



REVIEW OF COMPUTER-BASED METHODS FOR ARCHAEOLOGICAL CERAMIC SHERDS RECONSTRUCTION

REVISIÓN DE LOS MÉTODOS COMPUTERIZADOS PARA LA RECONSTRUCCIÓN DE FRAGMENTOS ARQUEOLÓGICOS DE CERÁMICA

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Highlights:

- The traditional manual method for reassembling sherds is very time-consuming and costly; it also requires a great deal effort from skilled archaeologists in repetitive and routine activities.
- Computer-based methods for archaeological ceramic sherds reconstruction can help archaeologists in the above-mentioned repetitive and routine activities.
- In this paper, the state-of-the-art computer-based methods for archaeological ceramic sherds reconstruction are reviewed, and some recommendations for future researches are proposed.

Abstract:

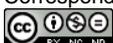
Potteries are the most numerous finds found in archaeological excavations; they are often used to get information about the history, economy, and art of a site. Archaeologists rarely find complete vases but, generally, damaged and in fragments, often mixed with other pottery groups. By using the traditional manual method, the analysis and reconstruction of sherds are performed by a skilled operator. Reviewed papers provided evidence that the traditional method is not reproducible, not repeatable, time-consuming and its results have great uncertainties. To overcome the aforementioned limits, in the last years, researchers have made efforts to develop computer-based methods for archaeological ceramic sherds analysis, aimed at their reconstruction. To contribute to this field of study, in this paper, a comprehensive analysis of the most important available publications until the end of 2019 is presented. This study, focused on pottery fragments only, is performed by collecting papers in English by the Scopus database using the following keywords: "computer methods in archaeology", "3D archaeology", "3D reconstruction", "automatic feature recognition and reconstruction", "restoration of pottery shape relics". The list is completed by additional references found through the reading of selected papers. The 53 selected papers are divided into three periods of time. According to a detailed review of the performed studies, the key elements of each analyzed method are listed based on data acquisition tools, features extracted, classification processes, and matching techniques. Finally, to overcome the actual gaps some recommendations for future researches are proposed.

Keywords: Computer methods in archaeology; 3D archaeology; 3D reconstruction; automatic feature recognition and reconstruction; restoration of pottery shape relics

Resumen:

Las cerámicas son los hallazgos más numerosos encontrados en las excavaciones arqueológicas; a menudo se usan para obtener información sobre la historia, la economía y el arte de un sitio. Los arqueólogos rara vez encuentran jarrones completos; en general, están dañados y en fragmentos, a menudo mezclados con otros grupos de cerámica. El análisis y la reconstrucción de fragmentos se realiza por un operador experto mediante el uso del método manual tradicional. Los artículos revisados proporcionaron evidencias de que el método tradicional no es reproducible, no es repetible, consume mucho tiempo y sus resultados generan grandes incertidumbres. Con el objetivo de superar los límites anteriores, en los últimos años, los investigadores han realizado esfuerzos para desarrollar métodos informáticos que permitan el análisis de fragmentos arqueológicos de cerámica, todo ello destinado a su reconstrucción. Para contribuir a este campo de estudio, en este artículo, se presenta un análisis exhaustivo de las publicaciones disponibles más importantes hasta finales de 2019. Este estudio, centrado únicamente en fragmentos de cerámica, se realiza mediante la recopilación de artículos en inglés de la base de datos Scopus, utilizando las siguientes palabras clave: "métodos informáticos en arqueología", "arqueología 3D", "reconstrucción 3D", "reconocimiento y reconstrucción automática de características", "restauración de reliquias en forma de cerámica". La lista se completa con referencias adicionales que se encuentran a través de la lectura

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de documentos seleccionados. Los 53 trabajos seleccionados se dividen en tres períodos de tiempo. Según una revisión detallada de los estudios realizados, los elementos clave de cada método analizado se enumeran en función de las herramientas de adquisición de datos, las características extraídas, los procesos de clasificación y las técnicas de correspondencia. Finalmente, para superar las brechas reales, se proponen algunas recomendaciones para futuras investigaciones.

Palabras clave: métodos informáticos en arqueología; arqueología 3D; reconstrucción en 3D; reconocimiento automático de características y reconstrucción; restauración de reliquias en forma de cerámica

1. Introduction

During archaeological excavations, a variety of potteries fragments (shortly called sherds) are excavated by archaeologists. These sherds provide valuable data relevant to the excavation site, such as periods, cultural groups, civilization, etc. (Kashihara, 2017; Son, Almeida, & Cooper, 2013), giving useful information to archaeologists to analyze data and to understand past life. Pottery is one of the most numerous artifacts in the excavations because of its resistance to atmospheric agents compared to other archaeological finds made with other materials.

Because of the large number of sherds and the existence of different shapes and sizes, the traditional manual method is costly and time-consuming (Di Angelo, Di Stefano, & Pane, 2017). It requires a great deal of effort for skilled archaeologists, involved in repetitive and routine activities (Brown et al., 2008; Di Angelo, Di Stefano, & Pane, 2018).

To overcome the above-mentioned problems, which limit both the number and the quality of the information provided by excavation, in the last two decades, the problem of reassembling fractured 3D objects has gained increasing importance and many different approaches have been developed (Kleber & Sablatnig, 2009; Rasheed & Nordin, 2015). These research activities were encouraged by the advent of low-cost acquisition systems with appropriate resolution and accuracy used in the archaeological field. The main goal of these studies is to increase efficiency together with reducing costs and working time by finding corresponding fragments and matching them. Such methods have the potential to help archaeologists on a large scale, assisting them with routine, repetitive, and time-consuming tasks. Commonly, the published methods include data acquisition and pre-processing, feature extraction, classification, and matching (Kleber & Sablatnig, 2009; Rasheed & Nordin, 2015). The information typically considered by the published methods for automatic reassembly is related to the shape and texture of fragments. Therefore, the quality of the recognition depends both on the quality of the clusterization (i.e. their cataloging) of the fragments and the quantitative information deduced from each one.

Due to the important implications that automatic methods of reconstruction can have in archaeology, in this paper, the performed studies in the field of computer-based methods for archaeological ceramic sherds reconstruction are reviewed. To enrich the proposed revision, methods for the extraction of sherds' quantitative information that has the potential to improve searches of matching of fragments are also analyzed. This review may contribute to providing to the reader essential references for understanding the actual challenges and developments in these areas; furthermore, some recommendations for future researches to overcome the actual gaps are proposed.

2. Literature review

In this section, the most important methods, published by the end of 2019, for reassembling artifact fragments are analyzed. It should be noted that this study focuses on pottery fragments only. The papers are collected by using in the Scopus database the following keywords: "Computer methods in Archaeology", "3D archaeology", "3D reconstruction", "Automatic feature recognition and reconstruction", "Restoration of pottery shape relics". Only papers in English were selected. Additional references, found through the reading of selected papers, completed the list. In total 53 papers were selected. The performed review of the related studies is divided into three periods of time.

2.1. Performed studies before 2000

During this period, most of the researchers were interested in procedures for drawing, archiving, storing, and retrieving the excavated fragments. However, some studies worked on the field of reassembling objects from their fragments. In the following, the most important studies are outlined.

Hall & Laffin (1984) proposed a new procedure to represent the profile of original ancient pottery. For this purpose, the authors utilized B-spline curves to approximate the profile of three-dimensional (3D) solids of revolution. In other words, the B-spline technique is used to generate object outlines and draw a profile of the pottery vessels. After the drawing profiles of 3D modeling of pottery, the results are saved to storage via computers for subsequent display and statistical analysis.

Sablating & Menard (1997) proposed two acquisition methods for archaeological finds that could help the archaeologist in his work. They focused on the acquisition methods to minimize errors in the output and to completely automate this process. To get a 3D-surface representation of a sherd, they tested the shape obtained by a structured light scanner. Then, they presented the outlooks for a computer-based automatic classification of archaeological finds. The classification is based on features such as excavation site, excavation layer, material, and color.

