger\,Y_d + 8\,g_3^2\,g_2^2 \\ & + \left(16\,g_3^2 - \frac{2}{5}\,g_1^2\right)\,\mathrm{Tr}(Y_d^\dagger\,Y_d) + \frac{6}{5}\,g_1^2\,\mathrm{Tr}(Y_e^\dagger\,Y_e) + \frac{4}{5}\,g_1^2\,Y_u^\dagger\,Y_u + \left(6\,g_2^2 + \frac{4}{5}\,g_1^2\right)\,Y_d^\dagger\,Y_d \\ & - \frac{16}{9}\,g_3^4 + \frac{8}{9}\,g_3^2\,g_1^2 + \frac{15}{2}\,g_2^4 + g_2^2\,g_1^2 + \frac{287}{90}\,g_1^4\Big]\end{aligned} \tag{2.20}$$

$$\beta^{(1)}_{Y_e} = Y_e \left[\, \mathrm{Tr}(3\,Y_d\,Y_d^\dagger + Y_e\,Y_e^\dagger) + 3\,Y_e^\dagger\,Y_e - 3\,g_2^2 - \frac{9}{5}\,g_1^2\right]$$

$$\begin{aligned}\beta^{(2)}_{Y_e} = Y_e \Big[& -3\,\mathrm{Tr}(3\,Y_d\,Y_d^\dagger\,Y_d\,Y_d^\dagger + Y_u\,Y_d^\dagger\,Y_d\,Y_u^\dagger + Y_e\,Y_e^\dagger\,Y_e\,Y_e^\dagger) - 3\,Y_e^\dagger\,Y_e\,\mathrm{Tr}(3\,Y_d^\dagger\,Y_d + Y_e^\dagger\,Y_e) \\ & - 4\,Y_e^\dagger\,Y_e\,Y_e^\dagger\,Y_e + \left(16\,g_3^2 - \frac{2}{5}\,g_1^2\right)\,\mathrm{Tr}(Y_d^\dagger\,Y_d) + \frac{6}{5}\,g_1^2\,\mathrm{Tr}(Y_e^\dagger\,Y_e) + 6\,g_2^2\,Y_e^\dagger\,Y_e \\ & + \frac{15}{2}\,g_2^4 + \frac{9}{5}\,g_2^2\,g_1^2 + \frac{27}{2}\,g_1^4\Big]\end{aligned} \tag{2.21}$$

This means that the two loop RGE for the gauge couplings (with only top quark contribution) are

$$\begin{aligned}\frac{dg_1}{dt} &= \frac{1}{6400\,\pi^4}\left[199\,g_1^5 + 5\,g_1^3(27\,g_2^2 + 528\,\pi^2 + 88\,g_3^2 - 26\,y_t^2)\right] \\ \frac{dg_2}{dt} &= \frac{g_2^3}{1280\,\pi^4}\left[9\,g_1^2 + 5(16\,\pi^2 + 25\,g_2^2 + 24\,g_3^2 - 6\,y_t^2)\right] \\ \frac{dg_3}{dt} &= -\frac{g_3^3}{1280\,\pi^4}\left[-11\,g_1^2 + 5(48\,\pi^2 - 9\,g_2^2 - 14\,g_3^2 + 4\,y_t^2)\right] \\ \frac{dy_t}{dt} &= \frac{y_t}{16\,\pi^2}\left(-\frac{13}{15}\,g_1^2 - 3\,g_2^2 - \frac{16}{3}\,g_3^2 + 6\,y_t^2\right) + \frac{1}{(16\,\pi^2)^2}\left\{\frac{y_t}{450}\left[2743\,g_1^4 + 3375\,g_2^4\right.\right. \\ & \left.\left. -800\,g_3^4 + 10\,g_1^2\,(45\,g_2^2 + 136\,g_3^2 + 54\,y_t^2) + 900\,g_2^2\,(4\,g_3^2 + 3\,y_t^2) - 100\,y_t^2\,(99\,y_t^2 - 72\,g_3^2)\right]\right\}\end{aligned} \tag{2.22}$$

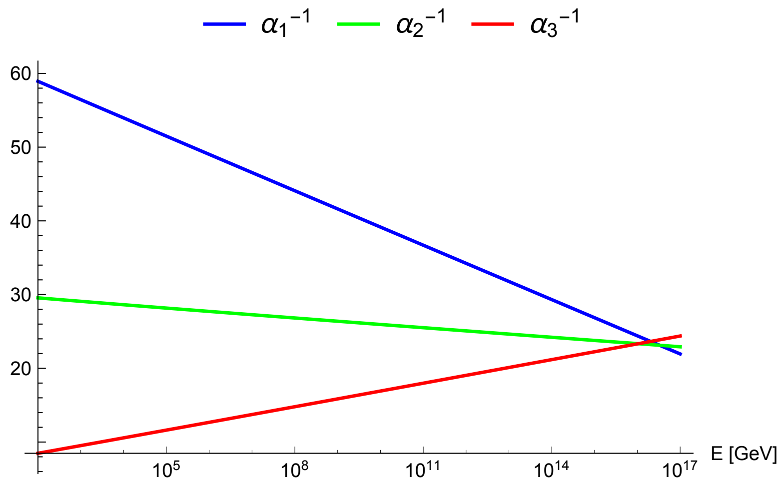


Figure 2.2: Two loop running for gauge couplings without Yukawa contributions.

For the moment, let us consider only the self renormalization of the gauge couplings at two loop without Yukawas, i.e. $Y_{u,d,e} \equiv 0$. With exactly the same procedure as in the one loop case, we show the running in Fig. 2.2, from which we get the estimate for the GUT scale and inverse gauge coupling

$$M_{GUT} = (3.326 \pm 0.093) \cdot 10^{16}\,\text{GeV}\,, \qquad \alpha_G^{-1} = 23.128 \pm 0.025 \tag{2.23}$$

and the corresponding value of α_3 at GUT scale is

$$\alpha_3^{-1}(M_{GUT}) = 23.87 \pm 0.16 \tag{2.24}$$

In the following we will study how the Yukawa contribution affects this result.

2.2 Top Yukawa Renormalization

The one loop RGE, provided by the first line of (2.19), reads

$$\frac{dy_t}{dt} = \frac{1}{16\,\pi^2}\,y_t\left(-\frac{13}{15}\,g_1^2 - 3\,g_2^2 - \frac{16}{3}\,g_3^2 + 6\,y_t^2\right) \tag{2.25}$$

We can solve for y_t by set different initial values at GUT scale, and then run back to the EW scale (or top mass scale). Recall that the top running mass at EW scale is related to its Yukawa coupling by

$$m_t(\mu \simeq m_t) = y_t(\mu \simeq m_t) \cdot v \cdot \sin\beta \tag{2.26}$$

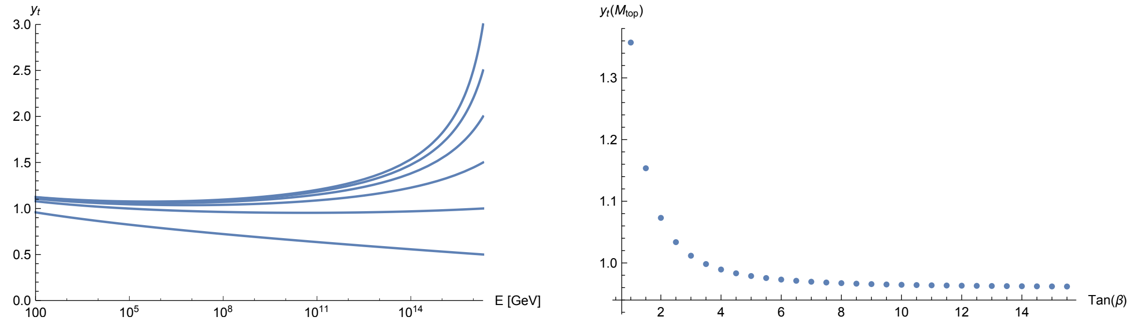


Figure 2.3: Left: Running for y_t in MSSM for different GUT scale values, from 0.5 to 3. Right: Top Yukawa coupling at EW scale as a function of $\tan\beta$.

where $v \simeq 174$ GeV is the Higgs vacuum expectation value (vev) and $\sin\beta$ is a free parameter we will constrain later. This mass is related to the measured pole mass $m_t^{pole} = 172.69 \pm 0.30$ GeV [61] by

$$m_t^{pole} = m_t^{run} \cdot \left\{ 1 + \frac{4}{3\,\pi}\,\alpha_s(\mu \simeq m_t) \right\} \tag{2.27}$$

which provides $m_t^{run} = (165.05 \pm 0.29)$ GeV. In Fig. 2.3 on the left we show the one loop running for the top Yukawa for different values at GUT scale. We see that, in the limit $y_t \gg 1$ at GUT scale, the corresponding EW value is $y_t(\mu \simeq m_t) \simeq 1.12$, which means that

$$\tan\beta \gtrsim 1.639 \tag{2.28}$$

so a typical values range $\tan\beta \gtrsim 2$ is perfectly working. Instead, the minimal value of y_t at EW scale is obtained in the limit $\tan\beta \to \infty$, where

$$y_t(\mu \simeq m_t)|_{min} = \frac{m_t^{run}}{v} = 0.95 \tag{2.29}$$

We show the behaviour of y_t at EW scale as a function of $\tan\beta$ on the right of Fig. 2.3; formally

$$y_t(\mu \simeq m_t) = \frac{m_t^{run}}{v} \cdot \frac{1}{\sin\beta} = \frac{m_t^{run}}{v} \cdot \frac{\sqrt{1+\tan^2\beta}}{\tan\beta} \tag{2.30}$$

By looking at equation (2.25), we see we can rewrite it in the form

$$\frac{d\ln y_t}{dt} = -\sum_{i=1}^{3} \frac{c_i}{B_i^{(1)}}\,\frac{d\ln g_i}{dt} + \frac{1}{16\,\pi^2}\,6\,y_t^2 \tag{2.31}$$

where $c_i = (\frac{13}{15}, 3, \frac{16}{3})$ and $B_i^{(1)}$ are defined in equation (2.18). Here we used also the fact that, from equation (2.1):

$$g_i^2 = \frac{16\,\pi^2}{B_i^{(1)}}\,\frac{d\ln g_i}{dt} \tag{2.32}$$

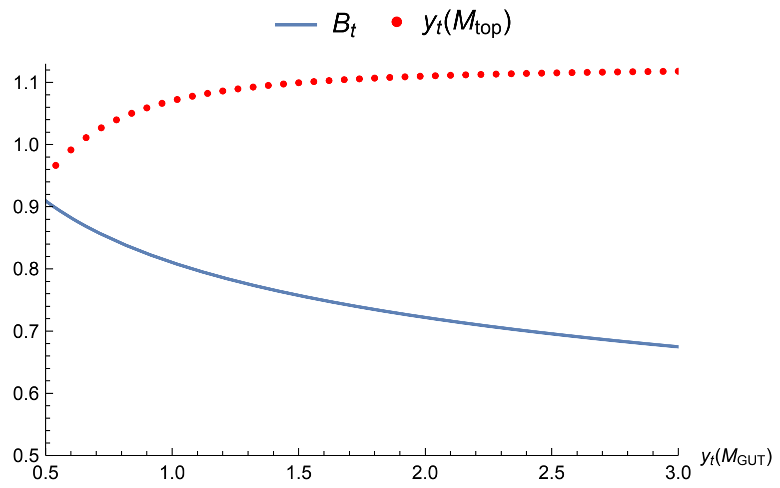


Figure 2.4: Renormalization factor $B_t(\mu)$ and $y_t(\mu)$ as a function of y_t at GUT scale for $\mu = m_t$.

Solutions to equation (2.31) can be written as [64]

$$y_t(\mu) = \overline{y}_t \, A_t(\mu) \, B_t^6(\mu) \tag{2.33}$$

where $\overline{y}_t$ is the value of y_t at GUT scale, and we defined the integrating factors

$$\begin{aligned} A_t(\mu) &= \exp\left[-\frac{1}{16\,\pi^2} \int_{\ln(M_{GUT})}^{\ln\mu} c_i \, g_i^2(\mu) \, d\ln\mu \right] = \prod_{i=1}^{3} \left[\frac{g_i(\mu)}{g_i(M_{GUT})} \right]^{-c_i/B_i^{(1)}} \\ B_t(\mu) &= \exp\left[\frac{1}{16\,\pi^2} \int_{\ln(M_{GUT})}^{\ln\mu} y_t^2(\mu) \, d\ln\mu \right] \end{aligned} \tag{2.34}$$

In Fig. 2.4 we show the behaviour of the renormalization factor B_t as a function of the GUT value $\overline{y}_t$ for $\mu \simeq 172$ GeV, i.e. the top mass, as well as the top Yukawa at EW scale as a function of $\bar{y}_t$. At this scale the other factor results to be $A_t \simeq 3.532$.
Alternatively, one can also solve equation (2.25) by using the ansatz

$$y_t(\mu) = Q(\mu) \, Y_t(\mu) \tag{2.35}$$

where $Q(\mu)$ is an unknown function which has to be determined, and $Y_t(\mu)$ is the solution of the associated homogeneous equation

$$16\,\pi^2 \, \frac{dY_t}{dt} = Y_t \left(-\frac{13}{15} \, g_1^2 - 3 \, g_2^2 - \frac{16}{3} \, g_3^2 \right) = -\sum_{i=1}^{3} c_i \, g_i^2 \, Y_t \tag{2.36}$$

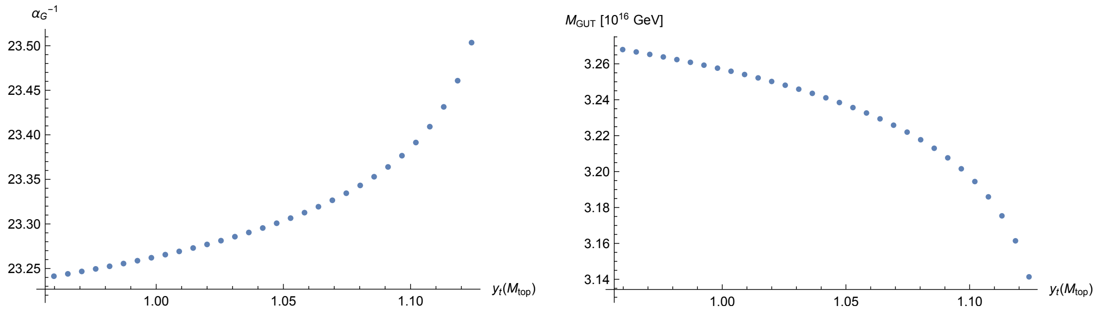


Figure 2.5: Left: Inverse α at GUT scale as a function of y_t at EW scale. Right: GUT scale as a function of y_t at EW scale.

whose solution is of course

$$\frac{Y_t(\mu)}{Y_t(\bar{\mu})} = \prod_{i=1}^{3} \left[\frac{g_i(\mu)}{g_i(\bar{\mu})}\right]^{-c_i/B_i^{(1)}} \tag{2.37}$$

which is nothing else than the $A_t(\mu)$ factor we defined in equation (2.34). Thus, by plugging (2.35) into (2.25) one gets an equation for the function $Q(\mu)$:

$$-\frac{1}{2}\,\frac{d(Q^{-2})}{dt} = \frac{6}{16\,\pi^2}\,Y_t^2 \tag{2.38}$$

which has solution

$$\frac{1}{Q^2(\bar{\mu})} - \frac{1}{Q^2(\mu)} = \frac{1}{16\,\pi^2}\int_{\ln\bar{\mu}}^{\ln\mu} 12\,Y_t^2(\mu)\,d\ln\mu \tag{2.39}$$

One can easily check that, as long as (2.39) holds, then (2.35) and (2.33) coincide.

