Quantized Sampled-Data Attitude Control of Ground Vehicles: An Event-Based Approach

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***Abstract*—Attitude control systems for ground vehicles have been an important topic in automotive research for decades, and have been extensively studied by resorting to classical continuous-time nonlinear design. Although this approach can incorporate saturation constraints and actuator dynamics in the design, the computed control laws are often approximated and applied within digital environments in absence of formal performance guarantees. In this letter, we present a quantized sampled-data approach to the vehicle attitude control problem. Starting from classical nonlinear design achieving tracking of prescribed trajectories in continuous time (emulation approach), we derive conditions for preserving the practical stability of the error dynamics by means of quantized sampled-data event-based controllers. Simulations performed in an non-ideal setting confirm the potential of the approach.**

***Index Terms*—Automotive control, event-triggered control, quantized control, sampled-data stabilization, tracking of nonlinear systems.**

I. Introduction

VEHICLE active control systems have been an important research topic in automotive industry for decades [1], [2]. A major goal in this context has been to improve safety and comfort of drivers and passengers [3], [4], while future studies will more and more evolve towards the goal of full autonomous driving (see, e.g., [5]), exploiting distance communication [6] and coordination [7] among vehicles.

Control of ground vehicles has been extensively studied by resorting to classical nonlinear design in continuous time (see, e.g., [8], [10]) or model predictive control (MPC) [11], [12]. Although such approaches can incorporate saturation constraints and actuators in the design [13], the designed methods are later applied within digital environments in absence of formal guarantees.

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To cope with this important issue, in this letter we apply to the vehicle control problem the theory of the *stabilization in the sample-and-hold sense* [14], providing the theoretical background for the preservation of the stability property in a semiglobal practical sense when the controller is applied in a digital setting by means of state sampling at discrete times and piecewise-constant actuations. Sampled-data stabilization is employed jointly with an *event-triggered* or *resource-aware* control approach [15], which is a recent successful framework allowing to preserve resources in control systems with limited computation and communication capabilities. In more detail, in the spirit of the *emulation approach* [16], we first construct a stabilizing control law in continuous time, and we later modify it by considering time sampling and state quantization [17], as typical non-idealities introduced by the digital environment, and the check of a triggering condition allowing to update the control law only when it is really necessary.

The contribution of this letter is twofold:

- A tracking control problem of ground vehicles is solved by means of a quantized event-based sampled-data controller, which is properly validated on an extended vehicle model to test the robustness of the design with respect to unknown dynamics and parameter uncertainties.
- To tackle the aforementioned problem, a novel result regarding the construction of event-based sampled-data controllers is proposed, which ensures semiglobal practical stability of the closed-loop error system (modeling the distance between plant and reference model), by also considering quantization in the design. Although motivated and applied in this letter to vehicle control, the methodology can be applied to general error systems.

The theoretical result presented in this letter builds upon some previous results by the same authors and developed for nonlinear time-delay systems. In particular, the work [18] addressed time-triggered quantized sampled-data stabilization, while the letters [19], [20] follow an event-triggered approach in absence of quantization. We here instead focus on the delay-free case and consider state quantization in a tracking problem, which is recast as a problem of stabilization of the origin of a nonlinear time-varying system. To the best of our knowledge, in this letter quantized event-based sampled-data controllers are developed and applied to the problem of vehicle attitude control for the first time.

This letter is organized as follows. Section II addresses the vehicle model and control design in continuous time. Section III reviews the quantized sampled-data setting and includes the main result of this letter. Section IV shows simulations in a non-ideal setting including a fully actuated vehicle with partial sampled-data observations. Section V concludes this letter with an outlook on ongoing and future work.

Notation: $\mathbb{R}$ denotes the set of real numbers, $\mathbb{Z}^+$ denotes the set of non-negative integer numbers and $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The symbol $|\cdot|$ stands for the Euclidean norm of a real vector. For a given positive integer n and a given positive real h, the symbol $\mathcal{B}_h^n$ denotes the subset $\{x \in \mathbb{R}^n : |x| \leq h\}$. Let us here recall that a continuous function $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ is: of class $\mathcal{P}_0$ if $\gamma(0) = 0$; of class $\mathcal{P}$ if it is of class $\mathcal{P}_0$ and $\gamma(s) > 0$, $s > 0$; of class $\mathcal{K}$ if it is of class $\mathcal{P}$ and strictly increasing; of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and unbounded. Throughout this letter, GUAS stands for Globally Uniformly Asymptotically Stable or Global Uniform Asymptotic Stability.

II. Vehicle Attitude Control in Continuous Time

A. Vehicle Model With 4 Independent Wheels

We consider a general vehicle model characterized by 4 independent wheels/tires [2] with the following equations

$$\frac{J_z}{\mu}\dot{\omega}_z = \left(F_{yfl} + F_{yfr}\right)\cos(\delta) l_f + M_z - \left(F_{yrl} + F_{yrr}\right) l_r + \left(F_{yfl} - F_{yfr}\right)\sin(\delta) t_v, \quad (1)$$

$$\frac{m}{\mu}(\dot{v}_y + v_x\omega_z) = \left(F_{yfl} + F_{yfr}\right)\cos(\delta) + \left(F_{yrl} + F_{yrr}\right). \quad (2)$$

In model (1)–(2), the time-varying quantities are: the yaw rate ω_z (expressed in rad/s); the lateral velocity v_y (m/s); the road wheel angle (rad)

$$\delta = \delta_{DRI} + \delta_c, \quad (3)$$

sum of the (measurable) driver angle δ_{DRI}, imposed by the driver by means of the steering wheel, and a control angle δ_c; the controlled yaw moment M_z (Nm); the lateral tire forces (N) F_{yij}, with $i = f, r$ (front/rear), $j = l, r$ (left/right).

