

When diffusion faces drift: consequences of exclusion processes for bi-directional pedestrian flows

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Abstract

Stochastic particle-based models are useful tools for describing the collective movement of large crowds of pedestrians in crowded confined environments. Using descriptions based on the simple exclusion process, two populations of particles, mimicking pedestrians walking in a built environment, enter a room from two opposite sides. One population is *passive* – being unaware of the local environment; particles belonging to this group perform a symmetric random walk. The other population has information on the local geometry in the sense that as soon as particles enter a visibility zone, a drift activates them. Their self-propulsion leads them towards the exit. This second type of species is referred here as *active*. The assumed crowdedness corresponds to a near-jammed scenario. The main question we ask in this paper is: *Can we induce modifications of the dynamics of the active particles to improve the outgoing current of the passive particles?* To address this question, we compute occupation number profiles and currents for both populations in selected parameter ranges. Besides observing the more classical faster-is-slower effect, new features appear

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as prominent like the non-monotonicity of currents, self-induced phase separation within the active population, as well as acceleration of passive particles for large-drift regimes of active particles.

Keywords: Simple exclusion process, two species stochastic dynamics, passive-active pedestrian flows, particles current, occupation numbers

2010 MSC: 82C20, 82C80

PACs: 05.10Ln, 05.90.+m, 63.90+t

1. Introduction

Scenarios of pedestrian flows in agglomerated urban environments, cellular membranes, glasses, supercooled liquids share an important feature – the dynamics takes place in a crowded environment with obstacles that are often active (i.e., not necessarily fixed in space and time). The management of the dynamics in these kinds of systems is far from being understood mainly due to the fact that the interplay between transport and particle–particle as well as particle–obstacle interactions is very complex; see, e.g., [1, 2, 3, 4] and references cited therein.

Exploring by means of computer simulations is an efficient tool for investigating the qualitative behavior of such kinds of systems and this is also the route we take here.

Depending on the level of observation, the modeling descriptions refer to micro-, meso-, macro-levels, or to suitable (multiscale) combinations thereof (see, e.g., [5, 6]). In this framework, we consider a microscopic approach based on a modification of the classical simple exclusion¹ process formulated for two different populations of interacting pedestrians (particles). Essentially, two populations of particles, mimicking pedestrians walking in a built environment, en-

¹A simple exclusion process refers to the stochastic motion of interacting particles on a lattice where the interaction is given by the exclusion (excluded “volume” constraints) property, i.e., two particles may not occupy the same site simultaneously. We refer the reader to [7] for rigorous considerations on the simple exclusion model and its hydrodynamic limits and to [8] for a basic modeling perspective.

ter a room from two opposite sides with the intention to cross the room and then exit from the door on the opposite side of the room.

Because of its own unawareness or lack of prior knowledge of the local environment, one population is *passive*, and hence, particles belonging to this population perform a random walk. On the other hand, the second population has information on the local geometry in the sense that as soon as particles enter a visibility zone, a drift activates them by sending them towards the exit door. This type of particles is referred here as *active*. The assumed crowdedness corresponds to a near-jammed scenario. To fix ideas, the number of occupied sites in the room is chosen to be of the order of the 60% from the total number of available sites².

In an evacuation due to an emergency situation (like fire and smoke propagating in the building), the *a priori* knowledge of the environment is certainly an advantage (cf., e.g., [10]). Hence, from this perspective, if a quick evacuation is needed, then the passive population has a disadvantage compared to the active population. We are wondering whether we can compensate at least partly this drawback, by managing intelligently the motion of the active population. In other words, the main question we ask in this paper is:

Q. Can we induce modifications of the dynamics of active particles to improve the outgoing current of passive particles?

The ingredients we have at our disposal are alterations either in the drift parameter of the active particles, or in their visibility zone by fine-tuning a parameter for a nonlocal interaction that activates the drift-towards-exit. It is worth noting that the latter feature is different from the nonlocal shoving of particles proposed in [11].

To address the above question, we compute occupation number profiles and currents for both populations in selected parameter ranges. Our numerical

²Museums in highly touristic cities are examples of crowded areas; compare, e.g., with the situation of Galleria Borghese in Rome as described in [9].

results exhibit the classical faster–is–slower effect (see, e.g., [12] for experimental evidence and [13] for numerical simulations exhibiting this effect using Helbing’s social force model) and point out as well new prominent features like the non–monotonicity of currents, the self–induced phase separation within the active population, as well as an acceleration of passive particles induced by a large drift and a large visibility zone of active particles.