By assuming that the pottery object is rotationally symmetric, Halir and Menard (1996) proposed an interesting method for diameters estimation of archaeological pottery sherds (Halir & Menard, 1996). In the proposed method, the sherd is manually oriented in the measurement area and illuminated by a laser plane. Halir & Flusser (1997) extended the approach (Halir & Menard, 1996) to a simple and robust profile estimator. In their work, the parameters including the diameters and the perimeters of the object are estimated based on the radius of one fragment. Also, the parallel arcs with different diameters and one axis are generated by using the intersections of the surface fragment with several parallel planes. To form a circular arc, the fragment is

placed in the correct orientation based on archaeologist opinion. Then, the intersection between the surface of the fragment and the projected laser plane generates the circular arc. The authors evaluated the method on both synthetic data and real pottery data; the results have indicated an error rate equivalent to 2-3 mm (Halir & Flusser, 1997).

An automatic reconstruction method is presented by Sablatnig *et al.* (1998), which uses the properties of the profile's curvature. For this task, the authors considered several shapes and partial information such as the rim, body, and base of the fragment. Their work consists of two main parts: first, the descriptive language procedure is used for the classification of unknown fragments, in which the profile is divided into primitive elements and stored. The fragments are classified by comparing their descriptive language with other fragments by calculating the graph similarity. In the second part, the reconstruction of the object is implemented based on the bottom-up strategy. This strategy is conducted according to the highest similarities between the graph and subgraph.

The reassembling of objects from their fragments becomes complicated if some parts are missing. To solve this problem, Üçoluk & Toroslu (1999) presented a new robust matching algorithm. For this purpose, first, the boundary curves are considered to represent the 3D surface fragment. Next, the curvature and torsion scalars are calculated from the discrete 3D boundary curve data as features. The features vector is provided at each discrete boundary curve point of the fragment. Finally, a Noise Tolerant algorithm is applied to match simulated broken objects. For this task, the proposed method matches and aligns the fragments of the object by comparing the broken surface boundary curves.

In most researches on archaeological finds, the researchers have proposed methods based on the fact that every ceramics or potteries are made on the wheel, and the horizontal section of broken fragments is in the form of a circular arc. Halif (1999) proposed an automatic approach to estimate the axis of rotation of archaeological pottery fragments based on geometrical properties. The proposed algorithm applied direct least-squares optimization and the M-estimator method to obtain a more robust estimation. Also, the authors applied an iterative refinement of the estimated rotation axis by a robust circle and line fitting for decreasing the effects of noise outliers and systematic errors on the estimation of the axis of rotation.

2.2. Performed studies from 2000 to 2009

In this period, the researchers have presented new automatic and semi-automatic reconstruction methods of archaeological pottery by applying new features such as texture, color in addition to geometric ones. These studies are summarized below.

Until then, computer-aided restoration of archaeological finds has focused on visualization and archiving of scanned objects, image processing and reconstruction of certain well-structured objects based on feature classification. However, few studies have been done to reconstruct objects from their fragments automatically. Papaioannou *et al.* (2000) introduced an approach based on surface morphology. This study is presented as the semi-automatic reconstruction of archaeological finds. In their approach, the description language method is

applied to classify the fragments by using geometrical information. In the next step, the authors utilized the depth buffer approach to estimate the fracture zones of the fragment-based on surface bumpiness. Next, the system chooses the least irregular sides for correct matching. Then, the matching error for all candidate facets for every pairwise of fragments is calculated. Finally, the full reconstruction is done on the pairs of candidate fragments that have small matching errors. The approach is implemented on synthetic and real data. The results have shown the method correctly assembles the 50% of fragments without material and structural constraints or user enforced selectively, and 90% with constraints and user intervention.

Cooper *et al.* (2001) proposed a framework for the automatic assembling of 3D pots by using a 3D model data of fragments. Their proposed approach consists of three steps: (i) generation of sherd-data starting from a description of the model; (ii) calculation of a probability measure for a first attempt arrangement of sherd data to represent a vase; (iii) aligning the sherd-data (such that the probability of this alignment and prior pot shape information is maximized). In assembling 3D sherd-data for estimating a mathematical model of a full pot, the Bayesian approach is used. The authors applied their approach by testing five fragments selected from Petra, Jordan.

Efforts have been made to describe and classify sherds based on the mathematical definition of shape and type. Schindler *et al.* (2001) have developed an automated classifying archaeological ceramic fragment system by depending on the profile. Therefore, they implemented several methods to interpolate and approximate the vessel profile by using B-spline. In their work, the profile sections are obtained automatically by a 3D-measurement system, which consists of the structured light and two laser techniques. Then, the authors combined several methods of approximation and interpolation of the closed curve by applying B-splines, since this function can provide a satisfactory approximation (Hlavackova-Schindler, Kampel, & Sablatnig, 2001).

Andrews & Laidlaw (2002) designed a computational framework for the automatic assembly of broken pottery vessels. Their research includes automatically comparing the features to reconstruct vessels from a pairwise of pottery fragments. The method starts by generating a set of match candidates for each pair using a proposed module. Then, the placement of each candidate is adjusted to maximize the ensemble likelihood and thereby improve the alignment of all features. For this task, a quasi-Newton algorithm for continuous function optimization is applied. Then, the resulting optimized matches are ranked according to their maximized ensemble likelihood values. Finally, a greedy strategy is used to select pair-wise matches. The procedure is tested on eight groups that included 16 fragments; the method achieved the correct reconstruction for only 13 valid pairwise matches.

Mara *et al.* (2002) developed a system for classification based on 3D-models of the sherds. For the 3D data acquisition of fragments, they used structured light. Next, the 3D-model is orientated according to the axis estimation performed by a plane-fitting and Hough-inspired method. The classification of the fragments depends on the profile shape, and multiple

measurements of the fragment, such as the diameter rim, diameter of the wall, height, and characteristic ratios. Therefore, in the next step, the fragment profile and the curvature points are estimated. Then, a set of characteristic points is defined on the profile by using segmentation rules based on curvature, tangency, and some invariants. The authors evaluated the proposed system for classification, a total of 70 real fragments. The results have shown that the system able to classify 62 fragments successfully.

In some methods, the classification is performed by using drawing and measuring techniques on a 2D profile.

Cao & Mumford (2002) proposed a robust approach to estimate the geometric structure from noisy 3D data of potteries with a symmetrical axis. In their approach, the symmetry rotational axis estimation is based on the property that for any point on a surface of revolution, the symmetry axis contains the center of the sphere of curvature corresponding to the parallel circles. To apply this property to real data, the symmetry rotational axis is determined through finding the line which minimizes the least-squares distance from the estimated centers to it. Then, the distance between each data point and the obtained axis is computed. Next, the profile curve is performed by a cubic spline fit. For assembling the pot from several fragments, the confidence bounds of the axis and the profile curve are considered as features. These features are computed by applying the bootstrap method.

Until then, the problem of reassembling an object from its fragments has never been addressed with an integrated computational approach. In other words, the proposed methods depended just on the fragments' geometric shapes and not on the recognition of specific features. Papaioannou et al. (2002) presented an approach to reconstruct the original objects from 3D sherds based on the fragments' geometric features. For this task, a matching error estimation algorithm based on the distance between the facing fragment sides is applied. The matching was performed directly through the plane between two arbitrary fragments, where the distance was calculated between the mutually-visible faces of the fragments, by utilizing the 3D points of the whole surfaces for fragments pair. Next, an error measure between the distances for both of the two faces of the fragments is computed. A global optimization algorithm is, then, applied with this error estimator as a cost function and material axis/surface overlap as constraints. The authors tested their approach on the digitized models of real fragments and the results of the fragments matching are achieved with constraints or without constraints.

Willis et al. (2003) presented a computerized method for estimating the axis/profile curve of an archaeological sherd based on axially-symmetric implicit polynomial surface models. In this method, the estimation of the axis/profile curve is performed by detailed statistical error analysis. Next, reconstructing the 3D fragments is done through computing the axially-symmetric algebraic surface. The authors applied their method by testing five fragments selected from Petra, Jordan. To enhance the robustness of their method, they used a bootstrap algorithm that included a set of information for each fragment, such as a covariance matrix for axis parameters.

