Published by IOP Publishing for SISSA

Received: *May 29, 2009*
Revised: *July 3, 2009*
Accepted: *July 3, 2009*
Published: *July 23, 2009*

Gravity modification with Yukawa-type potential: dark matter and mirror gravity

Zurab Berezhiani,[a,b] **Fabrizio Nesti,**[a,b] **Luigi Pilo**[a,b] **and Nicola Rossi**[a,b]

[a] *Dipartimento di Fisica, Università di L'Aquila, 67010 Coppito AQ, Italy*

[b] *INFN, Laboratori Nazionali del Gran Sasso, 67010 Assergi AQ, Italy*

E-mail: berezhiani@fe.infn.it, fabrizio.nesti@aquila.infn.it, luigi.pilo@aquila.infn.it, nicola.rossi@aquila.infn.it

Abstract: The nature of the gravitational interaction between ordinary and dark matter is still open. Any deviation from universality or the Newtonian law also modifies the standard assumption of collisionless dark matter. On the other hand, obtaining a Yukawa-like large-distance modification of the gravitational potential is a nontrivial problem, that has so far eluded a consistent realization even at linearized level. We propose here a theory providing a Yukawa-like potential, by coupling non-derivatively the two metric fields related respectively to the visible and dark matter sectors, in the context of massive gravity theories where the local Lorentz invariance is broken by the different coexisting backgrounds. This gives rise to the appropriate mass pattern in the gravitational sector, producing a healthy theory with the Yukawa potential. Our results are of a special relevance in the scenario of dark matter originated from the mirror world, an exact duplicate of the ordinary particle sector.

Keywords: Classical Theories of Gravity, Space-Time Symmetries

doi:10.1088/1126-6708/2009/07/083

Contents

1 Introduction

The problem of obtaining a Yukawa-like potential in a consistent theory of gravity is a nontrivial task and attempts in this direction date back to 1939 when Fierz and Pauli (FP) added a mass term m to the Lorentz-invariant action of the free spin-2 graviton [1]. Unfortunately, the resulting FP theory of massive gravity is unfit to be a consistent modification of GR because of the van Dam-Veltmann-Zakharov (vDVZ) discontinuity [2]: also in the limit $m \to 0$ the bending of light is 25% off the extremely precise experimental limit.

A further, theoretical, problem is that the fine-tuning needed to single out the ghost-free action at linearized level is spoiled by interactions and a sixth ghost-like mode starts to propagate, making the whole theory unstable [3] and unpredictable *below* some (unacceptably large) distance scale. The problem was reexamined in the framework of effective field theories realising that the reason behind the misbehavior of FP massive gravity is strong coupling of the scalar sector [4].

It has been shown that the sickness of the FP theory has its roots in the Lorenz invariance [5]. Indeed, retaining only rotational invariance, one can avoid the vDVZ discontinuity and the propagation of ghost-like states [6, 7] (for a different approach, see [8]). In these models, gauge invariance is broken by Lorentz-breaking mass terms, and the gauge modes that would start propagating, acquire a well behaved kinetic term, or do not propagate at all. What happens is that via Lorenz-breaking one can cure the discontinuity problem in the "spatial" sector while avoiding ghost-like propagating states.

In the context of bigravity [9], a suitable realization of a Lorenz-breaking (LB) massive phase of gravity can be obtained [10, 11]. In addition to our metric field $g_{1\mu\nu}$ and normal matter minimally coupled to $g_{1\mu\nu}$, described by the Lagrangian $\mathcal{L}_1$ (sector 1 in the following), one introduces a second metric tensor $g_{2\mu\nu}$ related to a hidden sector 2 (dark matter) with Lagrangian $\mathcal{L}_2$. Therefore, the visible and dark components are associated to separate gravitational sectors. The action of this theory consists of an Einstein-Hilbert term for each metric, plus an interaction term V

$$S = \int d^4x \left[\sqrt{g_1}\,\left(M_1^2 R_1 + \mathcal{L}_1\right) + \sqrt{g_2}\,\left(M_2^2\, R_2 + \mathcal{L}_2\right) + \epsilon^4\,\left(g_1 g_2\right)^{1/4} V(X)\right], \qquad (1.1)$$

where $M_{1,2}$ are the "Planck" masses of the two sectors and ϵ is some small mass scale which essentially will define the graviton mass through a see-saw type relation $m_g \sim \epsilon^2/M_P$, M_P being the experimental Planck mass (of course related to $M_{1,2}$). The metric determinants are denoted g_1 and g_2, and the interaction potential V among the two metrics is assumed to be non-derivative, so that it can always be taken as a scalar function of $X^\mu_\nu = g_1^{\mu\alpha} g_{2\alpha\nu}$. The invariance under diffeomorphisms is not broken; local Lorentz invariance, on the other hand, is spontaneously broken because in general there is no local Lorentz frame in which the two metric tensors $g_{1\mu\nu}$ and $g_{2\mu\nu}$ are proportional. Nonetheless, because each matter sector is minimally coupled to its own metric, the weak equivalence principle is respected and the breaking of local Lorentz invariance is transmitted only gravitationally. Once a flat rotationally invariant background is found, in the weak field limit a Lorentz breaking mass term for the gravitational perturbations emerges in a natural way from the expansion of the interaction term in the action [10].

One should point out that when V is absent the gauge symmetry is enlarged, because one can transform $g_{1\mu\nu}$ and $g_{2\mu\nu}$ by using two independent diffeomorphisms. On the other hand when V is turned on the symmetry is reduced to the common (diagonal) diffeomorphism group, corresponding to general covariance. As a result a massless graviton is always present, besides the massive excitations. As shown in [10], in the Lorentz breaking phase only tensors propagate, in particular in the vector and scalar sectors no time derivatives are present. The Newtonian potential is modified in the infrared, but the modification is not Yukawa-like. In fact, at linearized level, the deviation from a $1/r$ potential is a linearly growing term [7, 10].[1]

In order to have a massive phase with a Yukawa-like potential, bigravity must then be enlarged. In this paper we generalize the above construction and show that one can use a further rank-2 field $g_{3\mu\nu}$ as a Higgs field to achieve the Yukawa potential. The size of the fluctuations of $g_{3\mu\nu}$ is controlled by a 3rd "Planck" mass M_3 entering in its EH action. We will show that, in the limit $M_3 \gg M_{1,2}$, $g_{3\mu\nu}$ can be consistently decoupled and one is left with an effective bigravity theory with a Yukawa-like component of the gravitational potential. The tensor $g_{3\mu\nu}$ plays the role of a symmetry-breaking field, communicating the breaking of Lorentz invariance to $g_{1\mu\nu}$ and $g_{2\mu\nu}$ and thus introducing Lorentz-breaking mass terms to their fluctuations. The resulting phase of gravity features a Yukawa-modified static potential while still avoiding any propagation of ghosts and the vDVZ discontinuity, and this result survives in the limit of decoupling g_3.

This modification of gravity at large distances can then open new possibilities for the nature of dark matter. In the present paradigm the visible matter amounts only to about

[1] Such a term clearly breaks perturbativity at some large distance $r > r_{\rm IR}$, but remarkably this behavior is cured by the non-perturbative treatment [12] showing that the linear term is replaced by a non-analytic term r^γ.