This research was initiated in [14, 15], motivated by our intention to estimate the mean residence time of particles undergoing an asymmetric simple exclusion within a room in perfect contact with two infinite reservoirs of particles. Recent developments reported in [3] brought us to study drafting effects via the dynamics of mixed active–passive pedestrian populations in confined domains with obstacles and exit doors, which mimic a built complex environment.

2. Model description

We consider a square lattice $\Lambda := \{1, \dots, L\} \times \{1, \dots, L\} \subset \mathbb{Z}^2$ of side length L , where L is an odd positive integer number. Λ will be referred in this context as *room*. An element $x = (x_1, x_2)$ of the room Λ is called *site* or *cell*. Two sites $x, y \in \Lambda$ are said *nearest neighbor* if and only if $|x - y| = 1$. Let us call *horizontal* the first coordinate axis and *vertical* the second one. The words *left*, *right*, *up*, *down*, *top*, *bottom*, *above*, *below*, *row*, and *column* will be used accordingly. We call *door* the sets made of w_L and w_R pairwise adjacent in the left–most and right–most columns of the room Λ , respectively, symmetric with respect to its median row. This mimics the presence of two distinct doors on the left and right boundary of the room. The odd positive integers w_L, w_R smaller than L will be called *width* of the doors. Inside the room we define a rectangular driven zone, namely, the *visibility region* V , made of the first L_v left columns of Λ , with the positive integer $L_v \leq L$ called *depth* of the visibility region. By writing $L_v = 0$, we refer to the case in which no visibility region is considered.

We consider two different species of particles, i.e., *active* and *passive*, moving inside Λ (we shall sometimes use in the notation the symbols A and P to refer to

them). The dynamics will be defined so that the sites of the external boundary of the room, that is to say the sites $x \in \mathbb{Z}^2 \setminus \Lambda$ such that there exists $y \in \Lambda$ nearest neighbor of x , cannot be accessed by the particles. The state of the system is a *configuration* $\eta \in \Omega = \{-1, 0, 1\}^\Lambda$ and we say that the site x is *empty* if $\eta_x = 0$, *occupied by an active particle* if $\eta_x = 1$, and *occupied by a passive particle* if $\eta_x = -1$. The number of active (respectively, passive) particles in the configuration η is given by $n_A(\eta) = \sum_{x \in \Lambda} \delta_{1, \eta_x}$ (respectively, $n_P(\eta) = \sum_{x \in \Lambda} \delta_{-1, \eta_x}$), where $\delta_{\cdot, \cdot}$ is Kronecker's symbol. Their sum is the total number of particles in the configuration η .

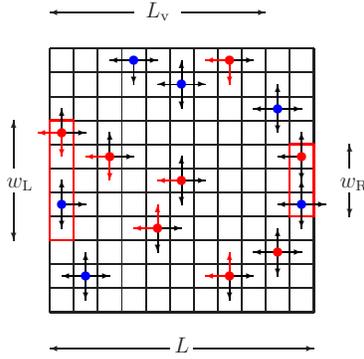


Figure 1: Schematic representation of our lattice model. Blue and red disks denote passive and active particles, respectively. The rectangles of sites delimited by the red contour denote the exit doors. Black and red arrows (color online) denote transitions performed with rates 1 and $1 + \varepsilon_1$ or $1 + \varepsilon_2$, respectively.

The interaction of particles inside the room is modeled via a simple exclusion random walk for two particle species undergoing two different microscopic dynamics. The passive particles enter through the left door and exit through the right door. They perform a symmetric simple exclusion dynamics on the whole lattice. Simultaneously, the active particles enter through the right door and exit through the left door. They perform a symmetric simple exclusion walk outside the visibility region, whereas inside such a region they experience also a drift pushing them towards the left door. In other words, the whole room

is *obscure*³ for the passive particles, while, for the active ones, only the region outside the visibility region⁴ is obscure. The model also includes two external particle waiting lists, each of which is designed to collect particles of a given species when these move out from the lattice Λ through their exit door and to reinsert them back on the lattice through their entrance door.