Kampel & Sablatnig (2003a) proposed a profile-based approach for reassembling pottery fragments. In this

approach, the original form of the vase is estimated through one fragment and the fragments are assembled without manual intervention. The authors focused on the estimation of the correct orientation and the profile of the fragment for the classification of the fragments. In the reconstruction stage, the authors evaluated the partial similarities of the profiles, and the pots were assembled based on the description of the data stored. The proposed approach is tested on 40 pottery fragments, and the results confirm 50% successful (Kampel & Sablatnig, 2003). In the same year, Kampel & Sablatnig (2003b) presented a prototypal system for the automatic storage of archaeological fragments. In their proposed system, the profiles related to archaeological fragments are used to effectively classify and reconstruct virtually sherds. The profiles are obtained by sectioning the fragment in the direction of the symmetry rotational axis. The authors have evaluated their reconstruction method on a larger test set taken from our 3D-mural test excavation site Sagalassos (Kampel & Sablatnig, 2003).

A semi-automatic approach is proposed by Melero et al. (2003) for the 3D reconstruction of Iberian vessels. For this purpose, they estimated the orientation of the fragment, computed the symmetry axis, and detected the profile of the sherd. They have implemented a software tool to carry out these tasks in a semiautomatic way. Their approach follows the same steps of the traditional procedure but using an interactive process that works with a virtual 3D model of the sherd. To estimate the correct orientation of a fragment, the authors used the genetic algorithms. The classification of the vessel fragments is done based on some features such as the estimation of the diameter at different heights, rim angle, the orientation of shape, extracting the profile, and drawing of the fragment. The 3D reconstruction of the vessel is generated by rotating the profile around the rotational axis.

One of the challenges in archaeological finds is that sometimes, the fragments are too small; hence estimating an accurate axis/profile-curve of a fragment is not obvious and may not even be possible. Furthermore, the break-curves (along which the surface breaks into pairs of fragments) may be eroded and chipped, so that the search space for reconstruction of these fragments can become huge. Willis & Cooper (2004) proposed an automatic system based on the Bayesian approach to overcome these challenges. Their method is done in three stages. First, 3D sherd-data is obtained by using a laser scanner. In the second stage, the fragment outer surface and break curves are approximated by mathematical models. Finally, optimal alignment is implemented by the Maximum Likelihood Estimation (MLE) method on break curves and outer surface. The proposed based on a Bayesian approach has three significant advantages: (i) it allows the user to combine different types of extracted information; ii) the search neglects unlikely configurations, and (iii) it is reasonable in terms of computational complexity for aligning break curves and sherd surfaces simultaneously. The result obtained was the assembling of 10 out of 13 fragments belonging to one vase.

So far, no integrated system has been proposed for data acquisition to reassemble sherds. Kampel & Sablatnig (2004) proposed a 3D method for the puzzling of archaeological fragments. This study develops an approach for the automated documentation of archaeological pottery, which also leads to a more

complete 3D-model out of multiple fragments. The proposed method has 4 four main tasks including 3D data acquisition, Orientation of the object, Classification of the object, Reconstruction. The authors used the Description method for the fragments' classification. The matching process has been done by primitives and relations among the fragments. The authors proposed an approach for a matching algorithm relying on the point-by-point distance between facing outlines. Matching results have been achieved in some fragments and failed in others.

The computation of the rotational axis is ambiguous when the surface of the fragment is too flat or too small. [Kampel and et al. \(2005\)](#) presented a new technique for fragments' orientation based on the rills on the inner side of the fragment. This method is similar to the traditional manual way of estimating the axis of rotation. In their method, first, the inner side of the fragment is defined. Then, the surface of the fragment is segmented into upper, medium, and lower curvatures. The orientation of the fragment is estimated, and the classification process is based on external points and estimates the oriented profile. The authors tested their method on 35 fragments with a small size and the lower curvature.

[Maiza & Gaildrat \(2005\)](#) proposed a method for automated classification of 3D archaeological fragments based on the technique of implicit surfaces. First, 3D data acquisition is done by the Konica Minolta VI-910 3D scanner. Next, axis and profile curves are obtained by carrying out a vertical cut of the object with a plane passing through the z-axis and perpendicular to the (x, y) plane. Finally, the distance between the tested model and the position of the specified fragment is computed. The features that are used in this study include the position and the orientation for each fragment. To obtain the best match, the distance between two pottery fragments is computed and then a genetic algorithm is applied for finding the perfect location based on profile computation.

The orientation of the sherd is done by estimation of the axis of rotation. In previous methods require the object to be symmetric and complete as well as including manual interaction for estimation of the axis of rotation. [Mara & Sablatnig \(2006\)](#) presented a method, which has been inspired by the manual method of archaeologists. Finally, the authors applied their method on synthetic fragments in different shapes and small sizes and real well-known vessels. The method had a small number of errors.

In many cases, reconstruction is driven by geometric features, while some other information such as color and texture were ignored. [Huang et al. \(2006\)](#) presented an approach for automatic reassembly of 3D fragments solids based on the type of patch-based surface features. In this work, the features are characterized by clusters and overlapping. Their approach includes four main steps: Data segmentation, Feature selection, Pairwise matching, and Multi-piece matching. First, they used a multi-scale edge extraction method for segmentation the surface of each fragment. Then, the graph-cut algorithm is used for partition faces to original and fracture faces. Next, the final features set is obtained robustly through a forward search algorithm. The matching process between the fragments of an object generates a set of possible matches. To final reconstruction, global multi-piece matching is computed, and a local multi-piece registration is performed simultaneously.

[Igwe & Knopf \(2006\)](#) presented an algorithm for enabling free-form shape reconstruction from digitized data of

fragmented pieces. They used unsupervised learning of the self-organizing feature map (SOFM) algorithm on the topological structure of the fragment for clustering the fragments automatically. Then, they established the lattice spherical mesh with triangular elements based on a 3D SOFM. To reconstruct, the largest fragment is selected to be the target and is assembled with the rest of the parts that have a similar geometry.

A solution to deal with the problems of reassembling fragments with thickness is presented by [Zhou et al. \(2007\)](#). Their approach covers the techniques of extraction of contours (as external and internal contour features) on solid objects with fractured surfaces. Next, polygonal arcs of the triangular shape of fragments are analyzed and a matching algorithm was implemented based on junction vertices. Finally, for the pottery reconstruction, the author used a binary tree algorithm, so that the fragments were represented as nodes. Since there are some seams and holes, as some data have been lost, a repair method has been investigated by authors.

[Brown et al. \(2008\)](#) present an inexpensive system for acquiring information of the shape on the side of the fragments, color, plaster surface texture, and surface roughness for small objects. In their system, first, a virtual 3D model of each fragment is captured with high-resolution color and texture information of the front surface. Then, matching candidates between pairs of fragments are obtained by computing the corresponding error at all possible orientations. The authors implemented their system on the specific problem of documenting and reconstructing fragments of wall paintings from the site of Akrotiri on the volcanic island of Thera. The results indicated that the proposed system to match the fragments has achieved high precision.

2.3. Performed studies from 2010 to 2019

With the advancement of technology, the use of machine vision systems and image processing in the field of object reconstruction is becoming more prominent. Also, it should be noted that the use of some new features such as morphological features besides geometric features has led to better results with high accuracy. In the following, the most important studies within the period 2010 to 2019 are highlighted.

[Zhou et al. \(2010\)](#) presented how to reconstruct pottery based on the rotation axis and profile. They used the laser scanner and structured light scanner to get information from the fragments' surface. Then, the authors proposed a method to estimate the rotation axis based on the Pottmann and optimization methods. Next, the full profile of the sherd is calculated by assembling adjacent fragments through matching features of the various fragments. Finally, the researchers used texture mapping for the purpose to display the pottery model in a much more realistic way.