2.2.1 Gauge Coupling Running Improved

Let us now study how much the top Yukawa coupling affects the previous two loop result. We simply solve the full coupled equations (2.22), by choosing as initial conditions for y_t different values in the range $(0.95 - 1.12)$, since as we seen in the previous section this is the range in which y_t (evaluated at EW scale) is constrained. We get the following range for the estimate of the GUT scale and the corresponding inverse α_G:

$$M_{GUT} = (3.14{\pm}0.11{-}3.27{\pm}0.12)\cdot 10^{16}\,\text{GeV}\,, \qquad \alpha_G^{-1} = (23.504{\pm}0.043{-}23.241{\pm}0.085) \tag{2.40}$$

resulting in a variation of the order $(1.7 - 5.6)\%$ with respect to the self renormalization

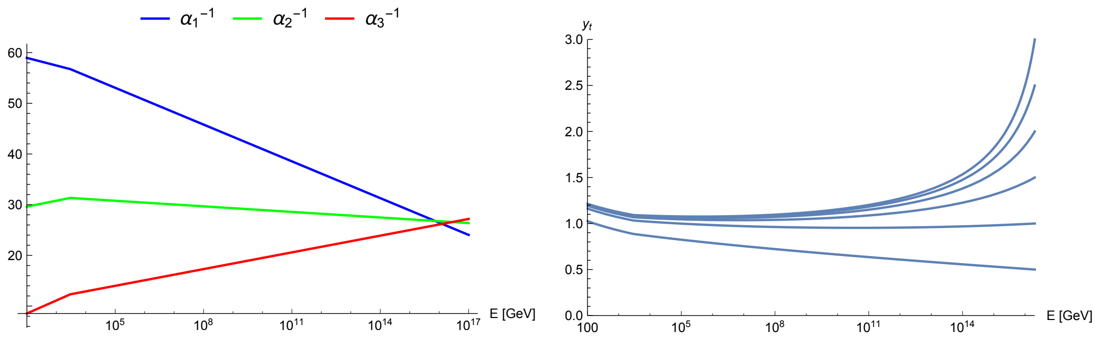


Figure 2.6: Left: Gauge coupling running for $\Lambda_S = 3$ TeV. Right: Running for y_t for different GUT scale values, from 0.5 to 3. From M_{GUT} to Λ_S we run with the supersymmetric beta, while below Λ_S we use the SM one.

case. The corresponding interval of values for α_3 at GUT scale results to be

$$\alpha_3^{-1}(M_{GUT}) = (23.94 \pm 0.13 - 24.09 \pm 0.16) \tag{2.41}$$

and we see that this is still worse than the one loop computation. We show both these results in Fig. 2.5.

2.3 Thresholds Effects

Let us now review the previous calculations by considering that MSSM beta functions (2.7) hold above a supersymmetry breaking scale [65]; below that scale we run with the usual SM renormalization group equations (2.5). Then, the value of the couplings at the breaking scale are taken as new initial conditions for the running with the supersymmetric RGE.

2.3.1 Common Threshold

We consider a common supersymmetry breaking scale for the 3 couplings of the order $\Lambda_S \simeq 3$ TeV. In this case we can proceed as in the first section by determining the GUT scale by the intersection of α_1 and α_2; this provides

$$M_{GUT} = (7.25 \pm 0.43) \cdot 10^{15}\,\text{GeV}\,, \qquad \alpha_{1,2}^{-1}(M_{GUT}) = \alpha_G^{-1} = 26.79 \pm 0.04 \tag{2.42}$$

Still, the estimated value of α_3 at GUT scale is in a worse agreement with the above value, since

$$\alpha_3^{-1}(M_{GUT}) = 25.93 \pm 0.17 \tag{2.43}$$

We show this running on the left of Fig. 2.6. Instead, below Λ_S the top Yukawa coupling obeys

$$\left.\frac{dy_t}{dt}\right|_{SM} = \frac{1}{16\,\pi^2}\,\beta^{(1)}_{y_t} = \frac{y_t}{16\,\pi^2}\left(-\frac{17}{20}\,g_1^2 - \frac{9}{4}\,g_2^2 - 8\,g_3^2 + \frac{9}{2}\,y_t^2\right) \tag{2.44}$$

while above Λ_S we recall that

$$\left.\frac{dy_t}{dt}\right|_{MSSM} = \frac{y_t}{16\,\pi^2}\left(-\frac{13}{15}\,g_1^2 - 3\,g_2^2 - \frac{16}{3}\,g_3^2 + 6\,y_t^2\right) \tag{2.45}$$

Now, we solve these equations by setting different values for y_t at GUT scale, then run back with the supersymmetric beta function to $\Lambda_S \simeq 3$ TeV, and then by using the value of y_t there as a new initial condition for a running with the SM beta function.
The results are shown on the right of Fig. 2.6; we can estimate the minimal value of $\tan\beta$ in this case by looking at the maximum value that y_t assumes at EW scale in this case. We get $y_t(\mu \simeq m_t) \lesssim 1.198$, which provides

$$\tan\beta \gtrsim 1.340 \tag{2.46}$$

2.3.2 Different Thresholds

However, we can proceed in an alternative way, by considering that the above thresholds are not equal for all the gauge couplings; this can be motivated by the following argument. We consider the one loop running for the 3 gaugino mass parameters M_i, which are given by [63]

$$\frac{dM_i}{dt} = \frac{2\,g_i^2}{16\,\pi^2}\,B_i^{(1)}\,M_i \tag{2.47}$$

where g_i are the gauge couplings and $B_i^{(1)}$ are the usual one loop beta function coefficients given by (2.7). We can solve equations (2.47) by requiring that, at GUT scale, the three parameters will be all equal to each other, and of order e.g. 1 TeV; we show the result in Fig. 2.7. Moreover, from the definition (2.9) we can write the one loop RGE for the gauge couplings in terms of α_i as

$$\frac{d\alpha_i}{dt} = \frac{B_i^{(1)}}{2\,\pi}\,\alpha_i^2 \tag{2.48}$$

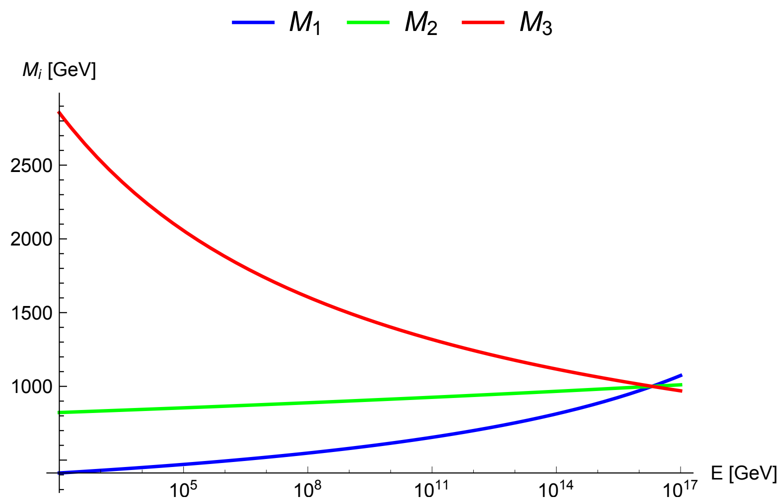


Figure 2.7: One Loop running for the gaugino mass parameters.

From (2.47) and (2.48) we can see that the ratio M_i/α_i is renorm-invariant

$$\frac{d}{dt}\left(\frac{M_i}{\alpha_i}\right) = 0 \tag{2.49}$$

so that we can use the known values of the couplings at some scale in order to get information about the gaugino mass parameters at lower energies; in particular, at EW scale we have

$$M_3 : M_2 : M_1 = 7 : 2 : 1 \tag{2.50}$$

Therefore, if at first approximation we neglect the slightly different values of M_1 and M_2 at low scale, we can study what happens if we consider 2 different supersymmetry thresholds, let's say Λ_W and Λ_S for the electroweak and strong sector, respectively. Naively, one could imagine that, if these scales are in a similar ratio as in (2.50), then the corresponding values of α_G^{-1} and α_3^{-1} at GUT scale should become again compatible. In order to see this, we first construct a table in which we calculate the GUT scale and the inverse α_G for different values of the Λ_W threshold.

Following our procedure described at the beginning of this chapter, the values of M_{GUT} and α_G will only depend on the electroweak threshold. The value of α_3 at GUT scale, instead, depends on both Λ_W and Λ_S; in the following table we show the numerical values of $\alpha_3^{-1}(M_{GUT})$ as a function of the independent thresholds, in order to see if our guess based on the above gaugino mass argument is justified.

Λ_W (TeV)	1	2	3	4
M_{GUT} (10^{15} GeV)	10.06 ± 0.60	8.18 ± 0.49	7.25 ± 0.43	6.66 ± 0.40
$\alpha_{1,2}^{-1}(M_{GUT}) = \alpha_G^{-1}$	26.01 ± 0.04	26.51 ± 0.04	26.79 ± 0.04	27.00 ± 0.04

Table 2.1: GUT scale and inverse coupling constant for different values of the electroweak supersymmetry threshold.

Λ_S (TeV) \ Λ_W (TeV)	1	2	3	4
1	25.39 ± 0.17	25.29 ± 0.17	25.24 ± 0.17	25.19 ± 0.17
2	25.83 ± 0.17	25.73 ± 0.17	25.68 ± 0.17	25.64 ± 0.17
3	26.09 ± 0.17	25.99 ± 0.17	25.94 ± 0.17	25.89 ± 0.17
4	26.27 ± 0.17	26.18 ± 0.17	26.12 ± 0.17	26.08 ± 0.17
5	26.42 ± 0.17	26.32 ± 0.17	26.26 ± 0.17	26.22 ± 0.17
6	26.53 ± 0.17	26.43 ± 0.17	26.38 ± 0.17	26.34 ± 0.17
7	26.63 ± 0.17	26.53 ± 0.17	26.47 ± 0.17	26.43 ± 0.17
8	26.72 ± 0.17	26.62 ± 0.17	26.56 ± 0.17	26.52 ± 0.17
9	26.79 ± 0.17	26.69 ± 0.17	26.63 ± 0.17	26.59 ± 0.17
10	26.86 ± 0.17	26.76 ± 0.17	26.70 ± 0.17	26.66 ± 0.17
11	26.92 ± 0.17	26.82 ± 0.17	26.76 ± 0.17	26.72 ± 0.17
12	26.97 ± 0.17	26.88 ± 0.17	26.82 ± 0.17	26.78 ± 0.17
13	27.02 ± 0.17	26.93 ± 0.17	26.87 ± 0.17	26.83 ± 0.17
14	27.07 ± 0.17	26.97 ± 0.17	26.92 ± 0.17	26.88 ± 0.17
15	27.12 ± 0.17	27.02 ± 0.17	26.96 ± 0.17	26.92 ± 0.17
16	27.16 ± 0.17	27.06 ± 0.17	27.00 ± 0.17	26.96 ± 0.17
17	27.20 ± 0.17	27.10 ± 0.17	27.04 ± 0.17	27.00 ± 0.17

Table 2.2: Inverse strong coupling constant at GUT scale for different combinations of supersymmetry thresholds.

In red we show the values which are compatible with the inverse couplings in Table 2.1, corresponding to supersymmetry thresholds that are consistent with Grand Unification, while in blue we put the results in the common threshold scenario discussed previously,

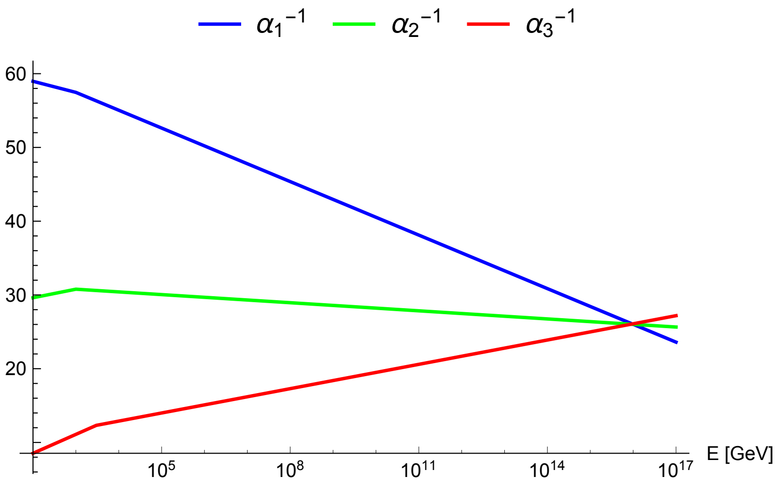


Figure 2.8: One Loop running for the inverse gauge couplings with different supersymmetry thresholds. $\Lambda_W = 1$ TeV for the electroweak sector, $\Lambda_S = 3$ TeV for the strong sector.

showing that they are further and further from being consistent with the corresponding α_G^{-1} as the EW thresholds increases.

From these calculations we see that there is room for Grand Unification with coherent values of the strong coupling constant at GUT scale if the ratio of the threshold energies is of the order of $3 - 5$. Indeed, if we evaluate (2.50) at TeV scale we get $M_3 : M_2 : M_1 \simeq 5.87 : 1.94 : 1$, which means $M_3/M_{1,2} \sim 3 - 5$.

In order to not take a too large values of Λ_S, we show as an example the quite reasonable case $\Lambda_W = 1$ TeV and $\Lambda_S = 3$ TeV in Fig. 2.8, where

$$\alpha_G^{-1} = 26.01 \pm 0.04 \qquad \text{and} \qquad \alpha_3^{-1}(M_{GUT}) = 26.09 \pm 0.17 \tag{2.51}$$

Below we complete Table 2.1 with the inverse strong coupling constant at GUT scale for the ratio $\Lambda_S/\Lambda_W \simeq 3.5$.

We see that the unification is perfectly consistent for all the values of the electroweak threshold.

2.4 Conclusions

We recalled how the gauge couplings and the (top) Yukawa coupling run under the renormalization group flow at one and two loop level. The analysis for the top Yukawa

Λ_W (TeV)	1	2	3	4
M_{GUT} (10^{15} GeV)	10.06 ± 0.60	8.18 ± 0.49	7.25 ± 0.43	6.66 ± 0.40
$\alpha_{1,2}^{-1}(M_{GUT}) = \alpha_G^{-1}$	26.01 ± 0.04	26.51 ± 0.04	26.79 ± 0.04	27.00 ± 0.04
$\alpha_3^{-1}(M_{GUT})$	26.09 ± 0.17	26.53 ± 0.17	26.73 ± 0.17	26.88 ± 0.17

Table 2.3: GUT scale and inverse coupling constant for different values of the electroweak supersymmetry threshold. The strong threshold is taken as $\Lambda_S = 3.5\,\Lambda_W$.

showed that y_t has to be strongly constrained at EW scale, which also provides a lower bound limit for $\tan\beta$; essentially models with moderate $\tan\beta \gtrsim 2$ are available, while the $\tan\beta = 1$ case is ruled out. Instead, on the gauge coupling side, we made multiple observations: the MSSM renormalization group running at two loop level make the one loop result definitely worse. We have been motivated to improve the calculation by taking into account threshold effects, where we consider a first running following the SM RGE below a supersymmetry breaking scale, and a supersymmetric model above. However, we have shown that, if this scale is the same for all the three gauge couplings, then the Grand Unification is absolutely inconsistent, since the values of α_3^{-1} at GUT scale and α_G^{-1} are in disagreement by at least 4σ. Then, by looking at the running of the gaugino mass parameters, we study the case in which the electroweak threshold is different from that of the strong coupling; we show finally that if these scales are in a similar ratio as the corresponding gaugino masses (e.g. at TeV scale) then α_3^{-1} at GUT scale and α_G^{-1} become compatible within less than 1σ; color is crucial in renormalization group running.

Chapter 3

Renormalization Group flows and Emergent Symmetries

We discuss the following proposition: Renormalization Group flow of quantum theory with a biased symmetry exhibits a fixed hypersurface at which the symmetry is exact. Such emergent symmetries may have important phenomenological implications, including supersymmetric models, gauge theories, and massive gravity.

3.1 Emergent symmetries

Ever since the seminal works by Noether [66], Weyl [67], Heisenberg [68] and Wigner [69], the role of symmetries in particle physics is ubiquitous. Symmetries provide the classification of particles, dictate conservation laws in their interactions, are instrumental in solving dynamical problems, etc. Some known symmetries have fundamental character, e.g. gauge symmetries of the Standard Model (SM). These are the precise symmetries of the Lagrangian used as postulated theoretical inputs when building the particle models. These symmetries can be broken spontaneously (by the vacuum state) but not explicitly. One also often deals with the controllable breaking of symmetries, such as spontaneous and explicit soft or anomalous breaking of symmetries. Some of the global symmetries emerge accidentally owing to the theoretical structures dictated by postulated fundamental symmetries (as e.g. baryon and lepton symmetries or isospin symmetry in the SM context) and in principle are *approximate* symmetries. Some other symmetries as conformal symmetry can be exact symmetries of the classical Lagrangian but are broken by the renormalization group flow of the dimensionless constants since the mass scale emerges due to dimensional transmutation.

Here we would like to discuss another, in our opinion important class of emergent symmetries. These are symmetries that manifest themselves only at some scales in *a priori* asymmetric theory. The central result of this chapter is the following proposition.

Consider a quantum field theory with a set of fields Φ *and parameters (coupling constants)* λ_k *which is described by an action* $S(\Phi, \lambda_k, \mu)$ *at a renormalization scale* μ*. Under the renormalization group (RG) evolution, the theory flows towards a fixed hypersurface given by* $f_i(\lambda_k) = 0$ *in the parameter space at which* β*-function of the constraint* f_i *vanishes and the theory exhibits an enhanced symmetry.*

The proof is rather straightforward. The enhanced symmetry under the constraints $f_i(\lambda_k) = 0$ on the theory parameters implies that the variation of the generating functional of the constrained theory $Z = \int D\Phi\, \delta[f_i(\lambda_k)] \exp[i\, S]$ vanishes under the symmetry transformations $\delta Z = 0$, i.e.

$$\delta S + \mathcal{A} = i \int d^d x\, c_i\, f_i(\lambda_k) \ , \tag{3.1}$$

where $\mathcal{A}$ is given by an anomalous variation of the functional measure, $\mathcal{A} = \ln \delta(D\Phi)$ and c_i are the auxiliary Lagrange multipliers that implement the constraints $f_i = 0$. This variation does not depend on the renormalization scale, that is

$$\frac{dc_i}{dt}\, f_i + c_i\, \frac{df_i}{dt} = 0 \ , \tag{3.2}$$

where $t = \ln \mu$. The last equation in turn implies

$$\beta_{f_i} = 16\, \pi^2\, \frac{df_i}{dt} = 0 \tag{3.3}$$

on the constraint hypersurface $f_i = 0$. Therefore, the constraint are fixed hypersurfaces of the RG equations.