The parameters of model (1)–(2) are: the vehicle mass m (kg); the vehicle inertia momentum J_z ($kg\ m^2$); the vehicle longitudinal velocity v_x (m/s), assumed constant in attitude control [2]; the distances l_f, l_r from the center of gravity to the front and rear axle (m); the vehicle axle track length t_v (distance between the centerline of two road wheels) (m); the tire–road friction coefficient μ (dimensionless). Lateral tire forces F_{yij}, with $i = f, r$ (front/rear), $j = l, r$ (left/right), depend on the front/rear slip angles (rad)

$$\alpha_{fj} = \delta - \tan^{-1}\left(\frac{v_y + l_f\omega_z}{v_x \mp t_v\omega_z}\right), \alpha_{rj} = -\tan^{-1}\left(\frac{v_y - l_r\omega_z}{v_x \mp t_v\omega_z}\right) \quad (4)$$

defined as the angles between directions and instantaneous velocities of the wheels (the minus sign is for $j = l$).

The functional form of tire force functions, with $i = f, r$ (front/rear), $j = l, r$ (left/right), is given by the Pacejka's magic formulas [21]

$$F_{yij}(\alpha_i) = F_{yij,\text{sat}} \sin\left(A_{y,ij} \arctan(B_{y,ij}\alpha_{ij})\right), \quad (5)$$

describing smooth functions with bounded derivatives, fitted from experimental data.

The actuations are characterized by first-order dynamics:

$$\dot{\delta}_c = -\frac{1}{\tau_\delta}\delta_c + \frac{1}{\tau_\delta}\delta_m, \quad (6)$$

$$\dot{M}_z = -\frac{1}{\tau_M}M_z + \frac{1}{\tau_M}T_c, \quad (7)$$

actuated by an Active Front Steer (AFS) and a Rear Torque Vectoring (RTV), imposing incremental road angles and yaw torques through the front and rear axle tire characteristics, respectively (see [4], [9]). In the previous equations, δ_m (rad) and T_c (Nm) are the physical inputs and τ_δ (s) and τ_M (s) are the time constants of the actuators.

B. Reduction to a Single-Track Model

The interested reader can verify (see [2], [9]) that Eqs. (1)–(7) can be easily reduced to a simpler model by disregarding the actuator dynamics (6)–(7) (by means of a quasi-steady-state approximation $\dot{\delta}_c = \dot{M}_z \simeq 0$, implying $M_z \simeq T_c$ and $\delta_c \simeq \delta_m$) and some minor terms, by exploiting small-angle trigonometric simplifications and finally putting together the contributions of each axle: $F_{yf} = F_{yfl} + F_{yfr}$, $F_{yr} = F_{yrl} + F_{yrr}$. This leads to the so-called *single-track* or *bicycle model*

$$\begin{cases} \dot{\omega}_z = \frac{\mu\left(F_{yf}(\alpha_{f0})l_f - F_{yr}(\alpha_r)l_r + \Delta_c l_f + M_z\right)}{J_z} \\ \dot{v}_y = -v_x\omega_z + \frac{\mu\left(F_{yf}(\alpha_{f0}) + F_{yr}(\alpha_r) + \Delta_c\right)}{m} \end{cases} \quad (8)$$

where the following AFS *virtual* control input is defined to represent the AFS contribution to the front lateral force:

$$\Delta_c = F_{yf}(\delta_c + \alpha_{f0}) - F_{yf}(\alpha_{f0}), \quad (9)$$

with the approximate slip angles being equal to

$$\alpha_{f0} = \delta_{DRI} - \frac{v_y + l_f\omega_z}{v_x}, \qquad \alpha_r = -\frac{v_y - l_r\omega_z}{v_x}. \quad (10)$$

We refer the interested reader to Fig. 1 in [25] for a graphical illustration scheme, which is here omitted for lack of space. By collecting the state variables within the state vector $x(t) = [x_1(t), x_2(t)]^T = [\omega_z(t), v_y(t)]^T$, and by highlighting the dependence of δ_{DRI} on the time t and on the state variables in (10), Eqs. (8) can be rewritten as follows:

$$\dot{x}(t) = f(t, x(t)) + Bu(t), \qquad t \geq 0 \quad (11)$$

with

$$f(t, x) = \begin{bmatrix} f_1(t, x) \\ f_2(t, x) \end{bmatrix} = \begin{bmatrix} \frac{\mu\left(F_{yf}(\alpha_{f0}(t,x))l_f - F_{yr}(\alpha_r(x))l_r\right)}{J_z} \\ -v_x x_1 + \frac{\mu\left(F_{yf}(\alpha_{f0}(t,x)) + F_{yr}(\alpha_r(x))\right)}{m} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & 0 \end{bmatrix} = \begin{bmatrix} \frac{\mu l_f}{J_z} & \frac{\mu}{J_z} \\ \frac{\mu}{m} & 0 \end{bmatrix} \quad u(t) = \begin{bmatrix} \Delta_c(t) \\ M_z(t) \end{bmatrix}$$

where $x(t) \in \mathbb{R}^n$ is the state vector ($n = 2$) and $u(t) \in \mathbb{R}^m$ is the input vector ($m = 2$).