JHEP07(2009)083

4% of the present energy density of the Universe while the fraction of dark matter is about 5 times bigger. Cosmological observations are consistent with the hypothesis of cold dark matter. On the other hand, the situation at the galactic scale is rather different. According to the CDM paradigm cold and collisionless dark matter is distributed, differently from the visible matter, along the galactic halos and is responsible for the anomalous behaviour of galactic rotational curves. However, in CDM the curves obtained using N-body simulations do not reproduce the observed rotational curves of small galaxies [13, 14]. The implicit assumption behind this scenario is that gravitational interaction between matter and dark matter is universal and Newtonian. If these hypotheses are relaxed our view and phenomenological modelling may be radically modified.

One of the intriguing possibilities is to consider dark matter as matter of a hidden gauge sector which is an exact copy of the ordinary particle sector, so that along with the ordinary matter: electrons, nucleons, etc. the Universe contains also the mirror matter as mirror electrons, mirror nucleons, etc. with exactly the same mass spectrum and interaction properties. Such a parallel sector, dubbed as mirror world [15], can have many interesting phenomenological and cosmological implications (for a review, see [16]). In particular, the baryon asymmetry in both sectors can be generated via out-of-equilibrium $B-L$ and CP violating processes between the ordinary and mirror particles [17] and it can naturally explain the proportion between the visible and dark matter fractions in the Universe. Such processes can be induced by some very weak interaction between ordinary and mirror sectors. The very same interaction can also induce mixing terms between neutral particles of the two sectors, as e.g. kinetic mixing for photons [18] or mass mixing in the case of the neutrinos and neutrons [19].

Mirror matter, dark in terms of ordinary photons, is coupled with ordinary matter through gravity and can be a viable candidate for dark matter. As it was shown in [20], the cosmological observations on large scale structure and CMB are consistent with the mirror dark matter picture. However, the essential problem emerges at the galaxy scales. It is difficult to understand how the mirror matter, being collisional and dissipative as normal matter, could produce extended galactic halos and thus explain the galactic rotational curves.

In this paper we show that this difficulty is overcome if mirroring is extended also to the gravitational sector as encoded in the action (1.1), normal and mirror matter have separate gravitational interactions associated with $g_{1\mu\nu}$ and $g_{2\mu\nu}$. A healthy Yukawa modification of the gravitational potential appears when Lorentz breaking is induced by the third metric $g_{3\mu\nu}$ (whose "Planck" mass M_3 is eventually taken to be much larger than the ordinary Planck mass M_P). Explicitly, the potential felt by a test particle of the type 1 (normal matter) at the distance r from a source is

$$\phi(r) = \frac{G}{2r}\left[(m_1 + m_2) + (m_1 - m_2)\; e^{-r/r_m}\right], \tag{1.2}$$

where G is the Newton constant, m_1 and m_2 are respectively the masses of the visible (type 1) and mirror (type 2) matter sources, and $r_m = m^{-1}$ is a Yukawa length scale. Hence, at small distances $r \ll r_m$, gravitational force between two sectors is not universal: normal and mirror matter effectively do not interact gravitationally . At distances $r \ll r_m$ a normal test mass interacts only with m_1 through the ordinary Newton potential. However, at large distances $r \gg r_m$ gravity becomes universal and Newtonian, a test particle feels both ordinary and dark matter and attributes to the source a total mass $(m_1 + m_2)$ with an effectively *halved* Newton constant $G/2$. The main result of this work is to reproduce the potential (1.2) in a consistent model of gravity.

This scenario can have interesting astrophysical implications. One can show [21] that the galactic rotational curves are reproduced even if dark matter[2] has a similarly "clumped" distribution as normal matter, as it is expected from its dissipative character.

The paper is organized as follows: in section 2 we review the linearized analysis of bigravity theories that will be used as building block for our model, in particular both the Lorentz breaking and Lorentz invariant phases are discussed. In section 3 we introduce the model and show how a Yukawa potential arises in the limit when the additional metric is decoupled. In section 4 we discuss our findings. Finally, in appendix A the general expression of the Yukawa-like potential is given, and appendix B contains the detailed expressions for the graviton mass matrices and an example of the interaction potential having all the required features.

2 Bigravity: a review of the linearized analysis

In bigravity generically one can find bi-flat SO(3) preserving vacuum solutions [10]:

$$\begin{aligned} \bar{g}_{1\mu\nu} &= \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1) \\ \bar{g}_{2\mu\nu} &= \hat{\eta}_{\mu\nu} = \omega^2 \mathrm{diag}(-c^2,1,1,1); \end{aligned} \tag{2.1}$$

we have set the speed of light in our sector (1) to one in natural units, whereas c is the speed of light in the hidden sector 2 and ω is a relative constant conformal factor. Once V is given, c and ω can be computed by solving the equations of motion following from (1.1), and if $c \neq 1$ Lorentz symmetry is broken. Consider the linearized theory obtained by expanding the total action (1.1) at quadratic level in the metric perturbations around the bi-flat background (2.1):

$$g_{1\mu\nu} = \eta_{\mu\nu} + h_{1\mu\nu}\,, \qquad g_{2\mu\nu} = \hat{\eta}_{\mu\nu} + \omega^2 h_{2\mu\nu}\,. \tag{2.2}$$

The gravitational perturbations $h_{1\,\mu\nu}$ and $h_{2\,\mu\nu}$ interact with matter 1 and 2 through their conserved EMTs, respectively $T_1^{\mu\nu}$ and $T_2^{\mu\nu}$. Since the background preserves rotations, it is convenient to decompose the perturbations $h_{a\mu\nu}$ $(a = 1, 2)$ according to irreducible SO(3) representations

$$\begin{aligned} h_{a00} = \psi_a\,, \qquad h_{a0i} = u_{ai} + \partial_i v_a\,, \\ h_{aij} = \chi_{aij} + \partial_i S_{aj} + \partial_j S_{ai} + \partial_i\partial_j\sigma_a + \delta_{ij}\,\tau_a\,. \end{aligned} \tag{2.3}$$

[2]For bigravity inspired interpretation of dark matter see [22].