More precisely, picked the positive integers N_A and N_B and set $N = N_A + N_P$, the dynamics is the continuous time Markov chain $\eta(t)$ on Ω with initial configuration $\eta(0)$ such that $n_A(\eta(0)) = N_A$ and $n_P(\eta(0)) = N_P$ and rates $c(\eta, \eta')$ defined as follows: Let $\varepsilon_1, \varepsilon_2 \geq 0$ be the *horizontal* and *vertical drift*; for any pair $x = (x_1, x_2), y = (y_1, y_2)$ of nearest neighbor sites in Λ we set $\epsilon(x, y) = 0$, excepting the following cases:

- $\epsilon(x, y) = \varepsilon_1$ if $x, y \in V$ and $y_1 = x_1 - 1$, namely, x and y belong to the visibility region and x is to the right with respect to y ;
- $\epsilon(x, y) = \varepsilon_2$ if $x, y \in V$ and $y_2 = x_2 + 1 \leq (L + 1)/2$, namely, x belongs to the bottom part of the visibility region and x is below y ;
- $\epsilon(x, y) = \varepsilon_2$ if $x, y \in V$ and $y_2 = x_2 - 1 \geq (L + 1)/2$, namely, x belongs to the top part of the visibility region and x is above y .

Next, we let the rate $c(\eta, \eta')$ be equal

- to 1 if η' can be obtained by η by replacing with 0 a -1 at the right door (passive particles leave the room);
- to $1 + \epsilon(x, y)$ if η' can be obtained by η by replacing with 0 a 1 at the left door (active particles leave the room);
- to $[N_A - n_A(\eta)]/m_R$ if the number of empty sites in the right door is $m_R > 0$ and η' can be obtained by η by adding a 1 at one of the empty sites of the right door;

³We refer the reader to [14], where the authors discuss the gregarious behavior of crowds moving in the dark.

⁴The concept of visibility region was introduced by the authors in [3].

- to $[N_A - n_A(\eta)]/m_L$ if the number of empty sites in the left door is $m_L > 0$ and η' can be obtained by η by adding a -1 at one of the empty sites of the left door;
- to 1 if η' can be obtained by η by exchanging a -1 with a 0 between two neighboring sites of Λ (motion of passive particles inside Λ);
- to $1 + \epsilon(x, y)$ if η' can be obtained by η by exchanging a $+1$ at site x with a 0 at site y , with x and y nearest neighbor sites of Λ (motion of active particles inside Λ);
- to 0 in all the other cases.

We stress that, at time t , the quantities $N_A - n_A(\eta(t))$ and $N_P - n_P(\eta(t))$ represent the number of active, and, respectively, passive particles that exited the room and entered their own waiting list at time t , whereas $m_L > 0$ and $m_R > 0$ are the number of empty sites of the left and right doors at time t .

The system will reach a stationary state, since passive particles exiting the domain via the right door are introduced back in one site randomly chosen among possible empty sites of the left door, while active particles leaving the system through the left door are introduced back also at one random site chosen among possible empty sites of the right door. The total number of active and passive particles in the room Λ is only approximatively constant during the evolution. It slightly fluctuates due to the fact that particles may enter waiting lists. On the other hand, the total number of particles N in the system (considering both the room and the waiting lists) is conserved.

In the study of this dynamics, the main quantity of interest are the stationary *outgoing fluxes* or *currents of active and passive particles* which are the values approached in the infinite time limit by the ratio between the total number of active and passive particles, respectively, that in the interval $(0, t)$ exited through the left and the right door and entered the waiting lists and the time t . In order to discuss and to understand the behavior of currents with respect to the model parameters, we shall also look at the active and passive particles

occupation number profiles, namely, we evaluate the stationary mean value of the occupation numbers δ_{1,η_x} and δ_{-1,η_x} of each site $x \in \Lambda$, where, we recall, $\delta_{\cdot,\cdot}$ is the Kronecker symbol.

3. Numerical results

We simulate the model introduced in Section 2 using the following scheme: at time t , we extract an exponential random time τ with parameter the total rate $\sum_{\zeta \in \Omega} c(\eta(t), \zeta)$ and set the time equal to $t + \tau$. We then select a configuration using the probability distribution $c(\eta(t), \xi) / \sum_{\zeta \in \Omega} c(\eta(t), \zeta)$ and then set $\eta(t + \tau) = \xi$.

We compute the currents by applying directly the definition given at the end of Section 2 and the occupation number profiles as follows: we run the dynamics for a sufficiently long time (order of 9×10^7 MC steps) so that the system reaches the stationary state and then we average the occupation numbers δ_{1,η_x} and δ_{-1,η_x} for each site of the room for the following 9×10^7 MC steps; we thus obtain a function of $x \in \Lambda$ taking values in the interval $[0, 1]$.