C. State Feedback

The *control problem* is to determine the input $u(t)$ for the asymptotic tracking of some bounded trajectories, determined by the driver's input, which are the solution of a reference generator in the general form (see, e.g., [9], [13])

$$\dot{x}_{\text{ref}}(t) = f_{\text{ref}}(t, x_{\text{ref}}) = \begin{bmatrix} f_{1,\text{ref}}(t, x_{\text{ref}}) \\ f_{2,\text{ref}}(t, x_{\text{ref}}) \end{bmatrix} = \begin{bmatrix} \frac{\mu(F_{yf,\text{ref}}(\alpha_{f0}(t,x_{\text{ref}}))l_f - F_{yr,\text{ref}}(\alpha_r(x_{\text{ref}}))l_r)}{J_z} \\ -v_x x_{1,\text{ref}} + \frac{\mu(F_{yf,\text{ref}}(\alpha_{f0}(t,x_{\text{ref}})) + F_{yr,\text{ref}}(\alpha_r(x_{\text{ref}})))}{m} \end{bmatrix} \quad (12)$$

with reference state vector $x_{\text{ref}}(t) = [x_{1,\text{ref}}(t), x_{2,\text{ref}}(t)]^T = [\omega_{z,\text{ref}}(t), v_{y,\text{ref}}(t)]^T$, which is assumed to have a globally asymptotically stable equilibrium point at the origin. In other words, we address the AFS control problem such that the tracking error $e = [e_1, e_2]^T = x - x_{\text{ref}}$ converges to zero asymptotically, uniformly, and for all initial conditions.

The time-dependent driver angle function $\delta_{DRI}(t)$ is assumed Lipschitz continuous in time, bounded by some $\delta_{\max} \in \mathbb{R}^+$ and known at all times t. In order to prove the stability of the error system, we here recall the following result (see [22, Ths. 4.8–4.9]).

Fact 1: A time-varying system described by

$$\dot{z}(t) = \bar{F}(t, z(t)), \quad (13)$$

with $\bar{F} : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ being locally Lipschitz, $\bar{F}(t, 0) = 0$ for all $t \in \mathbb{R}^+$, is 0-GUAS if there exists a continuously differentiable Lyapunov function $V : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^+$, functions α_1, α_2 of class $\mathcal{K}_\infty$ and α_3 of class $\mathcal{K}$, such that the following conditions hold for all $z \in \mathbb{R}^n$, for all $t \in \mathbb{R}^+$

$$\alpha_1(|z|) \le V(t, z) \le \alpha_2(|z|), \quad (14)$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial z}\bar{F}(t, z) \le -\alpha_3(|z|). \quad (15)$$

The following proposition establishes the stability of the closed-loop error system with the uniformly in time locally Lipschitz feedback law

$$u = k(t, e) = \begin{bmatrix} k_1(t, e) \\ k_2(t, e) \end{bmatrix} \quad (16)$$

with

$$k_1(t, e) = \frac{1}{b_{21}}\big(f_{2,\text{ref}}(t, x_{\text{ref}}(t)) - f_2(t, e + x_{\text{ref}}(t)) - \bar{k}_2 e_2\big) \quad (17)$$

$$k_2(t, e) = \frac{1}{b_{12}}\Big(f_{1,\text{ref}}(t, x_{\text{ref}}(t)) - f_1(t, e + x_{\text{ref}}(t)) - \bar{k}_1 e_1 - \frac{b_{11}}{b_{21}}\big(f_{2,\text{ref}}(t, x_{\text{ref}}(t)) - f_2(t, e + x_{\text{ref}}(t)) - \bar{k}_2 e_2\big)\Big) \quad (18)$$

and $\bar{k}_1, \bar{k}_2 > 0$.

Proposition 1: The time-varying error system

$$\dot{e}(t) = \dot{x}(t) - \dot{x}_{\text{ref}}(t) = F(t, e(t), u(t)) = f(t, e(t) + x_{\text{ref}}(t)) - f_{\text{ref}}(t, x_{\text{ref}}(t)) + Bu(t), \quad (19)$$

with reference signal $x_{\text{ref}}(t)$ defined in (12) and feedback law $u(t) = k(t, e(t))$ from (16)–(18), is 0-GUAS.

Proof: The proposition is proven by direct application of Theorem 1 to the closed-loop system

$$\dot{e}(t) = \bar{F}(t, e(t)) := F(t, e(t), u(t)) \quad (20)$$

by means of the time-invariant Lyapunov candidate

$$V(t, e) = (e_1^2 + e_2^2)/2, \quad (21)$$

satisfying condition (14) with $\alpha_1(e) = \alpha_2(e) = |e|^2$.

We define the directional derivative $D^+V : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ of the function V along the error dynamics F:

$$D^+V(t, e, u) := \frac{\partial V(t, e)}{\partial e} F(t, e, u), \quad (22)$$

and we observe that such a derivative along the closed-loop system dynamics is

$$\frac{\partial V}{\partial e}\bar{F}(t, e) = D^+V(t, e, k(t, e)) = -\bar{k}_1 e_1^2 - \bar{k}_2 e_2^2, \quad (23)$$

satisfying (15) with $\alpha_3(e) = \min\{\bar{k}_1, \bar{k}_2\}|e|^2$. This concludes the proof. ∎

III. The Sampled-Data Event-Triggered Approach

The vehicle error model in (19) falls within a more general class of systems in the form

$$\dot{e}(t) = F(t, e(t), u(t)), \quad (24)$$

with $e(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and function F being locally Lipschitz. In the sampled-data approach, the input signal is assumed to be piecewise-constant. We recall that the system (24) admits a unique locally absolutely continuous solution in a maximal time interval $[0, b)$, with $0 < b \le +\infty$ [22].

