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ABOUT THESE PROCEEDINGS

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A VIDEOGAME AS A TOOL TO ORCHESTRATE PRODUCTIVE MATHEMATICAL DISCUSSIONS

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We structure an educational activity with a videogame starting from the hypothesis that technological tools could support in collecting data on students' achievements and designing mathematical whole-class discussions. In this paper, we present and discuss an example from our case studies, with the aim of analysing the role played by the videogame in supporting productive discussion concerning relational thinking. We conducted a whole-class discussion, and we noticed that the videogame could be a valuable tool for structuring the discussion. Indeed, the log files allow us to follow students' achievements and difficulties in solving tasks, and the dedicated web interface permits students to upload their responses and share them immediately.

Keywords: Educational videogames, web interface, mathematical discussion, relational thinking.

INTRODUCTION

Carpenter et al. (2005) define relational thinking as "looking at expressions and equations in their entirety rather than as a process to be carried out step by step" (Carpenter et al., 2005, p. 54). Furthermore, they sustain that "relational thinking involves using fundamental properties of number and operations to transform mathematical expressions rather than simply calculating an answer following a prescribed sequence of procedures" (Carpenter et al., 2005, p. 54). To develop this kind of thinking, the authors recommend going beyond traditional arithmetic practices and considering elementary arithmetic concepts as a bridge to learn algebra. Involving students in the solution and subsequent discussion of particular tasks seems crucial for focusing them on relations and fundamental properties of arithmetic operations, rather than focusing exclusively on procedures for calculating answers. In tune with this statement, the authors suggest engaging students in solving true/false and open number sentences, which provide a flexible context for representing relations among numbers and operations. As literature states (e.g., Lampert, 2001), involving students in well-designed tasks is not enough, the role of the teachers is central in orchestrating productive discussions. Although Carpenter et al. (2003; 2005) mentioned the importance of teachers' and students' interactions, it seems that the role of teachers and how they could interact with their students are not clearly defined. Since, in most of their papers, the authors present examples of interviews with students and excerpts from discussions with the teacher, we think that classroom discussions are the core of the development of this kind of thinking. Recent research shows that teachers' prompt for relational thinking had an immediate effect on students' relational thinking (Lin et al., 2015).

The aim of this paper is to develop technological tools that could support orchestrating productive class discussions. Our hypothesis is that these tools could help in collecting data on students' achievement and in orchestrating productive discussions. With this aim, we design a set of materials for students (called "unplugged tabs") and a digital tool for those who will orchestrate the mathematical discussion (called "monitoring web interface"). In this paper, after introducing the theoretical frame, we will present an example from our case studies. This example is aimed at investigating the role played by the videogame in supporting productive discussion concerning relational thinking and focusing on how we plan and implement it.

THEORETICAL FRAMEWORK

To reach our goal, we need to define what we mean by designing productive mathematical discussions. Thus, it seems crucial to identify the strategies for orchestrating productive mathematical discussions (Stein et al., 2008).

Stein and colleagues (2008) design a pedagogical model of five practices for discussion facilitation. They constructed this model based on the hypothesis that teachers need to perform a set of practices to prepare themselves for discussions and gradually learn how to become better discussion facilitators over time (Stein et al., 2008). The authors describe the following five key practices:

- *anticipating* likely students' responses to cognitively demanding mathematical tasks,
- *monitoring* students' responses to the tasks,
- *selecting* particular students to present their mathematical responses,
- purposefully *sequencing* the student responses that will be displayed,
- helping the class make mathematical *connections* between different students' responses and between students' responses and the key ideas.

The authors view each of the practices as "drawing on the fruits of the practices that came before it" (Stein et al., 2008, p. 321); together, these practices help to make discussions more likely, and teachers will be able to use students' responses to advance the mathematical understanding of the class as a whole.

Anticipating Students' Mathematical Responses

The first practice consists in trying to imagine how students might mathematically approach the tasks that they will be asked to engage in. Anticipating students' answers involves "developing anticipation about how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that the teacher would like his or her students to learn." (Stein et al., 2008, pp. 322–323)

In activating this practice, the authors suggest teachers to draw both on their knowledge of particular students' mathematical skills and understandings and on their knowledge of the research literature about typical students' responses to the same or similar tasks. For this reason, the study by Carpenter and colleagues seems to be appropriate to this practice (see, for example, Carpenter et al., 2003): they present lots of examples that illustrate teaching and learning activities focusing on tasks, students and teachers.