For each perturbation $h_{a\mu\nu}$ one has a gauge invariant transverse traceless tensor χ_{aij}, two vectors and four scalars. The quadratic Lagrangian $\mathcal{L}$ reads

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm mass} + \mathcal{L}_{\rm src}\,, \tag{2.4}$$

$$\mathcal{L}_{\rm kin} = \frac{1}{4}\chi^t_{ij}\mathcal{K}(\mathcal{C}^2\Delta - \partial_t^2)\chi_{ij} - \frac{1}{2}w^t_i\mathcal{K}\Delta w_i + \phi^t\mathcal{K}\Delta\,\tau - \frac{1}{2}\tau^t\mathcal{K}\left(\mathcal{C}^2\Delta - 3\partial_t^2\right)\tau. \tag{2.5}$$

We have introduced a compact notation for the fluctuations: $h_{\mu\nu} = (h_{1\mu\nu}, h_{2\mu\nu})^t$, $\chi_{ij} = (\chi_{1ij}, \chi_{2ij})^t$ and the following 2×2 matrices: $\mathcal{C} = \mathrm{diag}(1, c)$, $\mathcal{K} = M_1^2\,\mathrm{diag}(1,\kappa)$ and $\kappa = M_2^2/M_1^2\omega c$. In the kinetic term coming from the expansion of EH terms, the fluctuations enter only through the gauge invariant combinations $\chi_{a\mu\nu}$, $w_i = u_i - \partial_t S_i$ and $\phi = \psi - 2\partial_t v + \partial_t^2\sigma$. Finally, $\mathcal{L}_{\rm src}$ describes the gravitational coupling to matter conserved sources

$$\mathcal{L}_{\rm src} = T^t_{0i}\,\mathcal{C}^{-1}W_i - T^t_{00}\frac{\mathcal{C}^{-3}}{2}\phi - T^t_{ii}\frac{\mathcal{C}}{2}\tau - T^t_{ij}\frac{\mathcal{C}}{2}\chi_{ij}\,. \tag{2.6}$$

Clearly $\mathcal{L}_{\rm kin}$ and $\mathcal{L}_{\rm src}$ are gauge invariant. The mass term $\mathcal{L}_{\rm mass}$ is produced by the expansion of the interaction potential V. For the bi-flat background (2.1) the mass term $\mathcal{L}_{\rm mass}$ has the following form

$$\mathcal{L}_{\rm mass} = \frac{\epsilon^4}{4}\Big(h^t_{00}\mathcal{M}_0 h_{00} + 2h^t_{0i}\mathcal{M}_1 h_{0i} - h^t_{ij}\mathcal{M}_2 h_{ij} + h^t_{ii}\mathcal{M}_3 h_{ii} - 2h^t_{ii}\mathcal{M}_4 h_{00}\Big) \tag{2.7}$$

and the explicit value of the mass matrices can be easily computed for any given V.

It is however crucial to realize that due to linearized gauge invariance the mass matrices have the following property [10]

$$\mathcal{M}_{0,1,4}\begin{pmatrix}1\\ c^2\end{pmatrix} = 0\,, \qquad \mathcal{M}_{1,2,3}\begin{pmatrix}1\\ 1\end{pmatrix} = 0\,, \qquad \mathcal{M}^t_4\begin{pmatrix}1\\ 1\end{pmatrix} = 0\,. \tag{2.8}$$

Thus general covariance forces the mass matrices to be at most of rank one.

Lorenz-Invariant (LI) phase. In this case an FP graviton mediates Yukawa-like potential. Indeed, when $c = 1$, two conditions in (2.8) coincides, allowing a non-zero $\mathcal{M}_1$ and all mass matrices are rank one and proportional:

$$\mathcal{M}_0 = \lambda_0\mathcal{P}, \quad \mathcal{M}_1 = \mathcal{M}_2 = \lambda_2\mathcal{P}, \qquad \mathcal{M}_3 = \mathcal{M}_4 = (\lambda_2+\lambda_0)\mathcal{P}\,, \quad \mathcal{P} = \begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}. \tag{2.9}$$

After introducing a canonically normalized graviton field $h^{(c)} = \mathcal{K}^{1/2}h$, the mass matrices can be diagonalized by a rotation of angle ϑ with $\tan\vartheta = \kappa^{1/2} = M_2/\omega M_1$; the spectrum consists of a massless and a massive graviton. This latter has a standard Lorentz-Invariant mass term and to avoid ghosts one has to choose $\lambda_0 = 0$, as a result the mass term has the Pauli-Fierz form. Both matter sectors interact with the massless graviton and mediates a force with a standard Newtonian potential. The massive graviton on the other hand mediates a Yukawa-like force and thus modifies the static potential at scales larger than m^{-1}, where $m = \epsilon^2\lambda_2^{1/2}|\sin\vartheta|/M_1$ is the graviton mass.

In the most interesting case, when $M_1 = M_2 = M$, $\omega = 1$ and so $\tan\vartheta = 1$, a static potential for a test particle of type 1, generated by point-like sources of mass m_1 (type-1) and m_2 (type-2) at the same point, is given by

$$\phi_{1\text{matter}}(r) = \frac{1}{32\pi M^2}\left[(m_1 + m_2) + \frac{4}{3}\, e^{-m\,r}\,(m_1 - m_2)\right]. \tag{2.10}$$

Therefore, the presence of the massive gravity state mediating a Yukawa-like force makes the effective Newton constant distance dependent: the Newton constant measured experimentally via the gravitational interaction between type-1 test bodies at small distances $r \ll m^{-1}$ should be identified with $G = G_{UV} = 7/96\pi M^2$. At large distances $r \gg m^{-1}$ it effectively becomes $G_{IR} = 1/32\pi M^2 = 3G/7$ and is universal for both type-1 and type-2 matter. On the other hand, at $r \ll m^{-1}$ the force between the type-1 and type-2 test masses is repulsive, with an effective Newton constant $G^{12}_{UV} = -G/7$; this is an indication of instability of the theory. However, more serious problem is the vDVZ discontinuity. The static potential felt by a photon is

$$\phi_{1\text{light}}(r) = \frac{1}{32\pi M^2}\left[(m_1 + m_2) + e^{-mr}\,(m_1 - m_2)\right]. \tag{2.11}$$

Therefore, for the light bending at distances $r \ll m^{-1}$ we have $G_{\text{light}} = 1/16\pi M^2$, and thus $G/G_{\text{light}} = 7/6$. This discrepancy is somewhat milder than in the FP theory where we have $G/G_{\text{light}} = 4/3$; anyway the deviation from the GR prediction $G/G_{\text{light}} = 1$ is unacceptably large and it is clearly excluded by the post-Newtonian gravity tests [23].

The problems can be softened if the two sectors are not symmetric, $M_1 \neq M_2$ and the mixing is small enough. In this case, the static potential respectively a massive test particle and for a photon of the type 1 reads

$$\phi_{1\xi}(r) = \frac{\cos^2\vartheta}{16\pi M_1^2}\left[(m_1 + m_2) + \xi\, e^{-m\,r}\left(m_1 \tan^2\vartheta - m_2\right)\right], \tag{2.12}$$

where $\xi = 4/3$ for a massive particle and $\xi = 1$ for light. Therefore, at small distance ($r \ll m^{-1}$) the Newton "constant" is $G_{UV} = G(1+4/3\tan^2\vartheta)$, at large distance ($r \gg m^{-1}$) it tends to G.

At small distances, the ratio of (2.12) defines the post-Newtonian parameter δ:

$$\delta = \lim_{m\to 0}\left[\frac{\phi_{1\xi}(\xi = 4/3)}{\phi_{1\xi}(\xi = 1)}\right]_{m_2=0} = 1 + \frac{1}{3}\sin^2\vartheta\,. \tag{2.13}$$

For GR $\delta = 1$ and the current light bending experiments constrain δ to be in the range $\delta = 1.0000 \pm 0.0001$ [23]. In the limit of vanishing graviton mass the well known vDVZ discontinuity [2] of Pauli-Fierz massive gravity emerges. In our case the mixing angle ϑ controls the size of the discontinuity.