[Toler-Franklin et al. \(2010\)](#) suggested a multi-feature approach to determine matches between small archaeological fragments. The authors used three types of information to extract features from the database: color maps, normal maps, and 3D meshes. In order to classify fragments, a machine learning method is used, so that the classifier is trained to score patch pairs based on the differences between the calculated properties of patches. The authors have tested their system on three datasets of fresco fragments. Their method achieved a correct

percentage of the features selected equal to 90%, and non-match 78%.

Most of the recently proposed methods fall into the category of boundary matching, which used the boundaries of the fragment for matching. [Cohen et al. \(2010\)](#) suggested a surface-based matching method which is derived from exploited surface markings. Their work includes four steps. In the first one, ceramic vessels are scanned using Konica Minolta Vivid 910 3D scanner. In the second step, all surface markings on the vessels are extracted through thresholding the color information of the markings. Then, a 3D convex hull is extracted for each surface marking. And finally, surface alignment is done using affine moment invariants that are constructed through the convex hull of surface markings.

[Karasik & Smilansky \(2011\)](#) proposed a method to classify pottery fragments by analyzing the morphological information of profiles. In their method, the profile of pottery fragments represented in terms of a distance function (radius, tangent, and curvature). The approach is performed in three main steps: first, the Principal Component Analysis (PCA) method is used to provide the most economical characterization of the correlations within the data. In the second step, the Cluster Analysis (CA) method is used to cluster. Finally, Discriminate Analysis (DA) tests the significance of the resulting typological classification. The proposed approach has been applied on 358 fragments from Iron Age at Tel Dor, Israel, and the results showed that the accuracy of the classification of fragments is 94.8%.

For the reconstruction of fractured archaeological artifacts, previous computer methods arranged scoring functions by considering several features for potential matches, such as color and geometric compatibility across fractured surfaces. Also, they usually considered only one or a few properties at once; therefore, they provided matching predictions with very low precision. [Funkhouser et al. \(2011\)](#) suggested an approach to sort predicted matches between pairs of fragments according to matching precision. Their method follows the observations of two papers:

- [Shin et al. \(2010\)](#), whose algorithm analyzed many properties in the matching in assembled frescoes;
- [Toler-Franklin et al. \(2010\)](#) proposed to considering some features of matching, including surface color, normal maps, and edge geometry using machine learning; examined the matching features, including surface color, normal maps, and edge geometry using machine learning.

The method proposed by authors used a set of examples to train a classifier, based on a multitude of computable properties such as contour and ribbon, junction angle, and others. After the classification model is built from the training set, the accurate probability is estimated for a new matching between a pair of the fragments. Then, a classifier was trained on the three datasets taken from different regions. The results show that it is possible to train the classifier for matching a dataset based on its properties and then used it for prediction matching from another set ([Funkhouser et al., 2011](#)).

[Belenguer & Vidal \(2012\)](#) presented a global registration technique for archaeological fragments reconstruction. The authors focused on setting up a 3D characterization format for fragments to accelerate the search process. In their technique, all heavy calculations are performed on

the Graphics Processing Unit (GPU). In this way, the complexity related to geometric transformations, visibility tests, and discretization operations is removed. Then, a search algorithm is applied to align fragments. Also, the hierarchical search technique is applied to obtain the optimal solution for the problem of storage in memory. The results indicated the efficiency of the proposed technique (250 times faster than exhaustive search) ([Belenguer & Vidal, 2012](#)).

In some cases, if the original whole pottery shape is unknown, 3D reconstruction cannot be performed. So, several possible combinations must be investigated and evaluated by experts based on their knowledge and experience to find patterns and shapes from the fragments. [Kashihara \(2012\)](#) presented a method to assemble archaeological fragments without any previous knowledge about patterns and mathematical models. In their method, a Genetic Algorithm (GA) computation is firstly used for finding a global solution. Next, the hill-climbing method is implemented for fine-tuning. Then, the silhouettes of an object are achieved by several cameras located at different angles. Finally, a matching step is accomplished among fragments based on the obtained silhouettes. The method validation is performed on a vase that consisted of five fragments.

Axially symmetric pots can be automatically reassembled from their fragments by using two important information: the existence of an ideal axis of symmetry and reassembly has to be performed coupling the break curves. [Son et al. \(2013\)](#) proposed an approach to automatically assembly fragments that are critical for other methods. These fragment types include those which are almost flat, chipped, and represented by very noisy data. For solving this problem, under the hypothesis that the object is axially symmetric, two methods evaluating the local and global solutions are applied. In their approaches, the accurate Axis of symmetry Profile Curve (APC) for each fragment is estimated by using circle templates. Then, the reconstruction step is done based on an APC based method by using the break-curve matching method. Results show that the system is robust to noise, bumps, and erosion. Also, the approach was able to reassemble three vases of 48 fragments in 10.56 hours.

In previous works, some problems in the reconstruction objects from fragments are still unresolved, for example when the size of fragments is very small. In this case, the axis cannot be uniquely determined, and the estimation of the axis has low accuracy. [Han & Hahn \(2014\)](#) proposed a method for axis detection considering multi-scale and principal curvatures constraints. In the same paper, they proposed a method for grouping sherds, based on the distribution of principal curvatures.

A new method to reconstruct pottery from archaeological fragments based on a polynomial function is presented by [Rasheed & Nordin \(2014\)](#). The proposed method consists of five: the first one is the image acquisition by the camera, followed by the pre-processing to improve the robustness of the extracted features. After that, for each fragment, the edge curve as a feature is extracted by the Canny filter. Next, the polynomial function algorithm is applied on the edge of fragments to obtain the vectors of coefficients. Finally, the classification process is conducted based on the correlation of the coefficients; then the best matching between a pairwise of pottery fragments is done according to the relationship of their coefficients.

Vendrell-Vidal & Sánchez-Belenguier (2014) proposed a global registration discrete method to reconstruct automatically the ancient artifacts from flat archaeological fragments (typically fresco fragments). At first, the 3D models of fragments are pre-processed to speed-up search. All heavy calculations are executed by the GPU in the pre-processing stage. Then, a cost function is defined to evaluate the quality of alignment based on a discrete sampling of the fragments. To ensure the convergence of the global solution, a hierarchical strategy is added. Theoretical and experimental evaluations of the proposed method have shown great performances.

A new computer-based algorithm to classify ancient pottery is proposed by Rasheed & Nordin (2015). The proposed technique consists of several steps. First, six images from different angles of each fragment are captured. To enhance the selection of the important features, the image segmentation algorithm is applied to separate the fragments from the background. Then, the image features were extracted using two fundamental procedures: the intersection of colors between the fragments, and the Gray Level Co-occurrence Matrix (GLCM) to extract color feature and four texture features (Contrast, Correlation, Entropy, and Homogeneity) respectively. To classify the fragments, the authors proposed the new algorithm based on color features and defined the Euclidean distance equation based on texture features. The performance of the algorithm is evaluated on a pottery database. This analysis has given a success rate of 95%.

Cohen *et al.* (2016) presented a complementary approach to reconstruct vessels virtually by aligning 3D scanned fragments. For this purpose, generic models of vessels are generated based on the experts' historical knowledge, the provenance of the artifact, and site location. Then, to produce a virtual reconstruction, the fragment with the surface's markings is aligned against a generic model using weighted moments. If there are no surface markings or if the fragment with markings cannot be aligned to a generic model, the mending of fragments is done based on anchor points on borders. The mending process is applied to pairs of fragments, without any global consideration. For symmetric vessels, this problem is solved by adopting the surface of revolution as a global constraint. The authors used the fragments excavated from the Independence National Historical Park (INHP) in Philadelphia, Pennsylvania.

Lucena *et al.* (2016) proposed a new method based on morphological data to help the archaeologist for profile classification. In this way, the authors proposed a decision support system by encoding morphological data on potteries profiles. To assign the most similar class to a given profile, they applied a pre-processing stage on profile. The profile is split into several parts (lip, neck, body, base, and handles), and then each of them is characterized by a vector obtained by sampling its morphological curves. Finally, Euclidean Distance is used as a similarity measure. This methodology is implemented on the cases from the upper valley of the Guadalquivir River (Spain) for measuring the similarity between ceramic profiles.