We introduce here the following assumption, which ensures the existence of a continuous-time control law stabilizing the closed-loop system.

Assumption 1: There exists a uniformly in time locally Lipschitz feedback $k : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^m$, which is a time-varying function of the error $e(t)$, such that the closed-loop error system (19) with the law $u(t) = k(t, e(t))$ is 0-GUAS.

Note that Assumption 1 holds for the vehicle error system (19) by virtue of Proposition 1, implying

$$\alpha_1(|z|) \le V(z) \le \alpha_2(|z|), \quad (25)$$

$$\frac{\partial V}{\partial e}F(t, e, k(t, e)) \le -\alpha_3(|e|) \quad (26)$$

for the time-invariant Lyapunov function in (21). Notice that, for general time-varying systems in the form (24), it is possible to use converse results (see, e.g., [22, Th. 4.16], [23, Th. 3.5]) implying, from Assumption 1, the existence of a Lyapunov function satisfying (14)–(15). Hence the results discussed later in this section for the case of vehicle control can be applied to more general tracking problems with time-varying error model in the form (24).

We recall here the notion of partition of $[0, +\infty)$ [14].

Definition 1: A partition $\pi = \{t_i\}_{i \in \mathbb{Z}^+}$ of $[0, +\infty)$ is a countable, strictly increasing sequence t_i, with $t_0 = 0$, such that $t_i \to +\infty$ as $i \to +\infty$. The diameter of π, denoted *diam*(π), is defined as $\sup_{i \ge 0} t_{i+1} - t_i$. The dwell time of π, denoted *dwell*(π), is defined as $\inf_{i \ge 0} t_{i+1} - t_i$. For any positive real $a \in (0, 1]$, $\delta > 0$, $\pi_{a,\delta}$ is any partition π with $a\delta \le dwell(\pi) \le diam(\pi) \le \delta$.

The real $a \in (0, 1]$, in Definition 1, is introduced in order to allow for non-uniform sampling [18], [19] and to rule out the possibility of Zeno behavior [20].

We now define input and state quantizer operators as:

$$[\cdot]_{\mu_u} : \mathbb{R}^m \to \mathcal{Q}_u, \qquad [\cdot]_{\mu_x} : \mathbb{R}^n \to \mathcal{Q}_x, \tag{27}$$

where $\mathcal{Q}_u$ and $\mathcal{Q}_x$ are suitable finite subsets of $\mathbb{R}^m$ and $\mathbb{R}^n$, respectively. These quantizers are characterized by the following implications (see [11], [12])

$$|u| \le U \Rightarrow |u - [u]_{\mu_u}| \le \mu_u \tag{28}$$

$$|x| \le E \Rightarrow |x - [x]_{\mu_x}| \le \mu_x \tag{29}$$

for some positive reals U, E, and μ_u, μ_x, called ranges and error bounds of the quantizers, respectively (see [18]).

Before introducing the main result, we consider the following more compact notation for the sake of readability:

$$e^x(t) = [x(t)]_{\mu_x} - x_{\text{ref}}(t) \qquad \bar{u}(t) = k(t, e(t)) \tag{30}$$

$$u^x(t) = k\big(t, e^x(t)\big) \qquad u^*(t) = \big[u^x(t)\big]_{\mu_u}. \tag{31}$$

In the following, the quantized sampled-data event-based controller is proposed. For given positive reals r, R, with $0 < r < R$, let E, $\bar{E}$, U, $\bar{U}$ be positive reals such that:

$$0 < r < R < E, \qquad \alpha_1(E) > \alpha_2(R), \tag{32}$$

$$\bar{E} = E + 1, \quad U = \sup_{e \in \mathcal{B}^n_{\bar{E}}} |\bar{k}(e)|, \quad \bar{U} = U + 1, \tag{33}$$

where functions α_1 and α_2 are defined in (25), and $\bar{k}(e)$ is a uniform-in-time upper bound of the continuous-time feedback law $k(t, e)$ in (17)–(18). The upper bound $\bar{k}(e)$ depends only on the error e, computed from the bounded functions f and f_{ref} involved in Eqs. (8) and (12), respectively.

Furthermore, for any $\sigma \in (0, 1)$, and any partition $\pi_{a,\delta}$ with $a \in (0, 1]$ and $\delta > 0$, define the event-based control law (see [19], [20], for the case without quantization)

$$u(t) = u^*(\tilde{t}_h), \quad \tilde{t}_h \le t < \tilde{t}_{h+1}, \quad h = 0, 1, \ldots, \tag{34}$$

and the sequence $\{\tilde{t}_h\}_{h \in \mathbb{Z}^+}$, defined, for $h \ge 0$, as

$$\tilde{t}_{h+1} = \min\big\{t > \tilde{t}_h | - D^+V(t, e^x(t), u^*(\tilde{t}_h)) + \sigma D^+V(t, e^x(t), u^*(t)) \le 0 \;\; t = t_j, \;\; j = 0, 1, \ldots\big\}, \tag{35}$$

with $\tilde{t}_0 = 0$. In Eq. (35), the map D^+V is defined in (22), and L and K are positive reals such that the following inequalities hold

$$|k(t_1, e_1) - k(t_2, e_2)| \le L(|t_1 - t_2| + |e_1 - e_2|); \tag{36}$$

$$|D^+V(t_1, e_1, u_1) - D^+V(t_2, e_2, u_2)| \le K(|t_1 - t_2| + |e_1 - e_2| + |u_1 - u_2|) \tag{37}$$

$\forall t_1, t_2$, $\forall e \in \mathcal{B}^n_{\bar{E}}$, $\forall u \in \mathcal{B}^m_{\bar{U}}$.