When $M_2 \gg M_1$, we have $\vartheta \to \pi/2$ and $\delta = 4/3$, unacceptably large. In this limit sector 2 is very weakly coupled, and the discontinuity is mainly shifted to sector 1, that approaches a normal Fierz-Pauli massive gravity.

Conversely when $M_2 \ll M_1$ we have $\vartheta \to 0$ and the discontinuity is shifted to sector 2; h_+ and h_- almost coincide with h_1 and h_2 and gravity is stronger in sector 2. The experimental bound on δ translates into $\vartheta \simeq 0.02$, that amounts to roughly $M_2 \simeq \vartheta M_1$. In this case, if m_2 is interpreted as dark matter, it gives a sizable contribution, increasing the gravitational force in the region $r \gtrsim m^{-1}$. Notice incidentally that for small r, the potential is repulsive. This result contradicts observations in the gravitationally bounded systems as cluster and galaxies, for this reason is ruled out.

Lorenz-Breaking (LB) phase. In this phase, $c \neq 1$, conditions (2.8) imply that $\mathcal{M}_1$=0 and for other masses one has

$$\mathcal{M}_0 = \lambda_0 \, \mathcal{C}^{-2} \, \mathcal{P} \, \mathcal{C}^{-2} \,, \quad \mathcal{M}_{2,3} = \lambda_{2,3} \, \mathcal{P} \,, \quad \mathcal{M}_4 = \lambda_4 \, \mathcal{P} \, \mathcal{C}^{-2} \,. \tag{2.14}$$

In this situation all the scalar and vector perturbations become non-dynamical [10]. The vanishing of $\mathcal{M}_1$ in the LB phase is the reason behind the absence of ghosts or tachyons appear, gravitons are the only propagating states. However, this is also the reason behind the absence of Yukawa-like gravitational potential. The resulting modification was studied in detail in [12] both at linear and non-linear level.

3 Effective Higgs phase

In order to find a phenomenologically viable Yukawa phase, we introduce one more rank-2 field g_3 which couples both metrics g_1 and g_2 and triggers LB:[3]

$$S = \int d^4x \left[\sqrt{g_1} \, \left(M^2 R_1 + \mathcal{L}_1 \right) + \sqrt{g_2} \, \left(M^2 \, R_2 + \mathcal{L}_2 \right) + M_3^2 \, \sqrt{g_3} R_3 \right.$$
$$\left. + \, \epsilon^4 \, \left(g_1 g_2 g_3 \right)^{1/6} V(g_1, g_2, g_3) \right] . \tag{3.1}$$

The only non-trivial tensors that can be formed are: $X_{12} = g_1^{-1} g_2$, $X_{13} = g_1^{-1} g_3$, $X_{23} = g_2^{-1} g_3$, that satisfy the identity $X_{13} = X_{12} X_{23}$. Therefore V can be taken as a scalar function of two of them.

We have also introduced in (3.1) a discrete symmetry under the exchange $1 \leftrightarrow 2$, so that the potential V is symmetric and the two sectors 1, 2 have equal Planck masses M. The third Planck mass will be eventually taken to infinity, $M_3 \gg M$, and the fluctuations of the third field will be effectively decoupled.

The first step is to find a suitable background. As for bigravity, we look for LB flat solutions of the form

$$\begin{aligned} \bar{g}_{1\mu\nu} &= \bar{g}_{2\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1) \\ \bar{g}_{3\mu\nu} &= \hat{\eta}_{\mu\nu} = \omega^2 \, \text{diag} \left(-c^2, 1, 1, 1 \right) , \end{aligned} \tag{3.2}$$

[3] In principle, any tensor condensate e.g. emerging via a strongly coupled hidden gauge sector can be also used for inducing the Lorentz-breaking background [24].

so that $\bar{X}_{12} = \mathbb{I}$ and $\bar{X}_{13} = \eta^{-1}\hat{\eta}$. The background (3.2) is a solution of the equations of motion (EOMs) if

$$\begin{aligned} V\mathbb{I} + 6\frac{\partial V}{\partial X_{21}} X_{21} + 6\frac{\partial V}{\partial X_{31}} X_{31} &= 0 \\ V\mathbb{I} + 6\frac{\partial V}{\partial X_{12}} X_{12} + 6\frac{\partial V}{\partial X_{32}} X_{32} &= 0 \\ V\mathbb{I} + 6\frac{\partial V}{\partial X_{13}} X_{13} + 6\frac{\partial V}{\partial X_{23}} X_{23} &= 0 \,, \end{aligned} \tag{3.3}$$

where $X_{ba} = X_{ab}^{-1}$. Using the 1↔2 exchange symmetry of the EOMs, the symmetric form of the ansatz (3.2) and the the identity

$$\frac{\partial V}{\partial X_{ab}} X_{ab} = -\frac{\partial V}{\partial X_{ba}} X_{ba} \,, \tag{3.4}$$

we have $\partial V/\partial X_{12} = 0$ and the EOMs reduce to

$$V = 0 \,, \qquad \frac{\partial V}{\partial X_{13}} = 0 \,. \tag{3.5}$$

These are three independent equations for the two parameters ω and c, thus one fine-tuning is needed for (3.2) to be a solution. This fine tuning is analogous to the cosmological constant in standard GR, and can be easily realized for instance by introducing a cosmological constant in sector 3.

Once a background solution is found, one can study small fluctuations around it, defined by

$$g_{1\mu\nu} = \eta_{\mu\nu} + h_{1\mu\nu} \,, \qquad g_{2\mu\nu} = \eta_{\mu\nu} + h_{2\mu\nu} \,, \qquad g_{3\mu\nu} = \hat{\eta}_{\mu\nu} + \omega^2 h_{3\mu\nu} \,. \tag{3.6}$$

The structure of the quadratic Lagrangian for the fluctuations is the same as in (2.4)–(2.6) except that now the tensor, vector, scalar and source fields all have 3 components, $h_{\mu\nu} = (h_{1\,\mu\nu}, h_{2\,\mu\nu}, h_{3\,\mu\nu})$. Also,

$$\mathcal{K} = \mathrm{diag}(M^2, M^2, M_3^2/\omega^2 c) \,, \qquad \mathcal{C} = \mathrm{diag}(1,\, 1,\, c) \,, \tag{3.7}$$

and $\mathcal{M}_i$ are 3×3 matrices entering the usual mass Lagrangian:

$$\mathcal{L}_{\text{mass}} = h_{00}^t \mathcal{M}_0 h_{00} + 2h_{0i}^t \mathcal{M}_1 h_{0i} - h_{ij}^t \mathcal{M}_2 h_{ij} + h_{ii}^t \mathcal{M}_3 h_{ii} - 2h_{ii}^t \mathcal{M}_4 h_{00} \,. \tag{3.8}$$

Diagonal diffeomorphisms invariance constrains the form of these matrices:

$$\mathcal{M}_{1,2,3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \mathcal{M}_4^T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \mathcal{M}_{0,1,4} \begin{pmatrix} 1 \\ 1 \\ c^2 \end{pmatrix} = 0 \,. \tag{3.9}$$