The presence of two species fighting to get to opposing exit doors reasonably reduces the intensity of the current that we would expect to measure if a single species were present in the room. We have tested this fact by running simulations for one single species with the some dynamics as the one defined above and with the same choice of parameters and we have found currents that are typically three or four time larger.

In the following discussion the parameters are $L = 30$ and $N_A = N_P = 280$ and all of the simulations are done starting the system from the same initial configuration chosen once for all by distributing the particles at random with uniform probability. With such a choice of the parameters the number of occupied sites in the room is of the order of the 60% of the total; indeed, our study, as explained in the introduction, aims at understanding the behavior of the model in a crowded regime.

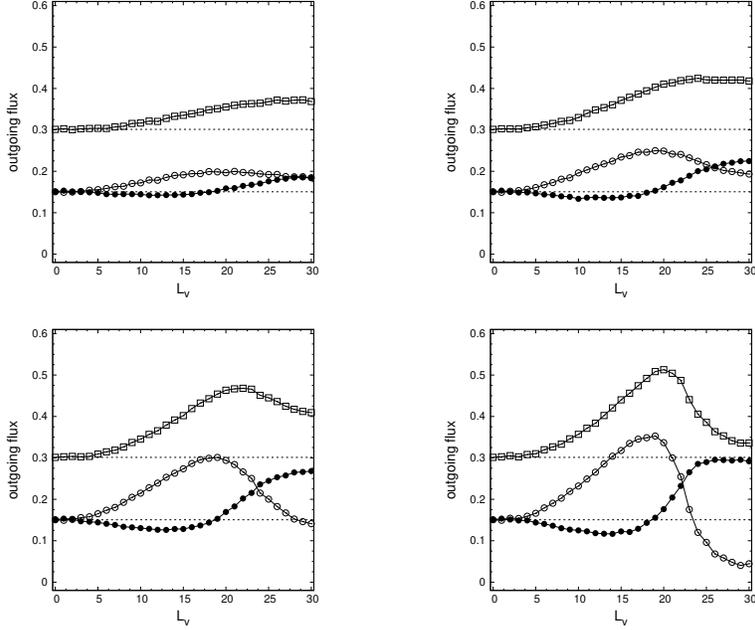


Figure 2: Stationary currents of active (empty circles) and passive particles (solid disks) and cumulative current (empty squares) as functions of L_v for $\varepsilon = 0.05, 0.1, 0.15, 0.2$ (lexicographical order). The black dashed lines are eye guides showing the value measured in the zero drift case.

3.1. The corridor model

In this subsection, we consider the model defined above assuming that the doors are as wide as the room, namely, $L = w_L = w_R$. In such case, it is rather natural to limit the discussion to the case $\varepsilon_2 = 0$, that is to say, to the case in which only the horizontal (longitudinal with respect to the position of the doors) component of the drift is considered. To simplify the notation, we shall also denote the longitudinal drift ε_1 simply by ε .

Among the different interesting results that we will discuss in this section, we want to single out a very peculiar behavior: since only active particles experience the drift, changes of the parameters L_v and ε act directly only on the active species. Nevertheless, due to the exclusion interaction between the two different species, also the passive particles behavior will be affected. In particular, we

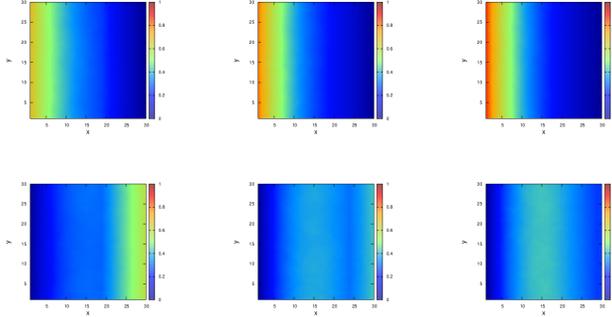


Figure 3: Occupation number profile of passive (top row) and active (bottom row) particles at stationarity for $\varepsilon = 0.15$ and $L_v = 20, 25, 30$ (from left to right).

will see that a robust increase of the visibility length L_v and the longitudinal drift ε will induce a significant increase of the passive particle current.

In Figure 2, we plot the passive and active particle currents for four different values of the drift, i.e., $\varepsilon = 0.05, 0.1, 0.15, 0.2$, when the visibility length L_v is varied from 0 to L . We note that the active particles current (open circles) increases with L_v up to some value where it attains a maximum. This effect is visible in all of the panels of Figure 2, but the position and size of the maximum changes. This effect is more prominent for the largest value of considered ε (see bottom right panel in Figure 2). Moreover, excepting the smallest value of the drift $\varepsilon = 0.05$, soon after the active particle current reaches its maximum, the passive and active particle currents intersect so that, for the largest value of the visibility length, the transport of passive particles becomes more efficient than that of active ones.