This observation has significant implications for our understanding of the role of biased symmetries. In effect, any *a priori* asymmetric theory exhibits an emergent symmetry, providing the symmetry-enhanced hypersurface exists. The emergent symmetries are a common feature of many condensed matter systems, while remain less explored in high energy physics. We believe they can provide new insights into fundamental problems. One such is the naturalness of physical theories, which according to common lore demands some enhanced symmetries [70]. These symmetries may be emergent rather than the fundamental feature of the theory. The emergent nature of spacetime symmetries such as relativistic invariance [71] and/or supersymmetry [72, 73] are other interesting venues for exploration. Finally, gauge symmetries may also be an emergent description in some energy domain of some asymmetric (perhaps entirely unconventional) theory [72, 74].

In what follows, we discuss a few examples that illustrate our proposition. While these examples are simplistic and are set for illustration purposes only, we hope different emergent symmetries discussed below can be incorporated into more elaborate realistic physics models.

3.2 Emergent global symmetries

It is commonly believed that global symmetries are incompatible with a theory of quantum gravity. Nevertheless, some global symmetries, such as the chiral symmetry of QCD, are instrumental in understanding low energy physics. It is conceivable to think that such global symmetries are emergent.

3.2.1 Two real scalars

As a simple example, let us consider first a toy model, first discussed in [72], which involves two real scalar fields ϕ_i $(i = 1, 2)$. It exhibits two discrete symmetries under independent sign reflections $\phi_1 \to -\phi_1$ and $\phi_2 \to -\phi_2$, and in addition a discrete exchange symmetry $\phi_1 \leftrightarrow \phi_2$. The most general Lagrangian including renormalizable interactions that respect these symmetries reads:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\left(\partial_\mu \phi_i\right)^2 - \frac{\lambda}{2}\left(\phi_1^4 + \phi_2^4\right) - \lambda' \phi_1^2 \phi_2^2 \\ &= \frac{1}{2}\left(\partial_\mu \phi_i\right)^2 - \frac{\lambda}{2}\left(\phi_1^2 + \phi_2^2\right)^2 - \left(\lambda' - \lambda\right) \phi_1^2 \phi_2^2 \qquad (3.4)\end{aligned}$$

Stability conditions require $\lambda > 0$ and $\lambda' > -\lambda$.

One can easily see that three particular adjustments of couplings lead to enhanced sym-

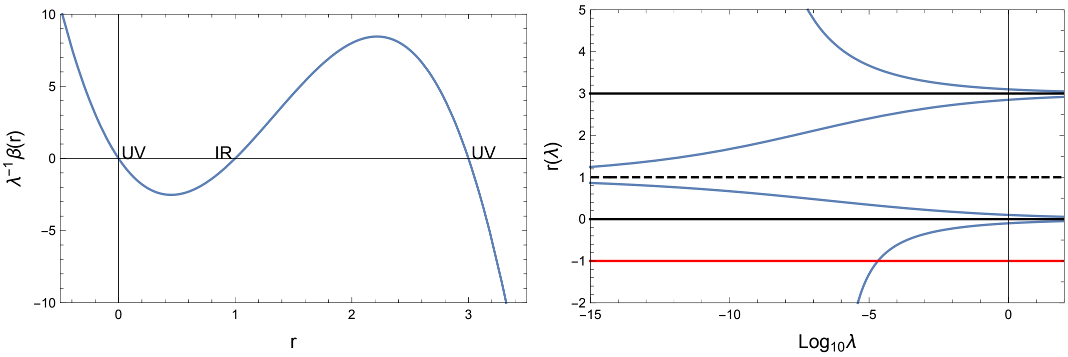


Figure 3.1: Left: Beta function for the ratio of couplings $r = \lambda'/\lambda$. Right: Running for the ratio for different initial conditions. The black solid lines correspond to the decoupling limit, while the dashed one to the enhanced symmetry line. The stability line is also represented in red.

metries in this theory. Namely, $\lambda' = 0$ and $\lambda' = 3\,\lambda$ corresponds to the decoupling limit[1], where the original Fock space branches out into two orthogonal Fock spaces with the associated doubling of symmetries (e.g., Poincare invariance). A more interesting case is $\lambda' = \lambda$ when the global $O(2)$ symmetry emerges. These limits of the theory are indeed seen in RG flows, by our proposition. The (one-loop) beta functions can be computed as:

$$\beta_{\lambda}^{(1)} = 36\,\lambda^2 + 4\,\lambda'^2\,, \qquad \beta_{\lambda'}^{(1)} = 24\,\lambda\,\lambda' + 16\,\lambda'^2 \tag{3.5}$$

For our purpose, it is sufficient to inspect the RG flow of the ratio of couplings, $r = \lambda'/\lambda$, which is governed by the following equation:

$$\frac{dr}{dt} \;=\; \frac{\beta_r^{(1)}}{(4\,\pi)^2} = \frac{1}{(4\,\pi)^2}\,\frac{\lambda\,\beta_{\lambda'}^{(1)} - \lambda'\,\beta_{\lambda}^{(1)}}{\lambda^2} = \frac{\lambda}{4\,\pi^2}\,r\,(r-1)\,(3-r)\,. \tag{3.6}$$

It is evident from Eq. (3.6) that $r = 0, 1$ and 3 are the RG fixed-points. The nature of these fixed points can be determined by inspecting the derivative of β_r near the respective fixed point. Considering the bounded from below potentials only ($\lambda > 0$ and $r > -1$), we see that $O(2)$ symmetric fixed-line $r = 1$ (i.e. $\lambda' = \lambda$) is an infrared (IR) fixed point; on the contrary, near the fixed points $r = 0$ (i.e. $\lambda' = 0$) and $r = 3$ (i.e. $\lambda' = 3\,\lambda$) $d\beta_r/dr$ is negative and hence these are UV fixed points, see Fig. 3.1 on the left. Now let us look

[1]For $\lambda' = 3\,\lambda$ the decoupling is manifest after changing the field variables: $\varphi_1 = (\phi_1 + \phi_2)/\sqrt{2}$, $\varphi_2 = (\phi_1 - \phi_2)/\sqrt{2}$.

at Fig. 3.1 on the right: these are the solutions (for different initial conditions) of the equation

$$\frac{dr}{dx} = \frac{r\,(r-1)\,(3-r)}{r^2+9}\,, \tag{3.7}$$

where $x = \ln\lambda$. This can be obtained by eliminating the $\ln\mu$ dependence from the RG equations.

As we can see, for $1 \leq r \leq 3$ at renormalization scale the theory flows towards the decoupling limit in the UV, while it reassembles itself in a global $O(2)$ symmetric theory in the IR, being $r = 1$ the enhanced symmetry line. Moreover, we see that for $r > 3$ or $r < 0$ at renormalization scale the theory still flows towards the decoupling limit, but the IR fixed lines will be no more an attractor; in the latter case we see also that the theory is no more stable, since the stability condition $r > -1$ will be sooner or later violated.

3.2.2 Two complex scalars

As a straightforward generalization, let us consider the following model for 2 complex scalar degrees of freedom ϕ_1 and ϕ_2; the most general potential consistent with a $U(1)_1 \times U(1)_2$ symmetry together with independent sign reflection and exchange $\phi_{1,2} \to \phi_{2,1}$ is

$$\begin{aligned} \mathcal{V} &= \frac{\lambda}{2}\,(\phi_1^\dagger\phi_1)^2 + \frac{\lambda}{2}\,(\phi_2^\dagger\phi_2)^2 + \lambda'\,(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\ &= \frac{\lambda}{2}\,\left(|\phi_1|^2 + |\phi_2|^2\right)^2 + (\lambda' - \lambda)\,|\phi_1|^2|\phi_2|^2 \end{aligned} \tag{3.8}$$

The one and two loop beta functions for these couplings read

$$\begin{aligned} \beta^{(1)}_{\lambda} &= 10\,\lambda^2 + 2\,\lambda'^2, & \beta^{(2)}_{\lambda} &= -60\,\lambda^3 - 10\,\lambda\,\lambda'^2 - 8\,\lambda'^3 \\ \beta^{(1)}_{\lambda'} &= 8\,\lambda\lambda' + 4\,\lambda'^2, & \beta^{(2)}_{\lambda'} &= -20\,\lambda^2\,\lambda' - 48\,\lambda\,\lambda'^2 - 10\,\lambda'^3 \end{aligned}$$

The RG flow for the ratio $r = \lambda'/\lambda$ is governed by the one loop equation

$$\frac{dr}{dt} = -\frac{\lambda}{8\,\pi^2}\,r\,(r-1)^2\,. \tag{3.9}$$

It is clear that the fixed points are $r = 0$ (corresponding to the decoupling limit) and $r = 1$ (corresponding to an enhanced global $U(2)$ symmetry). Although $r = 0$ results to be an UV fixed point (since the derivative of $\beta^{(1)}_r$ is negative) we need for a two loop contribution in order to determine the nature of the other fixed point, since it gives a null derivative

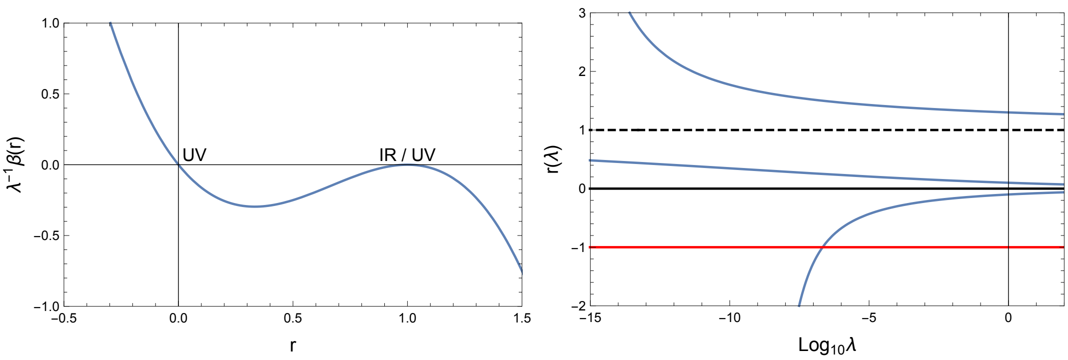


Figure 3.2: Left: Beta function for the ratio of couplings $r = \lambda'/\lambda$. Right: Running for the ratio for different initial conditions. The black solid line corresponds to the decoupling limit, while the dashed one to the enhanced symmetry line. The stability line is also represented in red.

at one loop[2]; in Fig. 3.2 we show the analogous results of the previous case. A simple calculation provides

$$\beta_r^{(2)} = 8\,\lambda^2\, r\,(r-1)\,(r^2+r-5)\,, \tag{3.10}$$

whose derivative is negative for $r = 1$, thus being an UV fixed point.
Moreover, $d\beta_r^{(2)}/dr|_{r=0} > 0$: one can notice that the two-loop contribution has the opposite behaviour with respect to the one loop case. At the perturbative level, this will not change the nature of the fixed point since this contribution will be very suppressed by the two loop phase space extra factor $16\,\pi^2$ with respect to the previous one[3].

3.2.3 Two Higgs doublets

Let us consider two complex doublets ϕ_1 and ϕ_2 of two different global symmetries $U(2)_1$ and $U(2)_2$. The general Lagrangian includes the terms

$$\begin{aligned}\mathcal{L} &= -\frac{\lambda}{2}\,(\phi_1^\dagger\phi_1)^2 - \frac{\lambda}{2}\,(\phi_2^\dagger\phi_2)^2 - \lambda'\,(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)\\ &= -\frac{\lambda}{2}\,\left(|\phi_1|^2+|\phi_2|^2\right)^2 - (\lambda'-\lambda)\,|\phi_1|^2|\phi_2|^2\end{aligned} \tag{3.11}$$

[2]Formally one can see that, for $0 < r < 1$ at renormalization scale, the theory reassembles itself into an $U(2)$ symmetric theory in the IR and flows towards the decoupling limit in the UV, while for $r > 1$ the theory is decoupled in the UV, but there is no IR fixed line.

[3]Indeed, the derivative of the full beta $\beta_r^{(1)} + \frac{1}{16\,\pi^2}\,\beta_r^{(2)}$ is still negative for $r = 0$.

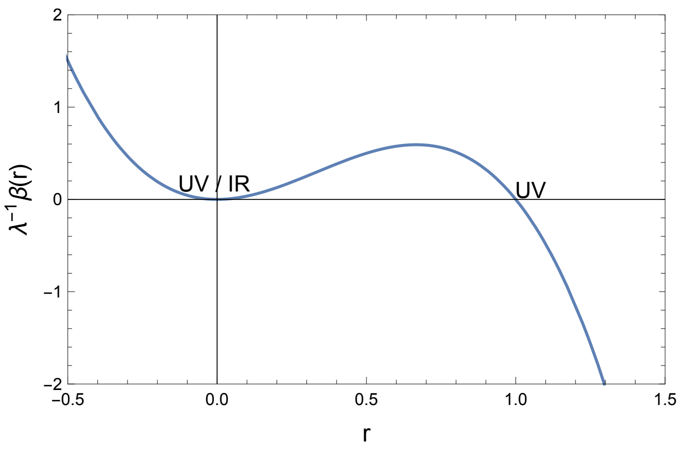


Figure 3.3: One loop beta function for the ratio of couplings for the 2 Higgs model. On the local minimum it is not possible to determine the nature of the fixed point; a higher loop level is needed.

where we again impose a discrete symmetry $\phi_1 \leftrightarrow \phi_2$ under the exchange of two $U(2)$ factors. Obviously, in the limit $\lambda' - \lambda \to 0$ this model acquires a larger symmetry $U(4)$. The one and two loop beta functions are

$$\beta^{(1)}_{\lambda} = 12\,\lambda^2 + 4\,\lambda'^2, \qquad \beta^{(2)}_{\lambda} = -78\,\lambda^3 - 20\,\lambda\,\lambda'^2 - 16\,\lambda'^3$$
$$\beta^{(1)}_{\lambda'} = 12\,\lambda\lambda' + 4\,\lambda'^2, \qquad \beta^{(2)}_{\lambda'} = -72\,\lambda^2\,\lambda' - 30\,\lambda\,\lambda'^2 - 12\,\lambda'^3$$

We can determine the nature of the fixed points by inspecting the RG flow of the ratio $r = \lambda'/\lambda$; at one loop level we get

$$\beta^{(1)}_r = -4\,\lambda\,r^2\,(r-1) \tag{3.12}$$

which has $r = 0$ and $r = 1$ as fixed points; the former corresponds to the decoupling limit. By evaluating the derivative of this beta

$$\frac{d\beta^{(1)}_r}{dr} = 4\,\lambda\,r\,(2 - 3\,r) \tag{3.13}$$

we see that it is negative for $r = 1$, being it an UV fixed point, while it is zero for $r = 0$; this means that we need to inspect the two loop corrections in order to determine the nature of the fixed point relative to the decoupling limit, see Fig. 3.3. Indeed

$$\beta^{(2)}_r = 2\,\lambda^2\,r\,(r-1)\,(8\,r^2 + 12\,r - 3)\,. \tag{3.14}$$

Evaluating the derivative of this beta with respect to r for $r = 0$ we get $d\beta_r^{(2)}/dr|_{r=0} > 0$; it is an IR fixed point. In summary, there is an UV attractive fixed point corresponding to an emergent global $U(4)$ symmetry.

3.3 Emergent supersymmetry

Supersymmetry is the unique non-trivial extension of Poincare invariance, which provides important insights into several theoretical and phenomenological aspects of high energy physics, such as strongly coupled theories or stability of hierarchies of scales. Therefore, it is interesting to study systems that exhibit emergent supersymmetry.