From these conditions and from the 1 ↔ 2 symmetry it follows that the matrices can be written in terms of the following combinations of projectors

$$\begin{aligned} \mathcal{M}_0 &= a_0 \mathcal{P}_{12} + b_0 \mathcal{C}^{-2} (\mathcal{P}_{13} + \mathcal{P}_{23}) \mathcal{C}^{-2} \\ \mathcal{M}_1 &= a_1 \mathcal{P}_{12} \\ \mathcal{M}_2 &= a_2 P_{12} + b_2 (\mathcal{P}_{13} + \mathcal{P}_{23}) \\ \mathcal{M}_3 &= a_3 \mathcal{P}_{12} + b_3 (\mathcal{P}_{13} + \mathcal{P}_{23}) \\ \mathcal{M}_4 &= a_4 \mathcal{P}_{12} + b_4 \, (\mathcal{P}_{13} + \mathcal{P}_{23}) \, \mathcal{C}^{-2}, \end{aligned} \tag{3.10}$$

where

$$\mathcal{P}_{12}=\begin{pmatrix}1&-1&0\\-1&1&0\\0&0&0\end{pmatrix}\qquad\mathcal{P}_{13}=\begin{pmatrix}1&0&-1\\0&0&0\\-1&0&1\end{pmatrix}\qquad\mathcal{P}_{23}=\begin{pmatrix}0&0&0\\0&1&-1\\0&-1&1\end{pmatrix},\tag{3.11}$$

and a_i, b_i are constants that depend on V.

Since we are interested in the gravitational potential we focus on the scalar sector. The quadratic Lagrangian for the scalars is

$$\begin{aligned}\mathcal{L}_{\text{scalars}}=&\ \phi^t\mathcal{K}^2\Delta\,\tau-\tau^t\frac{\mathcal{K}^2}{2}\left(\mathcal{C}^2\Delta-3\partial_t^2\right)\tau+\\&+\frac{1}{4}\Big[\psi\mathcal{M}_0\psi-2\Delta v\mathcal{M}_1 v-(\tau+\Delta\sigma)\mathcal{M}_2(\tau+\Delta\sigma)-2\tau\mathcal{M}_2\tau\\&\qquad\qquad+(3\tau+\Delta\sigma)\mathcal{M}_3(3\tau+\Delta\sigma)-2(3\tau+\Delta\sigma)\mathcal{M}_4\psi\Big]\\&-\phi\frac{\mathcal{C}^{-3}}{2}T_{00}-\tau^t\frac{\mathcal{C}}{2}T_{ii}\,.\end{aligned}\tag{3.12}$$

In order to disentangle the different fluctuations it is useful to a 'tilded' basis where the fluctuations are rotated:

$$[\psi,v,\sigma,\tau]=S\left[\tilde{\psi},\tilde{v},\tilde{\sigma},\tilde{\tau}\right],\qquad\tilde{\mathcal{M}}_i=S^t\mathcal{M}_iS\qquad S=\begin{pmatrix}-\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}&0\\\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}&0\\0&0&\frac{1}{\sqrt{2}}\end{pmatrix}.\tag{3.13}$$

In this basis the mass matrices take the block-diagonal form

$$\begin{aligned}\tilde{\mathcal{M}}_0&=\begin{pmatrix}2a_0+b_0&0&0\\0&b_0&-b_0/c^2\\0&-b_0/c^2&b_0/c^4\end{pmatrix},\qquad&\tilde{\mathcal{M}}_1&=\begin{pmatrix}4a_1&0&0\\0&0&0\\0&0&0\end{pmatrix},\\\tilde{\mathcal{M}}_{2,3}&=\begin{pmatrix}2a_{2,3}+b_{2,3}&0&0\\0&b_{2,3}&-b_{2,3}\\0&-b_{2,3}&b_{2,3}\end{pmatrix},\qquad&\tilde{\mathcal{M}}_4&=\begin{pmatrix}2a_4+b_4&0&0\\0&b_4&-b_4/c^2\\0&-b_4&b_4/c^2\end{pmatrix}.\end{aligned}\tag{3.14}$$

Because the $\mathcal{K}$ commutes with S, we see that in the new basis the system naturally splits into two: a single massive gravity sector and a bigravity sector encoded in the 2×2 sub-matrices in (3.14). The presence $\bar{g}_3$ induces in both sectors a Lorenz breaking mass pattern. The first sector can be analysed as in [6], while for the second the analysis of [10] applies. As a result, a consistent theory, free of ghosts and of instabilities at linearized level is possible. Indeed, in the single massive graviton sector, ghosts can be avoided if the relevant entry 1-1 in $\tilde{\mathcal{M}}_0$ vanishes. We have thus the condition:

$$a_0=-b_0/2\,.\tag{3.15}$$

The bigravity sector on the other hand is automatically free of ghosts as shown in [10] thanks to the vanishing of $\tilde{\mathcal{M}}_1$ in the relevant block.

At this point we can study in the new basis the static gravitational potential in each sector captured by the gauge invariant field $\tilde{\phi}_a=\tilde{\psi}_a-2\partial_t\tilde{v}_a+\partial_t^2\tilde{\sigma}_a$ $(a=1,2,3)$. It is convenient to define also the rotated and M^2-normalized sources $\tilde{t}_{\mu\nu}=S\,t_{\mu\nu}=S\,(T_{\mu\nu}/M^2)$.

JHEP07(2009)083

The field $\tilde{\phi}_1$ is separated from the bigravity sector and gives the Yukawa-like static potential. It turns out that in general $\tilde{\phi}_1$ is a combination of two Yukawa potentials, with two parametrically different mass scales (see appendix A for the details). Here, for notation simplicity, we consider the case where the scales coincide

$$\tilde{\phi}_1 = \frac{\tilde{t}_1}{2\Delta - m^2}\,, \qquad m^2 = 3(2a_4 + b_4)\frac{\epsilon^4}{M^2}\,. \tag{3.16}$$

In this sector, in addition to the propagating massive graviton (two polarizations) also a vector and a scalar propagate (respectively two and one degrees of freedom). All these fields are massive with mass given by the relative 1-1 entry of $\tilde{\mathcal{M}}_2$. The vector and the scalar can have well behaved properties, i.e. no ghosts when condition (3.15) is enforced. In [6] it was also argued that the scale of strong coupling is high enough, coinciding with $\Lambda_2 \simeq \sqrt{Mm}$, with m the characteristic mass scale in this sector.