Kotoula *et al.* (2016) presented a comparative analysis of manual and digital reconstruction. In the paper, three different semiautomatic approaches for fragments matching (MeshLab, Fragments Reassembler, and 3ds Max) are compared with each other as well as with

manual methods; the methods are compared on the base of their effectiveness in the alignment of fragments. Finally, an integrated strategy is proposed for semi-automatically assembling fragments to provide the best results with time efficiency. In this study, Faenza maiolica, black-glazed, Gnathian, and coarse ware ceramics were used as case studies. The results indicated that the proposed Fragments Reassembler is the most efficient approach for the alignment of fragments, and it showed the overall successful execution of semiautomatic reconstruction.

All previous techniques are affected by the problems introduced by the external wear and decay of the material during the exposure in the soil; it follows that many surface's characteristics of the sherd change. As a result, the efficiency of research methods based on external characteristics is decreased. Stamatopoulos *et al.* (2016) proposed a new method based on exploration, extraction, and utilization of all possible thickness information (Thickness Profile, TP) possibly included in each sherd and cannot be affected by the presence of harsh environmental conditions. The proposed methodology consists of three steps. In the first one, each fragment is fixed on a stable basis and photographed, from all sides and various angles. Then, a 3D model is set up by using specialized software. In the next step, the optimal TP of the 3D model of each fragment is extracted. Finally, the repetitive process is implemented between TPs for maximizing matching scores between possibly neighboring sherds. To evaluate the methodology, the specific vessels are intentionally broken and are used. The results indicate that reassembling with the TP method is successfully done in the analyzed test cases; it seems not to be affected by the above-mentioned problems (Stamatopoulos & Anagnostopoulos, 2016).

Kashihara (2017) presented a smart computer assistance system to reconstruct archaeological finds from some fragments. For this purpose, the method is based on a Real-Coded Genetic Algorithm (RCGA) to solve the positioning problem of a 3D restoration. The image features of 3D objects are considered for the GA process. So, the image features based on the Accelerated KAZE (AKAZE) method are computed. After approximating a global solution by RCGA, a hill-climbing method is applied to fine-tune the 3D positions. Simulation results indicate that the presented approach can efficiently adjust the positions of 3D fragments.

An automatic classification method for the digitization of pottery profile drawings is presented by Banterle *et al.* (2017). In this work, a structured description of the main geometric features is extracted and then, a 3D representation of each class is generated based on the geometric features. Next, these data are used to populate the reference database for the classification and to build a huge set of synthetic sherds. These synthetic sherds are employed to train the classification system. The authors implemented their methodology on three typologies of pottery: Roman amphorae, terra sigillata, and medieval pottery.

Rasheed *et al.* (2017) proposed a novel method to classify archaeological ancient pottery fragments based on the HSV color feature. The method starts with the conversion of images from their original RGB to HSV color. Then, a 2D median filtering algorithm is applied to remove noisy objects of various shapes and sizes. Next, each image is divided into six sub-blocks and

the HSV color feature is extracted by the mathematical method, similar to the one used in (Rasheed & Nordin, 2015). At the final stage, the fragments are classified by using a Self-Organization Map (SOM) method, which is an unsupervised learning process. The presented method is applied on several images of 2D ceramic fragments and achieves a value of 89.6% for classification of the fragments into similar groups (Rasheed, Nordin, Dakheel, Nados, & Maaroo, 2017).

Archaeologists usually identify some specific dimensional features, which consist of factors useful to retrieve the whole shape. This activity, manually performed by experts, is time-consuming, expensive, and affected by wide uncertainties. To solve these problems, Di Angelo et al. (2017) proposed a new automatic method that performs the dimensional characterization of archaeological pottery. First of all, a 3D high point density model of the sherd is obtained. Then, the proposed method consists of three main steps: (i) axis identification; (ii) features (geometrical and morphological) segmentation and recognition; (iii) dimensional feature evaluation. The authors have verified their proposed method in the extraction of dimensional features, by comparing the performances with those of traditional methods (Di Angelo et al., 2017). It is noteworthy that Di Angelo et al. (2018) conducted similar research to this study on an Olla and an Amphora as new case studies.

Since ancient potteries could have a lot of missing parts, only a limited number of fragments exist to reveal the basic design of them. Fragkos et al. (2018) proposed a methodology to construct missing fragments of an archaeological find. For this task, there are three main phases. In the first phase, the authors utilized the NextEngine 3D laser scanner and the ScanStudio software for the data collection. Next, the Geomagic Studio software is used to set up the 3D model. In the area where the missing fragments are supposed to be, sketches are created through reverse modeling. With these sketches, the missing fragments are designed. Finally, the digital fabrication of the missing part is reconstructed by the Additive Manufacturing technology.

In archaeological finds some ancient potteries present decorative elements, which are not axially symmetric surfaces. However, these potteries can be semantically significant and have specific geometric features useful to drive the assembling process. For this type of archaeological potteries, Di Angelo et al. (2018) proposed a robust methodology to automatically recognize not axially symmetric geometric features with a constant radius. This methodology consists of two main steps: (i) segmenting the constant radius geometric features, (ii) measuring the dimensional parameters of features. The segmentation step is conducted using a nonconventional logic suitable for exploring the object with a fuzzy sensitivity. The second step is performed by a robust fitting method applied to the segmented entities. The authors analyzed an Olla pottery for evaluating the proposed method.

A critical situation in the archaeological fragments' reconstruction occurs when this activity has to be made without knowing the whole pottery's shape. This knowledge helps to manage the presence of gaps due to losing parts of the artifact. To overcome this problem, Rasheed & Nordin (2018) presented an approach to

classify fragments and to reconstruct the original object from them. The classification is done based on color and texture features. For this task, first, the fragments are classified depending on the color with the proposed algorithm (Rasheed & Nordin, 2015). Then, the fragments are classified, a second time, based on the texture using the Euclidean distance. Next, the reconstruction step is implemented in four main phases: (i) Acquisition of 3D model; (ii) Feature (Edge and Slope) extraction; (iii) Recognizing by Neural Network; (iv) Aligning and Matching. Several experiments are conducted using the dataset obtained from the website (Ceramic Sherd Database, 2010) for evaluating the proposed approach. The results indicate that the proposed approach achieved an accuracy rate of 96.1% (Rasheed & Nordin, 2018).

Kalasarinis & Koutsoudis (2019) presented a pipeline to generate the missing parts of the fragmented vessels' main bodies. For this purpose, the authors used digital technologies such as 3D digitization, data analysis, processing, and additive manufacturing. First of all, they applied the Structure from Motion/Multiview Dense Stereovision (SfM/MVS) photogrammetric approach to generate the 3D virtual model of the artifact. Next, the main body missing shreds is analyzed by 3D modeling (Blender, 2018). Then, 3D data processing for the generation of the synthetic missing shreds is performed. The next step of the pipeline is the 3D manufacturing of the synthetic shreds with an FDM 3D printer. Finally, the post-processing of the manufactured shreds and their placement on the vessels are conducted. For this task, the support structures are removed using a scalpel knife with surgical blades, and also, the usage of small quantities of acetone is used to improve the surface quality. The authors evaluated the methodology by applying it to two ancient Greek vessels of the Hellenistic period. The results have indicated the great applicability of the approach for the restoration of the missing parts of potteries (Kalasarinis & Koutsoudis, 2019).

Sakpere (2019) applied a virtualization technique to reconstruct archaeological pottery using point cloud data. The point clouds are acquired through the multiple views of the pottery. Then, the pre-processing step (point cloud cleaning) is performed on the point clouds with CloudCompare software. This step includes segmentation, normal computation, down-sampling, and boundary point computation. Next, the key points are extracted from the point clouds as features by using PCA. In the next step, pairwise alignment of the point clouds is undertaken with correspondences between the key-points of point clouds. Then, global registration is implemented according to the Multiview Registration method that is proposed by Pulli (1999). Finally, Iterative Closest Point (ICP) algorithm is applied to refine the alignment for all the point clouds. This study is evaluated on the virtualization of a fractured oil lamp (Sakpere, 2019).