3.3.1 Emergent Wess-Zumino model

A theory, for having a possible supersymmetric realization, must contain an equal amount of fermionic and bosonic degrees of freedom. A simple system that exhibits this property is a Higgs-Yukawa model with a complex scalar field ϕ and a 2-component Weyl spinor ψ. The most generic Lagrangian containing only renormalizable interactions reads[4]:

$$\mathcal{L} = |\partial_\mu \phi|^2 + i\,\psi^\dagger\,\bar{\sigma}^\mu\,\partial_\mu \psi - \lambda\,(\phi^*\phi)^2 - y(\phi\,\psi^2 + \text{h.c.}) \tag{3.15}$$

where the Yukawa coupling constant y can be made real by a phase transformation of the fields. For $\lambda = y^2$ this theory exhibits an enhanced spacetime symmetry, the (on-shell) $\mathcal{N} = 1$ supersymmetry, when the Lagrangian can be obtained from the superpotential $W = (y/3)\,\Phi^3$ with Φ being a chiral superfield containing ϕ and ψ. We did not include in the Lagrangian (3.15) the mass term. The fermion mass term $m\,\psi^2$ is forbidden by $U(1)$ symmetry which in supersymmetric context corresponds to $U(1)_R$ symmetry. The scalar mass term $\mu^2\phi^\dagger\phi$ can be interpreted as a soft supersymmetry breaking term.
Let us examine RG flows of couplings, which are governed (at one loop) by the following β-functions:

$$\beta_y^{(1)} = 6\,y^3, \qquad \beta_\lambda^{(1)} = 20\,\lambda^2 + 8\,\lambda\,y^2 - 16\,y^4. \tag{3.16}$$

[4]For the sake of brevity, we choose to enforce an unbroken global $U(1)$ symmetry: $\phi \to e^{2\,i\,\alpha}\,\phi$, $\psi \to e^{-i\,\alpha}\,\psi$.

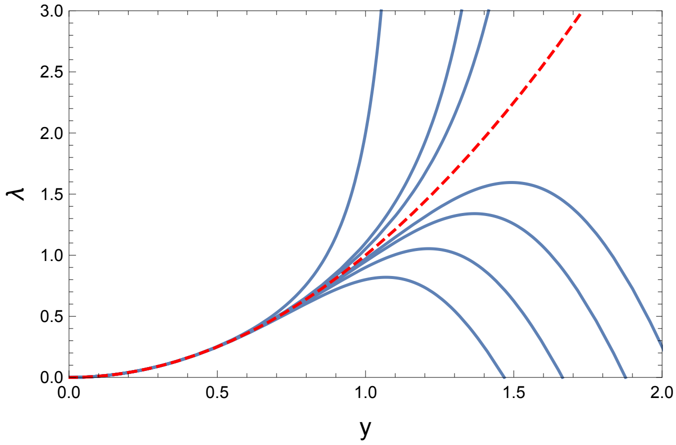


Figure 3.4: RG flows towards $\mathcal{N}=1$ supersymmetric IR fixed-line $\lambda = y^2$ (red dashed) in the Wess-Zumino model.

As in the previous section, it is sufficient to inspect the evolution of the ratio, $r = \lambda/y^2$. At one loop, we get

$$\frac{\beta_r^{(1)}}{16\,\pi^2} = \frac{dr}{dt} = \frac{y^2}{4\pi^2}\,(r-1)\,(4+5r)\,. \tag{3.17}$$

In the stable domain ($\lambda > 0$), the model indeed exhibits $\mathcal{N}=1$ supersymmetric fixed-line $\lambda = y^2$ (i.e. $r=1$), together with the decoupling fixed-point $y=0$. By taking the derivative of $\beta_r^{(1)}$ in (3.17) we get

$$\frac{d\beta_r^{(1)}}{dr} = 4\,y^2\,(10\,r-1)\,. \tag{3.18}$$

Evaluating this expression in $r=1$, we get a positive value. Therefore, the supersymmetric fixed-line is an IR attractor (see also Fig. 3.4).

Now we calculate also two-loop contributions:

$$\begin{aligned}\beta_y^{(2)} &= 4\,y^5 - 32\,y^3\,\lambda + 4\,y\,\lambda^2,\\ \beta_\lambda^{(2)} &= -240\,\lambda^3 - 80\,\lambda^2\,y^2 + 16\,\lambda\,y^4 + 256\,y^6.\end{aligned} \tag{3.19}$$

In the supersymmetric limit $\lambda = y^2 \equiv a$, one has

$$\beta_\lambda = 2\,y\,\beta_y\,. \tag{3.20}$$

This remains satisfied even at two-loop level: indeed, one obtains a unique beta function

$$\beta_\lambda^{(2)} = 2\,y\,\beta_y^{(2)} = -48\,a^6\,. \tag{3.21}$$

The Lagrangian (3.15) can be presented as

$$\mathcal{L} = \mathcal{L}(\Phi)_{\mathrm{WZ}} + \bar{\lambda}\,[(\Phi^\dagger\Phi)^2]_D\,, \tag{3.22}$$

where the first term corresponds to the supersymmetric Lagrangian of the Wess-Zumino model with $\Phi(\phi,\psi)$ being a chiral superfield, i.e. Lagrangian (3.15) with the scalar quartic coupling taken as $\lambda = y^2$. The second term can be considered as a (hard) supersymmetry breaking D-term. For β-function of corresponding coupling constant $\bar{\lambda} = \lambda - y^2$ we get

$$\begin{aligned} \beta^{(1)}_{\bar{\lambda}} &= \bar{\lambda}\,(48\,y^2 + 20\,\bar{\lambda}) \\ \beta^{(2)}_{\bar{\lambda}} &= -\bar{\lambda}\,(816\,y^4 + 808\,\bar{\lambda}\,y^2 + 240\,\bar{\lambda}^2) \end{aligned} \tag{3.23}$$

We see that supersymmetry breaking D-term in (3.22) disappears at low energies: $\bar{\lambda} = 0$ is an IR fixed point at two-loop level.

3.3.2 Emergent $\mathcal{N} = 1$ supersymmetric gauge theory

Here we consider a few models with gauge symmetries: first, we focus on a Yang-Mills theory, showing there the possibility of having an IR emergent supersymmetry; second, we emphasize that one cannot achieve the same situation if the starting point is a model with a $U(1)$ gauge symmetry only.

$SU(2)$ gauge model

Let us take a pure Yang-Mills model (with a gauge symmetry e.g. $SU(2)$ for simplicity) complemented by a two-component fermion χ in adjoint (triplet) representation, so that the number of bosonic and fermionic degrees of freedom are equal. The Lagrangian of this theory reads:

$$\mathcal{L}_0 = -\frac{1}{4}\,G_{a\,\mu\nu}\,G^{\mu\nu}_a + i\,\chi^\dagger\,\bar{\sigma}^\mu\,D_\mu\chi \tag{3.24}$$

where $D_\mu = \partial_\mu - i\,g\,A^a_\mu\,T_a$ is a covariant derivative, with T_a $(a = 1,2,3)$ being generators of $SU(2)$ in respective representation. Hence, this model contains only a gauge coupling constant g, with $\beta^{(1)}_g = -6\,g^3$.

This model automatically exhibits exact $\mathcal{N} = 1$ supersymmetry, as a consequence of the gauge symmetry: Lagrangian (3.24) describes $\mathcal{N} = 1$ super Yang-Mills theory. The mass term $\frac{m}{2}\big(\chi^2 + \mathrm{h.c.}\big)$, if any, can be interpreted as a soft supersymmetry breaking term.

Next, we consider a model that includes gauge interactions and multiple couplings. We add matter species as a Weyl fermion ψ and a scalar ϕ both in a doublet representation of $SU(2)$, so that the numbers of fermion and boson degrees of freedom are again equal. (In fact, such a toy theory is ill-defined since it has global $SU(2)$ anomaly [75], but we consider it first for the sake of simplicity). The most general interaction Lagrangian, besides the gauge interactions (3.24), contains the following terms with dimensionless coupling constants:

$$\mathcal{L} \;=\; -\frac{\lambda}{8}\left(\phi^\dagger\phi\right)^2 - \frac{1}{\sqrt{2}}\left(y\,\phi^\dagger\,\tau^a\,\psi\,\chi^a + \text{h.c.}\right) \tag{3.25}$$

where τ^a $(a = 1, 2, 3)$ are the Pauli matrices which act, in the second term, between the doublets ϕ and ψ. The Yukawa constant y can be rendered real and positive by the phase transformations. The vacuum stability condition implies $\lambda > 0$ for the quartic scalar coupling.

The one-loop beta functions of this model read:

$$\begin{aligned} &\beta_g^{(1)} = -\frac{11}{2}\,g^3\,, \qquad\qquad \beta_y^{(1)} = \frac{11}{4}\,y^3 - \frac{33}{4}\,g^2\,y\,,\\ &\beta_\lambda^{(1)} = 9\,g^4 - 9\,g^2\,\lambda - 20\,y^4 + 6\,y^2\,\lambda + 3\,\lambda^2\,. \end{aligned} \tag{3.26}$$

So beta functions of ratios $\bar{y} \equiv y/g$ and $\bar{\lambda} \equiv \lambda/g^2$ are

$$\begin{aligned} \beta_{\bar{y}}^{(1)} &= \frac{11}{4}\,g^2\,\bar{y}\,(\bar{y}^2 - 1)\\ \beta_{\bar{\lambda}}^{(1)} &= g^2\left(3\,\bar{\lambda}^2 + 2\,\bar{\lambda} + 6\,\bar{\lambda}\,\bar{y}^2 - 20\,\bar{y}^4 + 9\right) \end{aligned} \tag{3.27}$$

which exhibit the following fixed-line:

$$|\bar{y}| = \bar{\lambda} = 1 \tag{3.28}$$

It can be readily checked that this fixed line is an IR stable. The RG evolution is shown on the left of Fig. 3.5.

Equivalently, beta functions of the differences $\bar{y} \equiv y - g$ and $\bar{\lambda} \equiv \lambda - g^2$ are

$$\begin{aligned} &\beta_{\bar{y}}^{(1)} = \frac{11}{4}\,\bar{y}^2(3\,g + \bar{y})\\ &\beta_{\bar{\lambda}}^{(1)} = 3\,\bar{\lambda}\,(g^2 + 4\,g\,\bar{y} + 2\,\bar{y}^2 + \bar{\lambda}^2) - 2\,\bar{y}\,(34\,g^3 + 57\,g^2\,\bar{y} + 40\,g\,\bar{y}^2 + 10\,\bar{y}^3)\,. \end{aligned} \tag{3.29}$$

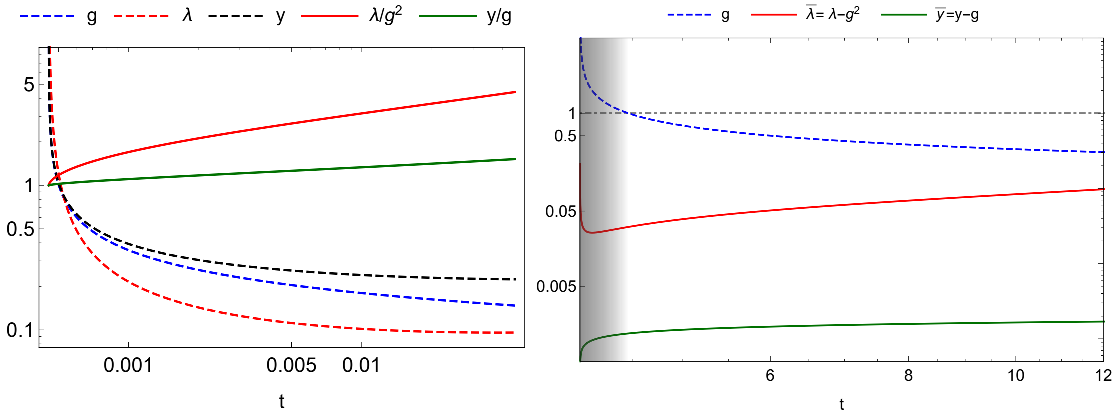


Figure 3.5: Left: IR convergence of the ratios of coupling constants of Lagrangian (3.25). Right: Running for both $\bar{\lambda} = \lambda - g^2$ and $\bar{y} = y - g$ along the perturbative range. The coupling $\bar{\lambda}$ starts to grow only in the perturbatively unreliable zone, denoted by the shaded area.

It follows that the values $\tilde{y} = \bar{\lambda} = 0$ are zeros of the betas. Notice that the determinant of the matrix

$$M_{ij} := \frac{d\beta^{(1)}_{g_i}}{dg_j} \quad \text{with} \quad g_i = (\bar{y}, \bar{\lambda}) \tag{3.30}$$

is zero on that fixed point. Therefore, at this level this point is not attractive, nor repulsive. The behavior of these couplings is on the right of Fig. 3.5. By the way, this is strongly dependent on the initial conditions for the renormalization group equations, in contrast with what happens for the ratios discussed above.

Using the Fierz identities for the Pauli matrices, Lagrangian (3.25) can be rewritten as

$$\mathcal{L} = -\frac{\bar{\lambda}\, g^2}{8}(\phi^\dagger \tau^a \phi)^2 - \frac{\bar{y}\, g}{\sqrt{2}}\left(\phi^\dagger\, \tau^a\, \psi\, \chi^a + \text{h.c.}\right) , \tag{3.31}$$

which shows that in the limit (3.28), i.e. $y = g$, $\lambda = g^2$, the theory tends to (on-shell) $\mathcal{N} = 1$ supersymmetric $SU(2)$ gauge theory of a vector supermultiplet (A^a_μ, χ^a) and chiral supermultiplet (ϕ, ψ) interacting via supergauge interactions with a coupling constant g.

Let now us generalize what discussed above by considering a model with a pair of doublets (ϕ_i, ψ_i), $i = 1, 2$. The Yukawa couplings, without losing generality, can be taken diagonal

$$-\frac{y_1}{\sqrt{2}}\, \phi_1^\dagger\, \tau^a\, \psi_1\, \chi^a - \frac{y_2}{\sqrt{2}}\, \phi_2^\dagger\, \tau^a\, \psi_2\, \chi^a + \text{h.c.} \tag{3.32}$$

with constants $y_{1,2}$ being real and positive.

The most general scalar potential reads

$$\mathcal{V}(\phi_1,\phi_2) = \frac{\lambda_1}{8}\,(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{8}\,(\phi_2^\dagger\phi_2)^2 - \frac{\lambda_3}{4}\,(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \frac{\lambda_4}{2}\,(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \tag{3.33}$$

$$+\frac{1}{4}\left[\lambda_5\,(\phi_1^\dagger\phi_2)^2 + \lambda_6\,(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7\,(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \text{h.c.}\right] \tag{3.34}$$

The constant λ_5 can be taken real; moreover, for the sake of simplicity, we can impose a discrete sign change symmetry $\phi_{1,2} \leftrightarrow -\phi_{1,2}$ together with the exchange symmetry $\phi_1 \leftrightarrow \phi_2$, to set $\lambda_1 = \lambda_2 = \lambda$, $\lambda_6 = \lambda_7 = 0$ and $y_1 = y_2 = y$ in the previous Lagrangians. These further assumptions will not affect our considerations.

One-loop beta functions are

$$\beta_g^{(1)} = -5\,g^3 \tag{3.35}$$

$$\beta_y^{(1)} = -\frac{33}{4}\,g^2\,y + \frac{13}{4}\,y^3 \tag{3.36}$$

and

$$\beta_\lambda^{(1)} = 9\,g^4 - 9\,g^2\,\lambda - 20\,y^4 + 6\,y^2\,\lambda + 3\,\lambda^2 + \lambda_3^2 - 2\,\lambda_3\,\lambda_4 + 2\,\lambda_4^2 + 2\,\lambda_5^2 \tag{3.37}$$

$$\beta_{\lambda_3}^{(1)} = -\,9\,g^4 - 9\,g^2\,\lambda_3 + 4\,y^4 + 6\,y^2\,\lambda_3 - \lambda_3^2 + \lambda\,(3\,\lambda_3 - 2\,\lambda_4) - 2\,\lambda_4^2 - 2\,\lambda_5^2 \tag{3.38}$$

$$\beta_{\lambda_4}^{(1)} = -\,9\,g^2\,\lambda_4 - 8\,y^4 + 6\,y^2\,\lambda_4 + 2\,\lambda_4^2 + \lambda\,\lambda_4 - 2\,\lambda_3\,\lambda_4 + 4\,\lambda_5^2 \tag{3.39}$$

$$\beta_{\lambda_5}^{(1)} = \lambda_5\,\left(6\,y^2 - 9\,g^2 + \lambda - 2\,\lambda_3 + 6\,\lambda_4\right)\,. \tag{3.40}$$