For the remaining bigravity sector the gravitational potential can be computed by solving the equations of motion as in [10]. The result is

$$\tilde{\phi}_2 = \frac{\tilde{t}_{200} + \tilde{t}_{2iii}}{2\Delta} + \mu^2\frac{\tilde{t}_{200}}{\Delta^2} \tag{3.17}$$

$$\tilde{\phi}_3 = -\mu^2\left(\frac{M}{M_3}\right)^2\frac{2c\omega^2\tilde{t}_{200}}{\Delta^2} \tag{3.18}$$

where

$$\mu^2 = \frac{\epsilon^4}{M^2}\left[b_2\frac{3b_4^2 + b_0(b_2 - 3b_3)}{2(b_4^2 + b_0(b_2 - b_3))}\right]\,. \tag{3.19}$$

When $M_3 \gg M$, the third sector has a sub-leading impact on the other gravitational potentials. In the limit $M_3 \to \infty$, the third sector decouples and g_3 just produces a LB fixed background $\hat{\eta}$. Going back to the original basis, the potentials are:

$$\begin{aligned}
\phi_1 &= \frac{t_{100} + t_{1ii} + t_{200} + t_{2ii}}{4\Delta} + \frac{t_{100} + t_{1ii} - t_{200} - t_{2ii}}{4\Delta - 2m^2} + \mu^2\frac{t_{100} + t_{200}}{2\Delta^2} \\
\phi_2 &= \frac{t_{100} + t_{1ii} + t_{200} + t_{2ii}}{4\Delta} - \frac{t_{100} + t_{1ii} - t_{200} - t_{2ii}}{4\Delta - 2m^2} + \mu^2\frac{t_{100} + t_{200}}{2\Delta^2} \\
\phi_3 &= -\mu^2\left(\frac{M}{M_3}\right)^2\frac{c\omega^2(t_{100} + t_{200})}{\Delta^2}\,.
\end{aligned} \tag{3.20}$$

The potentials $\phi_{1,2}$ contain a Newtonian term, a Yukawa-like term, and a linearly growing term, originating from μ^2/Δ^2.

This latter linear term is the same appearing in the bigravity case, as found in [10, 25]. It would invalidate perturbation theory at distances larger than $r_{IR}^{-1} \sim G\mu^2 M_*$ from a source M_* [10], but remarkably the full nonlinear solutions found in [12] shows that its linear growth is replaced by a non-analytic power $\sim r^\gamma$, where γ depends on the coupling constants in the potential. Moreover, in the full solution for a realistic star, also the magnitude of this new term is proportional to μ^2, therefore the effect can be eliminated by setting $\mu^2 = 0$. This can be achieved by simple fine-tuning, or by adopting a particular scaling symmetry of the potential, as discussed in [10]. We can thus obtain a pure Yukawa modification of the gravitational potential, by setting $\mu^2 = 0$, that here amounts to the condition $b_0 = -3b_4^2/(b_2 - 3b_3)$.

The analysis of vector modes is identical to that carried out in [6] for the single gravity sector and to the one of [10] for the bigravity one. In the single-gravity sector there is a vector state propagating with a nonlinear dispersion relation: at high energy its speed is $(2a_2 + b_2)/(2a_1 + b_1)$ and at low momentum it has a mass gap given by b_2/M^2. In the bigravity sector vector states do not propagate.

The analysis of tensor modes is similar and is best carried out in the original basis. In the limit of $M_3 \to \infty$, the equation of motion for the canonically normalized fields becomes:

$$\left[\begin{pmatrix} \Box & & \\ & \Box & \\ & & \widehat{\Box} \end{pmatrix} + \frac{1}{M^2}\begin{pmatrix} b_2 + a_2 & b_2 - a_2 & 0 \\ b_2 - a_2 & b_2 + a_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]\begin{pmatrix} \chi^c_{1\,ij} \\ \chi^c_{2\,ij} \\ \chi^c_{3\,ij} \end{pmatrix} = \begin{pmatrix} t_{1\,ij} \\ t_{2\,ij} \\ 0 \end{pmatrix} \tag{3.21}$$

where $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$, $\widehat{\Box} = \hat{\eta}^{\mu\nu}\partial_\mu\partial_\nu$ and we used the form of the projectors (3.10). We see that the massless spin two state decouples (it is a superimposition of mostly χ_3) and we are left with two massive gravitons, with two polarizations each, travelling at the normal speed of light. Their mass matrix can be diagonalized, and the resulting graviton masses are $m_{g1}^2 = (2a_2 + b_2)\epsilon^4/M^2$, $m_{g2}^2 = b_2\epsilon^4/M^2$.

As an explicit example, consider the simplest case of a potential quadratic in X_{12}, X_{13}, X_{23} plus two cosmological terms, satisfying the $1 \leftrightarrow 2$ exchange symmetry (taking, for simplicity $\omega = 1$):

$$\begin{aligned} V(g_1, g_2, g_3) = {} & \xi_0 + \xi_1\left(\mathrm{Tr}[X_{13}^2] + \mathrm{Tr}[X_{23}^2]\right) + \xi_2\,\mathrm{Tr}[X_{13}X_{23}] \\ & + \xi_3\left(\mathrm{Tr}[X_{13}]^2 + \mathrm{Tr}[X_{23}]^2\right) + \xi_4\,\mathrm{Tr}[X_{13}]\mathrm{Tr}[X_{23}] \\ & + \xi_5\left(\mathrm{Tr}[X_{12}]^2 + \mathrm{Tr}[X_{12}^{-1}]^2\right) + \xi_6\left(\mathrm{Tr}[X_{12}^2] + \mathrm{Tr}[(X_{12}^{-1})^2]\right) \\ & + \xi_7\left((\det X_{12})^{-1/6}(\det X_{13})^{-1/6} + (\det X_{12})^{1/6}(\det X_{23})^{-1/6}\right) \\ & + \xi_8(\det X_{13})^{1/6}(\det X_{23})^{1/6} \end{aligned} \tag{3.22}$$

The EOMs for a flat background require to solve for three constants (e.g. ξ_3, ξ_7, ξ_8) and then the coefficients of the projectors in the mass matrices a_i's and b_i are a function of the remaining coupling constants (see appendix B).

The no-ghost condition $b_0 = -2a_0$, the condition for the absence of the linear term $\mu^2 = 0$ and the condition for having a single Yukawa scale (see appendix A), can be solved for $\xi_{1,2,4}$ and we end up only four couplings eventually. The Yukawa scale m and the graviton masses m_{g1}^2 and m_{g2}^2, only depend on ξ_5 and ξ_6:

$$\begin{aligned} m^2 &= [p_0(c)\xi_5 + q_0(5)\xi_6]\frac{\epsilon^4}{M^2} \\ m_{g1}^2 &= [p_1(c)\xi_5 + q_1(c)\xi_6]\frac{\epsilon^4}{M^2} \\ m_{g2}^2 &= [p_2(c)\xi_5 + q_2(c)\xi_6]\frac{\epsilon^4}{M^2} \end{aligned} \tag{3.23}$$

JHEP07(2009)083

where $p_i(c)$'s and $q_i(c)$'s are given in appendix B.[4]

To summarize, in the limit where the third metric is decoupled the theory has two massive gravitons and the potential felt by a test particle of type 1 is:

$$\phi_1(r) = \frac{G\,m_1}{r}\left(\frac{1+e^{-mr}}{2}\right) + \frac{G\,m_2}{r}\left(\frac{1-e^{-mr}}{2}\right), \tag{3.24}$$

where $G = 1/16\pi M^2$. This shows that the vDVZ discontinuity is absent, and we have obtained the potential (1.2) while avoiding the troubles of the Lorentz-invariant FP theory.

4 Conclusions

Motivated by the interesting possibility to relax the assumption that dark and visible matter feel the same gravitational interaction, in this work we have addressed the possibility to obtain a Yukawa-like large-distance modification of the gravitational potential, while avoiding ghosts or classical instabilities.