This behavior is interesting and not trivial for two reasons: i) only active particles are driven and, hence, the parameters we play with directly act only on their dynamics; ii) when ε and L_v are increased, the active particle transport throughout the corridor is expected to become more and more efficient. To explain such an effect, we look at the stationary occupation profiles. In particular, we focus our attention to the case $\varepsilon = 0.15$, namely, we closely analyze the bottom left panel in Figure 2 and the corresponding occupation number profiles

plotted in Figure 3. In this figure, we have considered the cases $L_v = 20, 25, 30$ since the switch in the active and passive particle currents is observed around $L_v = 25$.

Looking at Figure 3, we note that for $L_v = 20$, passive and active particles mainly distributed close to their relative entrance doors, namely, passive particles on the left and active particle on the right. Nevertheless, a small depletion layer in the active particle distribution can be observed around $x = 20$, which is precisely the place where those particles enter the visibility regions. This is an expected behavior: active particle entering such a region from the right are accelerated towards the left and start to accumulate when they meet the passive particles standing at their left entrance. This behavior becomes more and more prominent when the visibility length is increased; see the panels corresponding to $L_v = 25, 30$, where the presence of the depletion region and the accumulation of the active particles in the middle of the room is more evident. As a consequence, when the visibility length is increased, the right entrance is no more occupied by active particles. This explains the observed increase in the passive particle current.

Similar occupation profiles are found for the other values of the longitudinal drift considered in Figure 2. Hence, the behavior of currents can be explained similarly. We do not report such pictures because they do not add anything new to the understanding of the behavior of our model.

In Figure 4, we consider larger values of ε for a fixed visibility length $L_v = 7, 15, 23, 30$. If L_v is smaller than 15, the passive and, respectively, active particle currents decrease and increase monotonically with respect to the drift $\varepsilon \in [0, 1]$. Moreover, to a high current of active particle it corresponds a small current of passive ones. This behavior is coherent with the occupation number profiles reported in Figure 5. We see an accumulation of active particles at the right door, which prevents high passive particle currents, as well as a rather spread distribution of passive particles in the left part of the room with minor accumulation at the entrance, which allows high active particle currents.

The scenario changes drastically for larger values of the visibility length, see

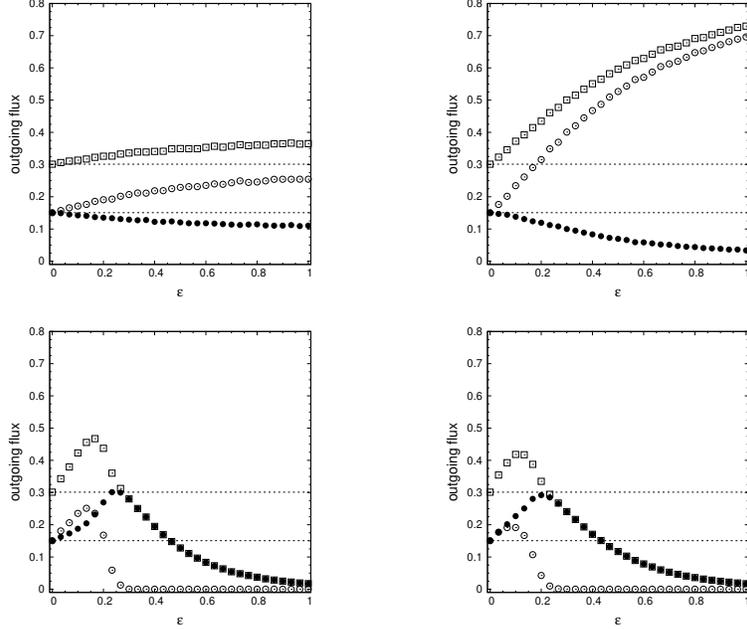


Figure 4: Stationary currents of active (empty circles) and passive particles (solid disks) and cumulative current (empty squares) as functions of ε for $L_v = 7, 15, 23, 30$ (lexicographical order). The black dashed lines are as in Figure 2

the bottom panels in Figure 4 and, in particular, focus the attention on the left one corresponding to $L_v = 23$. Currents are not anymore monotonic functions of the drift, and, at very low values of ε , the active particle current is higher than the passive particle one, as soon as the drift exceeds a certain value the latter overtakes the former. Consequently, we see again that by increasing the drift of active particles, the transport of passive ones is favored. This behavior can be explained as before referring to the occupation number profiles reported in Figure 6. The first three columns are similar to those shown in Figure 3 so that the phenomenon can be explained similarly. However, in the fourth column corresponding to $\varepsilon = 0.35$, a supplementary increase in the occupation number profile of the active particles in the middle region of the room is observed and this explains why for ε large also the passive particle currents becomes negligible. In this regime, a clogged configuration is eventually reached.