Defining the ratios $\bar{y} = y/g$, $\bar{\lambda}_i = \lambda_i/g^2$, the relative beta functions are

$$\beta_{\bar{y}}^{(1)} = \frac{13}{4}\,g^2\,\bar{y}\,\left(\bar{y}^2 - 1\right) \tag{3.41}$$

$$\beta_{\bar{\lambda}}^{(1)} = g^2\,\left(\bar{\lambda}_3^2 - 2\,\bar{\lambda}_3\,\bar{\lambda}_4 + 2\,\bar{\lambda}_4^2 + 3\,\bar{\lambda}^2 + \bar{\lambda} - 20\,\bar{y}^4 + 6\,\bar{\lambda}\,\bar{y}^2 + 9\right) \tag{3.42}$$

$$\beta_{\bar{\lambda}_3}^{(1)} = g^2\,\left(-\bar{\lambda}_3^2 + 3\,\bar{\lambda}_3\,\bar{\lambda} + \bar{\lambda}_3 - 2\,\bar{\lambda}_4\,(\bar{\lambda}_4 + \bar{\lambda}) + 4\,\bar{y}^4 + 6\,\bar{\lambda}_3\,\bar{y}^2 - 9\right) \tag{3.43}$$

$$\beta_{\bar{\lambda}_4}^{(1)} = g^2\,\left(\bar{\lambda}_4\,(-2\,\bar{\lambda}_3 + 2\,\bar{\lambda}_4 + \bar{\lambda} + 1) - 8\,\bar{y}^4 + 6\,\bar{\lambda}_4\,\bar{y}^2\right)\,, \tag{3.44}$$

which are null for $\bar{y} = \bar{\lambda}_i = 1$. The derivatives of $(\beta_{\bar{y}}, \beta_{\bar{\lambda}}, \beta_{\bar{\lambda}_3}, \beta_{\bar{\lambda}_4})$ with respect of $(\bar{y}, \bar{\lambda}, \bar{\lambda}_3, \bar{\lambda}_4)$ at the fixed point reads

$$\begin{pmatrix} 13\,g^2/2 & 0 & 0 & 0 \\ -68\,g^2 & 13\,g^2 & 0 & 2\,g^2 \\ 28\,g^2 & g^2 & 8\,g^2 & -6\,g^2 \\ -20\,g^2 & g^2 & -2\,g^2 & 10\,g^2 \end{pmatrix}\,, \tag{3.45}$$

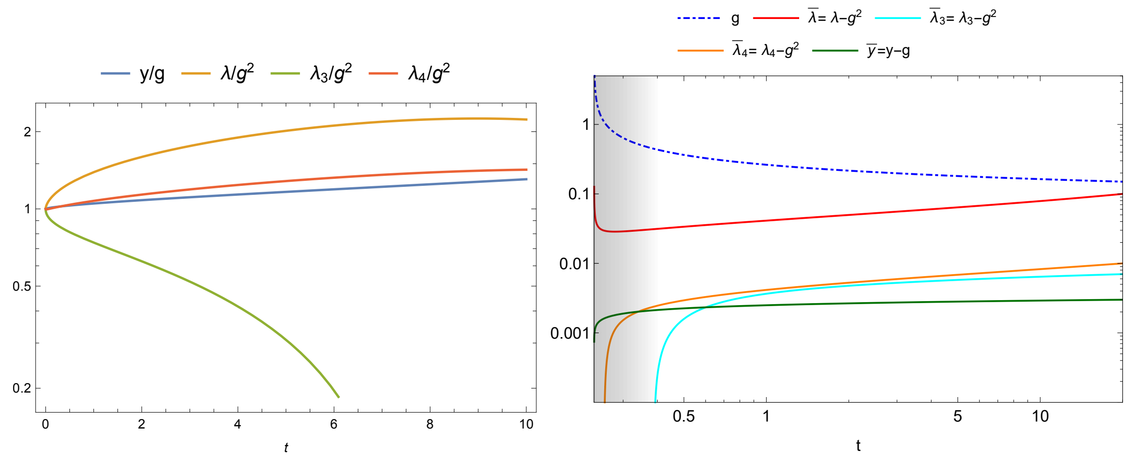


Figure 3.6: Left: Running for the ratios of coupling constants of Lagrangian (3.33). Right: Running for the differences. The shaded area denotes the perturbatively unreliable zone.

which has positive determinant, meaning that the fixed point is IR attractive.

We show in Fig. 3.6 (on the left) the running for the ratios. Moreover, in complete analogy with the discussion on the model with less coupling in (3.25), we also plot the differences $y-g$, $\lambda_i - g^2$ $(i=1,2,3,4)$ in Fig. 3.6 (on the right).

$U(1)$ gauge model

As anticipated in the beginning of this subsection, the case of an abelian gauge theory does not share with the Yang-Mills case the feature of having an attractive fixed point, corresponding to an enhanced supersymmetry. In what follows, we explicitly show this point.

Consider a $U(1)$ gauge-invariant theory with a pair of two-component fermions ψ_1 and ψ_2, a pair of scalar fields ϕ_1 and ϕ_2 which carry $U(1)$ charges $q_{\psi_1}=q_{\phi_1}=-q_{\psi_2}=-q_{\phi_2}=1$ and a zero-charge two-component fermion ξ. The interaction Lagrangian of the model reads:

$$\mathcal{L}=-\left(\frac{\lambda_1}{2}|\phi_1|^4+\frac{\lambda_2}{2}|\phi_2|^4+\lambda_3|\phi_1|^2|\phi_2|^2\right)-\sqrt{2}\,(y_1\,\xi\,\psi_1\,\phi_1^*+y_2\,\xi\,\psi_2\,\phi_2^*)$$
$$+\text{ gauge interaction terms }. \tag{3.46}$$

The one-loop beta functions for this model are:

$$\beta_g^{(1)} = 2\,g^3 \tag{3.47}$$

$$\beta_{y_1}^{(1)} = y_1\left(-3\,g^2 + 4\,y_1^2 + y_2^2\right) \tag{3.48}$$

$$\beta_{y_2}^{(1)} = y_2\left(-3\,g^2 + y_1^2 + 4\,y_2^2\right) \tag{3.49}$$

$$\beta_{\lambda_1}^{(1)} = 2\left(6\,g^4 - 6\,g^2\,\lambda_1 + 5\,\lambda_1^2 + \lambda_3^2 - 8\,y_1^4 + 4\,\lambda_1\,y_1^2\right) \tag{3.50}$$

$$\beta_{\lambda_2}^{(1)} = 2\left(6\,g^4 - 6\,g^2\,\lambda_2 + 5\,\lambda_2^2 + \lambda_3^2 - 8\,y_2^4 + 4\,\lambda_2\,y_2^2\right) \tag{3.51}$$

$$\beta_{\lambda_3}^{(1)} = 4\left(3\,g^4 - 4\,y_1^2\,y_2^2\right) + 4\,\lambda_3\left(-3\,g^2 + \lambda_1 + \lambda_2 + y_1^2 + y_2^2\right) + 4\,\lambda_3^2 \tag{3.52}$$

We focus on the running of ratios of couplings: $\bar{y}_1 = y_1/g$, $\bar{y}_2 = y_2/g$, $\bar{\lambda}_1 = \lambda_1/g^2$, $\bar{\lambda}_2 = \lambda_2/g^2$, $\bar{\lambda}_3 = \lambda_3/g^2$, whose beta functions read

$$\beta_{\bar{y}_1}^{(1)} = g^2\,\bar{y}_1\left(4\,\bar{y}_1^2 + \bar{y}_2^2 - 5\right) \tag{3.53}$$

$$\beta_{\bar{y}_2}^{(1)} = g^2\,\bar{y}_2\left(\bar{y}_1^2 + 4\,\bar{y}_2^2 - 5\right) \tag{3.54}$$

$$\beta_{\bar{\lambda}_1}^{(1)} = 2\,g^2\left(5\,\bar{\lambda}_1^2 - 8\,\bar{\lambda}_1 + \bar{\lambda}_3^2 - 8\,\bar{y}_1^4 + 4\,\bar{\lambda}_1\,\bar{y}_1^2 + 6\right) \tag{3.55}$$

$$\beta_{\bar{\lambda}_2}^{(1)} = 2\,g^2\left(5\,\bar{\lambda}_2^2 - 8\,\bar{\lambda}_2 + \bar{\lambda}_3^2 - 8\,\bar{y}_2^4 + 4\,\bar{\lambda}_2\,\bar{y}_2^2 + 6\right) \tag{3.56}$$

$$\beta_{\bar{\lambda}_3}^{(1)} = 4\,g^2\left(\bar{\lambda}_3^2 + \bar{y}_1^2\left(\bar{\lambda}_3 - 4\,\bar{y}_2^2\right) + \bar{\lambda}_3\left(\bar{\lambda}_1 + \bar{\lambda}_2 + \bar{y}_2^2 - 4\right) + 3\right) \tag{3.57}$$

The possible zeros are

$$|\bar{y}_1| = |\bar{y}_2| = 1\,, \quad \bar{\lambda}_1 = \bar{\lambda}_2 = |\bar{\lambda}_3| = 1 \tag{3.58}$$

$$|\bar{y}_1| = |\bar{y}_2| = 1\,, \quad \bar{\lambda}_1 = \bar{\lambda}_2 = \frac{7}{15}\,, \quad \bar{\lambda}_3 = \frac{5}{3} \tag{3.59}$$

Consider now the matrix of the derivatives

$$M_{ij} := \frac{d\beta_{g_i}^{(1)}}{dg_j} \quad \text{with} \quad g_i = (\bar{y}_1, \bar{y}_2, \bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3)\,.$$

Eq. (3.59) gives negative eigenvalues for M_{ij} (so IR unstable).

Inside Eq. (3.58), the only subcase giving positive eigenvalues for M_{ij} (thus being a IR stable fixed point) is

$$|\bar{y}_1| = |\bar{y}_2| = 1\,, \quad \bar{\lambda}_1 = \bar{\lambda}_2 = \bar{\lambda}_3 = 1\,, \tag{3.60}$$

while the eigenvalues have discord signs when, in the latter, one replaces $\bar{\lambda}_3 = -1$.

The latter would describe a fixed-line of the RG flow at which the theory exhibits an $\mathcal{N} = 1$ on-shell supersymmetry $-$ the $U(1)$ gauge field and the neutral fermion ξ are combined into $\mathcal{N} = 1$ gauge supermultiplet which couples to two chiral supermultiplets (ϕ_1, ψ_1) and (ϕ_2, ψ_2) via gauge interactions. However, it is neither UV attractive nor IR attractive due to the above signs discordance. The case with concord signs represents an emergent global $U(2)$ symmetry.

3.4 Emergent gauge symmetry

All the known fundamental interactions rely on the principle of local gauge invariance. This is the only known consistent and manifestly Lorentz-invariant description of quantum fields carrying spin ≥ 1. Yet, one may entertain the possibility that gauge theories are also emergent. This may be particularly true for gravity, for which the usual notions of locality and smooth spacetime manifolds fail at short scales.

3.4.1 Emergent $U(1)$ gauge theory

As a simple example of emergent local gauge theory consider a vector field A_μ coupled to a current j_μ. The theory is described by the following Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu A^\mu)^2 - \frac{g}{2}(\partial_\mu A_\nu)^2 - g' j_\mu \, A^\mu \ , \tag{3.61}$$

where $g(\mu)$ and $g'(\mu)$ are dimensionless running parameters defined at the renormalization scale μ. It is convenient to decompose the 4-vector potential as:

$$A_\mu = a_\mu + \frac{1}{\Lambda}\partial_\mu \phi \ , \tag{3.62}$$

where a_μ is a divergenceless 4-vector field, $\partial_\mu a^\mu = 0$, ϕ is a scalar field and Λ is an arbitrary parameter of mass dimension 1, which is introduced to measure ϕ in units of Λ. After rescaling the fields, $a_\mu \to \sqrt{g}\, a_\mu$ and $\phi \to \sqrt{g-1}\, \phi$, the Lagrangian (3.61) takes the form:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu a_\nu)^2 - \frac{g'}{\sqrt{g}} \, j_\mu \, a^\mu - \frac{1}{2\Lambda^2}\phi \, \Box^2 \, \phi + \frac{g'}{\Lambda \sqrt{g-1}} \phi \, \partial_\mu j^\mu \ . \tag{3.63}$$

The first two terms alone describe the usual quantum theory of a massless Abelian gauge field in the Lorenz gauge coupled to the current with the strength $g'/\sqrt{g}$. The last two terms describe the scalar field with a fourth-order derivative kinetic term, which thus carries two remaining degrees of freedom of the generic (non-gauge) 4-vector field. These degrees of freedom are removed if $g = 1$, the case where the theory becomes manifestly gauge invariant.

To show that $g = 1$ is a fixed point of the theory, we first note that any diagram, with ϕ in the internal legs, is finite, due to the $1/k^4$ behavior of the ϕ-propagator. In particular, $\phi - j_\mu$ coupling receives only finite corrections, and hence is independent of the renormalization scale, i.e.:

$$\frac{g'(\mu)}{\sqrt{g(\mu) - 1}} = \text{const.} \tag{3.64}$$

On the other hand, the $a_\mu j^\mu$ coupling is known to have the trivial fixed point at IR, i.e.:

$$\frac{g'(\mu)}{\sqrt{g(\mu)}} \stackrel{\mu \to 0}{\longrightarrow} 0. \tag{3.65}$$

The equations (3.64) and (3.65) then imply:

$$g(\mu) \stackrel{\mu \to 0}{\longrightarrow} 1 \ , \tag{3.66}$$

that is, the theory asymptotically becomes gauge invariant in the infrared. In the gauge invariant limit (3.66), longitudinal and ghost degrees of freedom become non-dynamical, and ϕ in the last term of Eq. (3.63) serves as a Lagrange multiplier field, which enforces the 4-current conservation, $\partial_\mu j^\mu = 0$.

3.4.2 Pauli-Fierz flow in generic massive spin-2 theory

The above discussion can be applied also to emergent non-linearly realized gauge symmetries. As an interesting example consider the diffeomorphism invariant linearized theory of the spin-2 field with the addition of a generic mass term:

$$\mathcal{L} \ = \ -\frac{1}{2}\, m^2 \left(h_{\mu\nu} h^{\mu\nu} - \alpha\, h^2 \right) + \beta\, h_{\mu\nu} T^{\mu\nu} + \text{gauge invariant terms} \ , \tag{3.67}$$

where $h = h^\mu_\mu$. For $\alpha = 1$ this theory exhibits non-linearly realized gauge invariance (linearized diffeomorphisms), owing to which there are five propagating degrees of freedom of massive spin-2 field [76]. For $\alpha \neq 1$ the theory is not gauge invariant and an additional scalar ghost degree of freedom appears in the spectrum. To see that the asymmetric theory indeed flows towards the $\alpha = 1$ Pauli-Fierz theory, let us split the tensor field into the gauge invariant transverse and traceless tensor $h^{\rm TT}_{\mu\nu}$ and the vector field ξ_μ:

$$h_{\mu\nu} = h^{\rm TT}_{\mu\nu} + \frac{1}{\Lambda} \partial_{(\mu} \xi_{\nu)} \ . \tag{3.68}$$

The Lagrangian (3.67) then becomes:

$$\mathcal{L} \;=\; -\frac{m^2}{\Lambda^2}\,(\partial_\mu \xi_\nu)^2 + \frac{\alpha}{2}\,\frac{m^2}{\Lambda^2}\,(\partial_\mu \xi^\mu)^2 - \frac{\beta}{\Lambda}\,\xi_\nu\,\partial_\mu T^{\mu\nu} + \text{ gauge invariant terms .}$$

The part of this Lagrangian involving the vector field ξ_μ is identical to the one in Eq. (3.61). Hence, applying the previous analysis, we conclude that the linearized massive spin-2 theory flows towards the ghost-free Pauli-Fierz theory in IR.

3.5 Outlook

The key result of this work is the proposition according to which emergent symmetries are directly related to RG fixed hypersurfaces in the parameter space of *a priori* asymmetric theories. We have illustrated the proposition with many simple models with emergent global symmetry, emergent supersymmetry, and emergent gauge symmetry. We would like to believe that these toy models can be expanded to realistic physical theories that address some important phenomenological problems.

The radiative stability of some measured parameters in particle physics and cosmology, most notably of the electroweak scale (i.e. the Higgs mass) and the cosmological constant, is believed to require certain (albeit approximate) symmetries at respective scales. The prime candidates for such symmetries are supersymmetry and scale invariance. It would be interesting to contemplate whether the relevant symmetries that ensure the radiative stability of these parameters are emergent rather than a fundamental feature of the theory. Emergent symmetries could potentially provide an important technical tool for addressing physical problems beyond the perturbative level. For example, several aspects of the dynamics in the strongly coupled regime can be understood within supersymmetric theories, while the phenomenologically relevant theory, Quantum Chromodynamics, does not expose such symmetry at the fundamental level. Would it be possible to understand the QCD confinement via emergent symmetries in the strongly coupled regime?

Finally, one may think of the gauge symmetries themselves, and most notably the diffeomorphism invariance of gravity, as emergent symmetries. Needless to say, that progress in any of these directions may result in a paradigm-changing discovery.

Chapter 4

Natural Axion from Gauge Family Symmetry

We develop a fermion mass model based on a gauge abelian horizontal symmetry. Once spontaneously broken, an accidental anomalous $U(1)_{PQ}$ is naturally set in the theory. What we want to show is that, without imposing anything more by hand, the corresponding axion is also of high quality, being the full potential compatible with the experimental bounds on the neutron electric dipole moment.