3. Results, analysis, and discussion

In previous sections, a comprehensive analysis of the most important available publications in the topic of fragment reconstruction is presented. In this section, key results are summarized and discussed.

The number of analyzed studies in each period is shown in Figure 1; it is evident that the number has increased over time. One of the reasons for this trend is the advancement of technology and the use of new computer-

based systems for cultural heritage. For example, improvements in data acquisition and measurement tools have led to the introduction of newer features that have helped to better classify and assemble fragments.

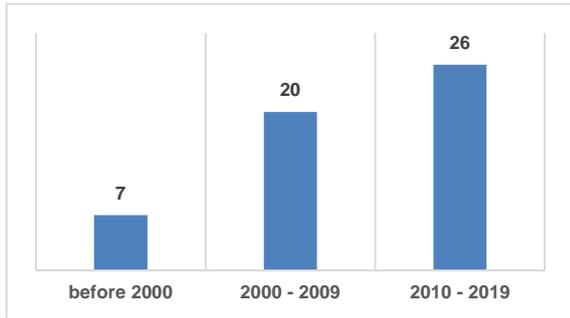


Figure 1: The number of analyzed studies in each period.

As already mentioned, in the period before 2000, most of the researchers were focused on the archiving, documentation, drawing, and storage of the excavated fragments (Hall, 1984; Halir & Menard, 1996; Halir & Flusser, 1997; Halir, 1999). With advancements in technology, the researchers have been able to overcome some of the issues for the automatic assembly of fragments, such as 3D model design, missing pieces, the asymmetric structure, lack of prior knowledge of the original profile, etc. (Cooper et al., 2001; Kashihara, 2012; Fragkos et al., 2018; Di Angelo, Di Stefano, & Pane, 2018).

According to the detailed review of the published methods in this study, an automatic computer-based method for archaeological potteries classification and reconstruction could consist of six steps (Figure 2):

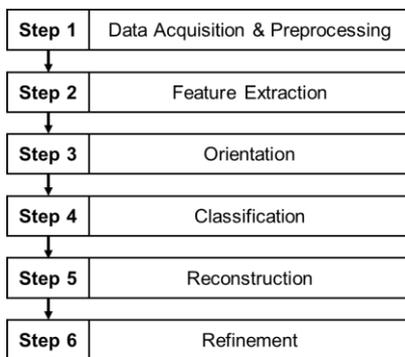


Figure 2: The main steps of the published computer-based methods.

Among these phases, the Orientation and Refinement are addressed in a few papers. Based on these considerations, Tables 1, 2 and 3 summarize the key elements of each four main phases (Data Acquisition & Pre-processing, Feature Extraction, Classification, and Reconstruction) of the analyzed methods, published before 2000 (Table 1), from 2000 to 2009 (Table 2) and from 2010 to 2019 (Table 3). In the following, some considerations are summarized for each of the six steps.

Step 1. Data Acquisition & Preprocessing

This step is the first step for implementing each computer-based method for archaeological potteries classification and reconstruction from fragments. For this

task, the researchers have used data capturing tools and pre-processing software. According to Tables 1, 2 and 3 up to 2002, the researchers have applied some camera to acquire the fragments profile (Halir & Menard, 1996; Hal & Flusser, 1997); afterward, the use of 3D laser scanner has become widespread, permitting to digitalize 3D models of objects. Some researchers have suggested alternative solutions: some cameras imaging objects from different angulations to create the 3D model (Kashihara, 2012; Stamatopoulos & Anagnostopoulos, 2016). The number of data acquisition technologies in different periods is shown in Figure 3: the use of a 3D scanner to set up a 3D model of objects is increasingly common. With available data, the papers where no reference is made to the technology used for the 3D model acquisition are counted.

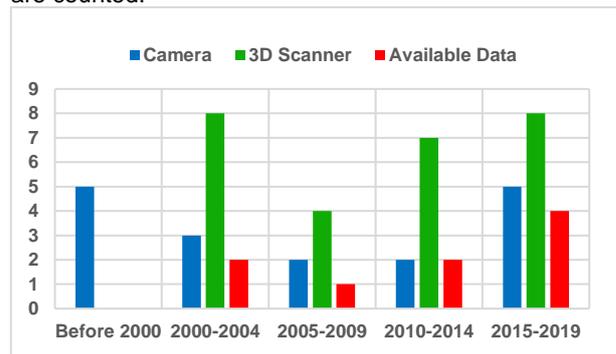


Figure 3: The number of uses of data acquisition approaches in different periods.

To obtain a manifold model analyzable by the following phases, the raw data from the acquisition have to be preprocessed. For this purpose, in the analyzed papers, some commercial software is used for denoising, cleaning, and tessellating (Kashihara, 2017; Sakpere, 2019).

Step 2. Feature Extraction

An important part of pattern recognition of an object to be classified is the feature extraction, where the feature identifiers (o feature vector) is a list of descriptions that includes sufficient information to identify a pattern (Rasheed & Nordin, 2018). In the literature, a variety of features were defined. These features are categorized in Figure 4: the most commonly used features are the rotational axis and geometric, profile, and color ones (Belenguer & Vidal, 2012; Karasik & Smilansky, 2011; Rasheed & Nordin, 2015). However, in recent years, more attention has been paid to morphological features, which are more difficult to recognize, being asymmetric structures, but are semantically significant and have specific geometric features useful to drive the assembling process (Di Angelo, Di Stefano, & Pane, 2018; Kalasarinis & Koutsoudis, 2019; Lucena et al., 2016).

Step 3. Orientation

The manual method for finding the orientation of sherds is generally performed by considering a part of the rim or the bottom. This method was considered by some researchers (Hal & Flusser, 1997; Halir, 1999). To avoid user intervention, some researchers presented procedures for the orientation of sherd through automatic estimation of the rotational axis (Brown et al., 2008; Kampel & Sablatnig, 2004; Mara & Sablatnig, 2006; Melero et al., 2003).

REVIEW OF COMPUTER-BASED METHODS FOR ARCHAEOLOGICAL CERAMIC SHERDS RECONSTRUCTION

Table 1: The key elements of each four main phases of the analyzed methods, published before 2000.

Year	Authors	Data Acquisition Tools	Features Extracted	Classification Methods	Matching Techniques
1984	Hall & Laffin	Light Pen / NEWEST Software	B-Spline Curves	-	-
1996	Sablatnig & Menard	Structured Light Technology	Excavation Site, Excavation Layer, Material and Color	Highest Similarities between Features	-
1996	Halir & Menard	Two CCD Cameras	Geometric Information (Diameter)	-	-
1997	Halir & Flusser	Two CCD Cameras	Geometric Information (Diameter & Perimeter)	-	-
1998	Sablatnig & Menard	Structured Light Technology	Profile (Rim, Body, Base)	Descriptive Language	Bottom-Up Strategy
1999	Ucoluk & Toroslu	Graphical User Interface (GUI)	Curvature & Torsion	Highest Similarities between Features Vectors	Noise Tolerant Algorithm
1999	Halir	Two CCD Cameras	Rotational Axis	-	-

Table 2: The key elements of each four main phases of the analyzed methods, published from 2000 to 2009.