The request to generate a Yukawa potential from a consistent theory led us to consider Lorenz-Breaking backgrounds in a suitable extended class of bigravity theories. For instance, bigravity while giving rise to a healthy Lorentz-Breaking massive phase, does not produce a Yukawa potential. Here we have generalized this picture and have shown that if an additional field g_3 is introduced, a Yukawa modification is allowed. The extra field g_3 can be harmlessly decoupled by freezing it to a Lorentz violating background configuration. The two remaining sectors represent two interacting massive gravities, of which one features a Yukawa potential. This pattern then leads to the desired modified gravity where standard matter (type 1) couples to all the graviton mass eigenstates.

On the technical side, the price to be paid to solve the problem is that two fine-tunings are needed, one to have a ghost-free spectrum at linear level, the other to avoid the linearly growing potential. The first one has be shown to follow (in single massive gravity theories) from extra unbroken partial diffeomorphisms invariance [26] and it would be interesting to extend that symmetry arguments also to the present model. The other can also be understood as the consequence of a scaling symmetry of the potential [10].

Let us note also that the theory presented here has three rank-two fields but only nine polarizations propagate (and are well behaved at quadratic level: three spin-2 with two polarizations each, one spin-1 with two polarizations and one scalar). On the other hand one may expect that the total number of propagating modes would be 22 = 3×10 - 4×2 (twice the number of diagonal diffeomorphisms), and the nine missing modes will probably propagate at non-linear level. The real non-perturbative question, to be addressed in a future work, is then at which scale these non-linear effects would show up.

The resulting setup, featuring two separate metric fields for the visible and dark matter, allows to consider also collisional and dissipative dark matter, as mirror matter and the potential generated by the ordinary and dark matter sources of mass m_1, m_2 felt by ordinary

[4]When $\xi_6 = 0$ and all the masses above depend only on ξ_5, one can check that they are positive, for $1.41 \lesssim c \lesssim 2.05$.

matter is distance dependent as in (3.24).[5] The result is very different from the standard picture when normal and dark matters both feel an universal Newtonian gravity; in fact the potential (3.24) can be used to fit galactic rotational curves using similar density profiles for the visible and dark sectors, alleviating the problems of profile formation [21].

In order to grasp the basic idea take, for instance, a spiral galaxy made of two overlapped disks of type 1 and type 2 matter. Both types of matter are distributed along the disks according to a similar density profile $\sigma_{1,2}(r) = \sigma_{1,2} e^{-r/r_{1,2}}$, where $\sigma_{1,2}$ are the central densities and $r_{1,2}$ are the core radii of the two types of matter respectively. The velocity profile is given by $v(r) = \sqrt{a(r) r}$, where $a(r)$ is the centrifugal acceleration of a star rotating at distance r from the center, obtained by integrating over the disks the acceleration $g(|(\mathbf{r} - \mathbf{r}'|)$ due to a point-like source

$$\begin{aligned} a(r) &= \int_{\text{disk}} d^2 r' g(\mathbf{r} - \mathbf{r}') \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}, \\ g(\mathbf{r} - \mathbf{r}') &= \frac{G}{2} \frac{(\sigma_1(r') + \sigma_2(r'))}{|\mathbf{r} - \mathbf{r}'|^2} + \frac{G}{2} \frac{(\sigma_1(r') - \sigma_2(r'))}{|(\mathbf{r} - \mathbf{r}'|^2} \left(1 + \frac{|(\mathbf{r} - \mathbf{r}'|}{r_m}\right) e^{|(\mathbf{r} - \mathbf{r}'|/r_m}. \end{aligned} \tag{4.1}$$

From this expression, one can see that in a wide region around r_m the gravitation interaction between the two types of matter turns on enhancing the rotational velocity over the Keplerian fall which would have given $v(r) \propto r^{-1/2}$. At large distance $r \gg r_m$, the behaviour is once again Keplerian-like, but with a crucial difference that the force is due not only to the visible matter, but the total mass average $(M_1 + M_2)/2 > M_1$. It is interesting to note that the effective Newton constant relative to the type 1 - type 1 and type 2 - type 2 interactions is distance dependent: $G_N(r \ll m^{-1}) = G$ and $G_N(r \gg m^{-1}) = G/2$.

Finally, let us comment on the cosmological implications of our model. Though a detailed study is left for a future work, we do not expect a strong impact at cosmological distances; for instance, the modified potential and the presence of a new type of matter will not change the expansion of the universe. The scale of gravity modification is set by the inverse of the graviton mass, which is about 10 kpc in our model. As a result, at cosmological scales only the Newtonian-like mode is active with an halved Newton constant $G/2$ and type 2 matter should behave as cold dark matter. The observed Hubble parameter would imply for the total density of the universe a value twice bigger being the effective critical density halved, e.g. $\rho_{cr} = 3H_0^2/4\pi G$ instead of $\rho_{cr} = 3H_0^2/8\pi G$. Then to reproduce the ratio Ω_B/Ω_{DM}, the mirror matter density should be about 10 times bigger than the baryon density.

In conclusion, at linearized level a Yukawa modification of the Newtonian gravitational potential is possible and it opens up the possibility to have collisional dark matter, coupled to ordinary matter via a modified gravitational interaction in a physically nontrivial way.

[5] Let us remark also that the weak equivalence principle does not exclude the possibility of direct (non-gravitational) interactions between the normal (type 1) and dark (type 2) matter components. To the action (1.1), besides the interaction gravitational term V, the mixed matter term $\int d^4x (g_1 g_2)^{1/4} \mathcal{L}_{\text{mix}}$ can be added with the Lagrangian $\mathcal{L}_{\text{mix}}$ including for example, the photon kinetic mixing term $\varepsilon F_1^{\mu\nu} F_{2\mu\nu}$ [18] or the neutrino interaction terms [19]. This also makes possible the direct detection of dark matter via such interactions [27], with interesting implications.

JHEP07(2009)083

Acknowledgments

We thank D. Comelli for useful discussions. The work is supported in part by the MIUR grant for the Projects of National Interest PRIN 2006 "Astroparticle Physics", and in part by the European FP6 Network "UniverseNet" MRTN-CT-2006-035863.

A General Yukawa-like potential

The degrees of freedom in the single gravity sector consists of a metric fluctuation with mass term that we can write as

$$\mathcal{L}_{\text{mass}} = \frac{M^2}{2}\left(m_0^2 h_{00}h_{00} + 2m_1^2 h_{0i}h_{0i} - m_2^2 h_{ij}h_{ij} + m_3 h_{ii}h_{jj} - 2m_4^2 h_{00}h_{ii}\right). \tag{A.1}$$