We are now ready to state the main result of this letter, showing that there exist a sufficiently fast sampling and suitable accurate quantizations of the input/state channels ensuring the semiglobal practical stability for the closed-loop error system (19) (i.e., an *arbitrarily small tracking error* is reached within a guaranteed settling time starting from an *arbitrarily large initial error*) when the quantized sampled-data event-based control law (34)–(35) is applied.

Theorem 1: Let $\sigma \in (0, 1)$, $a \in (0, 1]$. Then, $\forall\, r, R \in \mathbb{R}^+$, $0 < r < R$, for any E, U satisfying (32)–(33), there exist positive reals δ, T, μ_u, μ_x, such that: for any partition $\pi_{a,\delta} = \{t_j,\ j = 0, 1, \ldots,\}$ of $[0, +\infty)$, for any input and state quantizers with error bounds μ_u, μ_x, and ranges U, E, respectively, for any $e_0 \in \mathcal{B}^n_R$, the solution of the error system (19) starting from e_0 with the quantized sampled-data event-triggered control law (34)–(35), with L and K satisfying (36)–(37), exists $\forall t \ge 0$ and, furthermore, satisfies:

$$|e(t)| \le E, \;\; \forall t \ge 0; \qquad |e(t)| \le r, \;\; \forall t \ge T. \tag{38}$$

Remark 1: We highlight that the control law in (34) is updated only when the *triggering condition*

$$-D^+V(t, e^x(t), u^*(\tilde{t}_h)) + \sigma D^+V(t, e^x(t), u^*(t)) \le 0 \tag{39}$$

holds. This condition only depends on the quantized state and on the quantized current and computed inputs and is not checked continuously in time but just at times t_j, $j = 0, 1, \ldots$, with a guaranteed minimum dwell-time $a\delta$ between two consecutive sampling instants, hence no continuous-time monitoring of the state variable is needed [20].

We highlight that quantized event-based sampled-data tracking for nonlinear systems with time-varying error model in the form (19) is a novel contribution of this letter.

Proof: Taking into account Assumption 1 and Theorem 1, let $V : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^+$ be a smooth function, α_1, α_2 be functions of class $\mathcal{K}_\infty$, and α_3 be a function of class $\mathcal{K}$, such that conditions (25)–(26) hold.

Let r, R, be any positive reals, $0 < r < R$. Let $a \in (0, 1]$ be arbitrarily fixed. Let $e_0 \in \mathcal{B}^n_R$, and let s_1, s_2, E be positive reals satisfying $s_2 < s_1 < r$ and $\alpha_1(r) > \alpha_2(s_1)$, and let E be a positive real satisfying the inequalities in (32)–(33), where the increased bounds $\bar{E}$, $\bar{U}$ are defined to account for the further uncertainty involved in the quantization of state and input, respectively. In particular, it can be readily seen that $x \in \mathcal{B}^n_E$ implies that $[x]_{\mu_x} \in \mathcal{B}^n_{\bar{E}}$, finally leading to $u^* \in \mathcal{B}^m_{\bar{U}}$ (see also [18]).

Taking into account that V is a smooth Lyapunov function, let M, L and K be positive reals such that conditions (36) (37) and the following inequality hold:

$$|F(t, e, u)| \le M, \tag{40}$$

$\forall t$, $\forall e \in \mathcal{B}^n_{\bar{E}}$, $\forall u \in \mathcal{B}^m_{\bar{U}}$. Let $\beta = \sigma\alpha_3(s_2)$. Let δ, μ_u, μ_x be positive reals such that:

$$0 < \delta \le 1, \quad s_2 + \delta M < s_1, \quad R + \delta M < E, \tag{41}$$

$$0 < \mu_u \le 1, \quad 0 < \mu_x \le 1, \quad \alpha_1(r) > \alpha_2(s_1) + \frac{2}{3}\beta\delta, \tag{42}$$

$$\frac{\beta}{3} \ge K(M\delta + (2 + \sigma)(\mu_u + (L + 1)\mu_x)). \tag{43}$$

Let us consider a partition $\pi_{a,\delta} = \{t_j\}_{j \in \mathbb{Z}^+}$, with $t_0 = 0$ (see Definition 1). Let $\{\tilde{t}_h\}$ be the sequence of event times defined in (35), and define the sequence $i_j = \max\{g \in \mathbb{Z}^+ | g \le j, t_g \in \{\tilde{t}_h\}\}$, $j = 0, 1, \ldots$, implying $i_0 = 0$ (the first triggering event is at time 0). From (34)–(35), we have

$$u(t) = u^*(t_{i_j}), \qquad t_j \le t < t_{j+1}, \; j = 0, 1, \ldots \tag{44}$$

Following the reasoning developed, e.g., in [18], [24], it can be proved that the solution exists in $[0, +\infty)$ ([18, Claim 1], see also [20]), and that $e(t) \in \mathcal{B}_E^n$, $t \geq 0$.