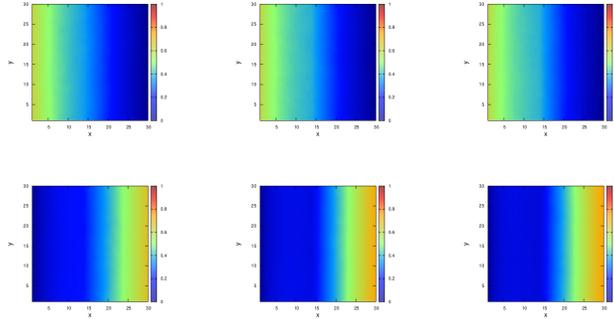


Figure 5: Occupation number profile of passive (top row) and active (bottom row) particles at stationarity for $L_V = 15$ and $\varepsilon = 0.2, 0.4, 0.48$ (from left to right).

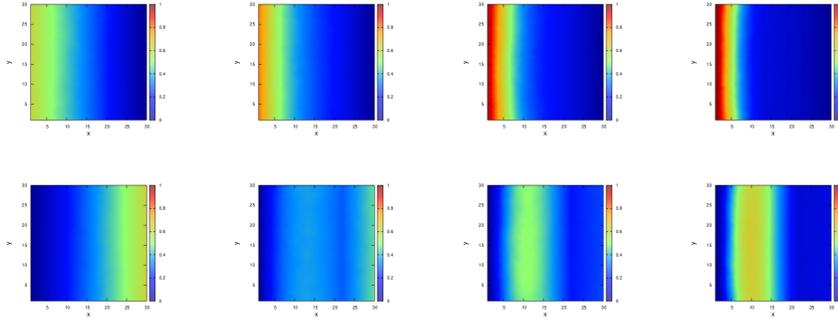


Figure 6: Occupation number profile of passive (top row) and active (bottom row) particles at stationarity for $L_V = 23$ and $\varepsilon = 0, 0.18, 0.25, 0.35$ (from left to right).

3.2. Effect of doors

In this section, we discuss the effect of the doors on the dynamics of our model. We consider the case in which the doors have equal widths $w_L = w_R = 14$, hence their capacity is reduced compared to the corridor model described in Subsection 3.1. Longitudinal and transversal (i.e., vertical) components of the drift, namely, ε_1 and ε_2 are chosen equal and will be simply denoted by ε .

Since the results are similar to those that we have found in the corridor case of Subsection 3.1, we do not repeat the discussion in detail. Instead, we bound ourselves to highlight the few key differences that can be observed.

Figures 7 and 8 are analogous to Figures 2 and 3. The only difference is the

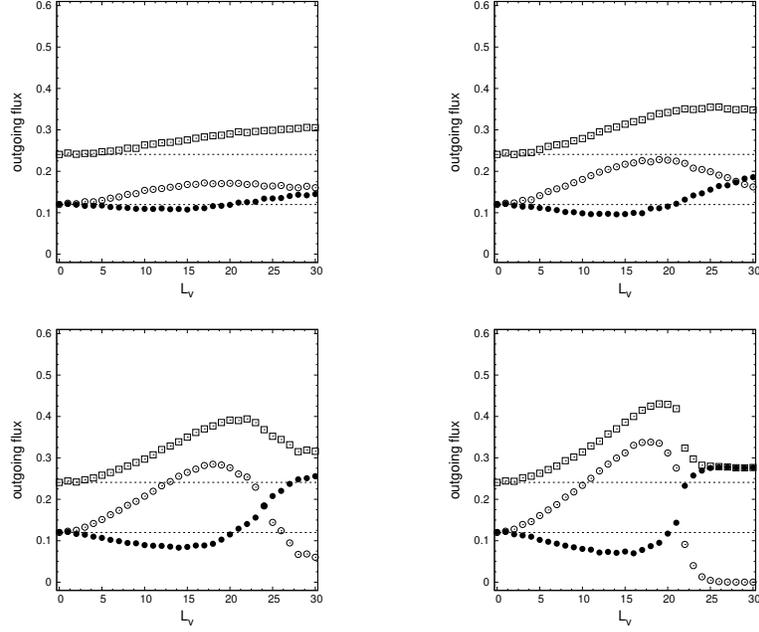


Figure 7: Stationary currents of active (empty circles) and passive particles (solid disks) and cumulative current (empty squares) as functions of L_v for $\varepsilon = 0.05, 0.1, 0.15, 0.2$ (lexicographical order). The black dashed lines are as in Figure 2