4.1 Introduction

One of the most intriguing challenge in physics beyond the Standard Model is the origin of the fermion mass and mixing textures. A typical approach to this issue is the introduction of a family, or horizontal, symmetry, whose spontaneously breaking at very high scale is triggered by the vacuum expectation value of some field Φ, usually called flavon. Then, the observed mass hierarchy between the fermion generations can be reproduced via the Froggatt-Nielsen (FN) mechanism [77, 78, 79, 80, 81], by integrating out a certain number of heavy vector-like fields, of mass scale M, named messengers; hierarchical pattern will be explained by suppressions of the form $(\langle\Phi\rangle/M)^n$, where n depends on the value of the horizontal charges of the fields. As a minimal extension of the MSSM able to generate a phenomenologically viable Yukawa textures, it turns out that the simplest $U(1)$ family group does the job, although it would be required to be anomalous [82, 83, 84, 85].

We consider here the possibility to have an abelian $U(1)$ horizontal symmetry, which is gauge; this implies that, by consistency, we have to introduce 2 independent flavon fields, whose vevs ratio will be used as expansion parameters instead of the old $\langle\Phi\rangle/M$. In this way the cutoff mass scale becomes dynamical, and can be chosen not so far with respect to the other, avoiding so an eventual extra hierarchy problem; moreover, the gauge nature of the FN symmetry can be the key to evade quantum gravity corrections which explicitly break global symmetries [86, 87, 88]. As a result, none of the 2 phases associated with the flavons will be physical; indeed, a specific linear combination of them will propagate as a physical degree of freedom, that is our axion, while the orthogonal one will be absorbed by the $U(1)$ gauge boson through an Higgs mechanism, providing it a mass term. We recall that, on the contrary of global symmetries, gauge symmetries cannot be explicitly broken, none anomalous; one can see that the MSSM spectrum induces gauge anomalies when charged under an anomalous $U(1)$ symmetry in the context of string compactifications. This model was widely used in the literature, where the flavon field get its vacuum expectation value from anomalous Fayet-Iliopoulos D-term, which should be close to reduced Planck/String scale [82, 89, 83, 84, 90, 91, 92], so that we need an extra fermionic spectrum or a Green-Schwartz (GS) mechanism in order to make it consistent [93]. One way to proceed is to add chiral spectator fermions at the family symmetry breaking scale; as a result, we do not need to introduce heavy vector-like fields to generate flavour hierarchies, since the ones which take care of anomalies, if chiral under family symmetry, are sufficient. We will discuss the case of an anomaly free $U(1)$ symmetry, by introducing two scalars, σ anf Φ, where one of the vacuum expectation values defines the cutoff scale of the FN operators, which can be lower than Planck scale; in this case the expansion parameter will be of the form $\langle\sigma\rangle/\langle\Phi\rangle$.

4.2 Two Scalars Model

In this section we investigate the scalar sector containing the flavon fields and then we show how to reproduce the observed mass hierarchy between the families by using these extra scalars and something else.

4.2.1 The flavon sector

Let us introduce 2 scalar fields σ and Φ, which are both charged under the horizontal $U(1)$ symmetry with charges q and Q respectively. Let us also assume that both these fields are singlets under $SU(5)$.

The horizontal symmetry will break after both σ and Φ acquire a non zero vacuum expectation value, therefore we can write

$$\sigma = \frac{v_1}{\sqrt{2}}\, e^{i\,\varphi_1/v_1}, \qquad\qquad \Phi = \frac{v_2}{\sqrt{2}}\, e^{i\,\varphi_2/v_2} \tag{4.1}$$

with $v_1/v_2 \equiv \varepsilon \sim 1/20$, which will be the expansion parameter we use to well reproduce the mass hierarchy between the fermion families. Moreover, it will turn out that only a particular combination of the phases φ_1 and φ_2 can propagate as a physical axion, while the orthogonal combination will have a vanishing kinetic term. In order to see this, let us consider that, under the gauge $U(1)$ symmetry, the 2 phases transform according with

$$\frac{\varphi_1}{v_1} \to \frac{\varphi_1}{v_1} + \alpha(x)\, q, \qquad\qquad \frac{\varphi_2}{v_2} \to \frac{\varphi_2}{v_2} + \alpha(x)\, Q \tag{4.2}$$

with $\alpha(x)$ gauge parameter. We then can eliminate, let's say, φ_2 by fixing a gauge in which $\alpha = -\varphi_2/Q\, v_2$; in this case the kinetic term has the form

$$\mathcal{L}_{kin} = |\partial_\mu \sigma|^2 + |\partial_\mu \Phi|^2 \tag{4.3}$$

can be diagonalized by using an orthogonal matrix to get

$$|\partial_\mu\, \sigma|^2 = \frac{1}{2\, \cos^2\theta}\, \partial_\mu\, \tilde{\varphi}_1\, \partial^\mu\, \tilde{\varphi}_1 \tag{4.4}$$

where

$$\tilde{\varphi}_1 = \cos\theta\, \varphi_1 + \sin\theta\, \varphi_2 \tag{4.5}$$

is the physical axion, while

$$\sin^2\theta = \frac{q^2\, v_1^2}{q^2\, v_1^2 + Q^2\, v_2^2}, \qquad \cos^2\theta = \frac{Q^2\, v_2^2}{q^2\, v_1^2 + Q^2\, v_2^2} \tag{4.6}$$

The orthogonal combination $\tilde{\varphi}_2 = \cos\theta\, \varphi_2 - \sin\theta\, \varphi_1$, as said before, will not propagate, since it is the eigenstate of the kinetic matrix corresponding to the eigenvalue zero.

A classical consequence of introducing an axion-like particle is the possible solution to the strong CP problem, that is, the smallness of the $\overline{\theta}$ parameter (constrained by the current bounds on the neutron electric dipole moment)

$$\overline{\theta} < 10^{-10} \tag{4.7}$$

If we have some scalar field ϕ whose vev breaks the $U(1)$ symmetry, then due to the non-renormalizability of gravity we have to include in the Lagrangian higher order operators like

$$\lambda\, e^{-i\,\delta}\, \frac{\phi^n}{M_{pl}^{n-4}} + \text{h.c.} \tag{4.8}$$

After the spontaneous breaking of $U(1)$, the axion potential reads

$$V(a) = -\Lambda^4 \cos\left(\frac{a}{f_a}\right) + 2\,\lambda \left(\frac{f_a}{M_{pl}}\right)^{n-4} f_a^4 \cos\left(n\,\frac{a}{f_a} - \delta\right) \tag{4.9}$$

where Λ is the QCD scale, and f_a is the axion decay constant, which is constrained from astrophysical and cosmological bounds to be of order

$$10^{10}\ \text{GeV} < f_a < 10^{12}\ \text{GeV} \tag{4.10}$$

We find that, in order for $\overline{\theta} = \langle a \rangle / f_a$ to be consistent with (4.7), we should require $n \gtrsim 12$. This constraint will come back when we will discuss anomalies, and it will provide an important consistency check on the operators we can build from the flavon fields.

4.2.2 The fermion sector

Before developing the Yukawa sector for fermion masses and mixings, let us very briefly recall how to fit the SM fields into representations of $SU(5)$, in order to fix the notation we will use in the following.

In the context of $SU(5)$ the known particles fit into an anti-fundamental and an antisymmetric representation

$$\overline{5}_i^{\alpha} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_i, \qquad 10_{\alpha\beta,\,i} = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}_i \tag{4.11}$$

where $\alpha, \beta = 1, \ldots, 5$ are $SU(5)$ indices, while $i = 1, 2, 3$ is a family index. Together with these there are of course the 2 Higgses $H \sim 5$ and $\overline{H} \sim \overline{5}$, which contain the SM Higgs doublet and an heavy color triplet

$$5_H = (T_1, T_2, T_3, H^+, H^0)^t \tag{4.12}$$

Since the horizontal symmetry we introduced is chiral, none of the Standard Model fermion can get a mass term, because it would not be $U(1)$ invariant. They can arise after the spontaneous breaking of the horizontal symmetry, starting from an effective theory whose Yukawa sector can be written in terms of the higher order operators

$$\mathcal{L} = a_{ij}^{(u)}\, 10_i \cdot 10_j \cdot H \cdot \left(\frac{\sigma}{\Phi}\right)^{n_{ij}^{(u)}} + a_{ij}^{(d)}\, \overline{5}_i \cdot 10_j \cdot \overline{H} \cdot \left(\frac{\sigma}{\Phi}\right)^{n_{ij}^{(d)}} + \frac{a_{ij}^{(\nu)}}{\Lambda}\, \overline{5}_i \cdot \overline{5}_j \cdot H^2 \cdot \left(\frac{\sigma}{\Phi}\right)^{n_{ij}^{(\nu)}} \tag{4.13}$$

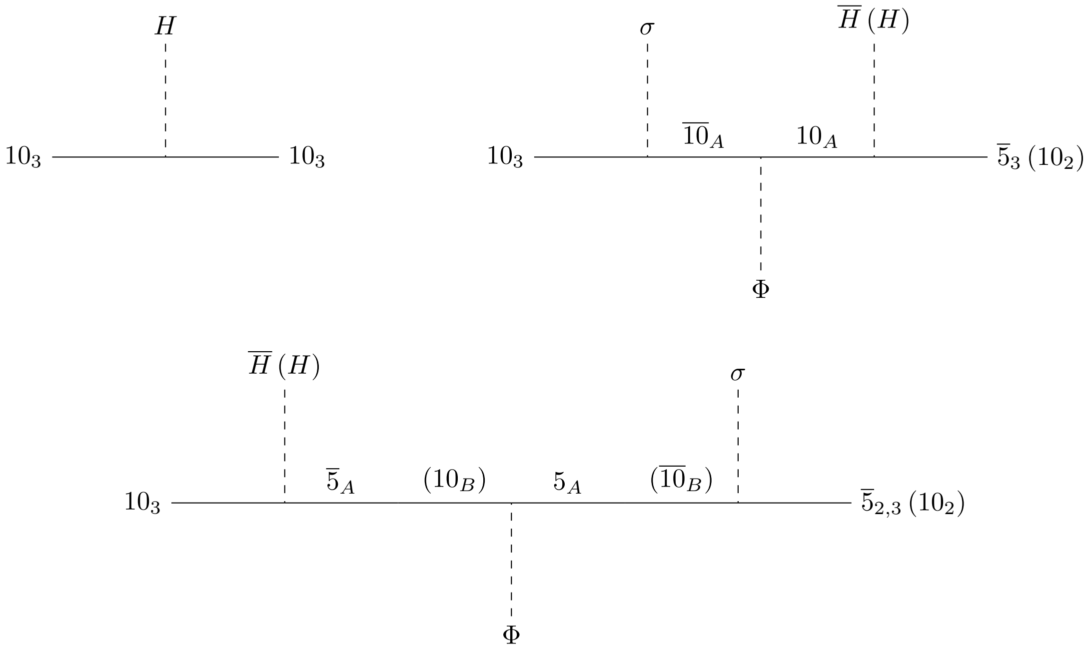
with $i, j = 1, 2, 3$ family indices and a_{ij} order one constants. The exponents n_{ij} depend on the horizontal charges of the fermions, which are generation dependent, resulting in a not diagonal coupling between the axion and the Standard Model fermions.

With our convention on the vacuum expectation values of the flavon fields, we observe that the mass hierarchy can be reproduced by effective Yukawa couplings of the form

$$Y_u \propto \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix}, \qquad Y_d \propto \begin{pmatrix} \varepsilon^3 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon & \varepsilon & \varepsilon \end{pmatrix} \tag{4.14}$$

which provide $y_t : y_c : y_u \simeq 1 : \varepsilon^2 : \varepsilon^4$ and $y_b : y_s : y_d \simeq 1 : \varepsilon : \varepsilon^2$. Indeed, (4.14) fix the values of the exponents in (4.13).

Those mass terms arise in the UV complete theory from integrating out some heavy fermions, analogous to the Froggatt-Nielsen messengers; however, in contrast with the original FN mechanism, in this case these heavy fermions are not vector-like under the $U(1)$ symmetry. This is due to the fact that in the former case we would have an expansion term of the form $(\sigma/M)^n$, with M being the mass scale of the integrated out FN messengers, while in our model this scale is replaced by a second flavon field, which couples to the heavy states and provides a mass term after the horizontal symmetry breaking. We find that the number of heavy families needed for the most general treatment is 4, which we denote as $(5 + \overline{5})_k$ and $(10 + \overline{10})_k$, with $k = A, B, C, D$ family index. Below we show some examples of diagrams in which these heavy messengers are involved.



Invariance under $U(1)$ of all the possible interactions determines the value of the horizontal charges of the SM fermions and flavons, as well as those of the heavy messengers; our results are summarized in the following tables:

Field	H	$\overline{H}$	σ	Φ	10_1	10_2	10_3	$\overline{5}_1$	$\overline{5}_2$	$\overline{5}_3$
$U(1)$ charge	0	0	q	Q	$2\,(Q-q)$	$Q-q$	0	$Q-q$	$Q-q$	$Q-q$

Field	10_A	$\overline{10}_A$	10_B	$\overline{10}_B$	10_C	$\overline{10}_C$	10_D	$\overline{10}_D$
$U(1)$ charge	$q-Q$	$-q$	0	$-Q$	$Q-q$	$q-2\,Q$	$2\,(q-Q)$	$Q-2\,q$

Field	5_A	$\overline{5}_A$	5_B	$\overline{5}_B$	5_C	$\overline{5}_C$	5_D	$\overline{5}_D$
$U(1)$ charge	$-Q$	0	$-q$	$q-Q$	$Q-2\,q$	$2\,(q-Q)$	$q-2\,Q$	$Q-q$

In the UV complete theory the heavy fermions couple to the SM fields and to themselves via the following Lagrangian

$$
\begin{aligned}
\mathcal{L}^{\text{heavy}} = \; & 10_A\,\overline{10}_A\,\Phi + 10_B\,\overline{10}_B\,\Phi + 10_C\,\overline{10}_C\,\Phi + 10_D\,\overline{10}_D\,\Phi + 5_A\,\overline{5}_A\,\Phi + 5_B\,\overline{5}_B\,\Phi \\
& + 5_C\,\overline{5}_C\,\Phi + 5_D\,\overline{5}_D\,\Phi + \overline{10}_A\,10_3\,\sigma + \overline{10}_B\,10_2\,\sigma + \overline{5}_1\,5_A\,\sigma + \overline{5}_2\,5_A\,\sigma + \overline{5}_3\,5_A\,\sigma \\
& + 10_B\,\overline{10}_A\,\sigma + 10_A\,\overline{10}_D\,\sigma + 10_C\,\overline{10}_B\,\sigma + \overline{10}_C\,10_1\,\sigma + 5_B\,\overline{5}_A\,\sigma + 5_A\,\overline{5}_D\,\sigma \\
& + \overline{5}_1\,5_D\,\Phi + \overline{5}_2\,5_D\,\Phi + \overline{5}_3\,5_D\,\Phi + 5_C\,\overline{5}_B\,\sigma + 10_A\,10_2\,H + 10_B\,10_B\,H + 10_3\,10_B\,H \\
& + 10_D\,10_1\,H + 10_C\,10_A\,H + \overline{5}_1\,10_A\,\overline{H} + \overline{5}_2\,10_A\,\overline{H} + \overline{5}_3\,10_A\,\overline{H} + \overline{5}_A\,10_3\,\overline{H} + \overline{5}_B\,10_2\,\overline{H} \\
& + \overline{5}_A\,10_B\,\overline{H} + \overline{5}_D\,10_A\,\overline{H} + \overline{5}_C\,10_1\,\overline{H} + \overline{5}_B\,10_C\,\overline{H} + \overline{10}_B\,10_3\,\Phi + \overline{10}_C\,10_2\,\Phi
\end{aligned}
\tag{4.15}
$$

with order 1 constants understood in front of each term. Solving the algebraic equations of motion for the heavy fields leads to the form (4.13).

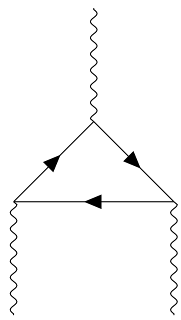
Let us come back briefly to the last term of (4.13) to discuss neutrino masses. In this lepton number violating term we can consider Λ as a singlet of $U(1)$ to get the following structure in ε for the Yukawa matrix

$$
Y_\nu \propto \begin{pmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \end{pmatrix} \tag{4.16}
$$

where we adopted a democratic approach since all the $\overline{5}$'s have the same horizontal charge.

4.3 Anomalies

Due to the fact that the horizontal symmetry we have introduced is also a gauge symmetry, we must require for it to be anomaly free. Therefore we have to look at triangle diagrams of the form



where at each vertex there could be either $SU(5)$ or $U(1)$ gauge bosons. Anomaly cancellation can be viewed as a requirement for vanishing d symbols

$$d^{abc} = \mathrm{Tr}\left(T_R^a \{T_R^b, T_R^c\}\right) = 0 \tag{4.17}$$

where T_R^a are the generators of the group in some representation R. It is well known that the $SU(5)^3$ anomaly cancels, since although for $\overline{5}$ and 10 representations separately we have a non zero anomaly coefficient, there exist the key result

$$A(\overline{5}) = -A(10) \tag{4.18}$$

which makes the total anomaly vanishing. So we have to look at mixed $SU(5) - U(1)$ anomalies, as well as the cubic $U(1)$ one.
Of course, as for any $SU(N)$ group, the mixed $SU(N) - U(1)^2$ anomaly always cancels, due to the tracelessness nature of the generators of $SU(N)$, so we are left with $U(1)^3$ and $SU(5)^2 - U(1)$ anomalies. By imposing that these vanish we will get important relations between the horizontal charges q and Q, as we will see.