Let $w(t) = V(e(t))$, where $e(t)$ is the solution of the closed-loop system described by the extended state equation (19) with control law (34)–(35). Then, for any fixed $t \in (t_j, t_{j+1}]$, $j \geq 0$, for some $t^\star \in [t_j, t]$, by virtue of the Mean Value Theorem for integrals, one can write:

$$\begin{aligned} w(t) - w(t_j) &\leq D^+V(t^\star, e(t^\star), u^*(t_{i_j}))(t - t_j) \\ &= D^+V(t^\star, e(t^\star), u^*(t_{i_j}))(t - t_j) + \sigma D^+V(t_j, e(t_j), \bar{u}(t_j))(t - t_j) \\ &\quad - \sigma D^+V(t_j, e(t_j), \bar{u}(t_j))(t - t_j) + D^+V(t_j, e(t_j), \bar{u}(t_{i_j}))(t - t_j) \\ &\quad - D^+V(t_j, e(t_j), \bar{u}(t_{i_j}))(t - t_j), \end{aligned} \tag{45}$$

and, by exploiting (26), (36), (37), (40), we get:

$$D^+V(t_j, e(t_j), \bar{u}(t_j)) \leq -\alpha_3(|e(t_j)|) \tag{46}$$

$$\begin{aligned} &|D^+V(t^\star, e(t^\star), u^*(t_{i_j})) - D^+V(t_j, e(t_j), \bar{u}(t_{i_j}))| \\ &\quad \leq KM\delta + K\mu_u + KL\mu_x \end{aligned} \tag{47}$$

$$D^+V(t_j, e(t_j), \bar{u}(t_{i_j})) - \sigma D^+V(t_j, e(t_j), \bar{u}(t_j)) \tag{48}$$

$$= \begin{cases} (1-\sigma)D^+V(t_j, e(t_j), \bar{u}(t_j)) & \text{if } i_j = j \quad (49) \\ D^+V(t_j, e(t_j), \bar{u}(t_{i_{j-1}})) & \\ -\sigma D^+V(t_j, e(t_j), \bar{u}(t_j)) & \text{if } i_j = i_{j-1} \quad (50) \end{cases}$$

We now analyze the two subcases in (49)–(50):

- if $i_j = j$ (trigger), from (49), it is seen readily that

$$(1-\sigma)D^+V(t_j, e(t_j), \bar{u}(t_j)) \leq 0 \tag{51}$$

- if $i_j = i_{j-1}$ (no trigger), the triggering condition in (39), evaluated at $t = t_j$ and $\tilde{t}_h = t_{i_{j-1}}$ is false, implying

$$\begin{aligned} &D^+V(t_j, e(t_j), \bar{u}(t_{i_{j-1}})) - \sigma D^+V(t_j, e(t_j), \bar{u}(t_j)) \\ &\quad < (1+\sigma)K(\mu_u + (L+1)\mu_x). \end{aligned} \tag{52}$$

As a consequence of (51)–(52), taking the worst case between (49) and (50), one gets (from (48)):

$$\begin{aligned} &D^+V(t_j, e(t_j), \bar{u}(t_{i_j})) - \sigma D^+V(t_j, e(t_j), \bar{u}(t_j)) \\ &\quad \leq (1+\sigma)K(\mu_u + (L+1)\mu_x). \end{aligned} \tag{53}$$

Hence, from (43) and (45)–(53), we obtain that, for any $j \in \mathbb{Z}^+$, for any $t \in (t_j, t_{j+1}]$, the following inequality holds:

$$w(t) - w(t_j) \leq -\sigma\alpha_3(|e(t_j)|)(t - t_j) + \frac{1}{3}\beta(t - t_j). \tag{54}$$

From here on, the proof continues as in the one of [19, Th. 4.1] (see also [18]), in order to prove that there exists T such that $e(t) \in \mathcal{B}_r^n$ for any $t \geq T$, with the time T given by $T = \frac{3\alpha_2(R)}{\beta a} + 1$. The interested reader can refer to steps from [19, eqs. (4.24)–(4.32)] with $k_2 = [\frac{3\alpha_2(R)}{\beta a\delta}] + 1$. This concludes the proof. ■

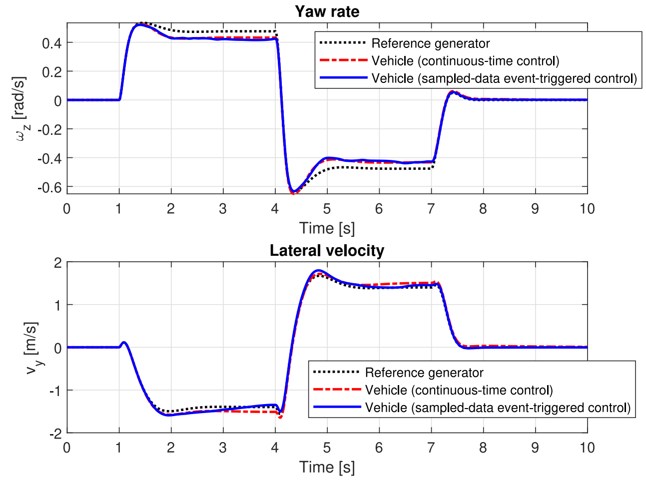

Fig. 1. State variables ω_z [rad/s vs s] (top panel) and v_y [m/s vs s] (bottom panel): reference generator (black dashed line), vehicle with continuous-time control (red dash-dotted line), vehicle with quantized sampled-data event-triggered control (blue solid line).