Year	Authors	Data Acquisition Tools	Features Extracted	Classification Methods	Matching Techniques
2000	Papaioannou et al.	Available 3D Model Database	Surface Information (Bumpiness) & Material	Descriptive Language	Simulated Annealing / Genetic-like Algorithm
2001	Cooper et al.	Available (Joukowsky, 1999)	Break-Curves	-	Maximum Likelihood Estimation / Bayesian approach
2001	Schindler et al.	Structured Light Technology	B-Spline Curves	Highest Similarities between Features	-
2002	Andrews & Laidlaw	Not Mentioned	Break-Curves	-	Evaluation of the probability a χ^2 -statistic / Greedy Strategy
2002	Mara et al.	3D Laser Scanner / Light Technology	Curvature & Rotational Axis	Highest Similarities between Features	-
2002	Cao & Mumford	Shape Grabber Laser Scanner	Curvature, Rotational Axis and B-Spline Curves	-	Weighted Least Squares Estimation / Bootstrap Method
2002	Papaioannou et al.	3D Scanner	Geometric Information (the Distance between mutually-visible faces)	-	Z-buffer Algorithm / Global Optimization Method
2003	Willis et al.	Shape Grabber 3D Laser Scanner	Rotational Axis & Break-Curves	-	Statistical Error Analysis / Bootstrap Method
2003a	Kampel & Sablatnig	Electronics Shape Snatcher Technology	Profile-Section (the Cross-Section of the fragment in the direction of the rotational axis of symmetry)	Profile-based Classification Strategy	-
2003b	Kampel & Sablatnig	Electronics Shape Snatcher Technology	Profile-Section (above defined)	profile-based Classification Strategy	Evaluation Similarities between Profiles
2003	Melero et al.	3D Scanner	Geometric Information (Diameter, Rim Angle), Rotational Axis and Profile-Section (above defined)	Genetic Algorithm	-
2004	Willis & Cooper	Laser Scanner	Profile (Sherd Outer Surface) & Break-Curves	-	Bayesian Approach / Maximum Likelihood Estimation
2004	Kampel & Sablatnig	Minolta Vivid 9i Laser Scanner	Profile (Rim, Body, Base)	Descriptive Language	Computing Point-by-Point Distance between Facing Outlines
2005	Kampel et al.	Structured Light Technology	Rotational Axis and Profile-Section (above defined)	-	Maximum Likelihood Estimation
2005	Maiza & Gaildrat	Available (Halir, 1999)	Rotational Axis and Profile-Section (above defined)	Minimum Distance between Features Vectors	Genetic Algorithm
2006	Mara & Sablatnig	3D Scanner	Rotational Axis and Profile-Section (above defined)	-	-
2006	Huang et al.	3D Scanner	Geometric Information (a Cluster of points on fracture surface), Color, Texture Information	-	Global Multi-piece Matching
2006	Igwe & Knopf	On-site Recording Media	Geometric Information (a Weight vector of data points of the fragments)	Self-organizing Feature Map (SOFM) Algorithm	Highest Similarities between Features
2007	Zhou et al.	FAST SCAN	Geometric Information (internal and external Contour of a solid object with fractured surfaces)	-	The matching algorithm was implemented based on Junction Vertices.
2008	Brown et al.	3D Scanner	Color & Texture Information	-	Computing the Exact Matching Error at all Possible Pairs

Table 3. The key elements of each four main phases of the analyzed methods, published from 2010 to 2019.

Year	Authors	Data Acquisition Tools	Features Extracted	Classification Methods	Matching Techniques
2010	Zhou et al.	3D Laser Scanner	Rotational Axis and Profile-Section (above defined)	-	Highest Similarities between Features
2010	Toler-Franklin et al.*	3D Scanner	Color, Normal Map and Texture Information	The similarity between the Properties of Patches	Machine Learning
2010	Cohen et al.	VIVID 910 3D Scanner	Surface Information (Convex Hull)	-	Absolute Affine Invariant Moments
2011	Karasik & Smilansky	Available (Gilboa et al., 2004)	Geometric Information (Radius & Tangent) and Curvature	Discriminate Analysis	-
2011	Funkhouser et al.	Available (Brown et al., 2008)	Geometric Information (Contour, Ribbon and Junction Angle) and Color	M5P regression trees	Machine Learning
2012	Belenguer & Vidal	3D laser scanner	Geometric Information (Distance between points on the edge of fracture surface)	-	GPU Depth Maps Technique / Global Hierarchical Search Algorithm
2012	Kashihara	Thirty Camera with a Calibration Mat	Surface Information (The polygonal Meshes of Fragments)	-	Genetic Algorithm / Hill-climbing Algorithm
2013	Son et al.	3D Laser Scanner	Break-Curves & Rotational Axis (Axis Profile Curve (APC))	-	Break Curve Matching
2014	Han & Hahn	3D Laser Scanner	Curvature, Rotational Axis	Hypothesis Test based on the circular structure of the object / Grouping based on Distribution of Principal Curvatures	-
2014	Rasheed & Nordin	Nikon Camera	The Edges of Fragment (defined as Coefficients that are extracted by Polynomial Function Algorithm)	Based on the Correlation Coefficient	Biggest Correlation Coefficient between two fragments
2014	Vendrell-Vidal & Sánchez-Belenguer	Konica Minolta Vivid 9i laser scanner	Geometric Information (Distance between points on the edge of fracture surface)	-	Discrete Global Registration Technique based on Cost Function / Hierarchical Search
2015	Rasheed & Nordin	Available on NEC Labs / Nikon Camera	Color (RGB) and Texture Information	Based on the Euclidean Distance and Color Similarity	-
2016	Cohen et al.	3D Laser Scanner	Color (Surface Markings) and Surface Information (Border Anchor Points)	-	Aligning Fragments to Generic Model / Mending based on Anchor Points
2016	Lucena et al.	Available the Database of 1133 Vessel Profiles	Morphological Information (Handles) and Profile (Lip, Neck, Body, Base)	Decision Support System	-
2016	Kotoula*	Computed Tomography Scanner	Color (Painted Design), Texture Information and Geometric Information (Shape)	-	MeshLab, Fragments Reassembler and 3ds Max (Softwares)
2016	Stamatopoulos et al.	Thirty Cameras	Profile-Section (above defined) and Thickness	-	Thickness Profile Matching Method / Human Interaction
2017	Di Angelo et al.	3D Laser Scanner	Geometric Information (Internal Wall, External wall, Rim, Base, Lip) and Morphological Information (Handle, Rib)	-	-
2017	Kashihara	Available (Kashihara, 2012)	Image Information (the distribution of pixel intensities within a scale-dependent neighborhood of each interest point)	-	Genetic Algorithm / Hill Climbing Method
2017	Banterle et al.	Already Available Paper Catalogs	Profile (Inner Wall, Outer Wall, Rim, Base), Morphological Information (Handle) and Rotational Axis	Machine Learning	-
2017	Rasheed et al.	Nikon camera	Color (HSV)	Self-Organization Map (SOM) Neural Network	-
2018	Fragkos et al.*	NextEngine 3D Laser Scanner	Surface Information (The Mesh 3D Model of Fragments), Geometric Information (Shape)	-	The Solidworks 3D CAD software / BCN3D Sigma Printer
2018	Di Angelo et al.	3D Laser Scanner	Geometric Information (Internal Wall, External wall, Rim, Base) and Morphological Information (Handle, Rib)	-	-
2018	Di Angelo et al.	3D Laser Scanner	Geometric Information (The detail feature of the constant radius (DFCR))	-	-
2018	Rasheed & Nordin	Primesense Carmine 1.09 3D Scanner	Color, Texture Information, Geometric Information (Slope) and Edge	Based on the Euclidean Distance and Color Similarity	Artificial Neural Networks / the Backpropagation Algorithm
2019	Kalasarinis & Koutsoudis	Canon Camera / Agisoft Photoscan Professional	Rotational Axis and Profile-Section (above defined)	-	-
2019	Sakpere	Line Laser, RGB Camera, Arduino UNO	Surface Information (The Key-points of Point-clouds)	-	Multiview Registration Method / Iterative Closest Point (ICP) Algorithm

* For the matching, the authors used software tools instead of an original algorithm.