Monitoring Student Responses

Monitoring students' responses means paying attention to the mathematical thinking in which students engage as they work on tasks. This practice is commonly done by walking between the stalls while students work. The goal of this practice is to identify the mathematical learning potential of particular strategies or representations used by the students. In tune with this aim, observations and thinking-aloud procedures offer opportunities for gathering knowledge about students' thinking and ways of solving tasks, and these opportunities can be enhanced through technology.

In the literature, the use of computers to follow and register the students' working is often emphasised; for example, software that record audio and screen or produces log files (which consist of a list of events carried out by students) are available for education. For the teacher, however, observing student recordings could be time-consuming, while analysing the log files (or observing the analysis produced by the software) could be a good compromise to enrich the practice of monitoring (Van den Heuvel-Panhuizen et al., 2011).

Purposefully Selecting and Sequencing Student Responses for Public Display

In these practices, teachers can select and then sequence particular students to share their work with the rest of the class. A typical way to select students' responses could be calling on specific students (or groups of students) or asking for volunteers to share with the class. The purposeful selection of students makes it more likely the mathematical ideas will be discussed by the class. Careful selection of students to present strategies could allow the ideas to be illustrated, highlighted, and then generalised.

After the selection of particular students' responses, teachers can then make decisions about how to sequence the students' presentations with respect to each other. Stein, Engle, Smith and Hughes (2008) present some examples: teachers could

- select the strategy used by most students and then those used by some of them;
- start with a particularly easy-to-understand strategy;
- begin with strategies that are based on common misconceptions or errors;
- relate or contrast right or wrong strategies.

The main goal of these two practices is to lead teachers to present in a particular sequence mathematical ideas to make a discussion more coherent and predictable.

A well-designed software could allow students to upload their responses and send them immediately to the teacher. Teachers can promptly read students' responses, select and then cluster/sequence some of them for sharing and analysis in the whole-class discussion (see, for example, FaSMEd project; e.g., Aldon et al., 2015). In this way, teachers could promote the comparison of different selected solutions.

Connecting Student Responses

Finally, teachers can help students to draw connections between the mathematical ideas that are reflected in the strategies and representations that they use. Stein et al. (2008) stress that having mathematical discussions consists of separate presentations of different ways to solve a particular problem: the main goal is to have student presentations build on each other to develop powerful mathematical ideas.

Research Aim

Starting from the theoretical framework presented by Stein and colleagues, we can better formulate the goal of this paper: exploring whether and how a videogame can facilitate in using the five key practices for designing productive mathematical discussions.

METHODOLOGY

In line with the theoretical framework just presented and with the goal of our research, we structured several activities in order to: involve students in tasks related to relational thinking, monitor students' responses and collect their productions to promote productive mathematical discussions.

Field Trial and Sample

We carry out a field trial divided into three activities: students play the "SuperFlat Math" videogame individually; then they work in small heterogeneous groups on unplugged tabs, and at the end, they participate in a whole-class discussion.

The sample is a grade 4 class from an Italian Primary school in Mantova. The class is composed of 13 students (five females and eight males). The study was carried out in January and February 2021 during school hours. Unfortunately, the field trial was interrupted by the pandemic situation: in February 2021, all Italian schools were mandatorily closed. We conducted the whole-class discussion remotely through the Google Meet platform available to the school.

SuperFlat Math Videogame

We use a skill and drill videogame called "SuperFlat Math", which was designed and developed by Prof. Leonardo Guidoni, from University of L'Aquila, starting from the free and open-source version of Minetest[1]. It is a sandbox videogame that enables primary and lower secondary school students to explore a blocky, procedurally generated 3D world. "SuperFlat Math" presents a list of mathematical tasks, and it provides a message on the correctness of the answers when the player gives the solution of each task. If the answer is correct, players gain points that could be converted into rewards.

The videogame also includes a web interface, which is designed to monitor the classroom's achievements by gathering information like access and play time, scores, number of correct answers, number of wrong answers, tasks and so on. The main goal of this system is to provide teachers/researchers with information that support anticipating and monitoring practices. By examining such web interface, teachers and researchers could immediately identify students' correct and wrong answers as well as the provided solutions to the tasks. Finally, students can upload their mathematical productions, tasks and other files to share them with teachers, researchers or mates. This feature can be useful for selecting and sequencing students' responses.