IV. Simulations

In this section, we will test on the extended model (1)–(7) the performance of the event-triggered controller designed on the reduced model (8)–(10). Simulations have been performed in MATLAB® by means of the `ode45` solver for nonstiff differential equations. The chosen parameters are the following ($j = l, r$): $A_{y,fj} = 1.81$, $B_{y,fj} = 7.2$, $F_{yfj,\text{sat}} = 4427$ N, $A_{y,rj} = 1.68$, $B_{y,rj} = 11$, $F_{yrj,sat} = 4197$ N, $m = 1550$ kg, $l_f = 1.17$ m, $l_r = 1.43$ m, $t_v = 1.53$ m, $J_z = 2300$ kg m^2, $\tau_m = 0.05$ s, $\tau_\delta = 0.05$ s, $\mu = 1$.

As a further non-ideality in addition to the unmodeled dynamics, we consider a realistic validation environment where the state is not fully available, but only sampled-data yaw rate measurements are taken and a state estimate is reconstructed. To this end, the sampled-data observer in [25] is used, with condition (39) being evaluated from the estimated state.

The state estimator in each sampling interval $[t_i, t_{i+1})$, $i \in \mathbb{Z}^+$, takes the following form [26]:

$$\begin{aligned} \dot{\hat{x}}(t) &= f(t, \hat{x}(t)) + Bu(t) \\ &\quad + e^{-\eta(t-t_i)}Q^{-1}(\hat{x}(t))K\big(y(t_i) - C\hat{x}(t_i)\big), \end{aligned} \tag{55}$$

where $K = \begin{bmatrix} -(\lambda_1 + \lambda_2) \\ \lambda_1\lambda_2 \end{bmatrix}$ assigns the eigenvalues $\lambda_1, \lambda_2 < 0$ of $(A - KC)$, with $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, to obtain convergence to zero of the estimation error $x(t) - \hat{x}(t)$, with $\eta > 0$ being a design parameter, and with the matrix $Q(x)$ defined in [25], Eqs. (13)–(15).

We consider a periodic sampling ($a = 1$) with $\delta = 0.05$, quantization values $\mu_u = 0.1$, $\mu_x = 0.05$, while the parameter σ affecting the triggering frequency is set to 0.3. We finally set $\lambda_1 = -4$, $\lambda_2 = -6$ and $\eta = 3$ in (55).

As a test maneuver, we consider a double step steer of 100° (at the steering wheel), given at time $t = 1$, starting from a straight-line condition with longitudinal velocity of 25 m/s (90 km/h). Fig. 1 shows the behavior of state variables, where the performance of the quantized sampled-data event-triggered

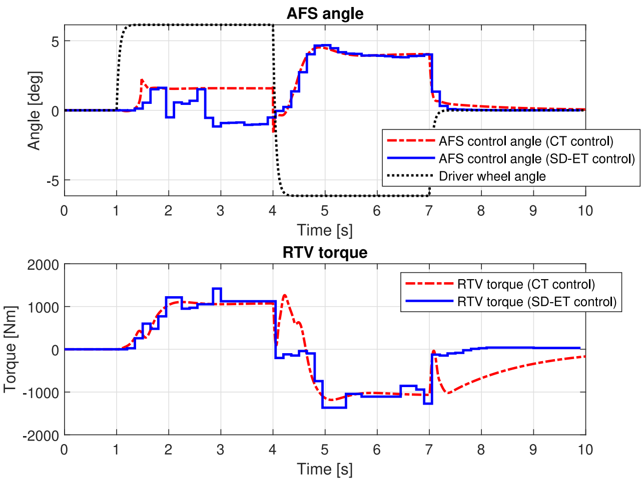

Fig. 2. Control inputs δ_c [*rad*] (top panel) and T_c [*Nm*] (bottom panel): vehicle with continuous-time (CT) control (red dash-dotted line), vehicle with quantized sampled-data event-triggered (SD-ET) control (blue solid line). The driver wheel angle is plotted in black dashed line in the top panel.

control is compared to the one exhibited by the continuous-time feedback in (16). Fig. 2 shows the control input in the two cases. As detailed in [9], the considered maneuver often leads to the saturation of the front axle in the transient and to the instability of an uncontrolled vehicle. The vehicle with sampled-data event-triggered controller shows a good state tracking, in spite of the considered non-idealities, and the loss of performance with respect to the continuous-time law is negligible, although the event-based control law is updated only in about 28% of the total number of sampling intervals.

V. Conclusion

In this letter, we presented a quantized sampled-data approach to the vehicle attitude control of ground vehicles. Starting from classical nonlinear design achieving tracking of prescribed trajectories in continuous time, we derived conditions for preserving the practical stability of the error dynamics by means of quantized sampled-data event-based controllers. Simulations performed in a non-ideal setting confirm the potential of the approach.

The method presented in this letter is not able to formally handle input saturation and state constraints in the current formulation. However, it would be possible to consider input constraints with the same methodology, at the cost that only regional practical stability (i.e., for a suitably small region of initial states) and not semiglobal practical stability can be ensured. This will be object of future work. Additional research effort will be devoted to the exploration of the output-feedback case and to the extensive validation of the controller on more realistic simulation environments.