In our case effectively $m_0 = 0$ and when $m_1 \neq 0$, there is an (healthy) propagating scalar degree of freedom (τ) as well as an healthy propagating vector [6]. The scalar perturbations obey the equations:

$$2\Delta\tau - m_4^2(\Delta\sigma + 3\tau) = t_{00} \tag{A.2}$$

$$2\partial_0\tau - m_1^2 v = \frac{1}{\Delta}\partial_0 t_{00} \tag{A.3}$$

$$2\partial_0^2\tau - m_2^2\Delta\sigma - m_2^2\tau m_3^2\Delta\sigma + 3m_3^2\tau - m_4^2(\phi + 2\partial_0 v - \partial_0^2\sigma) = \frac{1}{\Delta}\partial_0^2 t_{00} \tag{A.4}$$

$$2\Delta\phi - 2\Delta\tau + 2m_2^2\Delta\sigma = t_{ii} - \frac{3}{\Delta}\partial_0^2 t_{00}\,, \tag{A.5}$$

where $t_{\mu\nu} = T_{\mu\nu}/M^2$. The equations can be solved for ϕ to get the static Newtonian potential. One finds

$$\phi = \frac{(t_{00} + t_{ii})(\zeta_1 - 1)\zeta_2\Delta + [t_{ii} + t_{00}(3\zeta_1 - 1)\zeta_2]\zeta_2 m_4^2}{2(\zeta_1 - 1)\zeta_2\Delta^2 + (4\zeta_2 - 1)m_4^2\Delta - 3\zeta_2 m_4^4}\,, \tag{A.6}$$

with $\zeta_1 = m_3^2/m_2^2$ and $\zeta_2 = m_2^2/m_4^2$. The potential can be written as the sum of two Yukawa-like terms:

$$\phi = \frac{t_+}{2\Delta - m_+^2} + \frac{t_-}{2\Delta - m_-^2}\,, \tag{A.7}$$

where

$$t_\pm = \frac{1}{2}(t_{00} + t_{ii})\left(1 \pm \frac{1}{\nu}\right) \pm t_{00}\left(\nu - \frac{1}{\nu}\right)\,, \tag{A.8}$$

$$m_\pm^2 = m_4^2\,\frac{(4\zeta_2 - 1 \pm \nu)}{2\zeta_2(1 - \zeta_1)}\,, \qquad \nu = \sqrt{1 + 8\zeta_2(3\zeta_1\zeta_2 - \zeta_2 - 1)}\,. \tag{A.9}$$

Recall [6] that the conditions $\zeta_2 > 1/4$ and $\zeta_1 < 1$ ensure that the theory as no derivative instabilities neither in the UV nor in the IR. Moreover, if $\zeta_1 > (8\zeta_2^2 + 8\zeta_2 - 1)/24\zeta_2^2$, the two masses $m_\pm$ are real and positive, and the theory has no instabilities also at intermediate scales. Accordingly, the potential is the sum of two "genuine" Yukawa-like terms.

Finally, if $\zeta_1 = (1 + \zeta_2)/3\zeta_2$, then $\nu = 1$ and t_- vanishes, so that one is left with a single Yukawa potential:

$$\phi = \frac{t_{00} + t_{ii}}{2\Delta - 3m_4^2}\,. \tag{A.10}$$

B Explicit solution for potential (3.22)

After solving the EOMs (3.5), the coefficients that enter the masses for potential (3.22) can be written in terms of the coupling constants and read

$$
\begin{aligned}
a_0 &= -\frac{\xi_4 c^4}{2} - \frac{2\xi_1 c^2}{9} - \frac{\xi_0}{72} - \frac{c^2}{18}\left(9c^2+2\right)\xi_2 + \frac{35\xi_5}{9} + \frac{50\xi_6}{9} \\
b_0 &= -\frac{\xi_0}{72} - \frac{c^2\left(39-23c^2\right)\xi_1}{18\left(c^2+3\right)} - \frac{c^2\left(39-23c^2\right)\xi_2}{36\left(c^2+3\right)} - \frac{\xi_5}{9} - \frac{\xi_6}{9} \\
a_1 &= \frac{c^2\xi_2}{2} - 4\xi_5 \\
a_2 &= \frac{\xi_2}{2} - \xi_5 \\
b_2 &= \left(c^2-1\right)\xi_1 + \frac{1}{2}\left(c^2-1\right)\xi_2 \\
a_3 &= -\frac{2\xi_1 c^2}{9} - \frac{\xi_2 c^2}{9} - \frac{\xi_0}{72} - \frac{\xi_4}{2} + \frac{26\xi_5}{9} + \frac{50\xi_6}{9} \\
b_3 &= -\frac{\xi_0}{72} + \frac{\left(5c^2-6c\right)\xi_1}{18} + \frac{\left(5c^2-6\right)\xi_2}{36} - \frac{\xi_5}{9} - \frac{4\xi_6}{9} \\
a_4 &= -\frac{2\xi_1 c^2}{9} - \frac{\xi_2 c^2}{9} - \frac{\xi_4 c^2}{2} - \frac{\xi_0}{72} - \frac{\xi_5}{9} + \frac{50\xi_6}{9} \\
b_4 &= -\frac{\xi_0}{72} - \frac{\left(13c^4+3c^2\right)\xi_1}{18\left(c^2+3\right)} - \frac{\left(13c^4+3c^2\right)\xi_2}{36\left(c^2+3\right)} - \frac{\xi_5}{9} - \frac{4\xi_6}{9}
\end{aligned}
\tag{B.1}
$$

Finally, the functions p appearing in the graviton masses (3.23) can be written as

$$
p_0 = \frac{1}{C_2}\left[6\left(c^2\left(\left(3\left(1850c^8-7725c^6-31099c^4+154507c^2-92547\right)c^2+7C_1 - 168318\right)c^2-5C_1+117936\right)-18C_1\right)\right] \tag{B.2}
$$

$$
q_0 = \frac{1}{C_2}\left[12\left(c^2\left(\left(3610c^{10}-15312c^8-58045c^6+296415c^4-187461c^2 + 14C_1-115371\right)c^2-10C_1+88452\right)-36C_1\right)\right] \tag{B.3}
$$

$$
p_1 = -\frac{1}{C_3}\left[2\left(95c^{18}-5674c^{16}+21090c^{14}+95053c^{12}-447746c^{10} + 243567c^8+194157c^6-37C_1\left(6c^8-125117c^4+5c^2+42\right)\right)\right] \tag{B.4}
$$

$$
q_1 = -\frac{1}{C_3}\left[2\left(380c^{18}-7716c^{16}+22284c^{14}+123114c^{12}-529730c^{10}+334098c^6-148 + C_1\left(24c^8-205631c^4+20c^2+168238626\right)\right)\right] \tag{B.5}
$$

$$
p_2 = -\frac{\left(95c^{10}-324c^8-405c^6+378c^4+6C_1\right)C_4}{C_5} \tag{B.6}
$$

$$
q_2 = -\frac{4\left(95c^{10}-324c^8-405c^6+378c^4+6C_1\right)C_4}{C_5}, \tag{B.7}
$$

where

$$C_1 = c^4\left(5c^4 - 26c^2 + 21\right)\sqrt{13c^4 + 42c^2 + 9} \tag{B.8}$$

$$C_2 = (1404 - 1773c^2 + 6c^4 + 107c^6)(c^4(-2 + c^2)(-21 + 5c^2)) \tag{B.9}$$

$$C_3 = c^8(58968 - 117990c^2 + 62235c^4 - 4557c^6 - 3287c^8 + 535c^{10}) \tag{B.10}$$

$$C_4 = 2\left(c^4 + 2c^2 - 3\right) \tag{B.11}$$

$$C_5 = c^6(-29484 + 44253c^2 - 8991c^4 - 2217c^6 + 535c^8)\,. \tag{B.12}$$

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JHEP07(2009)083

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JHEP07(2009)083