4.3.1 $U(1)^3$ anomaly

The cancellation of this anomaly is equivalent to require that

$$\sum_i \dim(R_i) \cdot q^3(R_i) = 0 \tag{4.19}$$

for each representation R_i with horizontal charge $q(R_i)$; the sum extends of course over all the light $\overline{5}$ and 10, as well as over the heavy messengers. If we only look at the light degrees of freedom we get a contribution of $140\,(Q-q)^3$; in order to cancel that anomaly one could for sure think about introducing a certain number of extra fermions which must be $SU(5)$ singlets, in order to balance the contribution coming from the heavy messengers. Alternatively, and more elegantly, one can avoid this artifact and think about an hidden mirror sector [94, 95, 96, 97, 98], originally introduced in order to restore parity as a fundamental symmetry. In this context, for our particles being left handed (as described within the Standard Model) mirror sector is described by exactly the same physics for right handed particles, i.e. another copy of the Standard Model; if we denote with G the SM gauge group (or whatever extension of it like $SU(5)$, $SO(10)$ and so on) then the full theory will be described by the identical gauge factors $G \times G'$ with the identical particle contents, where the prime refers to mirror sector [99, 100]. Moreover, a discrete interchange symmetry between corresponding fields in the 2 sectors, so called *mirror parity*, implies that both particle sectors are described by the same Lagrangians, as well as all coupling constants have the same pattern, so that they have the same microphysics. In our case the hidden sector will have its own $SU(5)'$, that is also anomaly free, and fermions will have exactly opposite horizontal charges with respect to the ours, simply by mirror parity; this automatically account for the $U(1)^3$ anomaly cancellation. Also, in the mirror world hypothesis, the mirror baryon component emerges as a possible dark matter candidate, with specific cosmological implications [101, 102, 103, 104, 105, 106, 107]. For example, this hypothesis will introduce 2 new scales in the structure formation scenario, that are the Jeans scale of the mirror photon-baryon fluid and the Silk damping scale of mirror baryons [108, 109]: perturbations in the mirror fluid cannot grow before mirror photon decoupling, and if this occurs after the matter-radiation equality epoch, if mirror baryons are dark matter then one expects to see less structures on small scales with respect to the standard CDM scenario.

4.3.2 $SU(5)^2 - U(1)$ anomaly

The relation between the anomaly coefficients of $\overline{5}$ and 10 can be derived in this case in a quick way, by considering the symmetry breaking channel $SO(10) \to SU(5) \times U(1)_X$, where X is some abelian charge which is somehow related to $B - L$, with assignments (see [110, 111, 112] for more details)

$$\begin{aligned} 16 &= 1\,(5) + \overline{5}\,(-3) + 10\,(1) \\ 10 &= 5\,(-2) + \overline{5}\,(2) \end{aligned} \tag{4.20}$$

One can show that indeed this is anomaly free, so that the $\overline{5}$ and 10 anomaly coefficients are related by

$$A(10) = 3\,A(\overline{5}) \tag{4.21}$$

so that we have to require that

$$3 \sum_i q(10_i, \overline{10}_i) + \sum_i q(5_i, \overline{5}_i) = 0 \tag{4.22}$$

If we account for all the light fields and the set of heavy messengers, then we get

$$Q = -\frac{13}{3}\,q \tag{4.23}$$

This consistency condition can be used in the study of terms like (4.8) in our model; that is the cross term

$$\frac{\sigma^n\,\Phi^m}{M_{pl}^{n+m-4}} \tag{4.24}$$

which is $U(1)$ invariant only if $n\,q + m\,Q = 0$, and by using (4.23) we get

$$\frac{n}{m} = -\frac{Q}{q} = \frac{13}{3} \tag{4.25}$$

Being all integer numbers, anomaly cancellation implies that this kind of operator will be at least of order 16, and following the discussion made in the previous section this will not ruin the constraint (4.7).

4.4 Gauge coupling unification revised

In this section we briefly review the well known calculation for the gauge coupling unification in the Standard Model and in the MSSM; then we include our full set of heavy FN messengers above the axion scale to see what we get. The main idea is that the introduction of such heavy fields has not to break the asymptotic freedom of QCD, so that we should guarantee that the $SU(3)$ gauge coupling will not have a Landau pole before the Grand Unification scale.

If we denote the gauge couplings as g_i, $i = 1, 2, 3$, we can write the renormalization group equations for the parameters $\alpha_i = g_i^2/4\,\pi$ as

$$\frac{d}{dt}\,\alpha_i(t) = \frac{B_i^{(1)}}{2\,\pi}\,\alpha_i^2(t) \tag{4.26}$$

with $t = \ln(E/\mu)$ and μ renormalization scale. The coefficients $B_i^{(1)}$ for $U(1)$, $SU(2)$ and $SU(3)$ respectively are given by (see Chapter 2)

$$B_i^{(1)} = \left(\frac{4}{3}\,N_f + \frac{1}{10}\,N_H,\ \frac{4}{3}\,N_f + \frac{1}{6}\,N_H - \frac{22}{3},\ \frac{4}{3}\,N_f - 11\right) \tag{4.27}$$

with N_f families and N_H Higgs doublets. Solutions to (4.26) can be written as

$$\alpha_i(E) = \frac{\alpha_i(\mu)}{1 + \frac{B_i^{(1)}}{2\,\pi}\,\alpha_i(\mu)\,\ln\left(\frac{E}{\mu}\right)} \tag{4.28}$$

We deeply discussed this stuff in Chapter 2; in Fig. 4.1 we recall the standard running for SM and MSSM, respectively.

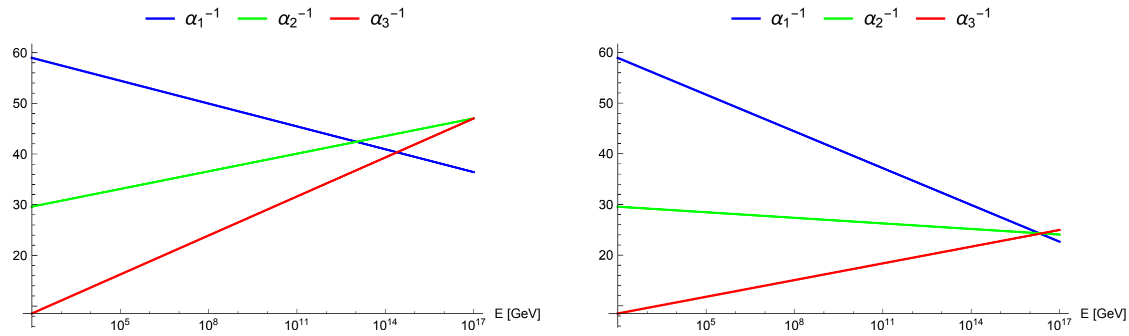


Figure 4.1: One Loop running for inverse gauge couplings. Left: Standard Model. Right: Minimal Supersymmetric Standard Model.

Now we can move on our model with the full set of FN messengers. These heavy species become relevant above the family symmetry breaking scale $v_2 \sim 10^{12}$ GeV. From the knowledge of the family symmetry breaking scale, one can obtain the PQ symmetry breaking scale v_1, which in turn is related to the axion decay constant f_a by the color anomaly factor [47]. In our case, $v_1/v_2 \sim 20$, while in order to get the axion decay constant we have to divide the PQ scale by the color anomaly factor, which we will see is of order 10; this means we are considering models with $f_a \gtrsim 10^{10}$ GeV.

Due to the extra degrees of freedom, from that scale the coefficients (4.27) will change with respect to the known results we gave in Chapter 2 since the number of Dirac fermions changes; what we get is

$$B^{(1)}_{i,\,SM} = \left(\frac{443}{30}, \frac{15}{2}, \frac{11}{3}\right) \tag{4.29}$$

and

$$B^{(1)}_{i,\,MSSM} = \left(\frac{113}{5}, 17, 13\right) \tag{4.30}$$

Although we find a similar value for the estimate of the GUT scale, in this scenario the supersymmetric model features a breakdown of the asymptotic freedom of QCD, unless the axion constant becomes unnaturally big, while for the non supersymmetric one we have a regular behavior of α_3. We show our results in Fig. 4.2.

As we can see, all the gauge couplings would blow up before reaching the GUT scale; this is due to the fact that maybe we have introduced too much heavy families to approach the mass hierarchy problem, and it could be that the complete set of messengers we adopted is not the minimal one we need. We explore this possibility in the next section.

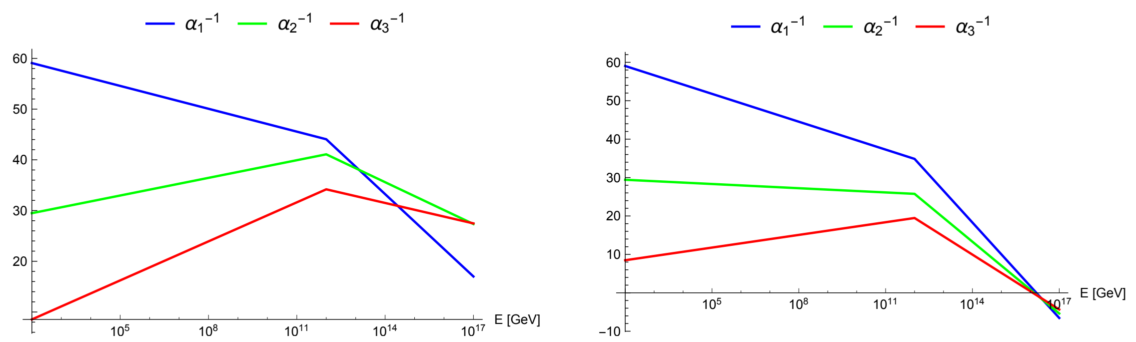


Figure 4.2: Gauge coupling running with heavy messengers contribution. Left: Standard Model running. Right: Minimal Supersymmetric Standard Model. In both cases the family symmetry scale is set to $v_2 \simeq 10^{12}$ GeV.

4.5 Can we use less messengers?

It turns out that, despite the full general UV theory (4.15), we could have achieved our goal with the introduction of only $3\,(10+\overline{10})+2\,(5+\overline{5})$. This is the very minimal set of FN messengers we need. The main differences in our discussion when less messengers are considered regard the anomalies cancellation and the gauge coupling unification; indeed, due to the different number of fermions participating in the theory, both (4.23) and the beta functions will change. We should check that also in this case the consistency conditions we put from the beginning will hold.

4.5.1 Anomalies

With the new set of messengers, of course the cubic $U(1)$ anomaly will always cancel for the exact same reasons we gave before. What we have to do is simply cancel the mixed $SU(5)^2-U(1)$ anomaly, by using (4.22) with the new fermion content. That equation provides

$$\frac{Q}{q}=\frac{12}{12-3\,n_{10}-n_{\overline{5}}}=12 \tag{4.31}$$

where 12 is the contribution in anomaly of the SM fermions, and n_{10} and $n_{\overline{5}}$ are the number of heavy messengers in representations 10 and $\overline{5}$ respectively. This has an important

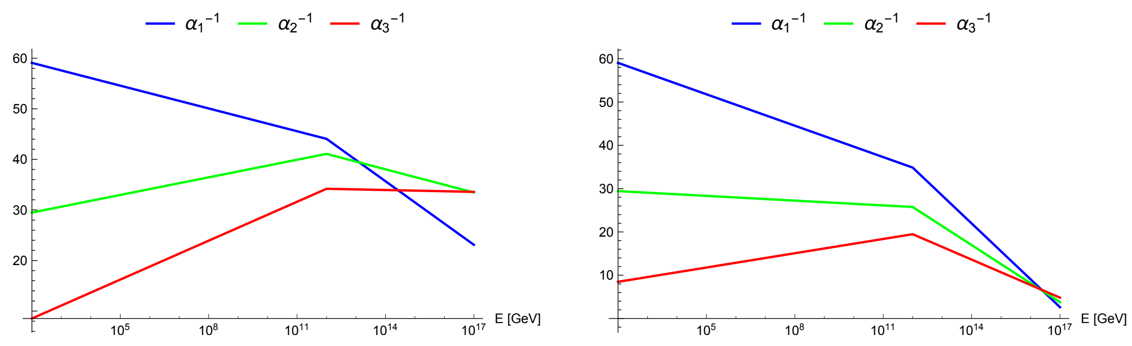


Figure 4.3: Gauge coupling running with heavy messengers contribution in the minimal scenario. Left: Standard Model running. Right: Minimal Supersymmetric Standard Model. In both cases the family symmetry scale is set to $v_2 \simeq 10^{12}$ GeV.

implication: the first higher order cross term operator we could write will be

$$\mathcal{L} = \lambda \frac{\sigma^{12}\,\Phi^\dagger}{M_{pl}^9} + \text{h.c.} \tag{4.32}$$

which has enough quality in order to be compatible with (4.7).

4.5.2 Gauge coupling unification

Of course, also the running of the gauge coupling will change, since we now have essentially half heavy Dirac fermions with respect to before. The new values of the beta function coefficients are

$$B^{(1)}_{i,\,SM} = \left(\frac{343}{30}, \frac{25}{6}, \frac{1}{3}\right) \tag{4.33}$$

and

$$B^{(1)}_{i,\,MSSM} = \left(\frac{88}{5}, 12, 8\right) \tag{4.34}$$

In Fig. 4.3 we show the new running of the gauge couplings. We see that in this case we are able to respect the asymptotic freedom of QCD, and also prevent whatever Landau pole for the gauge couplings before reaching the GUT scale; all this by taking a very reasonable value for the axion constant, while for the previous case we had problems with the supersymmetric theory unless a tuning for $f_a > 10^{12}$ GeV, a little bit in constrast with the observational constraints (4.10).

This discussion leads us to prefer this minimal scenario, instead of a full general UV theory.

Conclusions

Interesting aspects of physics beyond the Standard Model have been explored. We saw how the Grand Unified picture of the fundamental interactions is strongly linked to Supersymmetry, and in particular how the scale of SUSY breaking plays a crucial role; in this work I have only considered the possibility that the electroweak sector and the strong sector may have different thresholds, based on an argument concerning the running of gaugino masses, showing that at TeV scale the gluino mass parameter is slightly higher than the other two. Taking this difference into account allows for a surprisingly high quality unification for the gauge couplings. Of course this picture can be further improved: for example, here all the supersymmetric partners have been considered to have almost the same mass, resulting in a unique threshold for each coupling. One could consider the contribution of each superpartner separately in the calculation of all the beta functions in order to make smoother and smoother the behavior of the inverse couplings shown in Chapter 2. Another improvement can be done by considering low scale fragments from the spontaneous breaking of the Grand Unification group, similarly to what happened at the end of Chapter 4 in the case of the gauge horizontal symmetry; in these cases some heavy fermions will give a contribution to the beta function, resulting in a radical change of the gauge coupling running, and also in this case one can consider this 'heavy fermions threshold' as unique or not. Phenomenological observations such as proton decay and neutrino masses can help to give constraints on these intermediate scales in order to protect the Grand Unification, that is, to allow that the couplings meet before one of them blows up, see for example Fig. 4.2.

Two Loop effects are expected to be more and more suppressed with respect to the ones I discussed.

The role of accidental, or emergent symmetries discussed in Chapter 3 was at the origin of what I developed in the last chapter, that is, a model for fermion masses and mixing based on an abelian gauge horizontal symmetry; embedded in a Grand Unified context, the spontaneous breaking of that family symmetry provides a generation dependent Yukawa couplings for the low energy theory via

$$SU(5) \times U(1)_H \to SU(5) \to SU(3) \times SU(2) \times U(1)$$

The aim of that work was to address the family problem as well as the strong CP problem by a 'natural axion' solution, that is, a mechanism similar to the Peccei-Quinn one where the axion emerges accidentally, without imposing any ad hoc global symmetry. In the case I discussed the axion emerges as a pseudo Goldstone boson from the spontaneously breaking of a gauge symmetry, which has fundamental character. This was possible since the abelian horizontal symmetry, if local, need for 2 independent flavon fields in order to become non trivial, in order that a particular combination of their phases emerge as an axion; also here phenomenological observations help us to constrain some parameter of the model, in particular in the axion potential, which will have to be consistent with the upper bound on the θ parameter

$$\overline{\theta} \lesssim 10^{-10}$$

The local nature of this symmetry moreover implies that all the anomalies must cancel, and this reflects again on the parameters of the axion potential; what we saw in Chapter 4 is that the constraints arising from anomaly cancellation naturally satisfied the ones from neutron electric dipole moment observation, that is, they are naturally consistent with an axion solution to the strong CP problem. This is usually refereed to as an *high quality* axion.