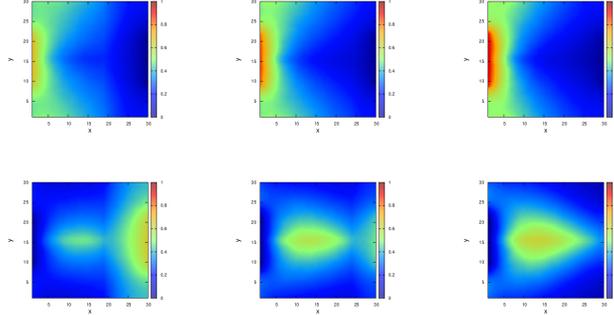


Figure 8: Occupation number profile of passive (top row) and active (bottom row) particles at stationarity for $\varepsilon = 0.15$ and $L_v = 21, 25, 30$ (from left to right).

shape of the region where particles accumulate which is strongly influenced by the presence of the door and by the presence of the transversal drift. The door gives the rounded shape to the occupation number profile of active particles

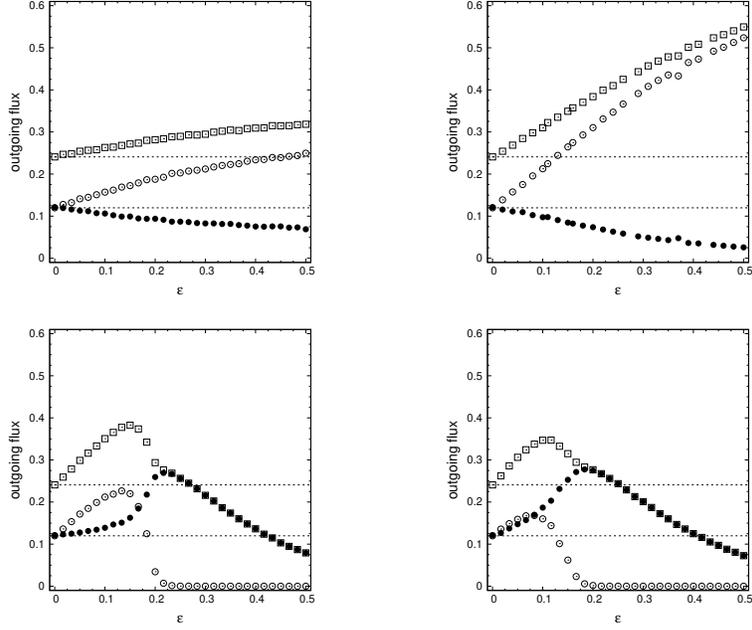


Figure 9: Stationary currents of active (empty circles) and passive particles (solid disks) and cumulative current (empty squares) as functions of ε for $L_v = 7, 15, 23, 30$ (lexicographical order). The black dashed lines are as in Figure 2

close to their entrance, whereas the transversal drift induces the formation of a “droplet” in the central region of the room.

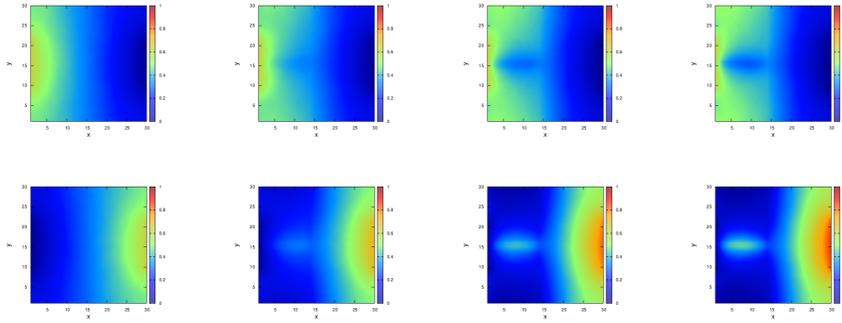


Figure 10: Occupation number profile of passive (top row) and active (bottom row) particles at stationarity for $L_v = 15$ and $\varepsilon = 0, 0.15, 0.35, 0.45$ (from left to right).