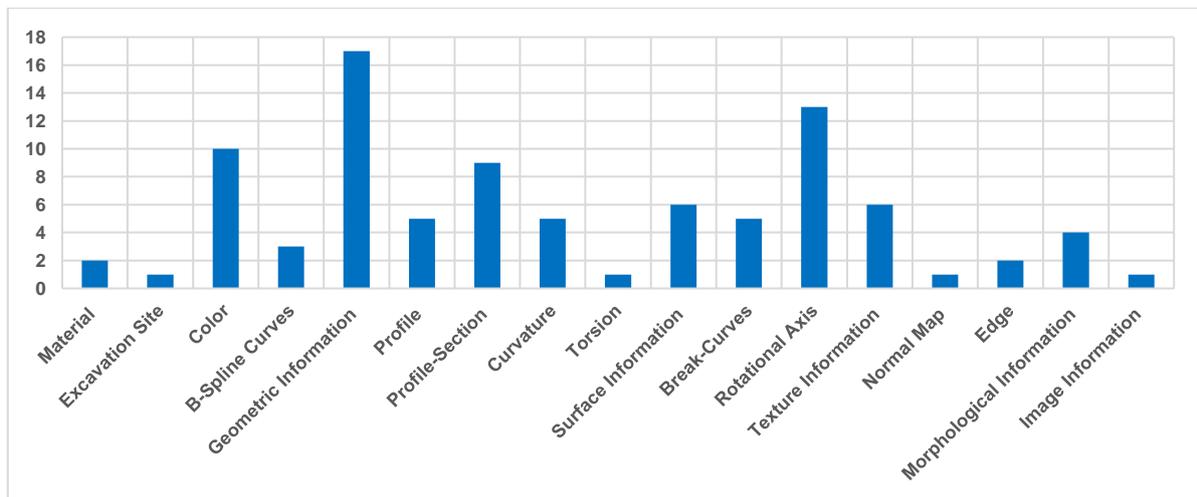


Figure 4: The defined features in the literature review.

Step 4. Classification

The comparison and classification of objects are one of the most important tasks for an archaeologist. Therefore, it would be interesting to provide automated methods to assist them in the classification process (Lucena et al., 2016). Regarding the classification step, the published methods can be classified as supervised and unsupervised. In the supervised methods, some information exists about the original objects and the classification is done according to specific classes (Igwe & Knopf, 2006; Rasheed et al., 2017). In the unsupervised ones, there is no previous information about the original objects. So, the classification is conducted based on the similarities between features that describe the fragments to be analyzed and the corresponding ones of pottery databases (Cohen et al., 2016; Maiza & Gaildrat, 2005; Rasheed & Nordin, 2018; Toler-Franklin et al., 2010). Figure 5 shows the usage percentages of the two classification approaches used in the analyzed methods.

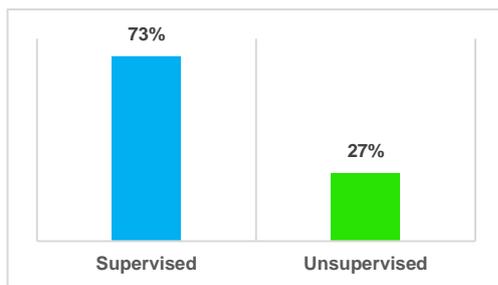


Figure 5: The usage percentages of the two classification approaches in the performed literature review.

With the advancement of data collection tools, the use of machine learning systems to classify is increasing. If the data is large enough, the system can train to automatically classify new data (Banterle et al., 2017; Funkhouser et al., 2011; Toler-Franklin et al., 2010).

Step 5. Reconstruction

After classifying the fragments, the reassembling step is carried out based on the similarity between feature vectors. Regarding general assembly procedures, solutions typically consist of two categories:

Local matching of fragments and global strategy for full assembly (Son et al., 2013). In the first category, most of the studies were discussed just about the reconstruction for the pairwise of fragments (Andrews & Laidlaw, 2002; Rasheed & Nordin, 2014, 2015). The methods belonging to the second category perform the full reconstruction of the object from their pieces (Cohen et al., 2016; Huang et al., 2006; Kashihara, 2017; Papaioannou et al., 2002).

Through the review of the mentioned studies, it was evident that an optimization algorithm was commonly used for matching fragments. The most used methods are the meta-heuristic (Genetic, Simulated Annealing, and Greedy strategy) and iterative methods (Andrews & Laidlaw, 2002; Cohen et al., 2016; Kashihara, 2012, 2017; Papaioannou et al., 2000). Some other researchers have proposed methods estimating the similarities between fragments based on the Bayesian approach and Maximum Likelihood Estimation (Cao & Mumford, 2002; Cooper et al., 2001; Kampel & Mara, 2005; Willis & Cooper, 2004). The use of the cost function has also been investigated (Papaioannou et al., 2002; Rasheed & Nordin, 2015). It should be noted that, in recent years, attention in using a machine learning system for matching is increasing (Banterle et al., 2017; Rasheed & Nordin, 2018). The used reconstruction methods in the analyzed literature are shown in Figure 6: the most commonly used approaches are Meta-heuristic optimization and Similarity Analysis.

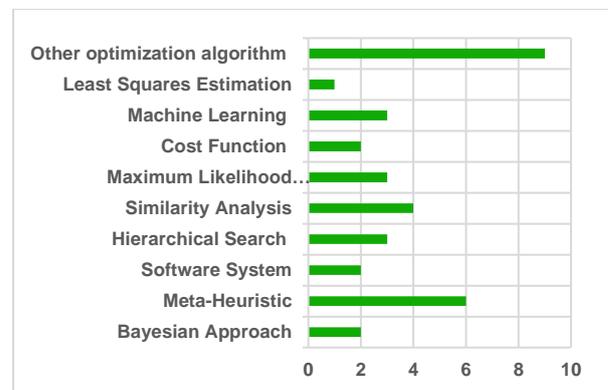


Figure 6: The used reconstruction methods in the literature review.

Step 6. Refinement

This step deals with the issues of missing pieces in the reconstruction of objects from their fragments. After the full restoration of the objects, some areas remain empty. Identifying, measuring, and retrieving these parts is one of the things that is done in this step. Few studies have been devoted to this topic (Fragkos et al., 2018; Kalasarinis & Koutsoudis, 2019). For example, Kalasarinis & Koutsoudis (2019) developed a method for 3D printing of missing fragments.

4. Conclusion and recommendations

Computer-based methods for the reconstruction of archaeological ceramic from sherds can help archaeologists to analyze data and to understand past life. Advancements in acquisition devices and computer systems have provided new tools for researchers to face the problems related to traditional manual methods. Consequently, in the last years, several different methods have been developed to analyze archaeological finds automatically. In this paper, a comprehensive analysis is presented in the most important available publications for reassembling pottery fragments by the end of 2019. The 53 papers in English from Scopus have been collected using some specific keywords and adding additional ones, through the reading of already selected papers. The detailed analysis of state-of-the-art here performed, whose key aspects are summarized in graphs and tables (useful for all researchers interested in the subject) also allows us to propose some recommendations for future research.

Commonly, the archaeological fragments are accumulated after they are collected from archaeological sites. Furthermore, the fragments modify the shape and the aspect since they are subjected for a long time to the abrasion and erosion. According to the proposed review, very little research has taken this problem into account, and often this is

not considered. Instead, to implement systems that can be used in the field, it would be useful to develop a system that is robust in the analysis of ceramic sherds noised, worn, encrusted, and chips.

Among all the main steps of the published computer-based methods, the data acquisition step remains the one that requires greater intervention by the operator. For this reason, it represents the bottleneck in the development of a fully automatic analysis method. Consequently, efforts should be spent on the development of an automated scanning system for large quantities of fragments.

With the advancement of data acquisition tools, the use of machine learning systems to classify is increasing. If the data is large enough, the system can train to classify new data automatically. This procedure easily is generalized to another collected data elsewhere. Data-mining techniques, artificial neural networks, and deep learning are some of the tools that researchers can use to classify and reconstruct, even when the original patterns are unknown.

In the literature review, the polynomial function algorithm is applied on the edge of fragments to obtain the vectors of coefficients. Then, these coefficients are used to classify fragments. Since the edges of fragments are almost always irregular, the use of nonparametric methods such as wavelet transformation is suggested, because these methods can better fit such curves.

Commonly, ancient potteries are extracted from excavations with missing parts; after the full reconstruction of the objects by using a computer-based method, some areas remain empty. Few studies have been dedicated to this topic and the proposed methods require user interaction during all steps of the process. In that regard, the research should be done on the development of methods suited for automatic identification and 3D printing of the missing fragments.

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