Of course also this field can be explored more and more by considering different horizontal groups, whose specific symmetry breaking pattern will have implications on the flavour violating coupling of the resulting axion, as well as on the picture of Grand Unification depending on all the intermediate scales are treated in the modified beta functions.

Appendix A

Poincaré group and Weyl spinors

Poincaré group corresponds to the symmetries of special relativity, which includes Lorentz transformations as well as spacetime translations; it acts on the spacetime coordinates as

$$x^\mu \to \Lambda^\mu_{\ \nu}\, x^\nu + a^\mu \tag{A.1}$$

where the Lorentz transformation $\Lambda \in SO(3,1)$ leaves the metric invariant

$$\Lambda_\mu^{\ \alpha}\, \eta_{\alpha\beta}\, \Lambda^\beta_{\ \nu} = \eta_{\mu\nu} \tag{A.2}$$

Generators of the Poincaré group are the Lorentz generators $M^{\mu\nu}$ together with the 4 momentum P^α; they satisfy the Poincaré algebra

$$\begin{aligned} [P^\mu, P^\nu] &= 0 \\ [M^{\mu\nu}, P^\alpha] &= i\,(P^\mu\, \eta^{\nu\alpha} - P^\nu\, \eta^{\mu\alpha}) \\ [M^{\mu\nu}, M^{\alpha\beta}] &= i\,(M^{\mu\beta}\, \eta^{\nu\alpha} + M^{\nu\alpha}\, \eta^{\mu\beta} - M^{\mu\alpha}\, \eta^{\nu\beta} - M^{\nu\beta}\, \eta^{\mu\alpha}) \end{aligned} \tag{A.3}$$

Moreover, we locally have a correspondence

$$SO(3,1) \simeq SU(2) \otimes SU(2) \tag{A.4}$$

The generators of spatial rotations J_i and Lorentz boosts K_i can be expressed as

$$J_i = \frac{1}{2}\, \epsilon_{ijk}\, M_{jk}, \qquad\qquad K_i = M_{0i} \tag{A.5}$$

which obey

$$\begin{aligned} [J_i, J_j] &= i\, \epsilon_{ijk}\, J_k \\ [J_i, K_j] &= i\, \epsilon_{ijk}\, K_k \\ [K_i, K_j] &= -i\, \epsilon_{ijk}\, J_k \end{aligned} \tag{A.6}$$

We can now define the linear combinations

$$J_i^{\pm} = \frac{1}{2}\,(J_i \pm i\,K_i) \tag{A.7}$$

which satisfy

$$\begin{aligned}
&[J_i^+, J_j^+] = i\,\epsilon_{ijk}\,J_k^+ \\
&[J_i^-, J_j^-] = i\,\epsilon_{ijk}\,J_k^- \\
&[J_i^+, J_j^-] = 0
\end{aligned} \tag{A.8}$$

Thus, we have shown that the Lie algebra for the Lorentz group has 2 commuting $SU(2)$ subalgebras; representations of $su(2) \oplus su(2)$ will determine representations of the Lorentz group.

On the other hand, there is the homeomorphism

$$SO(3,1) \simeq SL(2,\mathbb{C}) \tag{A.9}$$

whose basic representations are the fundamental one ψ_α, known as left handed Weyl spinor, and the conjugate one $\bar{\chi}_{\dot{\alpha}}$, known as right handed Weyl spinor.

We can define the generators of $SL(2,\mathbb{C})$ starting from $\sigma^\mu = (\mathbb{1}, \vec{\sigma})$ and $\bar{\sigma}^\mu = (\mathbb{1}, -\vec{\sigma})$ as

$$\begin{aligned}
(\sigma^{\mu\nu})_\alpha{}^\beta &= \frac{i}{4}\,(\sigma^\mu\,\bar{\sigma}^\nu - \sigma^\nu\,\bar{\sigma}^\mu)_\alpha{}^\beta \\
(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} &= \frac{i}{4}\,(\bar{\sigma}^\mu\,\sigma^\nu - \bar{\sigma}^\nu\,\sigma^\mu)^{\dot{\alpha}}{}_{\dot{\beta}}
\end{aligned} \tag{A.10}$$

which satisfy the Lorentz algebra. Under a finite Lorentz transformation with parameters $\omega_{\mu\nu}$, left handed and right handed spinors tranform respectively as

$$\begin{aligned}
\psi_\alpha &\to \exp\left(-\frac{i}{2}\,\omega_{\mu\nu}\,\sigma^{\mu\nu}\right)_\alpha{}^\beta\,\psi_\beta \\
\bar{\chi}^{\dot{\alpha}} &\to \exp\left(-\frac{i}{2}\,\omega_{\mu\nu}\,\bar{\sigma}^{\mu\nu}\right)^{\dot{\alpha}}{}_{\dot{\beta}}\,\bar{\chi}^{\dot{\beta}}
\end{aligned} \tag{A.11}$$

We define the product of 2 Weyl spinors as

$$\chi\,\psi = \chi^\alpha\,\psi_\alpha = \epsilon^{\alpha\beta}\,\chi_\beta\,\psi_\alpha = -\chi_\alpha\,\psi^\alpha \tag{A.12}$$

$$\bar{\chi}\,\bar{\psi} = \bar{\chi}_{\dot{\alpha}}\,\bar{\psi}^{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\,\bar{\chi}^{\dot{\beta}}\,\bar{\psi}^{\dot{\alpha}} = -\bar{\chi}^{\dot{\alpha}}\,\bar{\psi}_{\dot{\alpha}} \tag{A.13}$$

In particular

$$\psi\,\psi = \psi^\alpha\,\psi_\alpha = \epsilon^{\alpha\beta}\,\psi_\beta\psi_\alpha = \psi_2\,\psi_1 - \psi_1\,\psi_2 = 2\,\psi_2\,\psi_1 \tag{A.14}$$

being the components ψ_α anticommuting Grassmann numbers.

Appendix B

Basics of Supersymmetry

B.1 Supersymmetry algebra

In order to have a supersymmetric extension of the Poincaré algebra, we need to introduce the spinor generators Q_α and $\bar{Q}_{\dot{\alpha}}$, in representation $\left(\frac{1}{2},0\right)$ and $\left(0,\frac{1}{2}\right)$ of the Lorentz group, respectively[1], as well as the concept of graded Lie algebra; for operators O_a of a Lie algebra we have

$$O_a\, O_b - (-1)^{\eta_a\, \eta_b}\, O_b\, O_a = i\, C^e{}_{ab}\, O_e \tag{B.1}$$

where the gradings η_a take the values 0 or 1 for bosonic or fermionic operators, respectively. We already know tha algebra for Lorentz generators and translations (A.3); in order to have the full supersymmetry algebra we need to find the relations with the spinor generators we introduced above. One can verify that

$$\begin{aligned}
[Q_\alpha, M^{\mu\nu}] &= (\sigma^{\mu\nu})_\alpha{}^\beta\, Q_\beta \\
[Q_\alpha, P^\mu] &= [\bar{Q}^{\dot{\alpha}}, P^\mu] = 0 \\
\{Q_\alpha, Q_\beta\} &= 0 \\
\left\{Q_\alpha, \bar{Q}_{\dot{\beta}}\right\} &= 2\, (\sigma^\mu)_{\alpha\dot{\beta}}\, P_\mu
\end{aligned} \tag{B.2}$$

The last equation in turn implies that the action of 2 supersymmetry transformations is indeed a spacetime translation. Actually, we need also the commutator between a supersymmetry generator and some internal generator T_i; this usually vanishes, except for

[1] For the sake of simplicity, we consider here only the $\mathcal{N}=1$ SUSY case. The generalization to the case of extended supersymmetry is trivial.

some $U(1)$ automorphism of the supersymmetry algebra, known as R-symmetry:

$$Q_\alpha \to \exp(i\,\lambda)\,Q_\alpha, \qquad \bar{Q}_{\dot\alpha} \to \exp(-i\,\lambda)\,\bar{Q}_{\dot\alpha} \tag{B.3}$$

so that, if R is a $U(1)$ generator, then

$$[Q_\alpha, R] = Q_\alpha, \qquad [\bar{Q}_{\dot\alpha}, R] = -\bar{Q}_{\dot\alpha} \tag{B.4}$$

In the case of extended supersymmetry, we need to introduce additional labels $A, B = 1, \ldots, \mathcal{N}$ to the spinor generators; the algebra is the same of (B.2), except for

$$\begin{aligned} \left\{Q^A_\alpha, \bar{Q}_{\dot\beta\, B}\right\} &= 2\,(\sigma^\mu)_{\alpha\dot\beta}\, P_\mu\, \delta^A{}_B \\ \left\{Q^A_\alpha, Q^B_\beta\right\} &= \epsilon_{\alpha\beta}\, Z^{AB} \end{aligned} \tag{B.5}$$

where $Z^{AB} = -Z^{BA}$ are the central charges which commute with all the generators, and they are the main new ingredient of extended supersymmetries. Moreover, they form an abelian invariant subalgebra of internal symmetries.

B.2 Superspace and Superfields

Let us introduce a set of anticommuting Grassmann coordinates θ_α and $\bar\theta_{\dot\alpha}$; supersymmetry transformations are nothing less than translations along that directions in superspace. We will deal with objects called superfields $\Phi(X)$ which are functions of coordinates X in superspace and which have definite transformation properties under super Poincaré.

B.2.1 Chiral superfields

A chiral superfield Φ is defined by the property

$$\bar{\mathcal{D}}_{\dot\alpha}\,\Phi = 0 \tag{B.6}$$

where $\bar{\mathcal{D}}_{\dot\alpha}$ is the right handed supercovariant derivative; it is useful to introduce a chiral coordinate y^μ defined as

$$y^\mu = x^\mu + i\,\theta\,\sigma^\mu\,\bar\theta \tag{B.7}$$

so that the chiral superfield will only depend on y and θ; in the standard parametrization

$$\Phi(y, \theta) = \varphi(y) + \sqrt{2}\,\theta\,\psi(y) + \theta\theta\,F(y) \tag{B.8}$$

where φ represents the scalar part, ψ the some spin $1/2$ particle and F is an auxiliary field. We have also used the definition

$$\theta_\alpha \, \theta_\beta = \frac{1}{2} \, \epsilon_{\alpha\beta} \, (\theta\theta) \tag{B.9}$$

In terms of the original spacetime coordinates x^μ it takes the form

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) &= \varphi(x) + \sqrt{2} \, \theta \, \psi(x) + \theta\theta \, F(x) + i \, \theta \, \sigma^\mu \, \bar{\theta} \, \partial_\mu \varphi(x) \\ &- \frac{i}{\sqrt{2}} \, (\theta\theta) \, \partial_\mu \psi(x) \, \sigma^\mu \, \bar{\theta} - \frac{1}{4} \, (\theta\theta) \, (\bar{\theta}\bar{\theta}) \partial_\mu \partial^\mu \varphi(x) \end{aligned} \tag{B.10}$$

B.2.2 Vector superfields

A vector (or real) superfield $V(x, \theta, \bar{\theta})$ is defined by

$$V^\dagger = V \tag{B.11}$$

For such a superfield we can define a generalized gauge transformation

$$V \to V - \frac{i}{2} \, (\Lambda - \Lambda^\dagger) \tag{B.12}$$

with Λ chiral superfield, in order to gauge away some of the components of V. A common choice is the so called Wess-Zumino gauge, in which the superfield takes the form

$$V(x, \theta, \bar{\theta}) = (\theta \, \sigma^\mu \, \bar{\theta}) \, V_\mu(x) + (\theta\theta) \, (\bar{\theta}\bar{\lambda}(x)) + (\bar{\theta}\bar{\theta}) \, (\theta\lambda(x)) + \frac{1}{2} \, (\theta\theta) \, (\bar{\theta}\bar{\theta}) \, D(x) \tag{B.13}$$

The physical components of a vector superfield are then V_μ, corresponding to the gauge bosons, λ and $\bar{\lambda}$ corresponding to the gauginos and the auxiliary field D. One can also show that in this gauge

$$V^2 = \frac{1}{2} \, (\theta\theta) \, (\bar{\theta}\bar{\theta}) \, V^\mu \, V_\mu, \qquad V^{n+2} = 0 \;\text{ for all } n \geq 1 \tag{B.14}$$

Notice also that the Wess-Zumino gauge is not supersymmetric, since under supersymmetry the new vector superfields will not be of the form (B.13). However, under a combination of supersymmetry and generalized gauge transformation, we can end up with a vector field in Wess-Zumino gauge.

B.2.3 Abelian field strength superfield

For a non supersymmetric $U(1)$ gauge theory, a complex scalar field φ coupled to a gauge field V_μ via covariant derivative $D_\mu = \partial_\mu - i\, q\, V_\mu$ transforms as

$$\varphi(x) \to e^{i\,q\,\alpha(x)}\,\varphi(x), \qquad V_\mu(x) \to V_\mu(x) + \partial_\mu \alpha(x) \tag{B.15}$$

with charge q and local parameter $\alpha(x)$.
Under supersymmetry, we generalized these concepts to chiral superfields Φ and vector superfields V. Imposing the transformation properties

$$\Phi \to \exp(i\,q\,\Lambda)\,\Phi, \qquad V \to V - \frac{i}{2}\,(\Lambda - \Lambda^\dagger) \tag{B.16}$$

for some chiral superfield Λ, we find that the combination

$$\Phi^\dagger\, e^{2\,q\,V}\,\Phi \tag{B.17}$$

is gauge invariant.
Without supersymmetry, the abelian field strength is usually defined as

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \tag{B.18}$$

whose supersymmetric generalization is

$$\mathcal{W}_\alpha = -\frac{1}{4}\,(\bar{\mathcal{D}}\bar{\mathcal{D}})\,\mathcal{D}_\alpha V \tag{B.19}$$

which written in components takes the form

$$\mathcal{W}_\alpha(y,\theta) = \lambda_\alpha(y) + \theta_\alpha\, D(y) + (\sigma^{\mu\nu}\,\theta)_\alpha\, F_{\mu\nu}(y) - i\,(\theta\theta)\,(\sigma^\mu)_{\alpha\dot\beta}\,\partial_\mu \bar\lambda^{\dot\beta}(y) \tag{B.20}$$

B.2.4 Non abelian field strength superfield

As for the non supersymmetric case, the gauge degrees of freedom take values in the associated Lie algebra spanned by the generators T^a:

$$\Lambda = \Lambda_a\, T^a, \qquad V = V_a\, T^a, \qquad [T^a, T^b] = i\, f^{abc}\, T^c \tag{B.21}$$

Under the gauge transformation $\Phi \to e^{i\,q\,\Lambda}\,\Phi$, we want to keep $\Phi^\dagger\, e^{2\,q\,V}\,\Phi$ invariant; however, the non-commutative nature of Λ and V enforces a non linear transformation law for the vector superfield. Indeed, since

$$\exp(2\,q\,V) \to \exp(i\,q\,\Lambda^\dagger)\exp(2\,q\,V)\exp(-i\,q\,\Lambda) \tag{B.22}$$

it follows that

$$V \to V - \frac{i}{2}\,(\Lambda - \Lambda^\dagger) - \frac{i\,q}{2}\,[V, \Lambda + \Lambda^\dagger] + \cdots \tag{B.23}$$

Of course, also the field strength superfield needs to be modified in the non abelian case. Recall that the field strength $F_{\mu\nu}$ of a non supersymmetric Yang-Mills theory transforms as $U\,F_{\mu\nu}\,U^{-1}$ under unitary transformations; in a similar way, we get a gauge covariant quantity as

$$\mathcal{W}_\alpha = -\frac{1}{8\,q}\,(\bar{\mathcal{D}}\bar{\mathcal{D}})\,(\exp(-2\,q\,V)\,\mathcal{D}_\alpha \exp(2\,q\,V)) \tag{B.24}$$

which transforms as

$$\mathcal{W}_\alpha \to e^{i\,q\,\Lambda}\,W_\alpha\,e^{-i\,q\,\Lambda} \tag{B.25}$$

under (B.22).

Finally, one can express that field strength in Wess-Zumino gauge explicitly as

$$\begin{aligned} \mathcal{W}^a_\alpha(y,\theta) &= -\frac{1}{4}\,(\bar{\mathcal{D}}\bar{\mathcal{D}})\,\mathcal{D}_\alpha\,(V^a(y,\theta,\bar\theta) + i\,f^{abc}\,V^b(y,\theta,\bar\theta)\,V^c(y,\theta,\bar\theta)) \\ &= \lambda^a_\alpha(y) + \theta_\alpha\,D^a(y) + (\sigma^{\mu\nu}\,\theta)_\alpha\,F^a_{\mu\nu}(y) - i\,(\theta\theta)\,(\sigma^\mu)_{\alpha\dot\beta}\,D_\mu\bar\lambda^{a\dot\beta}(y) \end{aligned} \tag{B.26}$$

where

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + q\,f^{abc}\,V^b_\mu\,V^c_\nu \\ D_\mu\bar\lambda^a &= \partial_\mu\bar\lambda^a + q\,f^{abc}\,V^b_\mu\,\bar\lambda^c \end{aligned} \tag{B.27}$$

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