Figures 9–11 are analogous to Figures 4–6. It appears that active particles

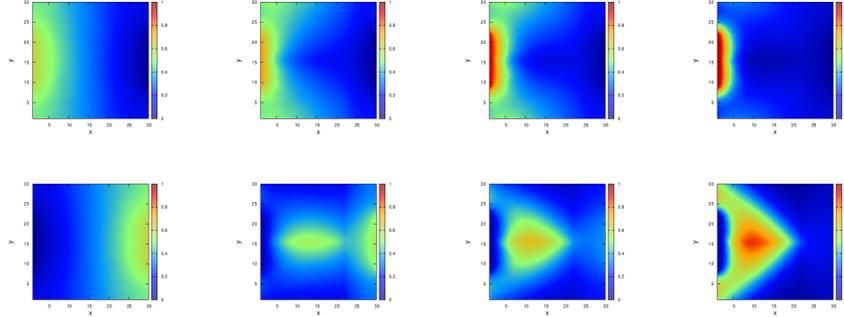


Figure 11: Occupation number profile of passive (top row) and active (bottom row) particles at stationarity for $L_V = 23$ and $\varepsilon = 0, 0.15, 0.2, 0.3$ (from left to right).

separate in two distinct groups. It looks to be a self-induced phase separation within the own population. The shape of the droplet is very much affected by the geometry of the room and the size of the door.

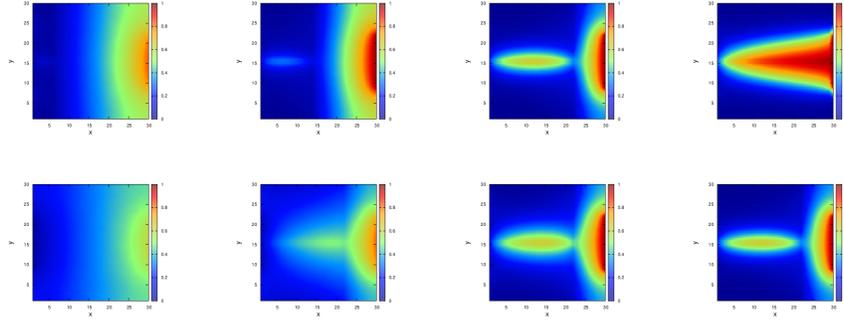


Figure 12: Occupation number profile at stationarity in a simulation without passive particles. Top row: $\varepsilon = 0.8$ and $L_V = 7, 15, 23, 30$ (from left to right). Bottom row: $L_V = 23$ and $\varepsilon = 0, 0.15, 0.5, 0.8$ (from left to right).

As a final comment, we report that, as we already mentioned at the very beginning of Section 3, we have performed some simulations of the model in absence of passive particles, namely, when just active particles are present in the lattice. The behavior of the current that we find is absolutely standard: the current increases monotonically both with respect to the drift and to the visibility length. Nevertheless, we find interesting to show some occupation number

profiles in Figure 12, indeed to transverse component of the drift, even in this case particles tend to accumulate in the central part of the room. Moreover, depending on the visibility length, they can form a central droplet detached from the inlet right door.

4. Conclusion

Based on the results detailed in Section 3, we see that if active particles undergo a non-zero drift and the visibility zone is sufficiently large, then the outgoing flux of passive particles improves. This is essentially the answer to the question Q. posed in the introduction. It is due to the fact that, in this regime, active particles move quickly far from their entrance door. In this way their entrance door becomes a free exit for the passive particles. The dynamics is still slow mainly because active particles succeed to jam around the center part of the room, slowing down the overall dynamics.

The population of active particles segregates in two different structures: one is an agglomeration located in the proximity of the entrance door, the other is a droplet in the center of the visibility zone. This is a consequence of the combined action of longitudinal and transversal drifts. On the other hand, if the transversal drift is not active, we still have an agglomeration in the central part of the visibility region, as we have observed in the corridor model. Its shape is not anymore a droplet but a vertical strip.

The fact that the flux of passive particles can be controlled via the active particle dynamics has been observed for a specific geometry and for a specific dynamics. The same kind of analysis can be done for concrete urban geometries, multiple populations of pedestrians, and different dynamics, providing potentially useful information for large crowd management.

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