Proposed Activities

"SuperFlat Math" is divided into several games on different mathematical topics, such as Fractions, Equalities, Operations, Number line, Prime numbers and so on. Each game is composed of several minigames which are composed of a set of short puzzles at increasing levels of difficulty. To access the videogame, each student must have a personal account, which is also used to track his or her progresses in the web interface. Specifically, we consider two games: Parkour (Figure 1) and Swimming pools (Figure 2), on the mathematical topic of Equalities.

Figure 1: screenshot by Parkour minigame

Figure 2: screenshot by Swimming pools minigame

The Parkour minigames is of a perilous uphill path, which presents number sentences or expressions with two possible solutions. As already explained, Parkour minigames are at increasing levels of difficulty: the first half of minigames present number sentences in which students should find the correct solution, whereas the second half contains equivalence between two expressions.

In Swimming pools minigames, there is a pool full of number blocks from 0 to 100: on one side of the pool, there is an open number sentence. The player has to find the correct number block in the pool to complete the sentence. As in the first activity, these minigames are at increasing levels of difficulty: the first half of the minigames contains open number sentences with two operations, whereas the second half has expressions with parenthesis and two or more different operations.

Unplugged Tabs

Drawing on the work of Carpenter and colleagues (2003), we design unplugged tabs including true/false and open number sentences (as in "SuperFlat Math") to engage students in relational thinking by focusing them on specific properties and ways of thinking about number operations (Table 1). For each task, we ask students to justify their answers so that teachers and researchers could understand their way of reasoning. For example, $38 + 47 = 47 + 38$ focuses on the commutative property of addition. Students might figure out that the number sentence is true by carrying out the addition on each side of the equal sign, but more commonly, they immediately conclude that the sentence is true because the order of the numbers has been changed. This can lead to a discussion of whether this relation generalises to all numbers and whether it is true for other operations.

Examples of true/false sentences	Examples of open number sentences
$8 = 3 + 5$	$25 + 32 = 27 + \cdots$
$3 \times 4 = 3 \times 4 + 3$	$8 \times 3 + 8 = 8 \times ..$
$2 \times 3 \times 5 = 6 \times 5$	$2 \times \ldots \times 7 = 14 \times 5$

Table 1: example of tasks in the unplugged tabs

RESULTS AND DISCUSSION

In the first part of the field trial, all students play the videogame. The web interface shows parameters like play time and scores (Figure 3).

ID	Last access	Task 1: a+b+c=...	Task 2: a+b+...=c	Task 3: a+b=...+c	Task 4: a+...=b+c
S.1	02 Mar 2021 10:05	2/3 66% Score: 60 Detail	1/1 100% Score: 30 Detail	1/1 100% Score: 30 Detail	1/1 100% Score: 30 Detail
S.2	02 Mar 2021 10:41	1/6 16% Score: 30 Detail	1/1 100% Score: 30 Detail	–	–

Figure 3. Screenshot of log files analyses in the web interface

The software allows us to know what and how many tasks students solved, to discover correct and wrong answers and the number of attempts. In Figure 3, we see that student S.1 made three attempts for solving task 1 giving two correct answers; for the same task, student S.2 made six attempts with only one right answer. The web interface also permits to have a detailed list of all the answers given by students.

Such information could be useful for the anticipating and monitoring strategies. The tasks in "SuperFlat Math" are presented in increasing levels of difficulty, and so the first ones could be useful for the anticipating practice then the last ones for the monitoring strategy. However, these collected data does not highlight the mathematical thinking in which students engage as they work on tasks. On the one hand, the web interface gives us an updated snapshot of students' performances; but on the other hand, it does not provide information about their mathematical processes.

The goal of the monitoring and anticipating practices is to identify the mathematical learning potential of particular strategies or representations used by the students, and so their mathematical thinking should not be ignored. For this reason, walking between the stalls while students work seems to be the best strategy for monitoring students, but the web interface snapshot could be useful to rapidly select which students to observe without walking around randomly.

Since students' strategies are not visible from the web interface and circulating between the stalls is not always possible, we administer to small groups of students the unplugged tabs, in which we require them to motivate each answer. We create small heterogeneous groups according to their game scores: in the same group, we include students with high, low and medium scores. We promote teamwork to encourage students to share their strategies and make autonomous mathematical discussions in small groups. Once they solve tabs, students use the web interface to share them with us. The possibility to instantly share the answer allows us to reach a big amount of information without waiting that all students finish to solve their unplugged tabs.