References

[1] R. Rajamani, *Vehicle Dynamics and Control* (Mechanical Engineering Series). New York, NY, USA: Springer–Verlag, 2006.
[2] M. Guiggiani, *The Science of Vehicle Dynamics: Handling, Braking, and Ride of Road and Race Cars*. Dordrecht, The Netherlands: Springer, 2014.
[3] G. Burgio and P. Zegelaar, “Integrated vehicle control using steering and brakes,” *Int. J. Control*, vol. 79, no. 2, pp. 162–169, 2006.
[4] D. Bianchi, A. Borri, G. Burgio, M. D. D. Benedetto, and S. D. Gennaro, “Adaptive integrated vehicle control using active front steering and rear torque vectoring,” *Int. J. Veh. Auton. Syst.*, vol. 8, nos. 2–4, pp. 85–105, 2010.
[5] L. Claussmann, M. Revilloud, D. Gruyer, and S. Glaser, “A review of motion planning for highway autonomous driving,” *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 5, pp. 1826–1848, May 2020.
[6] S. Wei, Y. Zou, X. Zhang, T. Zhang, and X. Li, “An integrated longitudinal and lateral vehicle following control system with radar and vehicle-to-vehicle communication,” *IEEE Trans. Veh. Technol.*, vol. 68, no. 2, pp. 1116–1127, Feb. 2019.
[7] J. Rios-Torres and A. A. Malikopoulos, “A survey on the coordination of connected and automated vehicles at intersections and merging at highway on-ramps,” *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 5, pp. 1066–1077, May 2017.
[8] L. R. Ray, “Nonlinear state and tire force estimation for advanced vehicle control,” *IEEE Trans. Control Syst. Techn.*, vol. 3, no. 1, pp. 117–124, Mar. 1995.
[9] A. Borri, D. Bianchi, M. D. D. Benedetto, and S. D. Gennaro, “Optimal workload actuator balancing and dynamic reference generation in active vehicle control,” *J. Franklin Inst.*, vol. 354, no. 4, pp. 1722–1740, 2017.
[10] A. Tahouni, M. Mirzaei, and B. Najjari, “Novel constrained nonlinear control of vehicle dynamics using integrated active torque vectoring and electronic stability control,” *IEEE Trans. Veh. Technol.*, vol. 68, no. 10, pp. 9564–9572, Oct. 2019.
[11] H. Peng, W. Wang, Q. An, C. Xiang, and L. Li, “Path tracking and direct yaw moment coordinated control based on robust MPC with the finite time horizon for autonomous independent-drive vehicles,” *IEEE Trans. Veh. Technol.*, vol. 69, no. 6, pp. 6053–6066, Jun. 2020.
[12] G. Chen, J. Yao, H. Hu, Z. Gao, L. He, and X. Zheng, “Design and experimental evaluation of an efficient MPC-based lateral motion controller considering path preview for autonomous vehicles,” *Control Eng. Pract.*, vol. 123, Jun. 2022, Art. no. 105164.
[13] D. Bianchi, A. Borri, B. Castillo-Toledo, M. D. D. Benedetto, and S. Di Gennaro, “Smart management of actuator saturation in integrated vehicle control,” in *Proc. 50th IEEE Conf. Decis. Control Eur. Control Conf.*, 2011, pp. 2529–2534.
[14] F. H. Clarke, Y. S. Ledyaev, E. D. Sontag, and A. I. Subbotin, “Asymptotic controllability implies feedback stabilization,” *IEEE Trans. Autom. Control*, vol. 42, no. 10, pp. 1394–1407, Oct. 1997.
[15] W. P. M. H. Heemels, K. H. Johansson, and P. Tabuada, “An introduction to event-triggered and self-triggered control,” in *Proc. 51st IEEE Conf. Decis. Control*, 2012, pp. 3270–3285.
[16] D. Nesic, and D. Liberzon, “A unified framework for design and analysis of networked and quantized control systems,” *IEEE Trans. Autom. Control*, vol. 54, no. 4, pp. 732–747, Apr. 2009.
[17] D. Liberzon, “Quantization, time delays, and nonlinear stabilization,” *IEEE Trans. Autom. Control*, vol. 51, no. 7, pp. 1190–1195, Jul. 2006.
[18] M. D. Ferdinando, P. Pepe, and A. Borri, “On practical stability preservation under fast sampling and accurate quantization of feedbacks for nonlinear time-delay systems,” *IEEE Trans. Autom. Control*, vol. 66, no. 1, pp. 314–321, Jan. 2021.
[19] P. Pepe, “On stability preservation under sampling and approximation of feedbacks for retarded systems,” *SIAM J. Control Optim.*, vol. 54, no. 4, pp. 1895–1918, 2016.
[20] A. Borri and P. Pepe, “Event-triggered control of nonlinear systems with time-varying state delays,” *IEEE Trans. Autom. Control*, vol. 66, no. 6, pp. 2846–2853, Jun. 2021.
[21] H. Pacejka, *Tyre and Vehicle Dynamics*. Oxford, U.K.: Elsevier Butterworth–Heinemann, 2005.
[22] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River, NJ, USA: Prentice Hall, 2000.
[23] I. Karafyllis, and Z. P. Jiang, *Stability and Stabilization of Nonlinear Systems*. London, U.K.: Springer, 2011.
[24] F. H. Clarke, “Discontinuous feedback and nonlinear systems,” *IFAC Proc. Vol.*, vol. 43, no. 14, pp. 1–29, 2010.
[25] A. Borri, “A nonlinear sampled-data observer of vehicle lateral velocity,” *IEEE Control Syst. Lett.*, vol. 1, no. 2, pp. 244–249, Oct. 2017.
[26] F. Cacace, A. Germani, and C. Manes, “An observer for a class of nonlinear systems with time varying observation delay,” *Syst. Control Lett.*, vol. 59, no. 5, pp. 305–312, 2010.

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