All the answers to tabs are correct, but justifications and group strategies differ. For example, concerning the following task: $\mathbf{25 + 32 = 27 + \cdots}$, some groups reveal the use of relational thinking in the justification: "We have chosen 30 because 25+2=27 and it [27] is what I already have, then on the other side, I took off 2 from 32, and I add it to 25, that results 27 and 32-2 equals 30 and I put it [30] beside 27."

Conversely, other students use computational strategies. For instance, another group writes: "We have chosen 30 because we've computed 25+32=57 and so 27+30=57."

The analysis of these answers allows us to select and then sequence them for designing the whole class discussion. We start by selecting the computational strategies, and then we sequence them with the ones that reveal relational thinking. The web interface permits to rapidly collect students' responses, so we have the possibility to quickly plan the whole-class discussion while students are still engaged with tabs.

We conduct a mathematical discussion based on the anticipating (by using the web interface) and monitoring practices (by using the web interface and the unplugged tabs). According to our data, we select and sequence students' answers following these criteria:

1. we make sure to select at least one response from each group in order to allow all students to participate in the discussion;
2. we choose those correct answers that reveal different strategies by also checking selected groups' achievements in the videogame;
3. we first present the answers that show computational strategies and then those revealing relational thinking so as to reflect on similarities and differences.

Unfortunately, due to the pandemic situation, we interrupt the field trial. For logistical reasons, we conduct the discussion remotely. Nevertheless, all students participated in the whole-class discussion and those who showed computational strategies in the tabs discussed with their mates about relational strategies.

CONCLUSION

We structure activities with a videogame starting from the hypothesis that technological tools could support teachers/researchers in collecting data on students' achievement and in planning and then orchestrating productive discussions. We present and discuss an example from our case studies, with the aim of analysing the role played by the videogame in supporting the design of a discussion concerning relational thinking.

The goal of this paper is to explore whether and how a videogame can facilitate the planning of mathematical discussions using the five key strategies (Stein et al., 2008). Concerning our research aim, we notice that the videogame could be a valuable tool for the following three reasons. first, the web interface makes the anticipating and monitoring practices possible by uploading the log files, through which we follow students' achievements and wrong/correct answers. Second, the log files do not show students' strategies and ways of thinking, so we need also to walk between the stalls while students work. However, the web interface data are useful to rapidly select which students to observe without walking around randomly. Finally, the web interface also makes the selecting and sequencing practices faster: the ability to retrieve single items speeds up and facilitates the discussion orchestration while students are still working on tabs.

Thus, the web interface seems to be a helpful tool for promoting four of the five key practices described by Stein and colleagues (2008) and for this reason, we suppose that technological tools could provide an added value compared with activities in a paper and pencil environment. All students participate actively in unplugged tabs and the discussion. During the activities, the teacher states that almost all students enjoy playing the videogame and the subsequent activities.

Therefore, it seems that the videogame plays both a motivational and a methodological role. Reflecting on log files and retrieving the tabs is essential, because otherwise, we would carry out the discussion by calling on students randomly, without considering their progress. Without these tools, the collection of data concerning students' performances, strategies or ways of thinking would be more time-consuming. So, the time between play time, tabs activities and the discussion would be too long, because we would need a lot of time to collect and organise students' data.

There are some important aspects not explored in this paper that are crucial for future research. We describe the methodological advantages of a videogame in planning and orchestrating mathematical discussions, but we do not consider its mathematical effectiveness in terms of development of relational thinking. We present a first reflection on our tools and materials by trying them out in small class activities. The future research will be based on successive cycles of design, observation, analysis and redesign of classroom sequences (Design-Based Research Collective, 2003) in order to design a set of materials for students and teachers. The analysis of mathematical effectiveness of such materials and tools needs a longer period and a greater set of activities involving more than one class and one teacher. It will also be interesting to study the usability of these materials and tools by teachers without our support.

Finally, we do not consider the role of feedback provided by the videogame. The scores inform students whether their answers are correct or not; in addition, the message returned by the videogame when students give a wrong answer could allow him or her to reflect on his or her strategy. In Figure 3, students made some attempts in the first task (S1 made three attempts; S2 made six), whereas they gave the correct answer in the next tasks. This suggests that the videogame could be used as a tool to support students in formative assessment practices (e.g., Aldon et al., 2015).

The activities described in this paper is conducted face-to-face, but they could also be carried out remotely. This is another interesting feature of technological tools that could be explored in future research: they offer the possibility to design intriguing activities to support teaching and learning mathematics in different scenarios.

NOTES

[1] https://www.minetest.net/

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